# Kernel Based Approaches for Change-Point Detection — Report 1
# Spectral density & Peridogram

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# 1 Spectral density

## 1.1 Linear combination of sinusoidals

Suppose we have a time series defined as

$$X_t = \sum_{j=1}^{k} (A_j \cos(\omega_j t) + B_j \sin(\omega_j t)), \qquad 0 < \omega_1 < \cdots < \omega_k < \pi \tag{1}$$

The time series is given in Figure 1 The timeseries is generated with values

$$\begin{aligned} K =& 2 \\ \omega =& [1.57079633, 0.78539816], \\ A =& [-0.31201389 - 1.04898091] \quad \text{and} \\ B =& [-0.33909457 - 0.1755208]. \end{aligned}$$

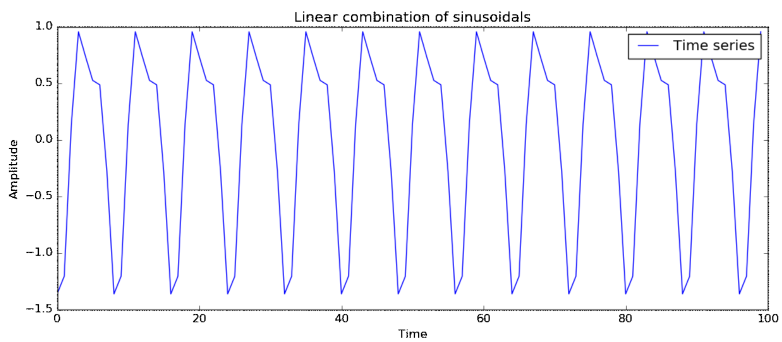


Figure 1: Timeseries generated by Linear combination of sinusoidals.

The Auto Covariance Function (ACVF) of this time series is given by

$$\gamma(h) = \sum_{j=1}^{k} \sigma^2 \cos(\omega_j h) \tag{2}$$

as defined in [1].
The ACVF as a function of $h$ is shown in Figure 2
Also the spectral density is given by:

$$F_j(\lambda) = \begin{cases} 0.0 & \text{if} \quad \lambda < -\omega_j, \\ 0.5 & \text{if} \quad -\omega_j \le \lambda < \omega_j, \\ 1.0 & \text{if} \quad \lambda \ge \omega_j. \end{cases} \tag{3}$$

The spectral density of the function for $h \in (-\pi, \pi)$ is shown in Figure 3.

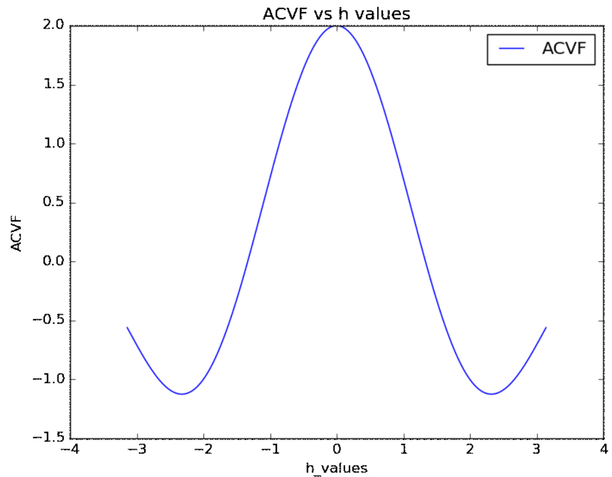


Figure 2: ACVF for the timeseries defined in Figure 1.

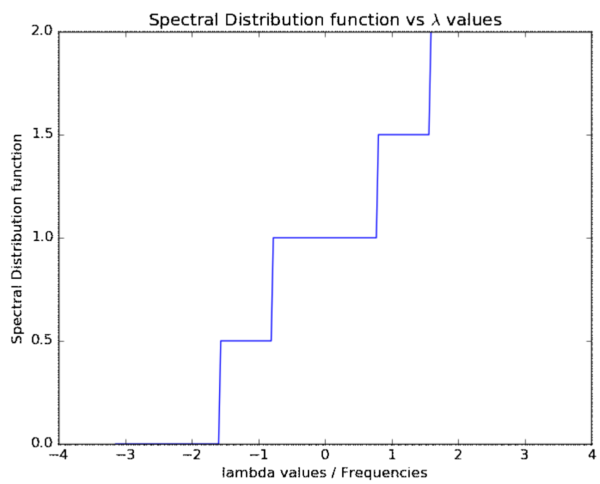


Figure 3: Spectral density for the timeseries defined in Figure 1.

