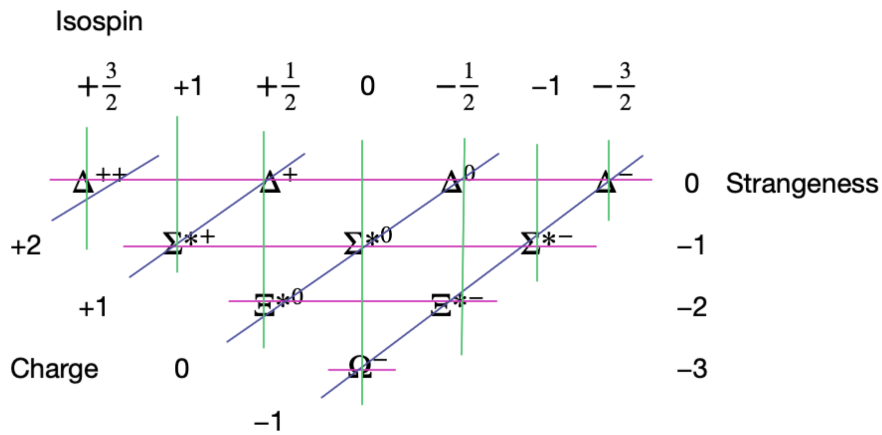
[25.4] Check that the charge values, indicated by the superfixes in the first table, come out right.

Answer: Charge values are easily confirmed by substituting the charge values from the table below for each particle

Quark	Charge Q	Isospin I	Isospin I3	Strangeness
u	⅔	½	+½	0
d	–⅓	½	–½	0
s	–⅓	0	0	-1



uuu uud udd ddd

uus uds dus

uss dss

sss

Constraints:

1. The strong force conserves: spin, charge, isospin, strangeness, color, baryon number
2. The wave functions of hadrons must be overall antisymmetric with the swapping of any two quarks
3. Observed particles must be color neutral: RYB, $\bar{R}\bar{Y}\bar{B}$, $R\bar{R}$, $Y\bar{Y}$, $B\bar{B}$, + linear combinations
4. The wave functions of hadrons must be antisymmetric in color with the swapping of any two quarks. Therefore, the spin, isospin, strangeness part of the wave function must be symmetric
5. Cannot directly observe individual quarks or color
6. One can observe particle spin, isospin, charge, strangeness, and baryon number

If the quarks behaved as fermions, then we would get antisymmetry under interchange of the quarks, not symmetry, which is inconsistent with this picture.[25.5]

[25.5] Explain this more completely, using the 2-spinor index description for the quark spins, as described in §22.8, and using a new 3-dimensional 'SU(3) index' which takes 3 values u, d, s.

The Baryon decuplet wave functions (without color) are:[1]

$$|\Delta^{++}\rangle = u_\uparrow u_\uparrow u_\uparrow$$

$$|\Delta^{+}\rangle = \frac{1}{\sqrt{3}}(d_\uparrow u_\uparrow u_\uparrow + u_\uparrow d_\uparrow u_\uparrow + u_\uparrow u_\uparrow d_\uparrow)$$

$$|\Delta^{0}\rangle = \frac{1}{\sqrt{3}}(d_\uparrow d_\uparrow u_\uparrow + d_\uparrow u_\uparrow d_\uparrow + u_\uparrow d_\uparrow d_\uparrow)$$

$$|\Delta^{-}\rangle = d_\uparrow d_\uparrow d_\uparrow$$

$$|\Sigma^{*+}\rangle = \frac{1}{\sqrt{3}}(s_\uparrow u_\uparrow u_\uparrow + u_\uparrow s_\uparrow u_\uparrow + u_\uparrow u_\uparrow s_\uparrow)$$

$$|\Sigma^{*0}\rangle = \frac{1}{\sqrt{6}}(s_\uparrow d_\uparrow u_\uparrow + s_\uparrow u_\uparrow d_\uparrow + d_\uparrow s_\uparrow u_\uparrow + d_\uparrow u_\uparrow s_\uparrow + u_\uparrow s_\uparrow d_\uparrow + u_\uparrow d_\uparrow s_\uparrow)$$

