Foundations of Machine Learning — Homework Assignment 1

Anirudhan J Rajagopalan
N18824115
ajr619

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C. Support Vector Machines

1

Installed the software from[2]. The installed version of software is also checked into github at[1].

2

See the following command:

```
$ ./svm-scale -s splice_noise_train.txt.range \
> splice_noise_train.txt > splice_noise_train.txt.scale
$ ./svm-scale -r splice_noise_train.txt.range \
> splice_noise_test.txt > splice_noise_test.txt.scale
```

3

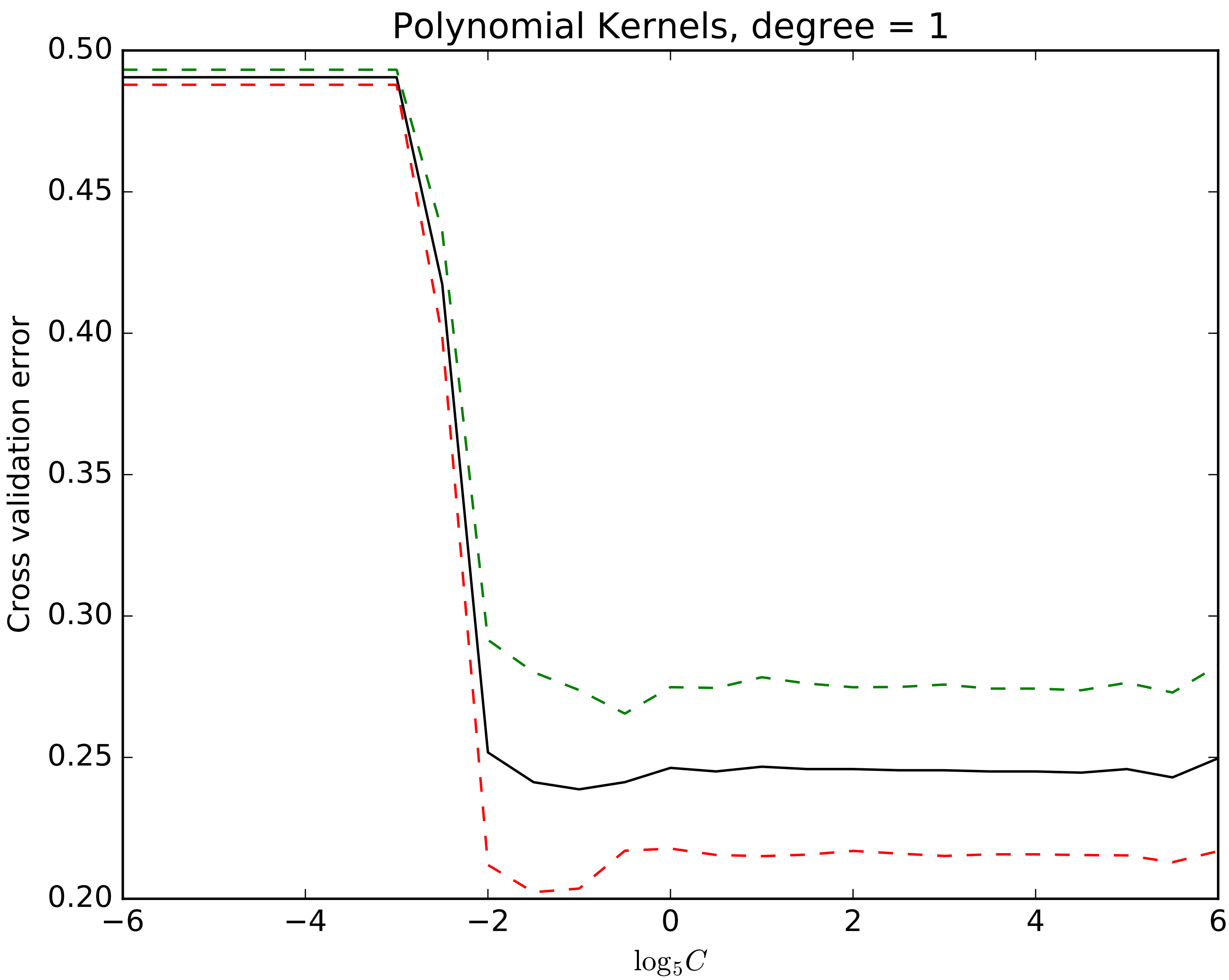
Run training and test script[3] by editing the KERNEL_DEGREE parameter for each value of d = 1, 3, 5.

```
$ python cross_validation.py > deg1.out # KERNEL_DEGREE = 1
$ python cross_validation.py > deg3.out # KERNEL_DEGREE = 3
$ python cross_validation.py > deg5.out # KERNEL_DEGREE = 5
```

