Thm WLLN: - says the tail prob'y of the empirical mean $\xrightarrow{n\to\infty} 0$

Let $X_1, X_2, \ldots, X_n$ be iid r.v.'s with finite mean $\mathbb{E}X_i = \mu$.
Then $\forall \epsilon > 0$,

$$\lim_{n\to\infty} P(|\bar{X} - \mu| \geq \epsilon) = 0.$$

Proof: Apply Chebyshev's inequality:

CE: $P(|\bar{X} - \mathbb{E}[\bar{X}]| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon}$.

So, $P(|\bar{X} - \mu| \geq \epsilon) \leq \frac{var(\bar{X})}{\epsilon^2}$

$$\boxed{P(|\bar{X} - \mu| \geq \epsilon) = \frac{var(X)}{n\epsilon^2}}$$

$\longrightarrow 0$ as $n \to \infty$ for any $\epsilon > 0$.

Why "weak"?

Jensen's Inequality

Ex., Recall the variance of a r.v. is positive.

$\therefore$ $Var(X) = \mathbb{E}(X - \mathbb{E}[X])^2$

$= \mathbb{E}[X^2 - 2X\mathbb{E}[X] + \mathbb{E}[X]^2]$

$= \mathbb{E}X^2 - (\mathbb{E}X)^2$

> 0

$\therefore$ $\mathbb{E}[X^2] > (\mathbb{E}[X])^2$

- defining $g(x) = x^2$ we have:

$$\mathbb{E}[g(x)] \geq g(\mathbb{E}[x])$$

$\rightarrow$ this turns out to hold for general convex g.