~~Example~~:

Useful fact: MGF of sums of r.v.

Say $Y = X_1 + X_2 + \cdots + X_n$ where X_i are indep. but can have different distn's.

Then $M_Y(s) = \mathbb{E}[e^{sY}]$

$= \mathbb{E}\, e^{s(X_1 + \cdots X_n)}$

$= \mathbb{E}\, e^{sX_1} \cdot e^{sX_2} \cdots e^{sX_n}$

$= \mathbb{E} e^{sX_1}\, \mathbb{E} e^{sX_2} \cdots \mathbb{E} e^{sX_n}$ ($\because$ independent)

$$\boxed{M_Y(s) = M_{X_1}(s) \cdot M_{X_2}(s) \cdots M_{X_n}(s)}$$

· iff the X_i are iid:

$$\boxed{M_Y(s) = M_X(s)^n}$$

Example: Binomial(n,p) - n coin flips $\bar{c}$ with weight p $(0,1)$ outcomes.

$X = X_1 + X_2 + X_3 + \cdots + X_n$

Where each $X_i \sim$ Bernoulli(p) : 1 $\bar{c}$ prob. p

0 $\bar{c}$ prob. $q = 1-p$.

$\rightarrow M_{X_i}(s) = p \cdot e^{s \cdot 1} + q e^{s \cdot 0}$

$= pe^s + q$

$\therefore$ $\boxed{M_X(s) = (pe^s + q)^n}$

· check $\frac{d}{ds} M_X^{n=1}(s)\Big|_{s=0} = pe^s\Big|_{s=0} = p$ = the mean, as expected.