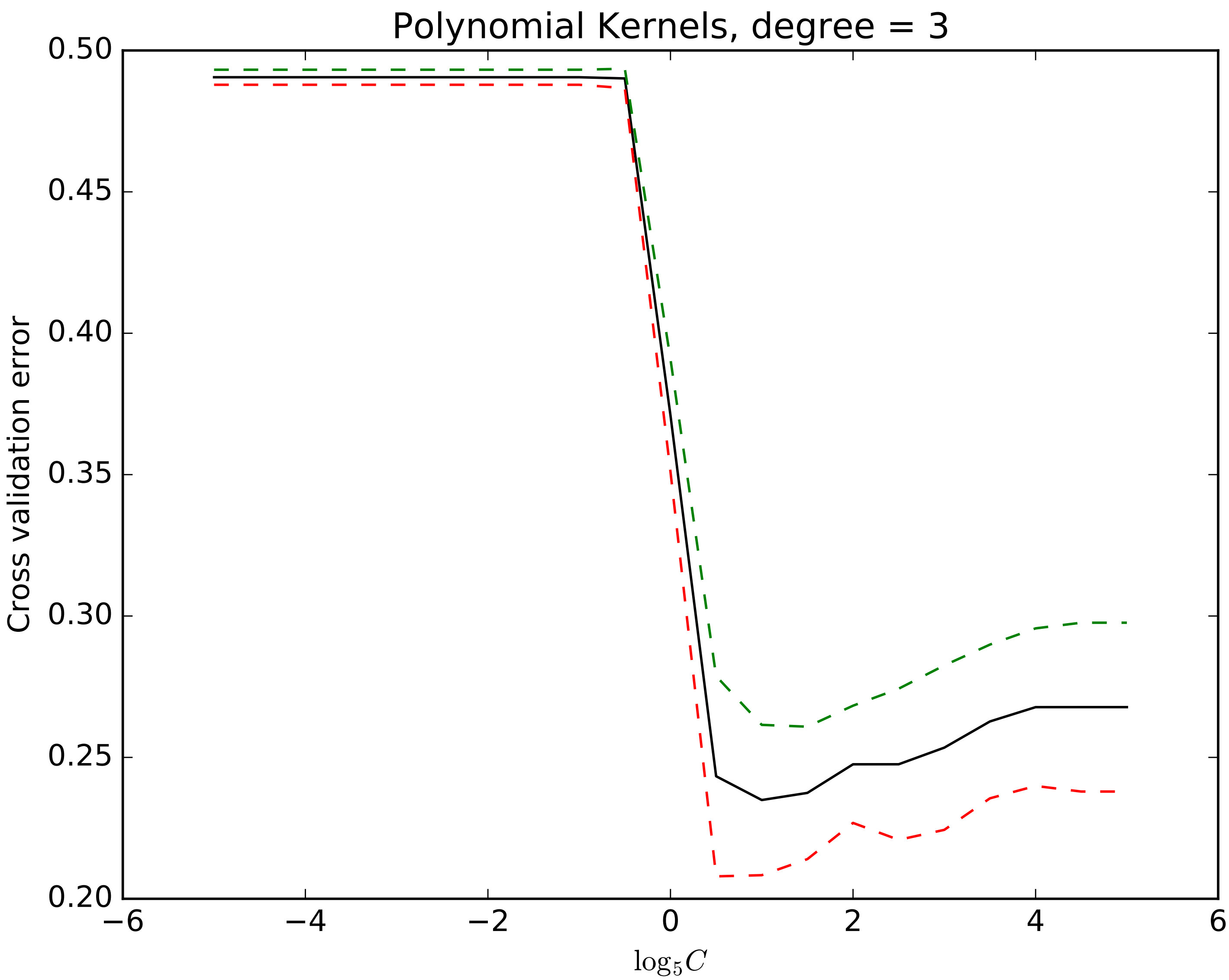
Filter the parameter and accuracy information from the run logs (deg1.out, deg3.out and deg5.out) by the command below.

```
$ cat deg1.out | grep OUR | cut -d' ' \
> -f2,3,4,5,6 > deg1.out.filtered
$ cat deg3.out | grep OUR | cut -d' ' \
> -f2,3,4,5,6 > deg3.out.filtered
$ cat deg5.out | grep OUR | cut -d' ' \
> -f2,3,4,5,6 > deg5.out.filtered
```

