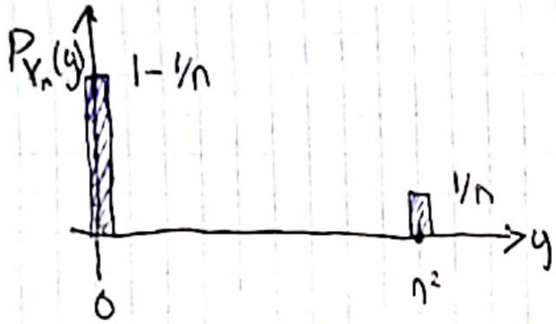
Example 1. - discrete r.v.



$P(Y_n=0) = 1-1/n$

$P(Y_n=n^2) = 1/n$

→ most prob. concentrated at 0 but still a small prob. at large value.

- intuitively you'd think $Y_n \xrightarrow[n\to\infty]{ip} 0$.
- check the def^n:
  - choose $\epsilon > 0$ & find $P(|Y_n - 0| \geq \epsilon)$.
  - see if it $\to 0$ as $n \to \infty$.

  → clearly $P(|Y_n - 0| \geq \ ) = P(Y_n \geq \epsilon)$

  $= \frac{1}{n}$ for n large enough (

  $\longrightarrow 0$ as $n \to \infty$

So, $Y_n \xrightarrow[n\to\infty]{ip.} 0$

- So, what is $\mathbb{E}[Y_n]$ as $n \to \infty$?

$\mathbb{E}[Y_n] = 0 \cdot P(Y_n = 0) + n^2 \cdot P(Y_n = n^2)$

$= n^2 \cdot \frac{1}{n}$

$= n$

$\longrightarrow \infty$ as $n \to \infty$.

convergence i.p. $\not\longrightarrow$ convergence in expectation

"fat tails"

- cip has to do $\bar{c}$ the bulk of the prob. → only cares if the tail → 0
- expectation is sensitive to outliers

  → the tail prob. may be small but it's assigned to large values.