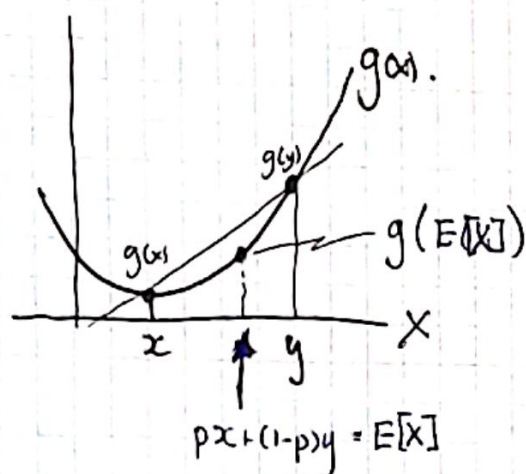


Def: A fcn in one variable is convex if the line segment connecting any two pts. on the graph lie completely above the graph. (8)

## Convex Function:



- not a proof but. makes it easy to remember.

- say we have a pmf on the two values  $x$  &  $y$ .

$x$  c prob.  $p$

$y$  c prob.  $q = 1 - p$ .

$$\rightarrow E[X] = px + qy$$

- lies btwn  $x$  &  $y$

$px + qy$  = convex comb. of  $x$  &  $y$ . =  $E[X]$

- now look at  $g(x)$  &  $g(y)$ .

- their convex sum =  $pg(x) + qg(y)$  lies on the line connecting  $(x, g(x))$  &  $(y, g(y))$  in the graph.

$$\begin{array}{l} \rightarrow (x, g(x)) \rightarrow (px, pg(x)) \\ (y, g(y)) \rightarrow (qy, qg(y)) \end{array} \left\{ \begin{array}{l} \text{convex} \\ \text{comb.} \end{array} \right. \begin{array}{l} (px + qy, pg(x) + qg(y)) \\ \parallel \\ E[X] \quad E[g(X)] \end{array}$$

~~note~~  $(px, pg(x))$

• by the def. of a convex fcn the line connecting the two pts lies above the fcn.

$$\therefore \boxed{g(E[X]) \leq E[g(X)]} \text{ for convex } g(x).$$

$\rightarrow$  generalizes to multiple variables, discrete or conts in the obvious way.

$\rightarrow$  called Jensen's Inequality.