Convergence of Random Variables.

- we all get concepts of convergence in high school & first year calculus.

$\Rightarrow$ how to apply these to sequences of r.v.'s ?

Recall WLLN's:

For any $\epsilon > 0$, $P(|\bar{X}_n - \mu| \geq \epsilon) \to 0$ as $n \to \infty$

- we'd like to say the "$\bar{X}_n$ converges to μ"
- need to define the word converges.
  → use the WLLN's! to motivate the defn.

Defn: Convergence in Probability.

A sequence Y_n converges in probability to a scalar a if:

$$\text{For any } \epsilon > 0, \lim_{n \to \infty} P(|Y_n - a| \geq \epsilon) = 0.$$

- write $Y_n \xrightarrow{ip} a$.
- so the WLLN's states: $\bar{X}_n \xrightarrow[n \to \infty]{ip} \mu$

A Picture: compare to ordinary idea of convergence of a sequence of numbers.

- recall we say a sequence: $a_n \to a$ when a_n gets arbitrarily close and stays close to a eventually.