

OR

$$P(|\bar{X}_{1000} - p| \geq 0.01) \leq \text{something "small"}$$

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• to get a handle on the rhs recall Chebyshev's Inequality

$$P(|Y - \mathbb{E}[Y]| \geq \epsilon) \leq \frac{\sigma_Y^2}{\epsilon^2}$$

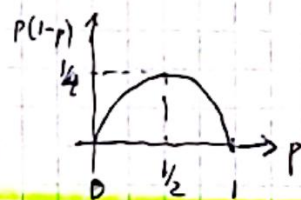
$$\text{Here } Y = \frac{X_1 + X_2 + \dots + X_n}{n} = \bar{X}_n$$

$$\mathbb{E}[Y] = p$$

$$\sigma_Y^2 = \text{Var}(Y) = \frac{\sigma_{X_i}^2}{n}$$

$$\text{So, } P(|\bar{X}_{1000} - p| \geq 0.01) \leq \frac{\sigma_{X_i}^2}{n \epsilon^2} \leftarrow \text{recall } \sigma_{\text{Bern.}}^2 = p(1-p).$$

$$P(|\bar{X}_{1000} - p| \geq 0.01) \leq \frac{p(1-p)}{10 \cdot 10^{-4}} = p(1-p) \leq \frac{1}{4}$$



* So, we can say if we sample 10,000 people the
* error being more than 1% occurs with less than 25% prob.

→ if we want the prob. of an error greater than 0.01 to be less than 5% → take $n = 50,000$.

→ so the prob. of a large error (> 0.01) less than 5% requires $n = 50,000$ sample size.

→ you get a trade off in parameters:

$$P(|\bar{X}_n - p| \geq \epsilon) \leq \frac{1}{4n\epsilon^2}$$

$$\text{or: } P(|\bar{X}_n - p| \geq \epsilon) \leq \delta \quad \text{where } \delta = \frac{1}{4n\epsilon^2}$$

Shop signs:

- good
- cheap
- fast

⇒ Pick Two