Chernoff Bound

- another (tighter) bound on tail prob's of a distⁿ.

→ we used first & second moments (mean & var.) to get tail prob's. → this uses all of them.

Defⁿ: the n'th moment of a r.v. X is:

$$\mathbb{E}[X^n]$$

The n'th central moment is:

$$\mathbb{E}\left[(X-\mathbb{E}X)^n\right]$$

Defⁿ: the moment generating function of a r.v. X is a function $M_X(s)$:

$$\boxed{M_X(s) \equiv \mathbb{E}\left[e^{sX}\right]}$$

Why would you do this → lets us read all the moments of X from the coeff's of the Taylor Exp
→ just take derivatives.

- recall $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

∴ X is real:

$$e^{sX} = \sum_{k=0}^{\infty} \frac{(sX)^k}{k!} = \sum_{k=0}^{\infty} \frac{s^k}{k!} X^k$$

k'th-moment!

$$M_X(s) = \mathbb{E}\left[e^{sX}\right] = \sum_{k=0}^{\infty} \frac{s^k}{k!} \mathbb{E}[X^k]$$

So given the function $M_X(s)$, we can read off the k'th moment.

$$\mathbb{E}[X^k] = \frac{d^k}{ds^k} M_X(s)\Big|_{s=0}.$$