- In Markov inequality why does the r.v. have to be strictly positive?

- Look at the chain of inequalities for general $X \in \mathbb{R}$, which could be negative.

$\Rightarrow$ $$\mathbb{E}[X] = \int_{-\infty}^{\infty} x\,p(x)$$

$$= \underbrace{\int_{-\infty}^{0} x\,p(x)}_{\text{I}} + \underbrace{\int_{0}^{a} x\,p(x)}_{\text{II}} + \underbrace{\int_{a}^{\infty} x\,p(x)}_{\text{III}}.$$

$\rightarrow$ replacing $x \to a$ in the integrand of III still gives the inequality:

$$\mathbb{E}[X] \geq \underbrace{\int_{-\infty}^{0} x\,p(x)}_{\text{I}} + \underbrace{\int_{0}^{a} x\,p(x)}_{\text{II}} + \underbrace{a \cdot \int_{a}^{\infty} p(x)}_{\text{III}}$$

$$\mathbb{E}[X] \geq \underbrace{\int_{-\infty}^{0} x \cdot p(x)}_{\text{I}} + \underbrace{a \int_{a}^{\infty} p(x)}_{\text{III}}.$$ remove II for next inequality

$\rightarrow$ this is the step that gets us into trouble if $p(x) > 0$ for $x < 0$.

$\Rightarrow$ We want $\boxed{\mathbb{E}[X] \geq a \cdot \int_{a}^{\infty} p(x)}$ (*) for the inequality.

- but if X can be < 0 then I is negative and we can no longer claim (*).

e.g. If $e \geq a+b$ we can only claim $e \geq b$ if $a = 0$.

or in numbers: $5 > 4 + 0.9$ & so $5 > 4$ & $5 > 0.9$

But $5 > -20 + 17$ but $5 \not> 17$ $\Rightarrow$ a negative term ruins it.