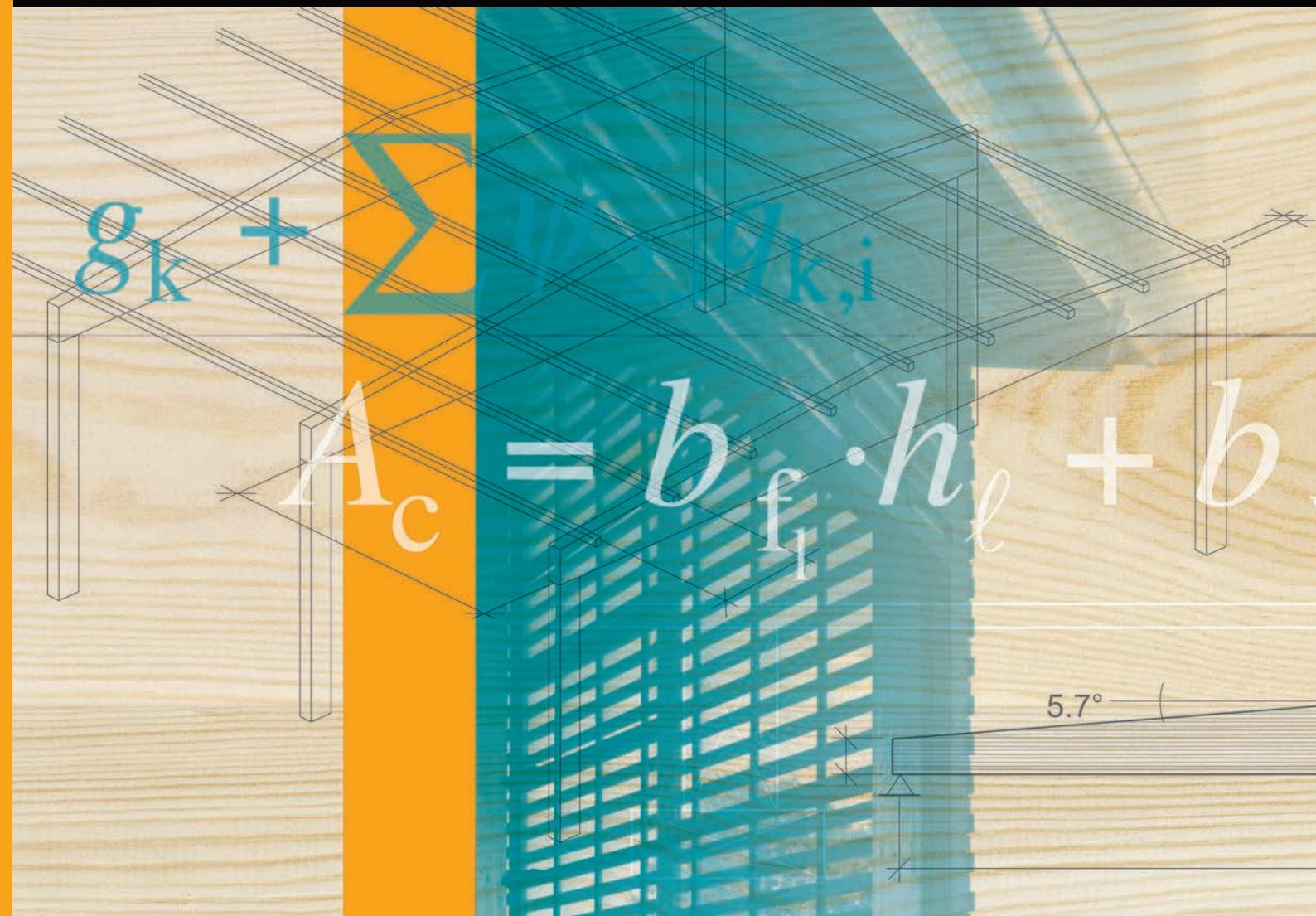


Design of timber structures

Examples

Volume 3





Design of timber structures Volumes 1–3 are adapted to Eurocode 5, Eurocode 0 and Eurocode 1.

- Volume 1: Structural aspects of timber construction
- Volume 2: Rules and formulas according to Eurocode 5
- Volume 3: Examples

Preface

This is the third revised edition of *Design of timber structures Volume 3, Examples* published in 2015. Rules and standards change in pace with the development of society, hence a publication of this type needs to be reviewed regularly. Contrary to earlier English editions, the current version is not adapted to the Swedish national choices in Eurocode 1995. Instead, the original Eurocode 1995 is referred to, in order to simply adaptation to different national parameters in the country using the book.

The book series *Design of timber structures Volume 1–3* has been produced to make it easier for structural designers to calculate timber structures and it is adapted to Eurocodes. It is being used for higher education at universities and institutes.

The book series *Design of timber structures Volume 1–3* includes *Volume 1, Structural aspects of timber construction* as well as *Volume 2, Rules and formulas according to Eurocode 5*. All three books are available in English and Swedish. Since the books are available in both languages and due to the nuanced content, our goal is that they will play a role for many users on different skill levels. Notice that the Swedish decimal holder „,” was kept throughout all the text, instead of the English “.”. All photos are taken in Sweden, unless otherwise indicated.

The solved examples in *Volume 3* has been compiled by the authors of each chapter in *Volume 1* respectively. The authors are Roberto Crocetti, Helena Lidelöw, Annika Mårtensson and Bert Norlin. Sven Thelandersson at the Faculty of Engineering at Lund University has been responsible for supervision and editing. The solutions to the examples are based on the rules in EN 1995-1-1, but should not be regarded as an official interpretation of them. Professional application must be based on the original documents published by CEN with corresponding national application documents.

More information about wood, glulam and timber construction can be found at www.swedishwood.com.

Stockholm, April 2022

Patrice Godonou
Swedish Wood

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The examples in this publication are numbered according to the equivalent chapters in *Design of timber structures – Volume 1*. (Chapter 1, 2, 8 and 9 do not contain design sections.)

Design in the ultimate limit state (ULS)

Example 3.1: Design of a straight timber joist

A simply supported rectangular joist is subjected to a design load of $q_{\text{dim}} = 3,3 \text{ kN/m}^2$ consisting of permanent and medium-term loads. The clear span ℓ is 4,5 m and the joists are spaced at 0,6 m centres.

The joist is part of an indoor floor. Service class 1 and safety class 2.

Design the joist in structural timber of strength class C24, for bending moment and shear load.

Solution

The characteristic value of bending strength is $f_{m,k} = 24 \text{ MPa}$.

The modification factor for load duration and service classes can be taken from *Volume 2: section 3.2*, with load duration class M and service class 1 the factor $k_{\text{mod}} = 0,8$.

Material factor for structural timber, $\gamma_M = 1,3$.

Size factor for depth greater than 150 mm, $k_h = 1,0$ (*Volume 2: section 3.3*).

It can often be reasonable to let the factor k_h adopt the value of 1,0 even for smaller sizes of the joist.