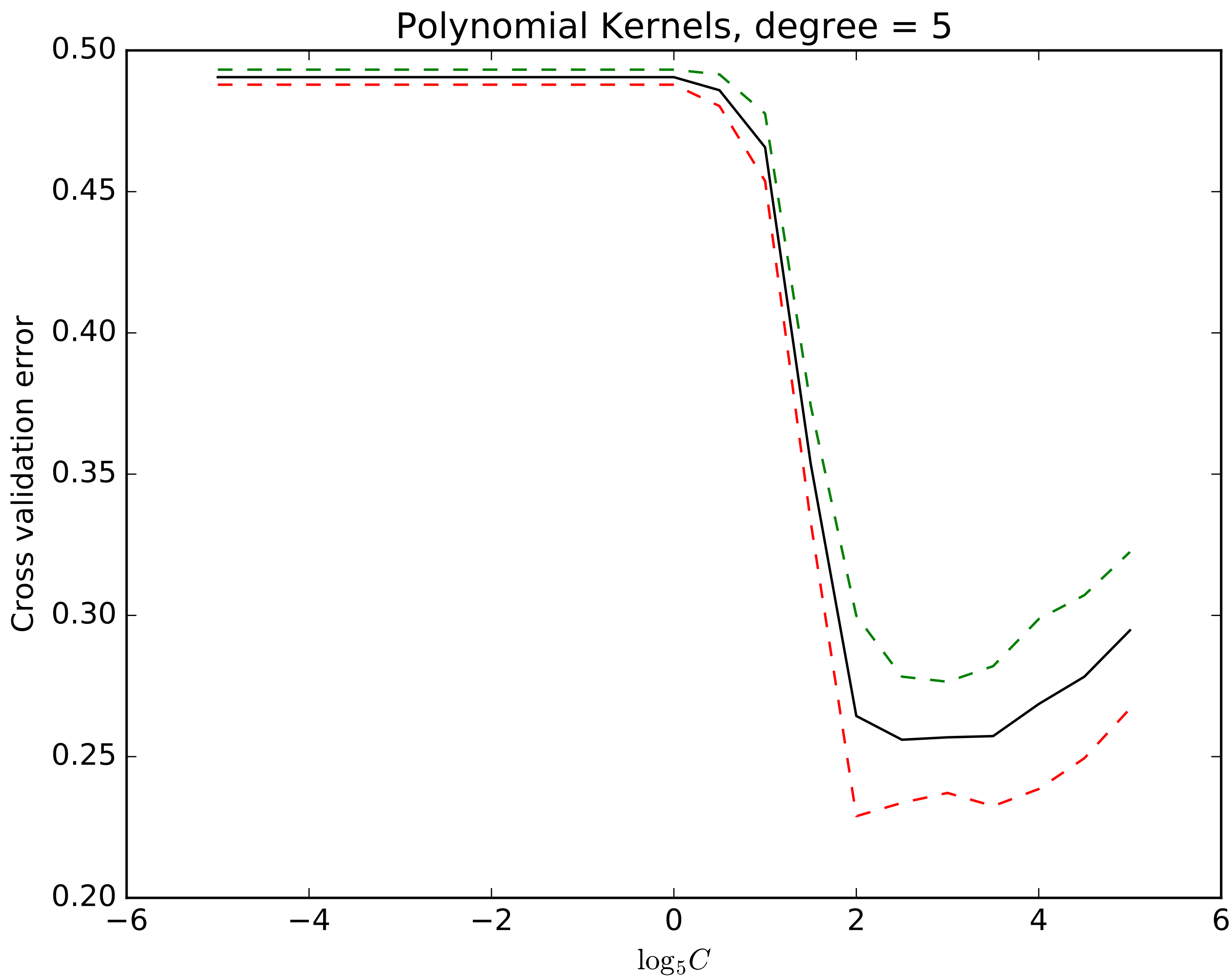
Use plotter.py[4] to create plots from the output values for KERNEL_DEGREE values 1, 3, 5. The output will be saved as deg1.pdf, deg3.pdf and deg5.pdf. All the three plots are embedded below one after the other.



