

# Deep Learning — Homework 2

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# 1 More Backpropagation

## 1.1 Backpropagation through a DAG of modules

Defining the vectors  $o = (o_{max}, o_{min})$ ,  $x = (x_1, x_2)$  and  $\frac{\partial E}{\partial x} = (\frac{\partial E}{\partial x_1}, \frac{\partial E}{\partial x_2})$  the first step in back propagation is:

$$\frac{\partial E}{\partial o} = \frac{\partial E}{\partial y} \frac{\partial y}{\partial o} = (\frac{\partial E}{\partial y}, \frac{\partial E}{\partial y})$$

The second step:

$$\frac{\partial E}{\partial i} = \frac{\partial E}{\partial o} \frac{\partial o}{\partial i} = (\frac{\partial E}{\partial y}, \frac{\partial E}{\partial y}) \begin{bmatrix} \frac{\partial max(i_1, i_2)}{\partial i_1} & \frac{\partial max(i_1, i_2)}{\partial i_2} \\ \frac{\partial min(i_1, i_2)}{\partial i_1} & \frac{\partial min(i_1, i_2)}{\partial i_2} \end{bmatrix} = (\frac{\partial E}{\partial y}, \frac{\partial E}{\partial y}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (\frac{\partial E}{\partial y}, \frac{\partial E}{\partial y})$$

Finally:

$$\frac{\partial E}{\partial x} = \frac{\partial E}{\partial i} \frac{\partial i}{\partial x} = (\frac{\partial E}{\partial y}, \frac{\partial E}{\partial y}) \begin{bmatrix} \frac{\partial \sigma_1}{\partial x_1} & \frac{\partial \sigma_1}{\partial x_2} \\ \frac{\partial \sigma_2}{\partial x_1} & \frac{\partial \sigma_2}{\partial x_2} \end{bmatrix} = (\frac{\partial E}{\partial y}, \frac{\partial E}{\partial y}) \begin{bmatrix} \frac{e^{-x_1}}{(1+e^{-x_1})^2} & 0 \\ 0 & \frac{e^{-x_2}}{(1+e^{-x_2})^2} \end{bmatrix}$$

That is:

$$\begin{aligned} \frac{\partial E}{\partial x_1} &= \frac{\partial E}{\partial y} \frac{e^{-x_1}}{(1+e^{-x_1})^2}, \\ \frac{\partial E}{\partial x_2} &= \frac{\partial E}{\partial y} \frac{e^{-x_2}}{(1+e^{-x_2})^2} \end{aligned}$$

## 1.2 Batch Normalization

1: Using the chain rule we have:

$$\frac{\partial E}{\partial x_k} = \frac{\partial E}{\partial y_k} \frac{\partial y_k}{\partial x_k} + \frac{\partial E}{\partial E(x_k)} \frac{\partial E(x_k)}{\partial x_k} + \frac{\partial E}{\partial \sigma^2(x_k)} \frac{\partial \sigma^2(x_k)}{\partial x_k} =$$

and so:

$$\frac{\partial E}{\partial x_k} = \frac{\partial E}{\partial y_k} \frac{1}{\sqrt{\sigma^2(x_k)}} + \frac{\partial E}{\partial E(x_k)} \frac{1}{n} + \frac{\partial E}{\partial \sigma^2(x_k)} \frac{2}{n} \sum_{i=1}^n (x_{k_i} - E(x_k)) \quad (1)$$

where we used the definition of  $E(x_k)$  and  $\sigma(x_k)$  to calculate their derivatives.

And where:

$$\begin{aligned} \frac{\partial E}{\partial E(x_k)} &= \sum_{i=1}^n \frac{\partial E}{\partial y_{k_i}} \frac{\partial y_{k_i}}{\partial E(x_k)} + \frac{\partial E}{\partial \sigma^2(x_k)} \frac{\sigma^2(x_k)}{\partial E(x_k)} = \sum_{i=1}^n \frac{\partial E}{\partial y_{k_i}} \frac{-1}{\sqrt{\sigma^2(x_k)}} + \frac{\partial E}{\partial \sigma^2(x_k)} \left( \frac{-2}{n} \sum_{i=1}^n (x_{k_i} - E(x_k)) \right) \\ \frac{\partial E}{\partial \sigma^2(x_k)} &= \sum_{i=1}^n \frac{\partial E}{\partial y_{k_i}} \frac{\partial y_{k_i}}{\partial \sigma^2(x_k)} = \sum_{i=1}^n \frac{\partial E}{\partial y_{k_i}} \left[ -\frac{1}{2\sqrt{(\sigma^2(x_k))^3}} (x_{k_i} - E(x_k)) \right] \end{aligned}$$

**2:** Adding a learnable bias we have:

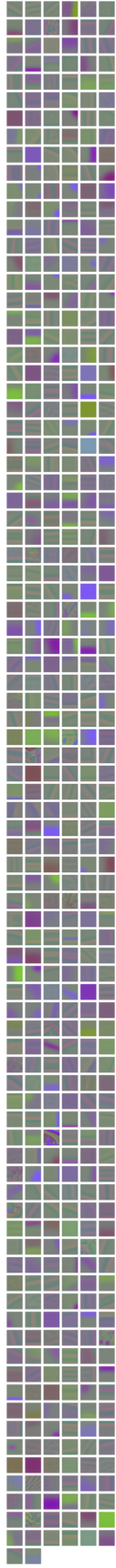
$$y_k = \frac{x_k - E(x_k)}{\sqrt{\sigma^2(x_k)}} + \epsilon$$

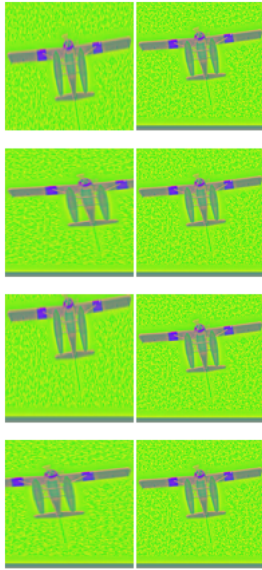
The back propagation will be the same of (1). And in addition we have the parameter  $\epsilon$  for which:

$$\frac{\partial E}{\partial \epsilon} = \sum_{i=0}^n \frac{\partial E}{\partial y_{k_i}} \frac{\partial y_{k_i}}{\partial \epsilon} = \sum_{i=0}^n \frac{\partial E}{\partial y_{k_i}}$$



## 2 Visualization





### 3 Kaggle