## 1.2 Observation and Questions

1. An important observation is that we didn't use the samples to find ACVF since we know $\omega$, and $\lambda$.

2. The value of sigma which is not used for generating the time series is used for calculating ACVF.

## 2 Peridogram

We can find the peridogram of a time series given by samples $x_1, \ldots, x_n$ by

$$I_n(\lambda) = \frac{1}{n} |\sum_{t=1}^{n} x_t e^{-it\lambda}|^2 \tag{4}$$

The peridogram of the time series defined given in Figure 1 is given in Figure 4

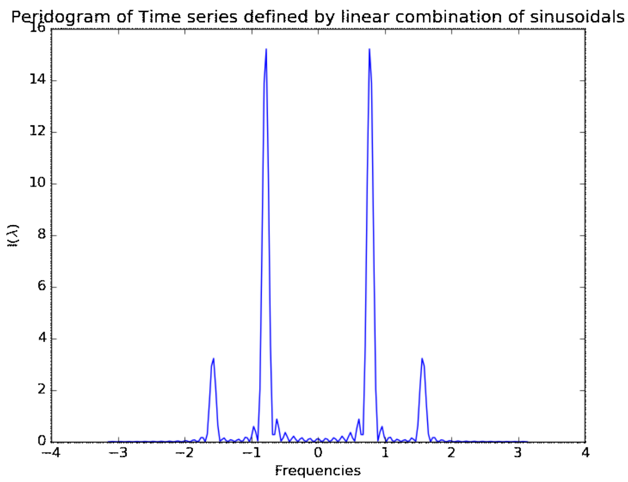


Figure 4: Peridogram for the timeseries defined in Figure 1.

As can be clearly seen from Figure 4, the peridogram has high values of $I$ which corresponds to the frequencies that make up the signal.

We also ran several other experiments using time series generated by stacking multiple sinusoidal signals with varying frequencies. Below is a time series which is a combination of sinusoidals with frequencies$(1, 2, 3)$ stacked one after the other along with gaussian noise.

The peridogram for this time series is calculated for values of lambda ranging from $(-\pi, \pi)$.

We also foudn the peridogram by using a sliding window of width 25 over the time series.

## References

[1] Peter J Brockwell. *Introduction to Time Series and Forecasting, Second Edition.* New York, Springe, 2002.

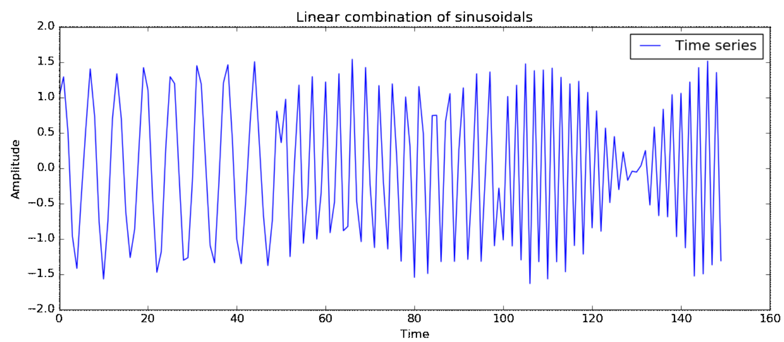


Figure 5: Time series formed by concatenating 50 samples generated by linear combination of sinusoidals of frequencies 1, 2, and 3. The change points are at time t = 50, and 100.

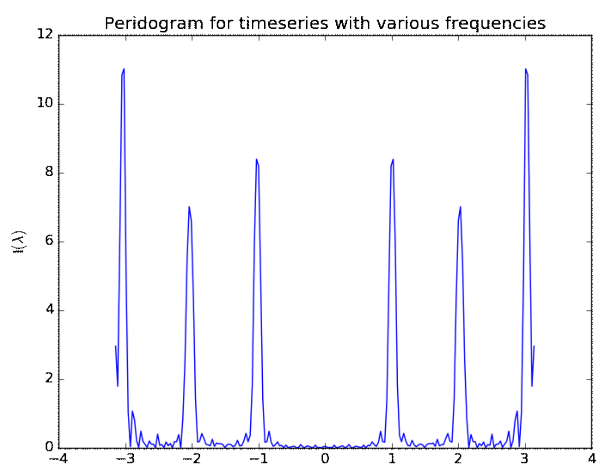


Figure 6: Peridogram for time series shown in Figure **??**.