Kernel Based Approaches for Change-Point Detection — Report 1
Spectral density & Peridogram

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1 Spectral density

1.1 Linear combination of sinusoidals

Suppose we have a time series defined as

$$X_t = \sum_{j=1}^{k} (A_j \cos(\omega_j t) + B_j \sin(\omega_j t)), \qquad 0 < \omega_1 < \cdots < \omega_k < \pi \tag{1}$$

The time series is given in Figure 1 The timeseries is generated with values

$$\begin{aligned} K =& 2 \\ \omega =& [1.57079633, 0.78539816], \\ A =& [-0.31201389 - 1.04898091] \quad \text{and} \\ B =& [-0.33909457 - 0.1755208]. \end{aligned}$$

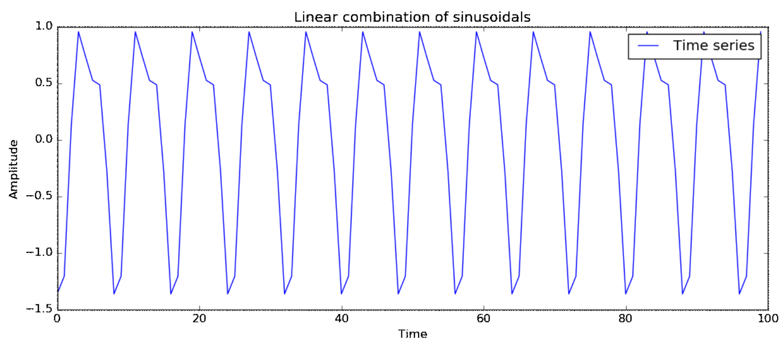

Figure 1: Timeseries generated by Linear combination of sinusoidals.

The Auto Covariance Function (ACVF) of this time series is given by

$$\gamma(h) = \sum_{j=1}^{k} \sigma^2 \cos(\omega_j h) \tag{2}$$

as defined in [1].
The ACVF as a function of h is shown in Figure 2
Also the spectral density is given by:

$$F_j(\lambda) = \begin{cases} 0.0 & \text{if} \quad \lambda < -\omega_j, \\ 0.5 & \text{if} \quad -\omega_j \leq \lambda < \omega_j, \\ 1.0 & \text{if} \quad \lambda \geq \omega_j. \end{cases} \tag{3}$$

The spectral density of the function for $h \in (-\pi, \pi)$ is shown in Figure 3.

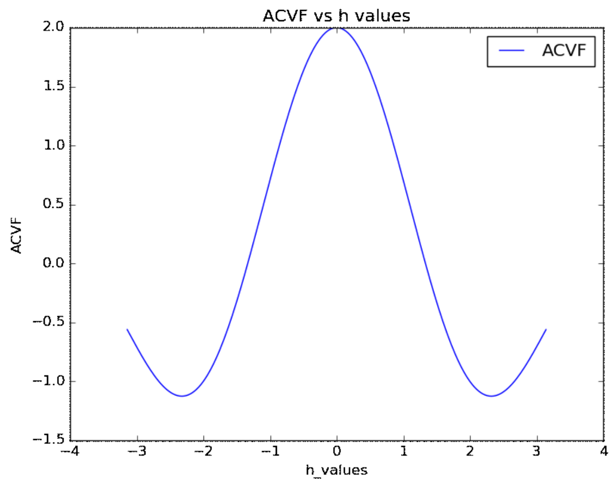

Figure 2: ACVF for the timeseries defined in Figure 1.

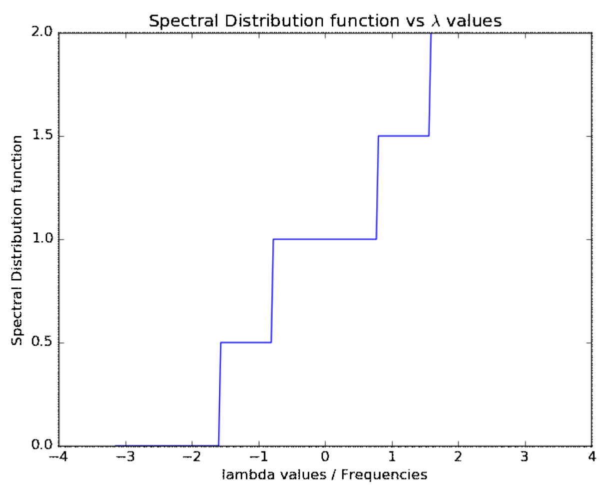

Figure 3: Spectral density for the timeseries defined in Figure 1.