Design value of the bending strength:

$$f_{m,d} = k_h \frac{k_{\text{mod}} f_{m,k}}{\gamma_M} = 1,0 \frac{0,8 \cdot 24}{1,3} = 14,8 \text{ MPa}$$

Design value of the load:

$$q_{\text{dim}} = 3,3 \text{ kN/m}^2$$

$$q_d = 0,6 \cdot 3,3 = 2,0 \text{ kN/m}$$

Design value of the bending moment:

$$M_{Ed} = q_d \cdot \ell^2 / 8 = 2,0 \cdot 4,5^2 / 8 = 5,1 \text{ kNm}$$

The required section modulus of the joist is given by:

$$W_{\text{eff}} = M_{Ed} / f_{m,d}$$

$$W_{\text{eff}} = \frac{M_{Ed}}{f_{m,d}} = \frac{5,1 \cdot 10^{-3}}{14,8} = 345 \cdot 10^{-6} \text{ m}^3$$

A common used width of a timber joist in Sweden is 45 mm. If this is chosen as the width b the required depth $h = 214 \text{ mm } (\sqrt{6 \cdot W_{\text{eff}}})/b$. A standard section is chosen 45 × 220 mm. This requires that the joist is braced against lateral torsional buckling by for instance a floor graded particleboard.

Check the shear capacity of the joist.

The design value of the shear force:

$$V_{Ed} = q_d \cdot \ell / 2 = 2,0 \cdot 4,5 / 2 = 4,5 \text{ kN}$$

Characteristic value of the shear strength:

$$f_{v,k} = 4,0 \text{ MPa}$$

Thereby is the design value of the shear strength given by:

$$f_{v,d} = \frac{k_{\text{mod}} f_{v,k}}{\gamma_M} = \frac{0,80 \cdot 4,0}{1,3} = 2,46 \text{ MPa}$$

The design value of the shear capacity is given by:

$$V_{Rd} = \frac{2}{3} A \cdot f_{v,d} \quad \text{kN}$$

The effective width b_{ef} is given by:

$$b_{\text{ef}} = k_{\text{cr}} b$$

where $k_{\text{cr}} = 0,67$ for structural timber C24. This gives the following for the design value of shear capacity:

$$V_{Rd} = \frac{2}{3} \cdot k_{\text{cr}} A \cdot f_{v,d} = \frac{2}{3} \cdot 0,67 \cdot 0,045 \cdot 0,220 \cdot 2,46 \cdot 10^3 = 10,9 \text{ kN}$$

- This shows that $V_{Ed} < V_{Rd}$, that is the joist has sufficient shear capacity.

Remark: Some way to further reduce V_{Ed} is possible, see Volume 2: Chapter 6.

Example 3.2: Bearing strength for the support a of timber joist

For the joist in *example 3.1* in this *Volume*, check the bearing strength at the end supports. The bearing length has been restricted to 66 mm at each end.

The check for bearing strength is given by:

$$\sigma_{c,90,d} \leq k_{c,90} \cdot f_{c,90,d}$$

The value of the bearing stress is given by:

$$A_{ef} = b \cdot (\ell_b + 0,03) = 0,045 \cdot (0,066 + 0,03) = 0,00432 \text{ m}^2$$

$$\sigma_{c,90,d} = \frac{P}{A_{ef}}$$

where b is the width of the beam, ℓ_b is the actual bearing length and $P = F_{c,90,d}$ is the force at the support.

$$b = 45 \text{ mm}$$

$$\ell_b = 66 \text{ mm}$$

$$P = \frac{q_d \ell}{2} = 4,5 \text{ kN} \quad (\text{according to example 3.1, page 5})$$

Thus the bearing stress is:

$$\sigma_{c,90,d} = \frac{F_{c,90,d}}{A_{ef}} = \frac{4,5 \cdot 10^3}{0,00432 \cdot 10^6} = 1,04 \text{ MPa}$$

The characteristic value of the compressive strength is $f_{c,90,k} = 2,5 \text{ MPa}$.

The design value of the compressive strength is ($k_{mod} = 0,8$ and $\gamma_M = 1,3$ according to *example 3.1*):

$$f_{c,90,d} = \frac{k_{mod} f_{c,90,k}}{\gamma_M} = \frac{0,8 \cdot 2,5}{1,3} = 1,54 \text{ MPa}$$

The factor $k_{c,90}$ is given by *Volume 2: section 5.2*:

$$k_{c,90} = 1,5$$

The check is then given by:

$$\sigma_{c,90,d} = 1,04 \text{ MPa} \leq k_{c,90} \cdot f_{c,90,d} = 1,5 \cdot 1,54 = 2,3 \text{ MPa}$$

- This shows that the bearing capacity at the end supports is sufficient.

The example also shows that the fact that only a minor part of the beam is subjected to compression, allows quite high stresses compared to when a larger area is subjected to compression. If the joist is supported by a timber top plate, the bearing capacity of that of course also have to be checked.

Example 3.3: Capacity of a notched timber beam

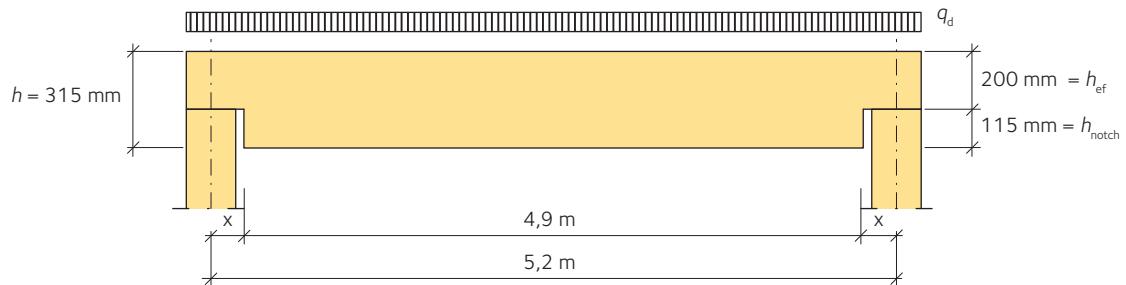


Figure 3.1

Check the capacity of the beam in figure 3.1 with regard to the ultimate limit state.

The beam is made of glulam GL30c and the cross section is 90×315 mm. Service class 1, safety class 3 and load duration class M.

The design load has been determined to $q_d = 5,1$ kN/m.

The design bending moment is:

$$M_{Ed} = \frac{q_d \ell^2}{8} = \frac{5,1 \cdot 5,2^2}{8} = 17,2 \text{ kNm}$$

The design shear force is:

$$V_{Ed} = \frac{q_d \ell}{2} = \frac{5,1 \cdot 5,2}{2} = 13,3 \text{ kN}$$

The characteristic value of the bending strength is $f_{m,k} = 30$ MPa.

The modification factor for load duration and service classes can be taken from *Volume 2: section 3.2*, with load duration class M and service class 1 the factor $k_{mod} = 0,8$.

