

Kernel Based Approaches for Change-Point  
Detection — Report 1  
Spectral density & Peridogram

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# Spectral density

## 1.1 Linear combination of sinusoidals

Suppose we have a time series defined as

$$X_t = \sum_{j=1}^k (A_j \cos(\omega_j t) + B_j \sin(\omega_j t)), \quad 0 < \omega_1 < \dots < \omega_k < \pi \quad (1)$$

The time series is given in Figure 1 The timeseries is generated with values

$$\begin{aligned} K &= 2 \\ \omega &= [1.57079633, 0.78539816], \\ A &= [-0.31201389 - 1.04898091i] \quad \text{and} \\ B &= [-0.33909457 - 0.1755208i]. \end{aligned}$$

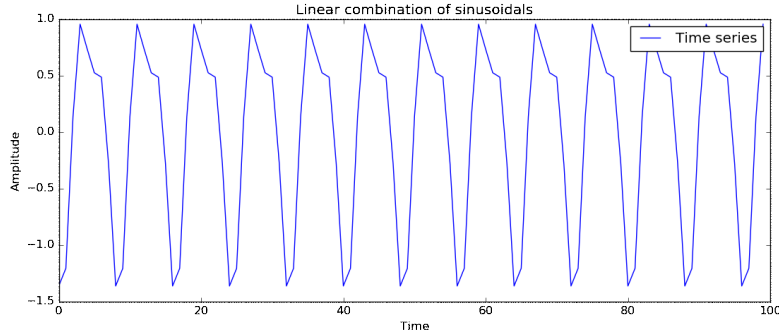


Figure 1: Timeseries generated by Linear combination of sinusoidals.

The Auto Covariance Function (ACVF) of this time series is given by

$$\gamma(h) = \sum_{j=1}^k \sigma^2 \cos(\omega_j h) \quad (2)$$

as defined in [1].

The ACVF as a function of  $h$  is shown in Figure 2

Also the spectral density is given by:

$$F_j(\lambda) = \begin{cases} 0.0 & \text{if } \lambda < -\omega_j, \\ 0.5 & \text{if } -\omega_j \leq \lambda < \omega_j, \\ 1.0 & \text{if } \lambda \geq \omega_j. \end{cases} \quad (3)$$

The spectral density of the function for  $h \in (-\pi, \pi)$  is shown in Figure 3.

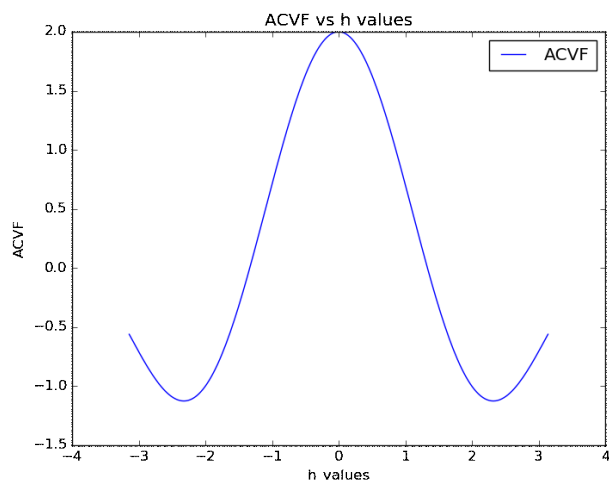


Figure 2: ACVF for the timeseries defined in Figure 1.

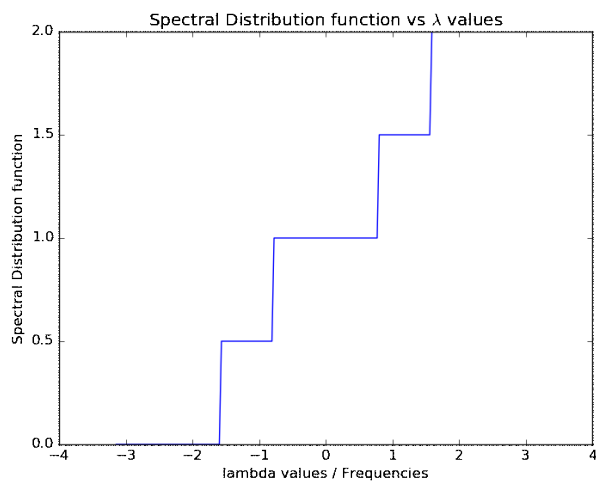


Figure 3: Spectral density for the timeseries defined in Figure 1.

