Converse result: "impossibility", you can not do better than ...
Achievability: non-constructive, show a code exists.

ASYMPTOTIC EQUIPARTITION THEORY

Existence.

Thur. Apr. 24/25

- follows from **Law of Large Numbers**:
  - Let $X_1, X_2, \ldots$ be an iid sequence with a finite mean $\mathbb{E}_x[X] = \mu < \infty$
  - Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ be the running sample mean for $n = 1, 2, \ldots$
  - The $\{\bar{X}_n\}_{n=1}^{\infty}$ form a sequence of r.v.'s themselves.
  - The sequence $\bar{X}_1, \bar{X}_2,$ converges to the distribution mean, μ, in the sense: For all $\epsilon > 0$,

$$\lim_{n\to\infty} \mathbb{P}\left(|\bar{X}_n - \mu| > \epsilon\right) = 0 \quad \text{or} \quad \bar{X}_n \xrightarrow{\text{ip}} \mu \text{ in probability}$$

  - the probability the sample mean diverges from the pop^ goes to zero as $n \to \infty$. - the sample mean is unlikely to be far off from the true mean, μ.
  - this is precisely why "taking the average" is a good estimate of the true pop^ mean.