1 review

Converting reps from linear operators to matrices.

If $g \in G$ for a group G and $|i\rangle$ is a complete orthonormal basis for the space in which $D(g)$ acts as a linear operators, then by inserting a cmplete set of states using

$$1 = \sum_j |j\rangle \langle j|D(g)|i\rangle \equiv \sum_j |j\rangle \left[D(g)\right]_{ji} \tag{1}$$

we get the connection between the linear operator and matrix forms of a representation. Here $D^{(\mu)}$ is a representation matrix for the irreducible representation μ of dimension n_μ, n_G is the order of the group, g stands for the group elements, and the hermitian conjugate $D^\dagger$ of a matrix D is obtained by transposing the rows and columns and replacing each element of the matrix by its complex conjugate. Character Theorems

Traces χ of the representation matrices satisfy the orthonormality and completeness relations

$$\sum_i \frac{n_i}{n_G} \chi_\mu^*(i) \chi^\nu(i) = \delta_\mu^\nu \tag{2}$$

$$\frac{n_i}{n_G} \sum_\mu \chi^\mu(i) \chi_\mu^{*j}(i) = \delta_i^j \tag{3}$$

A representation of a finite group is irreducible only if

$$\sum_i n_i |\chi_i|^2 = n_G \tag{4}$$

The number of times a_ν the the irrep ν occurs in this reduction is

$$a_\nu = \sum_i \frac{n_i}{n_G} \chi_\nu^*(i) \chi(i) \tag{5}$$

2 Exercise 15

Consider the representation $\gamma^{(4)}$. We have $n_1 = 1$, $n_2 = 2$, and $n_3 = 3$. The above is true because these are the sizes of the three conjugacy classes. We have $\chi_1(1) = 1$, $\chi_1(2) = 1$, $\chi_1(3) = 1$. The above values are just the traces of $\gamma^{(1)}$ for each of the three conjugacy classes. We have $\chi_2(1) = 1$, $\chi_2(2) = 1$, $\chi_2(3) = -1$. The above values are the traces of $\gamma^{(2)}$ for each of the three conjugacy classes. We have $\chi_3(1) = 2$, $\chi_3(2) = -1$, $\chi_3(3) = 0$. The above values are the traces of $\gamma^{(3)}$ on each of the three conjugacy classes. We have $\chi(1) = 3$, $\chi(2) = 0$, $\chi(3) = 1$, where $\chi = \chi^{(4)}$. These are the values of $\chi^{(4)}$ on each of the conjugacy classes.

$$\begin{aligned}
a_1 &= \frac{1}{6}[(1)(1)(3) + (2)(1)(0) + (3)(1)(1)] &&= 1, \\
a_2 &= \frac{1}{6}[(1)(1)(3) + (2)(1)(0) + (3)(-1)(1)] &&= 0, \\
a_3 &= \frac{1}{6}[(1)(2)(3) + (2)(-1)(0) + (3)(0)(1)] &&= 1,
\end{aligned}$$

Thus the content is the sum of $\gamma^{(1)}$ and $\gamma^{(3)}$ since $a_2 = 0$.