- note Markov used mean of distⁿ (1'st order moment) $\frac{\mu}{a}$

Chebyshev " variance " " (2'nd order). $\frac{\sigma^2}{b^2}$ ⑥

→ can we use higher orders.?

Yes → Chernoff Bound (tightest). → uses moment generating fcn.

→ Go to 6.5

e.g. $X \sim$ Binomial(n,p).

For $p = 1/4$: $P(X \geq \frac{3n}{4}) \leq \cancel{\frac{2}{3}}\ \frac{1}{3}$ Markov.

$P(X \geq \frac{3n}{4}) \leq \cancel{\frac{4}{n}}\ \frac{1}{3n}$ Chebyshev.

$P(X \geq \frac{3n}{4}) \leq \cancel{\left(\frac{16}{27}\right)^{\frac{n}{4}}}\ 3^{-n/2}$ Chernoff..

from 6.8

Law of Large Numbers: weak version.

Defⁿ: For iid r.v.'s $X_1, X_2 \cdots X_n$, the sample mean is

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}.$$

- note, $\bar{X}$ is itself a r.v. $\bar{c}$ a mean & variance,

mean: $\mathbb{E}[\bar{X}] = \frac{\mathbb{E}X_1 + \mathbb{E}X_2 + \cdots + \mathbb{E}X_n}{n}$

$= \frac{n \cdot \mathbb{E}[X]}{n}$

$= \mathbb{E}[X].$

variance: recall for iid $var(X+Y) = var X + var Y$

$var(aX) = a^2 var X$

$$Var(\bar{X}) = \frac{Var(X_1) + \cdots + Var(X_n)}{n^2} = var\left(\frac{X_1 + X_2 + \cdots + X_n}{n}\right)$$

$$= \frac{n \cdot Var(X)}{n^2}$$

$$= \frac{var(X)}{n}.$$

⑦

⑫