Material factor for glulam, $\gamma_M = 1,25$.

Size factor for depths $231 \text{ mm} \leq h < 600 \text{ mm}$, $k_h = (600/315)^{0,1} = 1,07$ (*Volume 2: section 3.3*).

Design value of the bending strength:

$$f_{m,d} = k_h \frac{k_{mod} f_{m,k}}{\gamma_M} = 1,07 \frac{0,8 \cdot 30}{1,25} = 20,5 \text{ MPa}$$

namely the bending moment is limited by:

$$M_{Rd} = f_{m,d} W = 20,5 \cdot 10^3 \cdot \frac{0,09 \cdot 0,315^2}{6} = 30,5 \text{ kNm}$$

namely the moment capacity is sufficient, provided that the beam is not subject to lateral buckling.

Characteristic value of the shear strength $f_{v,k} = 3,5 \text{ MPa}$.

Thereby is the design value of the shear strength given by:

$$f_{v,d} = \frac{k_{\text{mod}} f_{v,k}}{\gamma_M} = \frac{0,80 \cdot 3,5}{1,25} = 2,24 \text{ MPa}$$

The design value of the shear capacity is given by:

$$V_{\text{Rd}} = \frac{2}{3} A \cdot f_{v,d} \text{ kN}$$

The value of A should be determined on the basis of b_{ef} for a structural element subjected to bending moment. The effective width b_{ef} is given by:

$$b_{\text{ef}} = k_{\text{cr}} b$$

where $k_{\text{cr}} = 0,67$ for glulam, not exposed to precipitation and solar radiation. This gives the following for the design value of shear capacity:

$$V_{\text{Rd}} = \frac{2}{3} k_{\text{cr}} A \cdot f_{v,d} = \frac{2}{3} \cdot 0,67 \cdot 0,09 \cdot 0,315 \cdot 2,24 \cdot 10^3 = 28,4 \text{ kN}$$

- This shows that $V_{\text{Ed}} < V_{\text{Rd}}$ that is the beam has sufficient shear capacity.

Now the notched areas have to be checked. In *Eurocode 5: section 6.5.2*, it is stated that the following should be verified (also see *Volume 2: section 8.3*):

$$\tau_d = \frac{1,5 V_{\text{Ed}}}{b_{\text{ef}} h_{\text{ef}}} \leq k_v f_{v,d}$$

where h_{ef} is the reduced depth of the beam in the notch and k_v is a reduction factor defined as follows for beams notched on the same side as the support.

$$k_v = \min \left\{ \frac{1}{\sqrt{h} \left(\sqrt{\alpha(1-\alpha)} + 0,8 \frac{x}{h} \sqrt{\frac{1}{\alpha} - \alpha^2} \right)} \right. \\ \left. k_n \left(1 + \frac{1,1 i^{1,5}}{\sqrt{h}} \right) \right\}$$

where:

- i is the notch inclination.
- h is the beam depth in mm.
- x is the distance from the line of action of the support reaction to the corner of the notch, in mm.

$$\alpha = \frac{h_{\text{ef}}}{h}$$

$$k_n = 6,5 \text{ for glulam.}$$

Example 3.3: Capacity of a notched timber beam

Width of the beam	$b = 90 \text{ mm}$
Depth of the beam	$h = 315 \text{ mm}$
Notch depth at each end of beam	$h_{\text{notch}} = 115 \text{ mm}$
Notch inclination	$i = 0$
Effective beam depth at each end	$h_{\text{ef}} = 200 \text{ mm}$
Ratio of h_{ef}/h	$\alpha = h_{\text{ef}}/h = 0,63$
Length of the notch from the centre line of the end support	$x = 150 \text{ mm}$

$$k_v = \min \left\{ \frac{1}{\sqrt{315} \left(\sqrt{0,63(1-0,63)} + 0,8 \cdot \frac{150}{315} \sqrt{\frac{1}{0,63} - 0,63^2} \right)}} = 0,41 \right.$$

The shear stress is:

$$\tau_d = \frac{1,5 \cdot 13,3 \cdot 10^3}{0,67 \cdot 0,09 \cdot 0,2} = 1,65 \text{ MPa}$$

while the reduced shear strength is given by:

$$k_v f_{v,d} = 0,41 \cdot 2,24 = 0,92 \text{ MPa}$$

- This means that the notched beam not has a sufficient capacity.

Further calculations give that it could be possible to have a notch corresponding to 16 percent of the total depth, in order to still have the sufficient shear capacity. Alternatively the notches could be reinforced for example with wood screws.

Example 3.4: Lateral torsional buckling check of a glulam beam

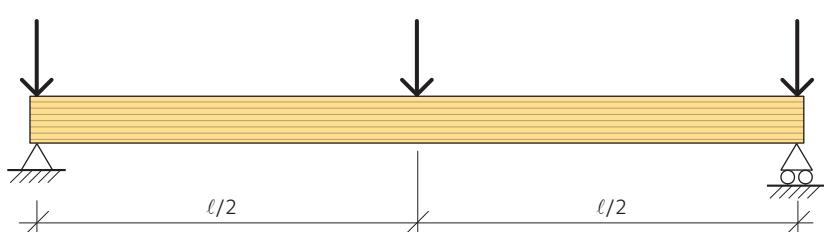
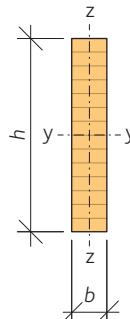


Figure 3.2



A simply supported beam is loaded with point loads (coming from columns standing on the beam). The point loads (combination of permanent and medium term duration) together with the self-weight of the beam causes a design bending moment $M_{Ed} = 74,6 \text{ kNm}$ in the middle of the beam and a design shear force $V_{Ed} = 15,8 \text{ kN}$ at the end of the beam. The beam length is $\ell = 10 \text{ m}$. The beam has the dimensions $h = 630 \text{ mm}$ and $b = 115 \text{ mm}$ and is made of glulam GL30c.