$$|\Sigma^{*-}\rangle = \frac{1}{\sqrt{3}}(s_\uparrow d_\uparrow d_\uparrow + d_\uparrow s_\uparrow d_\uparrow + d_\uparrow d_\uparrow s_\uparrow)$$

$$|\Xi^{*0}\rangle = \frac{1}{\sqrt{3}}(s_\uparrow s_\uparrow u_\uparrow + s_\uparrow u_\uparrow s_\uparrow + u_\uparrow s_\uparrow s_\uparrow)$$

$$|\Xi^{*-}\rangle = \frac{1}{\sqrt{3}}(s_\uparrow s_\uparrow d_\uparrow + s_\uparrow d_\uparrow s_\uparrow + d_\uparrow s_\uparrow s_\uparrow)$$

$$|\Omega^{-}\rangle = s_\uparrow s_\uparrow s_\uparrow \quad .$$

All these wave functions would need to flip sign under interchange of any two quarks and all these wave functions would vanish, e.g., $u_\uparrow u_\uparrow u_\uparrow = -u_\uparrow u_\uparrow u_\uparrow = 0$.

[1] Walter Greiner and Bendt Mülller, Quantum Mechanics Symmetries, p. 274-275

This arrangement comes about when we consider that the spin is now 1/2, so we can think of two of the quark spins as parallel and one of them antiparallel. It turns out that there are just two linearly independent ways of arranging this for the quark composition uds that is represented at the centre (corresponding to the pair Σ^0 and Λ^0); there is none at all for uuu, ddd, and sss, which explains the hexagonal rather than a triangular array; and there is just one for each of the rest.[25.6]

> [25.6] See if you can explain all this in some appropriate detail. Care is needed for the treatment of the 2-spinor spin indices, if you wish to use them. An antisymmetry in a pair of them allows that pair to be removed (as when representing a spin 0 state in terms of a pair of spin $\frac{1}{2}$ particles, as in §23.4). Yet there is a (hidden) symmetry also, because there are only two independent spin states for each quark.

The wave functions for the Σ^0 and Λ^0 are tricky because the spin/flavor pieces cannot be factored. Both particles have zero z component of isospin, but the Λ^0 has total isospin of 0 and the Σ^0 has total isospin of 1. Without the color indices, the wave functions are:[2]

$$
\begin{aligned}
|\Lambda^0_\uparrow\rangle = \frac{1}{\sqrt{12}}\cdot\{&|d_\uparrow s_\uparrow u_\downarrow\rangle + |s_\uparrow d_\uparrow u_\downarrow\rangle + |u_\downarrow d_\uparrow s_\uparrow\rangle + |u_\downarrow s_\uparrow d_\uparrow\rangle + |d_\uparrow u_\downarrow s_\uparrow\rangle \\
&+ |s_\uparrow u_\downarrow d_\uparrow\rangle - |d_\downarrow s_\uparrow u_\uparrow\rangle - |s_\uparrow d_\downarrow u_\uparrow\rangle - |u_\uparrow d_\downarrow s_\uparrow\rangle \\
&- |u_\uparrow s_\uparrow d_\downarrow\rangle - |d_\downarrow u_\uparrow s_\uparrow\rangle - |s_\uparrow u_\uparrow d_\downarrow\rangle\} \quad .
\end{aligned}
$$

$$
\begin{aligned}
|\Sigma^0_\uparrow\rangle = \tfrac{1}{6}\cdot\{&2|d_\uparrow u_\uparrow s_\downarrow\rangle + 2|u_\uparrow d_\uparrow s_\downarrow\rangle + 2|s_\downarrow d_\uparrow u_\uparrow\rangle + 2|s_\downarrow u_\uparrow d_\uparrow\rangle \\
&- |d_\uparrow u_\downarrow s_\uparrow\rangle - |u_\uparrow d_\downarrow s_\uparrow\rangle - |s_\uparrow d_\downarrow u_\uparrow\rangle - |s_\uparrow u_\downarrow d_\uparrow\rangle \\
&- |d_\downarrow u_\uparrow s_\uparrow\rangle - |u_\downarrow d_\uparrow s_\uparrow\rangle - |s_\uparrow d_\uparrow u_\downarrow\rangle - |s_\uparrow u_\uparrow d_\downarrow\rangle \\
&+ 2|d_\uparrow s_\downarrow u_\uparrow\rangle + 2|u_\uparrow s_\downarrow d_\uparrow\rangle - |d_\uparrow s_\uparrow u_\downarrow\rangle - |u_\uparrow s_\uparrow d_\downarrow\rangle \\
&- |d_\downarrow s_\uparrow u_\uparrow\rangle - |u_\downarrow s_\uparrow d_\uparrow\rangle\}
\end{aligned}
$$

With the color indices, each term above becomes six separate terms for a total of 72 terms for the Λ^0 and 108 terms for the Σ^0.

