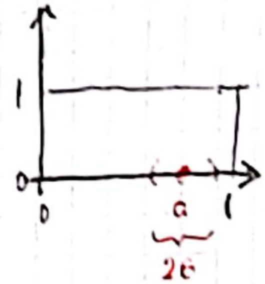
Example 2



i) Choose $X_i \sim$ iid uniform on $[0,1]$

- are the X_i convergent to a value?

No. Choose any $a \in [0,1]$

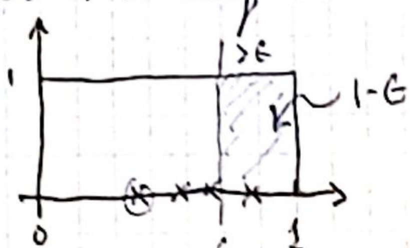
No matter what ϵ interval you place around it there is alway a finite prob. of $1-2\epsilon$ that sequence values will fall outside it.

ii) Choose a related r.v.:

$Y_n = \min\{X_1 X_2 \cdots X_n\}$ the minimum of the first n values of the sequence.

- notice Y_n can only decrease

$Y_{n+1} \leq Y_n$



- since it is bounded below by 0.

it seems reasonable to think it converges to 0.

Check:

① guess $a=0$

② pick some $\epsilon > 0$.

③ evaluate $P(|Y_n - a| > \epsilon)$.

$\rightarrow P(|Y_n - 0| > \epsilon) = P(Y_n \geq \epsilon)$ ← need to show $\rightarrow 0$ as $n\rightarrow\infty$

Two cases: i) $\underline{\epsilon > 1}$: $P(Y_n \geq 1^+) = \underline{0}$ ✓

$\underline{0 < \epsilon \leq 1}$ ii) $P(Y_n \geq \epsilon) = P(X_1 \geq \epsilon)\, P(X_2 \geq \epsilon) \cdots P(X_n \geq \epsilon)$. ($\because$ iid)

$= (1-\epsilon)^n$ ← $1-\epsilon < 1$

$\rightarrow 0$ as $n \rightarrow \infty$

$\therefore Y_n = \min\{X_1 \cdots X_n\} \xrightarrow[n\rightarrow\infty]{i.p.} 0$.