Multiplet Reference: https://en.wikipedia.org/wiki/Multiplet

A multiplet is the state space for 'internal' degrees of freedom of a particle. Examples: spin, color, isospin, hypercharge. The state space is a vector space which carries the action of a group of continuous symmetries.

Multiplets are described via representations of a Lie group or its corresponding Lie algebra.We are interested in irreducible representations (irreps, for short). In finite dimensions, there is a correspondence bewen representations of a Lie group and representations of it's Lie algebra, so we can use them interchangeably.

In mathematics, the homomorphism is often referred to as the representation, whereas in physics, the vector space is often referred to as the representation.

For an irreducible representation, an n-plet refers to an n dimensional irreducible represenation. Generally a group may have non-isomorphic representations of a given dimension. SU(2) has exactly one irreducible repesentation of dimension n for each non-negative integer n.

The group SU(2) is important in section 3.3 of Guidry. The study of representations of SU(2) is fundamental to the study of representations of semi-simple Lie groups.

Reference: https://en.wikipedia.org/wiki/Representation_theory_of_SU(2)

In the above Wikipedia page, look at the section on Weights and the structure of the representation. The first sentence has a reference to the following result from the Hall book on Lie Groups:

Let π be an irreducible representation of $sl(2, C)$ Lemma 4.33 Let u be an eigenvector of $\pi(H)$ with eigenvalue $\alpha \in C$. Then we have

$$\pi(H)\pi(X)u = (\alpha + 2)\pi(X)u \tag{1}$$

Thus either $\pi(X)u = 0$ or $\pi(X)u$ is an eigenvector for $\pi(H))$ with eigenvalue $\alpha + 2$. Similarly,

$$\pi(H)\pi(Y)u = (\alpha - 22)\pi(X)u \tag{2}$$

so that either $\pi(Y)u = 0$ or $\pi(Y)u$ is an eigenvector for $\pi(H)$ with eigenvalue $\alpha - 2$. Proof of lemma We know that $[\pi(H), \pi(X)] = \pi([H, X]) = 2\pi(X)$. Thus

$$\begin{aligned}\pi(H)\pi(X)u &= \pi(X)\pi(H)u = 2\pi(X)u \\ &= \pi(X)(\alpha u) + 2\pi(X)u \\ &= (\alpha + 2)\pi(X)u.\end{aligned}$$

Look at Reference: https://en.wikipedia.org/wiki/Angular_momentum_operator

A connected Lie group is called semisimple if its Lie algebra is a semisimple Lie algebra, i.e. a direct sum of simple Lie algebras. A simple Lie algebra is a Lie algebra that is non-abelian and contains no nonzero proper ideals.

Reference: https://en.wikipedia.org/wiki/Simple_Lie_algebra