## 1.2 Observation and Questions

1. An important observation is that we didn't use the samples to find ACVF since we know  $\omega$ , and  $\lambda$ .
2. The value of sigma which is not used for generating the time series is used for calculating ACVF.

## 2 Peridogram

We can find the peridogram of a time series given by samples  $x_1, \dots, x_n$  by

$$I_n(\lambda) = \frac{1}{n} \left| \sum_{t=1}^n x_t e^{-it\lambda} \right|^2 \quad (4)$$

The peridogram of the time series defined given in Figure 1 is given in Figure 4

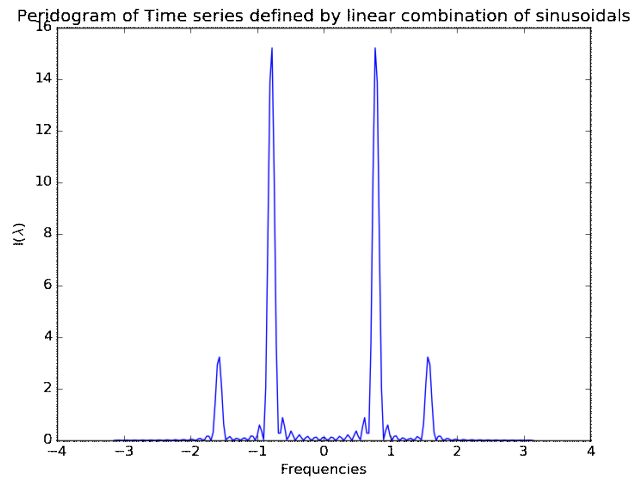


Figure 4: Peridogram for the timeseries defined in Figure 1.

As can be clearly seen from Figure 4, the peridogram has high values of  $I$  which corresponds to the frequencies that make up the signal.

We also ran several other experiments using time series generated by stacking multiple sinusoidal signals with varying frequencies. Below is a time series which is a combination of sinusoidals with frequencies(1,2,3) stacked one after the other along with gaussian noise.

The peridogram for this time series is calculated for values of lambda ranging from  $(-\pi, \pi)$ .

We also found the peridogram by using a sliding window of width 25 over the time series.

## References

- [1] Peter J Brockwell. *Introduction to Time Series and Forecasting, Second Edition*. New York, Springe, 2002.

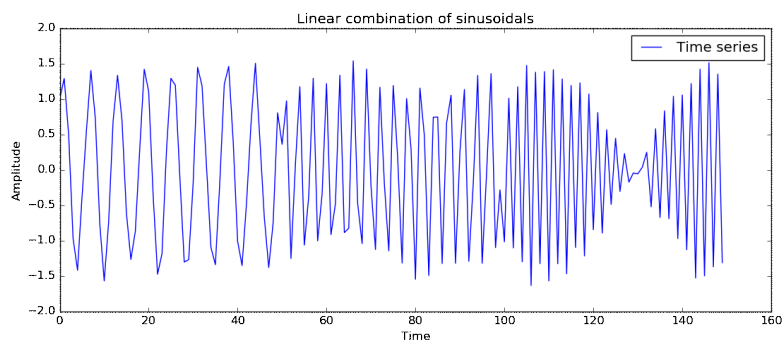


Figure 5: Time series formed by concatenating 50 samples generated by linear combination of sinusoids of frequencies 1, 2, and 3. The change points are at time  $t = 50$ , and 100.

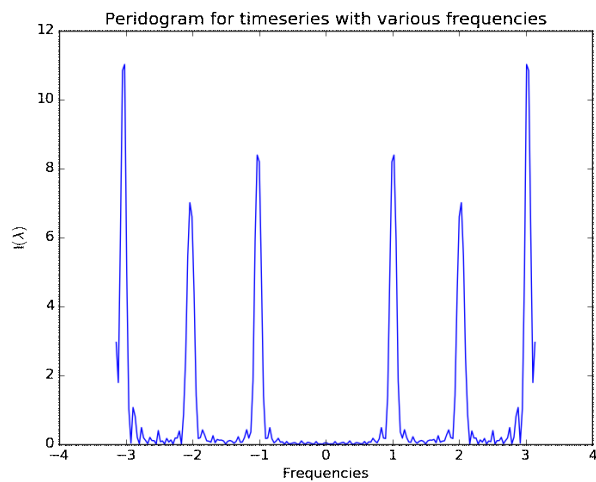


Figure 6: Periodogram for time series shown in Figure 5.