Check if the beam dimension is enough in service class 1 and safety class 3. (Include the effect of lateral torsional buckling)

3.4.1 Geometric properties of the beam

Width of the beam	$b = 115 \text{ mm}$
Depth of the beam	$h = 630 \text{ mm}$
Clear span of the beam	$\ell = 10\,000 \text{ mm}$
Section modulus of the beam about the $y-y$ axis	$W_y = \frac{b \cdot h^2}{6}$ $W_y = 7,61 \cdot 10^6 \text{ mm}^3$

3.4.2 Glulam properties

Glulam GL30c:

Characteristic bending strength	$f_{m,k} = 30 \text{ N/mm}^2$
Characteristic shear strength	$f_{v,k} = 3,5 \text{ N/mm}^2$
Characteristic compression strength perpendicular to the grain	$f_{c,90,k} = 2,5 \text{ N/mm}^2$
5 th percentile modulus of elasticity parallel to the grain	$E_{0,05} = 10,8 \text{ kN/mm}^2$
5 th percentile modulus of shear parallel to the grain	$G_{05} = 0,54 \text{ kN/mm}^2$
Mean modulus of elasticity parallel to the grain	$E_{0,mean} = 13,0 \text{ kN/mm}^2$
Mean shear modulus	$G_{mean} = 0,65 \text{ kN/mm}^2$
Mean density of the beam	$\rho_m = 430 \text{ kg/m}^3$
Volume 2: table 3.1	$\gamma_M = 1,25$
Material factor for glulam	

3.4.3 Modification factors

Factor for medium duration loading M and service class 1 (Volume 2: table 3.2)	$k_{mod} = 0,8$
Size factor for depth greater than 600 mm (Volume 2: section 3.3)	$k_h = 1,0$
Lateral stability of the beam (Volume 2: Chapter 4)	k_{crit}
Effective length of the beam — adopt case for the most critical condition — the point load (concentrated load) at mid-span (Volume 2: table 4.1)	$\ell_{ef} = 0,8\ell$ $\ell_{ef} = 8 \text{ m}$
Critical bending stress (Volume 2: Chapter 4 or Eurocode 5: Equation 6.32)	$\sigma_{m,crit} = \frac{0,78 \cdot b^2}{h \cdot \ell_{ef}} E_{0,05}$ $\sigma_{m,crit} = \frac{0,78 \cdot 115^2}{630 \cdot (0,8 \cdot 10000 + 2 \cdot 630)} \cdot 10800 = 19,1 \text{ N/mm}^2$
Relative slenderness for bending (Volume 2: Chapter 4 or Eurocode 5: Equation 6.30)	$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,crit}}}$ $\lambda_{rel,m} = 1,25$
Lateral stability factor (Volume 2: Chapter 4 or Eurocode 5: Equation 6.34)	$k_{crit} = \begin{cases} 1 & \text{if } \lambda_{rel,m} \leq 0,75 \\ 1,56 - 0,75 \lambda_{rel,m} & \text{if } 0,75 < \lambda_{rel,m} \leq 1,4 \\ \frac{1}{\lambda_{rel,m}^2} & \text{if } \lambda_{rel,m} > 1,4 \end{cases}$ $k_{crit} = 0,62$

3.4.4 Bending capacity

The design load case at the ULS is the result of a combination of self weight of the beam plus the combined permanent and variable point loads at mid span:

Design bending moment	$M_{Ed} = 74,6 \text{ kNm}$
Design bending strength	$f_{m,y,d} = 19,20 \text{ MPa}$
Design bending strength taking lateral torsional buckling effect into account (Volume 2: Chapter 4)	$M_{Rd} = f_{m,y,d} \cdot W_y \cdot k_{\text{crit}}$ $M_{Rd} = 90,6 \text{ MPa}$

Bending capacity is satisfactory for glulam, 115 x 630 GL30c.

3.4.5 Shear capacity

The design load case will be the result of combination of self weight of the beam plus the combined permanent and variable point loads at mid span:

Design value for end shear force	$V_d = 15,8 \text{ kN}$
Modification factor for shear (crack factor)	$k_{cr} = 0,67$
Effective width for shear	$b_{ef} = k_{cr} \cdot b$ $b_{ef} = 77 \text{ mm}$
Design shear stress (Volume 2: Chapter 6)	$\tau_d = \frac{1,5 \cdot V_d}{b_{rf} \cdot h} = \frac{1,5 \cdot 15,8 \cdot 10^3}{0,077 \cdot 0,63} = 0,48 \text{ MPa}$
Design shear strength (Volume 2: section 3.1)	$f_{v,d} = 2,24 \text{ MPa}$

- The beam dimension is sufficient.

Example 3.5: Design of a double tapered beam (ULS and SLS)

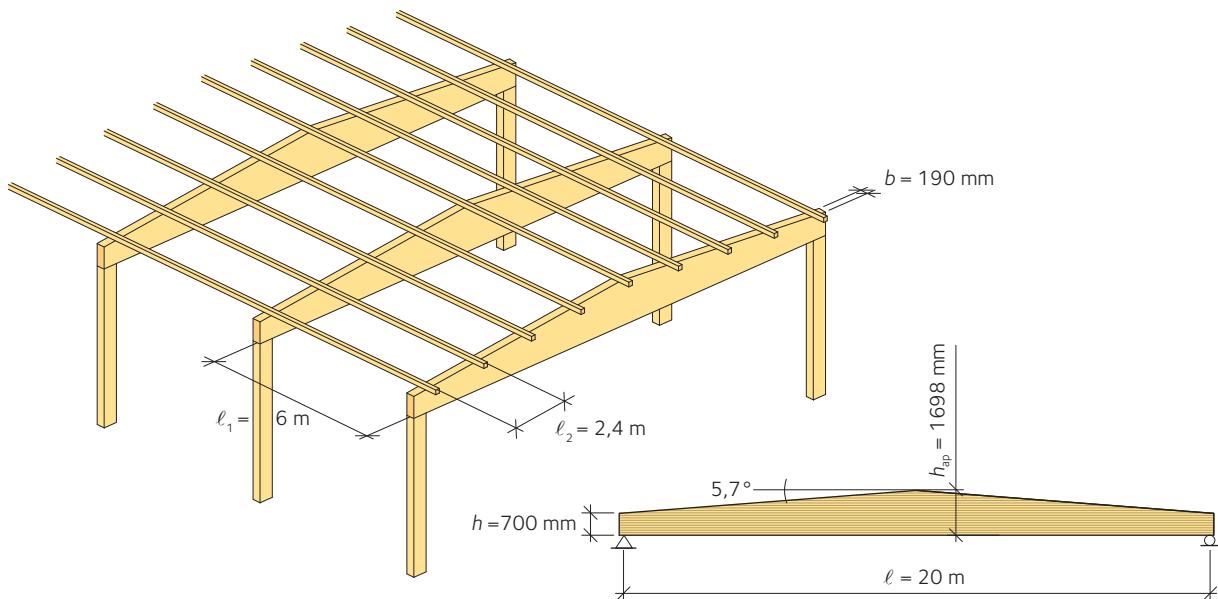


Figure 3.3 Geometry of the beam

Table 3.1

Load type	Uniformly distributed load [kN/m ²]	Uniformly distributed load [kN/m]
Beam self weight		$g_{1k} = 1,10$
Roof dead load	0,6	$g_{2k} = 3,96$
Snow	1,5	$s = 9,42$

Loads acting on the beam

The loads considered for the design of the double tapered beam are the following: beam self weight, roof dead load and snow load. The wind load can be neglected. The centre-to-centre distance between the primary beams is $\ell_1 = 6 \text{ m}$. However, the influence area for a beam is considered 10 percent larger than $\ell \times \ell_1$, to take into account the effect of the continuity of the purlins over the primary beams. The snow load is reduced to take into account the roof shape factor $\mu = 0,8 + 0,8 \cdot (5,7 / 30) = 0,952$. Here as an approximation on the safe side we calculate with the snow load on the leeward side as uniformly distributed along the whole beam length.

