Foundations of Machine Learning — Homework Assignment 1

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C. Randomized Halving

1. Psuedo code

Algorithm 1 Randomized Halving

$H_1 \leftarrow H$
for t $\leftarrow$ 1 to T **do**
    $RECEIVE(x_t)$
    $r_t \leftarrow \frac{\sum_{i:y_{t,i}=1} 1}{|H_t|}$
    $p_t \leftarrow 1$
    if $r_t \leq \frac{3}{4}$ **then**
        $p_t \leftarrow [\frac{1}{2} \log_2 \frac{1}{1-r_t}]$
    $\hat{y_t} \leftarrow GetRandomNumberWithProbability([1,0],[p_t, 1-p_t])$
    $RECEIVE(y_t)$
    if $\hat{y_t} \neq y_t$ **then**
        $H_{t+1} \leftarrow \{C \in H_t : C(x_t) = y_t\}$
return H_{T+1}

2. Prove $\forall t \geq 1, E[\mu] \leq \frac{\phi_t - \phi_{t+1}}{2}$

Given: Potential function: $\phi_t = \log_2 | H_t |$ and $\mu_t = 1_{y_t \neq \hat{y_t}}$

Proof:

We are only considering the case when the predicted value $\hat{y_t}$ is not equal to the received value y_t. The value of expectation can be written as

$$\begin{aligned} E[\mu_t] =& p_t * 1 + (1 - p_t) * 0 \\ =& p_t * 1 \end{aligned}$$

The probability of predicting 1 by the randomized algorithm is the probability of making a mistake since we are only considering the cases in which we make mistakes ($\mu_t = 1_{y_t \neq \hat{y_t}}$)

Therefore,

$$E[\mu_t] = p_t$$

3. Expected number of mistakes.

4. [Bonus Question]