1.2 Observation and Questions

1. An important observation is that we didn't use the samples to find ACVF since we know ω, and λ.

2. The value of sigma which is not used for generating the time series is used for calculating ACVF.

2 Peridogram

We can find the peridogram of a time series given by samples $x_1, \ldots, x_n$ by

$$I_n(\lambda) = \frac{1}{n}|\sum_{t=1}^{n} x_t e^{-it\lambda}|^2 \tag{4}$$

The peridogram of the time series defined given in Figure 1 is given in Figure 4

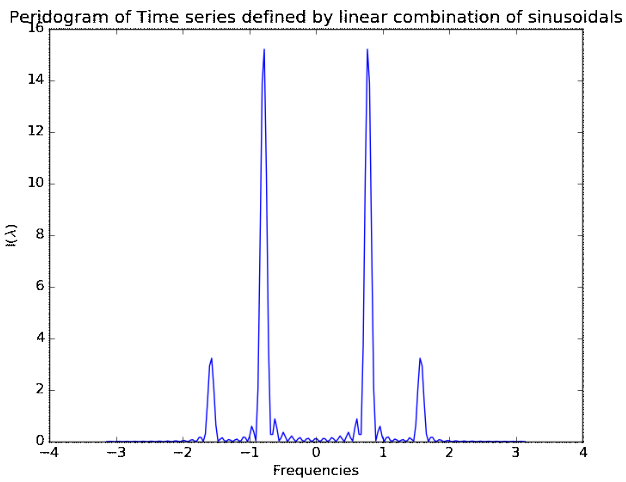

Figure 4: Peridogram for the timeseries defined in Figure 1.

As can be clearly seen from Figure 4, the peridogram has high values of I which corresponds to the frequencies that make up the signal.

We also ran several other experiments using time series generated by stacking multiple sinusoidal signals with varying frequencies. Below is a time series which is a combination of sinusoidals with frequencies$(1, 2, 3)$ stacked one after the other along with gaussian noise.

The peridogram for this time series is calculated for values of lambda ranging from $(-\pi, \pi)$.

We also foudn the peridogram by using a sliding window of width 25 over the time series.

References

[1] Peter J Brockwell. *Introduction to Time Series and Forecasting, Second Edition.* New York, Springe, 2002.

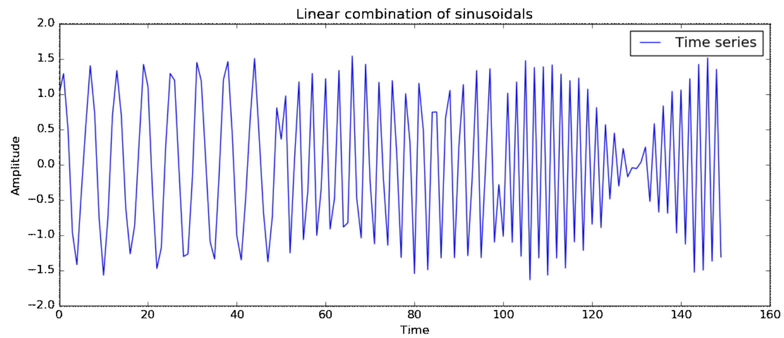

Figure 5: Time series formed by concatenating 50 samples generated by linear combination of sinusoidals of frequencies 1, 2, and 3. The change points are at time t = 50, and 100.

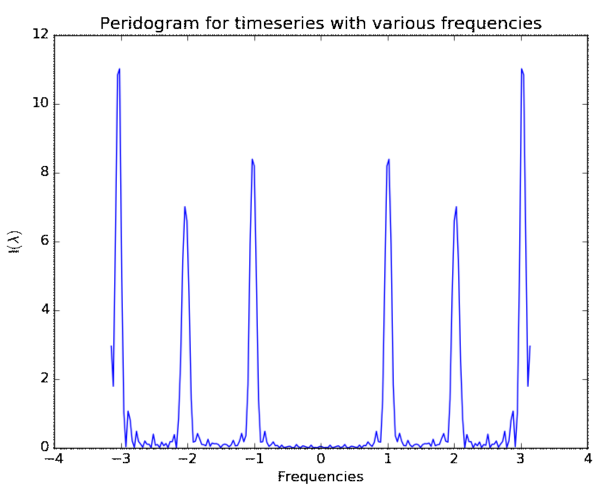

Figure 6: Peridogram for time series shown in Figure 5.