Load combinations

The beams are assumed to be indoors, in a heated environment. Therefore, they are characterized by an environment in which relative humidity very seldom, if ever, exceeds 65 %. Thus, the service class can be assumed to be 1.

Table 3.2

Safety class	Service class	Load combinations [kN/m]		Load duration	K_{mod}	K_{def}
Serviceability limit state (SLS)						
-	1	$g_k = (g_{1k} + g_{2k}) = 5,1$			-	0,6
-	1	$s = 9,42$			-	0,6
Ultimate limit state (ULS)						
3 →	1	$g_d = 0,85 \cdot 1,35 \cdot (g_{1k} + g_{2k}) = 5,8$	permanent	0,6	-	
3 →	1	$q_d = 0,85 \cdot 1,35 \cdot (g_{1k} + g_{2k}) + 1,5 \cdot s = 19,93$	medium-term	0,8	-	

Material

The material used for this structure is glulam GL30c ($\gamma_M = 1,25$, $k_{mod} = 0,8$).

3.5.1 Bending at critical section ($x = x_0$) and at mid-span ($x = \ell/2$)

a. Determination of stresses

For a symmetrical double tapered beam with constant uniformly distributed load q_d , the location of the critical cross section – that is the abscissa where the maximum bending stress occurs – can be calculated as follows:

$$x_0 = \frac{\ell \cdot h}{2 \cdot h_{ap}} = \frac{16800 \cdot 700}{2 \cdot 1538} = 3823 \text{ mm}$$

The corresponding depth of the beam is:

$$h_0 = h + x_0 \cdot \tan 5,7^\circ = 700 + 3823 \cdot \tan 5,7^\circ = 1081 \text{ mm}$$

The bending moment at the critical cross section is:

$$M_0 = \frac{q_d \cdot x_0}{2} \cdot (\ell - x_0) = \frac{19,93 \cdot 3,82}{2} \cdot (16,8 \cdot 3,82) = 494 \text{ kNm}$$

The corresponding bending stress at the critical cross section is:

$$\sigma_{m,\alpha,d} = \sigma_{m,0,d} = \frac{6 \cdot M_0}{b \cdot h_0^2} = \frac{6 \cdot 494 \cdot 10^6}{215 \cdot 1081^2} = 11,8 \text{ MPa}$$

The bending moment at mid-span is:

$$M_{ap,d} = \frac{q_d \cdot \ell^2}{8} = \frac{19,93 \cdot 16,8^2}{8} = 703 \text{ kNm}$$

The corresponding bending stress at mid span:

$$\sigma_{m,d} = \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = \frac{6 \cdot 703 \cdot 10^3}{215 \cdot 1538^2} = 8,3 \text{ MPa}$$

At the mid-span (apex zone), the tensile bending stress shall be magnified by a factor k_ℓ to take into account of the fact that the depth of the beam is not constant, but it varies linearly and it has a singularity at the apex. The magnification factor k_ℓ increases with increasing roof slope and it can be obtained from Volume 2: section 8.2 or Eurocode 5: equation 6.43. For a slope $\alpha = 5,7^\circ \rightarrow k_\ell = 1,2$.

$$\sigma_{m,d} = k_\ell \cdot \sigma_{m,d} = 1,2 \cdot 8,3 = 10,0 \text{ MPa}$$

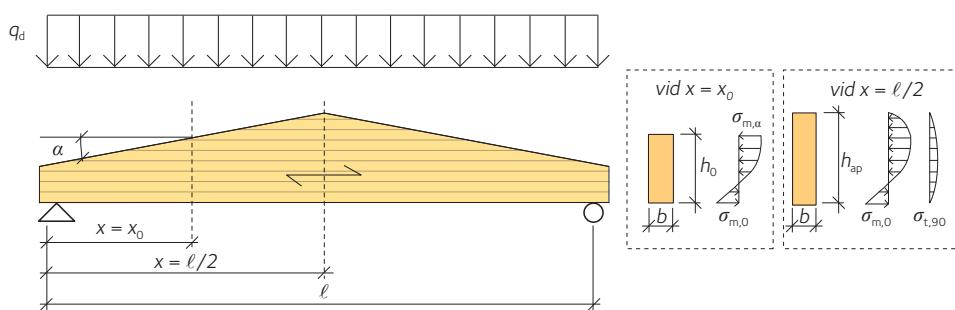


Figure 3.4

Table 3.3

Characteristics	Design values
Bending	$f_{m,d} = 19,2 \text{ MPa}$
Shear	$f_{v,d} = 2,2 \text{ MPa}$
Compression parallel to grain	$f_{c,0,d} = 15,7 \text{ MPa}$
Compression perpendicular to grain	$f_{c,90,d} = 1,6 \text{ MPa}$
Tension parallel to grain	$f_{t,0,d} = 12,5 \text{ MPa}$
Tension perpendicular to grain	$f_{t,90,d} = 0,32 \text{ MPa}$
Modulus of elasticity	$E_{0,mean} = 13\,000 \text{ MPa}$
Shear modulus	$G_{mean} = 650 \text{ MPa}$

Table 3.4

Position	$\sigma_{m,d}$ [MPa]	$f_{m,d}$ [MPa]	$k_{m,a}$	$f_{m,a,d}$ [MPa]	Utilization ratio
$x = x_0$	11,8	19,2	0,86	$19,2 \cdot 0,86 = 16,5$	$11,8 / 16,5 = 0,71$
$x = \ell / 2$	10,0	19,2	–	–	$10,0 / 19,2 = 0,52$

$\sigma_{m,d}$ = design bending stress; $f_{m,d}$ = design bending strength; $f_{m,a,d}$ = reduced design bending strength (tapered edge).

b. Verifications

At the tapered edge of the beam the bending strength must be reduced by a factor $k_{m,a}$ to take into consideration the effect of simultaneous action of compression parallel to grain, tension perpendicular to grain and shear. The reduction factor $k_{m,a}$ increases with increasing roof slope and it can be obtained from *Volume 1: figure 3.35*. For a slope $\alpha = 5,7^\circ \rightarrow k_{m,a} = 0,86$.

Lateral torsional buckling

Lateral torsional buckling in the double tapered beam (primary beams) may only occur between two adjacent purlins, under the condition that 1) the roof is braced in the transverse direction and 2) the purlins are adequately fastened to the primary beams. In that case, the buckling length can be assumed as the centre-to-centre distance between purlins, that is $\ell_2 = 2\,400$ mm. Within this distance, the depth of the cross section can be regarded as constant. The critical bending stress can be calculated according to *Volume 1: equation 3.34*:

$$\sigma_{m,crit} \approx \frac{0,78 \cdot b^2}{h_0 \cdot \ell_{ef}} \cdot E_{0,05} = \frac{0,78 \cdot 215^2}{1081 \cdot 2400} \cdot 10800 = 150 \text{ MPa}$$

The relative slenderness ratio for bending is defined in *Volume 1: equation 3.30* and the corresponding reduction factor k_{crit} is defined in *Volume 1: table 3.3*:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,crit}}} = \sqrt{\frac{30}{150}} = 0,45 < 0,75 \rightarrow k_{crit} = 1$$

- Since $\lambda_{rel,m} \leq 0,75$, full bending strength can be achieved without risk for lateral torsional buckling.

