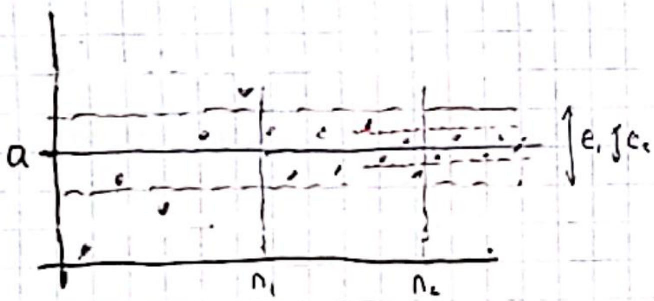
• Ordinary Sequence: $a_n \to a$ (a number):

"a_n gets arbitrarily close & stays close to a".

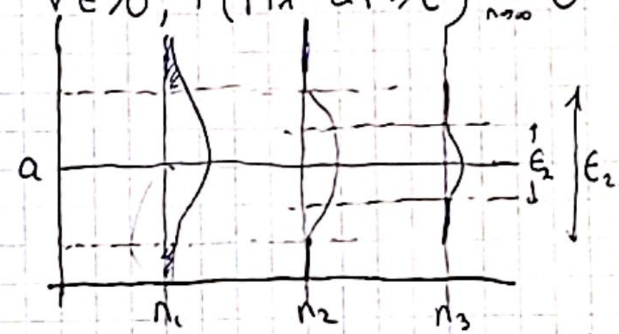


- for every $\epsilon > 0$, there exists N_0 such that for every $n \geq N_0$, we have $|a_n - a| \leq \epsilon$.

• Convergence in Probability Sequence: $Y_n \to a$

"the prob. Y_n falls outside a ϵ-band around a falls to 0."

$\forall \epsilon > 0,\ P(|Y_n - a| \geq \epsilon) \xrightarrow[n\to\infty]{} 0$



- almost all the prob. of Y_n eventually gets concentrated arbitrarily close to a.

Some Properties: of cip.

• parallel properties of convergence in sequences.

Suppose $X_n \xrightarrow{ip} a$, $Y_n \xrightarrow{ip} b$ in probability: (no assumption of indep. within the X_n's & Y_n sequence or with each other).

① If $g(\cdot)$ is continuous, then $g(X_n) \to g(a)$

new r.v.'s

② $X_n + Y_n \longrightarrow a + b$

• Note: Even if $X_n \xrightarrow{ip} a$, it does not mean

$\mathbb{E}[X_n] \longrightarrow a$

"convergence ip of rv's does not imply convergence of expectations"

(will see in example).