[2] Walter Greiner and Bendt Mülller, Quantum Mechanics Symmetries, p. 280 and 283.

This antisymmetry passes over to the quark states themselves, so that antisymmetry between individual (fermionic) quarks gets effectively converted into symmetry, in a three-quark particle.[25.7]

[25.7] Use indices to explain this comment, where there is a new 3-dimensional SU(3) *colour* index, in addition to a 3-dimensional flavour index of Exercise [25.5].

It's kind of obvious if the wave function has spin and flavor pieces which are now tensor producted with a totally antisymmetric color piece, the combined spin and flavor pieces better be symmetric under particle exchange in order for the overall wave function to be antisymmetric under particle exchange. Otherwise, if both pieces were antisymmetric, the minus signs would cancel out and the wave function would end up being symmetric

— — — — — — — —

We do not, for example, have three different versions of the Δ^+ particle, depending upon which colour the d-quark is in 'uud'. The antisymmetry in the colour degree of freedom, for actual free particles, ensures this.[25.8].

[25.8] Explain.

Again, in order for the wave function to be totally antisymmetric, if the color part is antisymmetric, the spin/flavor part needs to be symmetric and a linear combination of all quarks having all possible colors. The full wave function must have linear combinations of the three different d colored quarks. Specifically:

$$|\Delta^+\rangle = \frac{1}{\sqrt{3}}(d_\uparrow u_\uparrow u_\uparrow + u_\uparrow d_\uparrow u_\uparrow + u_\uparrow u_\uparrow d_\uparrow) \otimes (\mathrm{R} \wedge \mathrm{Y} \wedge \mathrm{B})$$

$$= \frac{1}{\sqrt{18}}(d_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{Y}}\, u_{\uparrow\mathrm{B}} + d_{\uparrow\mathrm{Y}}\, u_{\uparrow\mathrm{B}}\, u_{\uparrow\mathrm{R}} + d_{\uparrow\mathrm{B}}\, u_{\uparrow\mathrm{Y}}\, u_{\uparrow\mathrm{R}} - d_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{B}}\, u_{\uparrow\mathrm{Y}} - d_{\uparrow\mathrm{Y}}\, u_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{B}} - d_{\uparrow\mathrm{B}}\, u_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{Y}}$$

$$+u_{\uparrow\mathrm{R}}\, d_{\uparrow\mathrm{Y}}\, u_{\uparrow\mathrm{B}} + u_{\uparrow\mathrm{Y}}\, d_{\uparrow\mathrm{B}}\, u_{\uparrow\mathrm{R}} + u_{\uparrow\mathrm{B}}\, d_{\uparrow\mathrm{Y}}\, u_{\uparrow\mathrm{R}} - u_{\uparrow\mathrm{R}}\, d_{\uparrow\mathrm{B}}\, u_{\uparrow\mathrm{Y}} - u_{\uparrow\mathrm{Y}}\, d_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{B}} - u_{\uparrow\mathrm{B}}\, d_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{Y}}$$

$$+u_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{Y}}\, d_{\uparrow\mathrm{B}} + u_{\uparrow\mathrm{Y}}\, u_{\uparrow\mathrm{B}}\, d_{\uparrow\mathrm{R}} + u_{\uparrow\mathrm{B}}\, u_{\uparrow\mathrm{Y}}\, d_{\uparrow\mathrm{R}} - u_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{B}}\, d_{\uparrow\mathrm{Y}} - u_{\uparrow\mathrm{Y}}\, u_{\uparrow\mathrm{R}}\, u_{\uparrow\mathrm{B}} - u_{\uparrow\mathrm{B}}\, u_{\uparrow\mathrm{R}}\, d_{\uparrow\mathrm{Y}})$$