It is important to check the shear capacity for tapered beams due to the normally low beam depth at the supports. Yet this check is omitted in this example.

3.5.2 Tension perpendicular to the grain

a. Determination of stresses

The tensile stress perpendicular to the grain can be evaluated by multiplying the bending stress at mid-span by the factor k_p , which can be taken from *Volume 1: figure 3.38*:

$$\sigma_{t,90,d} = k_p \cdot \sigma_{m,d} = 0,02 \cdot 10,0 = 0,20 \text{ MPa}$$

Table 3.5

Position	$\sigma_{t,90,d}$ [MPa]	$f_{t,90,d}$ [MPa]	k_{vol}	$f_{t,90,d_red}$ [MPa]	Utilization ratio
$x = \ell/2$	0,20	0,32	0,64	$0,64 \cdot 0,32 = 0,205$	$0,20 / 0,205 \approx 0,97$

b. Verifications

The tension strength perpendicular to grain shall be reduced in order to take into account the volume effect. The volume of wood which is loaded in tension can be estimated as follows, see *Volume 1: table 3.4*:

$$V = b \cdot h_{ap}^2 = 215 \cdot 1538^2 \cdot 10^{-9} = 0,51$$

The reduction factor due to volume effect can be calculated according to *Volume 1: equation 3.53*:

$$k_{vol} = k_{dis} \cdot \left(\frac{0,01}{V} \right)^{0,2} = 1,4 \cdot \left(\frac{0,01}{0,51} \right)^{0,2} = 0,64$$

where k_{dis} is a factor that takes into account that the tension stress perpendicular to the grain is not uniformly distributed in the loaded timber volume V .

Here the need for reinforcement is just about necessary.

Additionally the shear capacity at the supports has to be verified.

This is of special importance for double tapered beams because of their reduced cross sectional height at the supports. By slopes 1/10 or more the shear stresses at the supports are often designing the beam dimensions.

3.5.3 Shear at the supports

Support reaction

$$R = 19,93 \cdot 16,8 / 2 = 167,4 \text{ kN}$$

Advantageously the shear force is given by applying the possible reduction according to *Eurocode 5: Section 6.1.7 (3)*. With purlins spaced at 2 400 mm centres a load on 1 200 mm of the primary beam goes directly to the support and does so not contribute to the shear stress.

$$V = 167,4 - 1,2 \cdot 19,93 = 143,5 \text{ kN}$$

$$\tau = 1,5 \cdot 143,5 \cdot 10^3 / (0,215 \cdot 0,7) = 1,43 \text{ MPa}$$

$$f_{v,d} = 0,67 \cdot 2,2 \cdot 10^6 = 1,47 \text{ MPa}$$

The utilization ratio is then $1,43 / 1,47 = 0,97$, what is acceptable.

3.5.4 Compression perpendicular to the grain at the supports

a. Determination of stresses

We assume that the column supporting the double tapered beam has a cross section 190×360 mm. The support length between beam and column is therefore $b \times \ell_b = 190 \times 360$ mm².

The compression stress perpendicular to the grain can be evaluated according to *Volume 2: section 5.2* or *Eurocode 5: section 6.1.5*:

$$\begin{aligned}\sigma_{c,90,d} &= \frac{0,5 \cdot q_d \cdot \ell}{A_{ef}} = \frac{q_d \cdot \ell}{2} \cdot \frac{1}{b \cdot (\ell_b + 30 \text{ mm})} = \\ &= \frac{19,93 \cdot 16800}{2} \cdot \frac{1}{215 \cdot (360 + 30)} = 2,00 \text{ MPa}\end{aligned}$$

Table 3.6

Position	$\sigma_{c,90,d}$ [MPa]	$f_{c,90,d}$ [MPa]	$k_{c,90}$	$f'_{c,90,d}$ [MPa]	Utilization ratio
$x = 0$	2,0	1,6	1,75	$1,75 \cdot 1,6 = 2,8$	$2,0 / 2,8 = 0,71$

b. Verifications

The compression strength perpendicular to grain can be magnified by factor $k_{c,90} = 1,75$. Because $g_k / q_k > 0,4$ it is not recommended to choose k_{mod} and γ_M as 1,0.

3.5.5 Deflection

For a double tapered beam, resting on supports and subjected to a uniformly distributed load q , the deflection w should be evaluated by the following equation, Piazza et al. (2005), see *Volume 1: section 3.6*:

$$w = \frac{5}{384} \cdot \frac{q \cdot \ell^4}{E_{0,\text{mean}} \cdot I_y} \cdot k_m + \chi \cdot \frac{1}{8} \cdot \frac{q \cdot \ell^2}{G_{\text{mean}} \cdot A} \cdot k_v$$

where $\chi = 1,2$, I_y and A are the moment of inertia and area of the cross section at the support respectively, whilst k_m and k_v are defined as:

$$\begin{aligned}k_m &= \left(\frac{h}{H} \right)^3 \cdot \frac{1}{0,15 + 0,85 \cdot h/H} = \\ &= \left(\frac{700}{1538} \right)^3 \cdot \frac{1}{0,15 + 0,85 \cdot 700/1538} = 0,18\end{aligned}$$

$$k_v = \frac{2}{1 + (H/h)^{2/3}} = \frac{2}{1 + (1538/700)^{2/3}} = 0,74$$

The instantaneous deflections are:

- $w_{g,\text{inst}}$ – due to self weight
- $w_{qs1,\text{inst}}$ – due to the variable action q_{s1} (snow load)

$$w_{g,inst} = \frac{5}{384} \cdot \frac{g_k \cdot \ell^4}{E_{0,mean} \cdot I_y} \cdot k_m + \chi \cdot \frac{1}{8} \cdot \frac{g_k \cdot \ell^2}{G_{mean} \cdot A} \cdot k_v = \\ = \frac{5}{384} \cdot \frac{5,1 \cdot 16800^4 \cdot 12}{13000 \cdot 215 \cdot 700^3} \cdot 0,18 + 1,2 \cdot \frac{1}{8} \cdot \frac{5,1 \cdot 16800^2}{650 \cdot 215 \cdot 700} \cdot 0,74 = 13,5 \text{ mm}$$

$$w_{qsl,inst} = \frac{5}{384} \cdot \frac{s \cdot \ell^4}{E_{0,mean} \cdot I_y} \cdot k_m + \chi \cdot \frac{1}{8} \cdot \frac{s \cdot \ell^2}{G_{mean} \cdot A} \cdot k_v = \\ = \frac{5}{384} \cdot \frac{9,42 \cdot 16800^4 \cdot 12}{13000 \cdot 215 \cdot 700^3} \cdot 0,18 + 1,2 \cdot \frac{1}{8} \cdot \frac{9,42 \cdot 16800^2}{650 \cdot 215 \cdot 700} \cdot 0,74 = 25,0 \text{ mm}$$