Polynomial Kernels, degree = 1
Cross validation error
0.50
0.45
0.40
0.35
0.30
0.25
0.20
−6
−4
−2
0
2
4
6
$\log_5 C$



Polynomial Kernels, degree = 3
Cross validation error
0.50
0.45
0.40
0.35
0.30
0.25
0.20
−6
−4
−2
0
2
4
6
$\log_5 C$



Polynomial Kernels, degree = 5
Cross validation error
0.50
0.45
0.40
0.35
0.30
0.25
0.20
−6
−4
−2
0
2
4
6
$\log_5 C$

Best values of c for polynomial kernels 1, 3, 5 are:

$$
\begin{aligned}
d &= 1 & c* &= 5^{-1.0} = 0.2 & \text{cv} - \text{err} &= 23.87\% \pm 3.5\% \\
d &= 3 & c &= 5^{1.0} = 5 & \text{cv} - \text{err} &= 23.49\% \pm 2.65\% \\
d &= 5 & c &= 5^{2.5} = 55.9017 & \text{cv} - \text{err} &= 25.59\% \pm 2.23\%
\end{aligned}
$$

The best C measured in the cross-validation set is $C^* = 5^{1.0}$ with degree $d^* = 3$ which gives an average error of $23.49\% \pm 2.65\%$

4

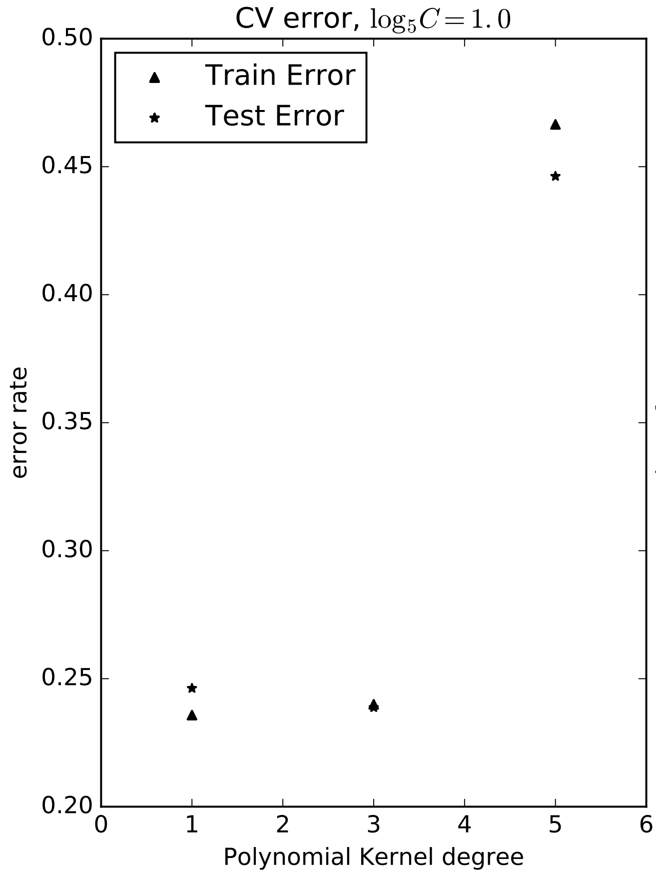
The average number of SV and BSV are taken across the cross validation sets. numMarginal represents the average number of marginal support vectors.

