Thm **Markov Inequality** - relate deviation to mean. (start here: 5 No pgs 1-4.)

Thur. Apr ~~18~~ 17/25

If X is any non-negative r.v., then

$$P(X \geq a) \leq \frac{\mathbb{E}[X]}{a}$$

Proof:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x\, p(x)\, dx$$

$$= \int_{0}^{\infty} x\, p(x)\, dx \quad \because X \geq 0$$

$$\geq \int_{a}^{\infty} x\, p(x)\, dx. \quad \text{for any } a \geq 0$$

$$\geq \int_{a}^{\infty} a\, p(x)\, dx \quad \because x > a \text{ in the integrated region}$$

$$= a \cdot P(X \geq a).$$

$$\therefore \boxed{P(X \geq a) \leq \frac{E[X]}{a}} \quad \text{for } a > 0.$$

⇒ See 5.5 for pos'ly discussion

Thm **Chebyshev Inequality**: - relates deviation to variance.

If X is any rv $\bar{c}$ finite variance (and so mean), then for any $k > 0$,

$$P(|X - E[X]| \geq k\sigma) \leq \frac{1}{k^2}.$$

Proof:

- define $Y = (X - \mathbb{E}[X])^2$.
- Y is non-negative (≥ 0.)
- apply Markov

$$P(Y \geq b^2) \leq \frac{E[Y]}{b^2}$$

- but $\mathbb{E}[Y] = \sigma^2$

$$\rightarrow P(|X - \mathbb{E}[X]| \geq b) \leq \frac{\sigma^2}{b^2}$$

→ let $b = k\sigma$.

$$\boxed{P(|X - \mathbb{E}[X]| \geq k\sigma) \leq 1/k^2}$$

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