Foundations of Machine Learning — Homework Assignment 1

Anirudhan J Rajagopalan
N18824115
ajr619

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D. Kernels

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Given: Kernel, K is defined by $K(x,y) = \sum_{i=1}^{N} \cos^n(x_i^2 - y_i^2)$ for all $(X,Y) \in \mathbb{R}^N \times \mathbb{R}^N$

Solution: We know that

$$\cos(x_i^2 - y_i^2) = \sin(x_i^2).\sin(y_i^2) + \cos(x_i^2).\cos(y_i^2) \tag{1}$$

This can be written as a dot product of two vectors

$$\phi(x_i) = \begin{bmatrix} \cos(x_i^2) \\ \sin(x_i^2) \end{bmatrix} \qquad \text{and} \qquad \phi(y_i) = \begin{bmatrix} \cos(y_i^2) \\ \sin(y_i^2) \end{bmatrix} \tag{2}$$

We know that if K can be written as $\langle \phi(x_i), \phi(y_i) \rangle$, then it is a PDS@.

Also, $\langle \phi(x_i), \phi(y_i) \rangle$ is a positive scalar. When a scalar is raised to a positive power (n in our case) and summed with N other positive scalar, we get a positive scalar as our answer. Hence

$$K(x,y) = \sum_{i=1}^{N} \cos^n(x_i^2 - y_i^2) \text{isPDS.}$$