

# Foundations of Machine Learning — Homework Assignment 1

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## D. Kernels

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Given: Kernel, K is defined by  $K(x, y) = \sum_{i=1}^N \cos^n(x_i^2 - y_i^2)$  for all  $(X, Y) \in \mathbb{R}^N \times \mathbb{R}^N$

Solution: We know that

$$\cos(x_i^2 - y_i^2) = \sin(x_i^2) \cdot \sin(y_i^2) + \cos(x_i^2) \cdot \cos(y_i^2) \quad (1)$$

This can be written as a dot product of two vectors

$$\phi(x_i) = \begin{bmatrix} \cos(x_i^2) \\ \sin(x_i^2) \end{bmatrix} \quad \text{and} \quad \phi(y_i) = \begin{bmatrix} \cos(y_i^2) \\ \sin(y_i^2) \end{bmatrix} \quad (2)$$

We know that if K can be written as  $\langle \phi(x_i), \phi(y_i) \rangle$ , then it is a PDS@.

Also,  $\langle \phi(x_i), \phi(y_i) \rangle$  is a scalar. When a scalar is raised to a positive power (n in our case) and summed with N other positive scalar, we get a positive scalar as our answer. Hence

$$K(x, y) = \sum_{i=1}^N \cos^n(x_i^2 - y_i^2) \text{ is PDS.}$$