With $\psi_{2,1} = 0,2$ (snow load) and $k_{def} = 0,6$ (service class 1), the final deflection is:

$$w_{fin} = w_{g,inst} \cdot (1 + k_{def}) + w_{q,inst} \cdot (1 + \psi_{2,1} \cdot k_{def}) = \\ = 13,5 \cdot (1 + 0,6) + 25 \cdot (1 + 0,2 \cdot 0,6) = 21,7 + 28,0 = 49,7 \text{ mm}$$

- This deflection corresponds to $\ell/338$ which is fully acceptable for an industrial building. For schools, stores and similar facilities with higher requirements, it can be considered to manufacture the beam with a precamber of say 40 mm.

Design of timber joints

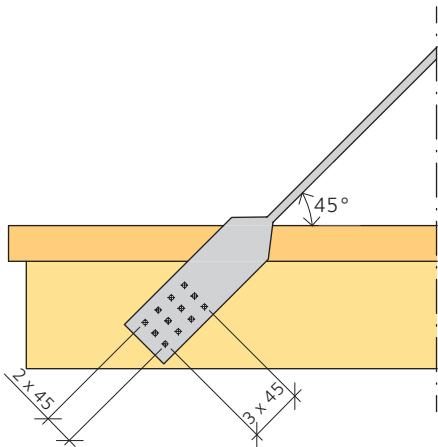


Figure 4.1

Example 4.1: Design of tie rod connection

Hint: Failure mode d) in Volume 2: figure 10.2 is governing the behaviour of the connection and the rope effect is not necessary to take into account.

A balcony in timber is at the front edge supported by a tie rod made of steel. This steel rod is attached to the floor beam of the balcony with 12 wood screws. The supporting floor beam has the dimension 115 × 270 mm. The wood screws have a length of 60 mm, a diameter of 8 mm and an ultimate strength of 410 MPa. For simplicity the effective diameter d_{ef} is here assumed to be equal the outer thread diameter d . This information is usually to be found in the declarations from the screw manufacturers. The thickness of the steel plate is 8 mm. The tensile resistance perpendicular to the grain of the floor beam is assumed to be sufficient to withstand the force from the tie rod.

Calculate the maximum load in the tie rod with respect to the capacity of the steel-to-wood connection.

Spacings and edge distances for the screws are assumed to be adequate.

Geometry

Steel thickness $t = 8 \text{ mm}$

Screw diameter $d_{ef} = d = 8 \text{ mm}$

Screw length $\ell_s = 60 \text{ mm}$

Screw length in the timber $t_1 = \ell_s - t = 52 \text{ mm}$

Angle between force and grain direction $\alpha = 45^\circ$

Number of wood screws $n = 12$

Capacity of the connection

Characteristic density, glulam GL30c $\rho_k = 390 \text{ kg/m}^3$

Wood screw strength $f_u = 410 \text{ MPa}$

Partial coefficient, connection $\gamma_M = 1,3$

Material factor for the connection
at the ULS, medium term load M,
service class 3 $k_{\text{mod}} = 0,65$

Yield moment wood screw $M_{y,Rk} = 0,3 f_u d_{\text{ef}}^{2,6}$

$$M_{y,Rk} = 2,74 \cdot 104 \text{ Nmm}$$

Embedding strength parallel fibers $f_{h,0,k} = 0,082(1 - 0,01d_{\text{ef}}) \rho_k$
 $f_{h,0,k} = 29,42 \text{ MPa}$

Softwood, correction factor $k_{90} = 1,35 + 0,015d_{\text{ef}} = 1,47$

Embedding strength in
the direction of the force $f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90} \sin^2 \alpha + \cos^2 \alpha}$
 $f_{h,\alpha,k} = 23,82 \text{ MPa}$

The connection is a one-sided connection with a thick steel plate,
that is failure mode c, d or e, see Volume 2: figure 10.2 and Volume 1:
table 4.3:

Failure mode c $F_{v,Rk,c} = \dots = 9,91 \text{ kN}$

Failure mode d $F_{v,Rk,d} = \dots = 4,83 \text{ kN}$

Failure mode e $F_{v,Rk,e} = \dots = 5,26 \text{ kN}$

Characteristic capacity $F_{v,Rk} = \min(F_{v,Rk,c}, F_{v,Rk,d}, F_{v,Rk,e})$
 $= 4,83 \text{ kN}$

Design capacity $F_{v,Rd} = \dots = 2,42 \text{ kN}$

Total capacity of the connection $F_v = n \cdot F_{v,Rd} = 29,0 \text{ kN}$

► The connection can withstand a force of 29 kN.

Example 4.2: Design of nailed connection in a Gerber system

This connection can advantageously be designed so that the force is being transferred via compression towards the horizontal parts of the steel, what also is the common procedure in practice. However in the example this have been ignored and focus is to show how nail design is done.

4.2.1 Problem description

The Gerber connection is placed in the outer bay of a beam with a span in the bay of 23 000 mm. The beam is loaded with an evenly distributed load.

- $q_d = 34,3 \text{ kN/m}$
- Load duration class M
- Nail material, $f_u = 600 \text{ MPa}$
- Material GL30c, $b \times h = 215 \times 1\,305 \text{ mm}$

The shear force to be transferred in the connection is calculated as:

$$V = 0,44q_d\ell = 0,44 \cdot 34,3 \cdot 23 = 347 \text{ kN}$$

4.2.2 Approach

The Gerber connection is always placed where the moment is (theoretically) zero, which means that the shear force is relatively large. The connection shall thus be designed so that no moment is transferred. A principal sketch is shown in figure 4.3.

In smaller Gerber connections it is possible to use a simple rectangular nail plate placed centrally in the beam. But for the large connection in figure 4.3, the steel must also be designed. Please refer to Eurocode 3 for further details on steel design.