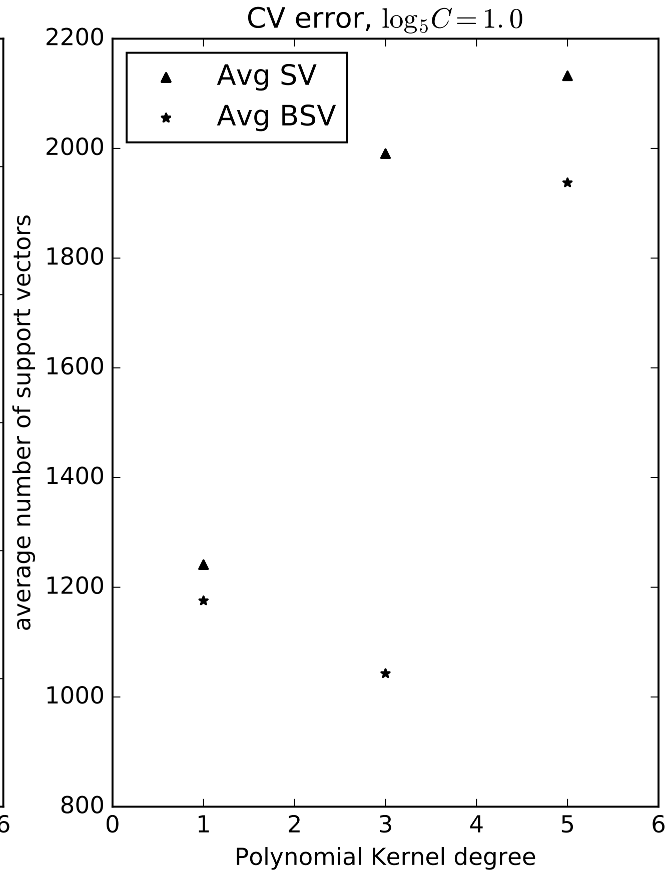
nSV and nBSV represent the number of support vectors and bounded support vectors in test set. Marginal represents the number of support vectors in test set.

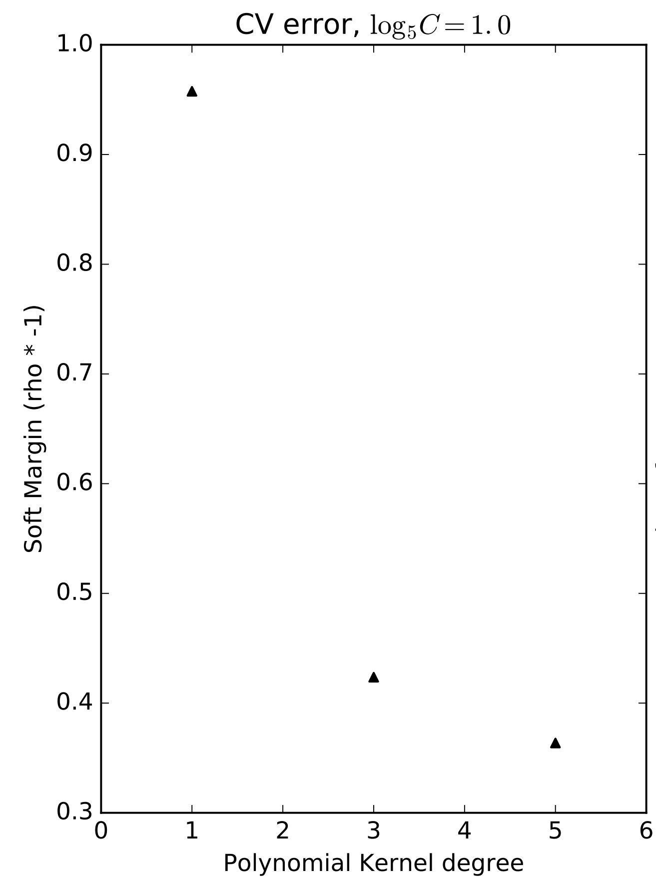
After manually populating d, cross-train accuracy, test accuracy, nSV, nBSV, rho, totalSV, totalBSV into a file[5], we run a script[6] to generate the plots as a pdf file.

