

## Example: Polling Problem

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- you have a referendum & a fraction  $p$  of the pop<sup>n</sup> is going to vote 'Yes'

Problem: conduct a poll & estimate  $p \rightarrow$  how many to poll?

- we could do the whole pop<sup>n</sup> but that's just conducting the referendum.

- randomly sample the pop<sup>n</sup> uniformly

- $i$ 'th person returns  $X_i = \begin{cases} 1 & \text{'Yes'} \\ 0 & \text{'No'} \end{cases}$

- note  $\mathbb{E}[X_i] = p$  (unknown).

- $\bar{X}_n = \frac{X_1 + X_2 + \dots + X_n}{n} = \text{fraction of 'Yes' in the sample.}$

- Your boss wants you to predict  $p$  but there will always be some error if  $n < N_{\text{pop}}$ .

- so he asks for an estimate  $\bar{c}$  a small error. Say,

$$|\bar{X}_n - p| < 0.01 \quad (1\%) \rightarrow \text{he wants this guaranteed!}$$

- let's say you sample  $n = 10,000$

- but you can't really meet the guarantee!

- say you accidentally polled every who say 'No'

$\Rightarrow$  there is a  $(1-p)^n$  chance of this happening

$\rightarrow$  you can't give a guarantee in absolute terms of an error. that it will be small

- instead you can offer this:

I cannot guarantee  $\bar{c}$  certainly the error will be small, but, I can guarantee that the error will be small  $\bar{c}$  high probability.