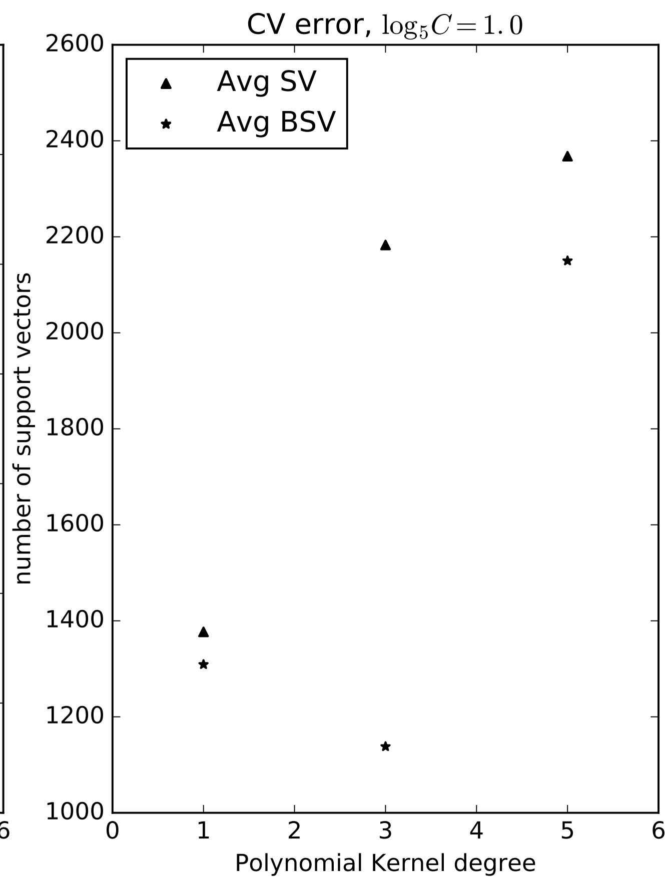
D	Avg Sv	Avg BSV	numMarginal	nSv	nBSV	marginal
1	1241.3	1175.4	65.9	1377	1309	68
3	1990.5	1042.7	947.8	2183	1138	1045
5	2132.4	1937.4	195.0	2368	2150	218

The plots generated are as shown below


CV error, $\log_5 C = 1.0$
Train Error
Test Error
error rate
Polynomial Kernel degree


CV error, $\log_5 C = 1.0$
Avg SV
Avg BSV
average number of support vectors
Polynomial Kernel degree


CV error, $\log_5 C = 1.0$
Soft Margin (rho * -1)
Polynomial Kernel degree


CV error, $\log_5 C = 1.0$
Avg SV
Avg BSV
number of support vectors
Polynomial Kernel degree

5

6

D. Kernels

1

Given: Kernel, K is defined by $K(x,y) = \sum_{i=1}^{N} \cos^n(x_i^2 - y_i^2)$ for all $(X,Y) \in \mathbb{R}^N \times \mathbb{R}^N$

Solution: We know that

$$\cos(x_i^2 - y_i^2) = \sin(x_i^2).\sin(y_i^2) + \cos(x_i^2).\cos(y_i^2) \tag{1}$$

This can be written as a dot product of two vectors

$$\phi(x_i) = \begin{bmatrix} \cos(x_i^2) \\ \sin(x_i^2) \end{bmatrix} \qquad \text{and} \qquad \phi(y_i) = \begin{bmatrix} \cos(y_i^2) \\ \sin(y_i^2) \end{bmatrix} \tag{2}$$

We know that if K can be written as $\langle \phi(x_i), \phi(y_i) \rangle$, then it is a PDS@.

Also, $\langle \phi(x_i), \phi(y_i) \rangle$ is a scalar. When a scalar is raised to a positive power (n in our case) and summed with N other positive scalar, we get a positive scalar as our answer. Hence

$$K(x,y) = \sum_{i=1}^{N} \cos^n(x_i^2 - y_i^2) \text{isPDS}.$$

References

[1] http://git.io/v80yn

[2] http://www.csie.ntu.edu.tw/~cjlin/libsvm/

[3] http://git.io/v80yY

[4] http://git.io/v80yk

[5] http://git.io/v8Edr

[6] http://git.io/v8EdH