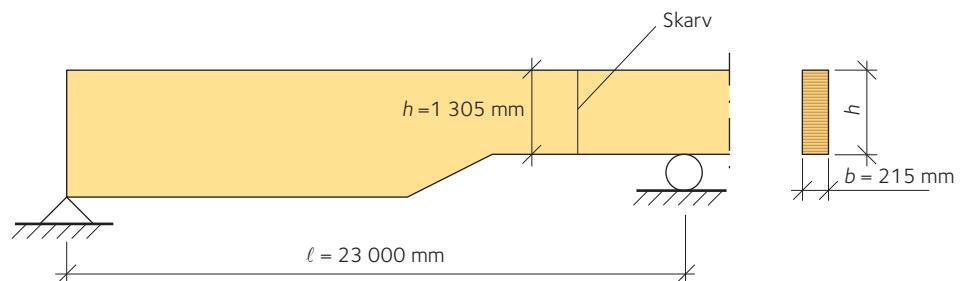


Figure 4.2 Geometry

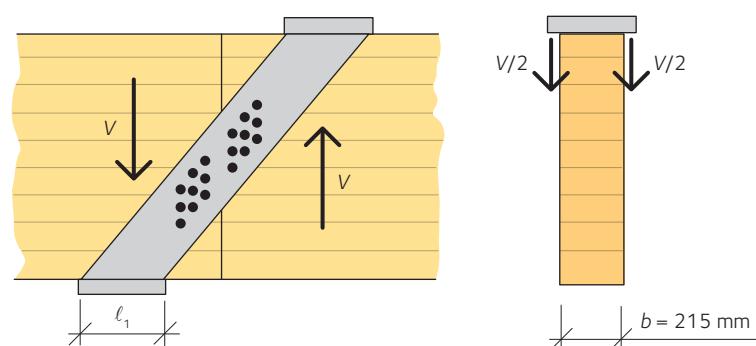


Figure 4.3 Principal solution for Gerber connection

4.2.3 Resistance of nails

Annular ring shanked nails of the dimension 40 – 4,0 mm are assumed. They have a head diameter of 6 mm. The connection will be a single shear connection steel-wood. The thickness of the steel plate is chosen as 5 mm, which means that $t_{\text{steel}} > d$ ($5 > 4$) and the nail has a fixed support in the steel plate. *Volume 1: table 4.3* contains the three failure modes that need to be calculated. The lowest of these three values will be the characteristic resistance for one nail. The embedding strength is established through using $\rho_k = 390 \text{ kg/m}^3$ in *Volume 1: equation 4.1*.

$$f_{h,k} = 0,082 \rho_k d^{-0,3} = 0,082 \cdot 390 \cdot 4^{-0,3} = 21,1 \text{ MPa}$$

The yield moment of the nail $M_{y,Rk}$ with $d = 4 \text{ mm}$:

$$M_{y,Rk} = 600 / 600 \cdot 270 \cdot 4^{2,6} = 9925 \text{ Nmm}$$

Somewhat simplified the formula for smooth, grooved nails has here been applied. In practice this value is usually obtained from each nail manufacturer.

4.2.3.1 Shear capacity

The thickness t_1 is the length of the nail minus the thickness of the steel plate; $t_1 = 40 - 5 = 35 \text{ mm}$:

$$F_{v,Rk} (\text{mode I}) = f_{h,k} t_1 d = 21,1 \cdot 35 \cdot 4 = 2954 \text{ N}$$

$$F_{v,Rk} (\text{mode II}) = 21,1 \cdot 35 \cdot 4 \cdot \left(\sqrt{\left(2 + 4 \cdot 9925 / (21,1 \cdot 4 \cdot 35^2) \right)} - 1 \right) = 1607 \text{ N}$$

$$F_{v,Rk} (\text{mode III}) = 2,3 \cdot \sqrt{(9925 \cdot 21,1 \cdot 4)} = 2105 \text{ N}$$

The lowest capacity of the three failure modes occurs for failure mode II, that is $F_{v,Rk} = 1,607 \text{ kN/nail}$. The design value is determined using Eurocode 5 with $k_{\text{mod}} = 0,8$ and $\gamma_M = 1,3$ (also see *Volume 2: section 3.1 and 3.2*):

$$F_{v,Rd} = 0,8 \cdot 1,607 / 1,3 = 0,989 \text{ kN/nail}$$

4.2.3.2 Axial capacity

The axial capacity is determined using *Volume 1: equations 4.14–4.16* and $t_{\text{pen}} = t_1 = 35 \text{ mm}$:

$$f_{ax,k} = 20 \cdot 10^{-6} \cdot 390^2 = 3,04 \text{ MPa} \Rightarrow F_{ax,Rk} = 3,04 \cdot 4 \cdot 35 = 426 \text{ N}$$

$$f_{\text{head},k} = 70 \cdot 10^{-6} \cdot 390^2 = 10,6 \text{ MPa} \Rightarrow F_{ax,Rk} = 10,6 \cdot 6^2 = 383 \text{ N}$$

$$F_{ax,Rk} = \min(426; 383) = 383 \text{ N} = 0,383 \text{ kN/nail}$$

In practice also these values are usually obtained from each nail manufacturer.

4.2.3.3 Fastener capacity

The fastener capacity is obtained by adding the shear and the axial capacities according to (where the second term in the right part is the rope effect):

$$F_{Rk} = 1,607 + 0,383 / 4 = 1,70 \text{ kN/nail}$$

The contribution from rope effect in relation to the shear capacity is only 6 percent and can therefore be completely counted, as the limit is 50 percent for annular ring shanked nails, see Volume 1: table 4.8.

The design capacity is determined using $k_{mod} = 0,8$ and $\gamma_M = 1,3$:

$$F_{Rd} = \frac{1,70 \cdot 0,8}{1,3} = 1,05 \text{ kN/nail}$$

4.2.4 Connection design

The number of nails needed on each side of the connection is:

$$n = \frac{347 / 2}{1,05} = 166 \text{ nails}$$

The nails should be placed as close to the neutral axis of the beams as possible to avoid eccentricity moments. The regulations for spacing parallel and perpendicular to grain needs to be followed, Volume 2: Section 10.4.2. For nailed joints, a distance of $5d$ is prescribed perpendicular to the grain and $10d$ parallel to the grain. A proposed placement of fasteners would be according to figure 4.4.

The size of the triangle where fasteners are to be placed is derived by assuring that 166 nails fit into the triangle without violating fastener spacing recommendations. To complete the design of the connection, the steel plates would need to be designed, refer to Eurocode 3 for steel design. Furthermore, the compression perpendicular to the grain of the wood must be checked as indicated in figure 4.4, see Volume 2: section 5.2.

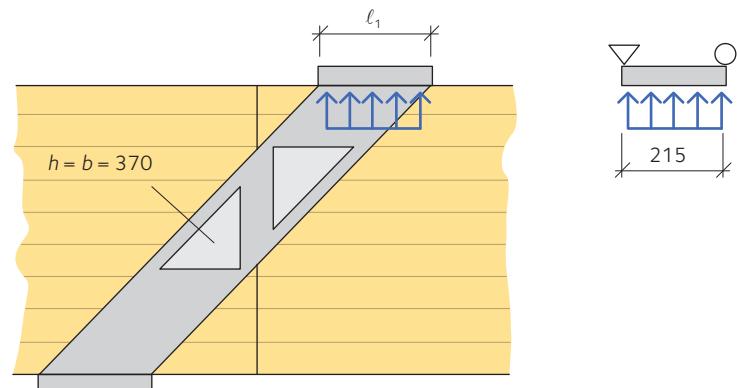


Figure 4.4

Example 4.3: Design of bolted connection in tension

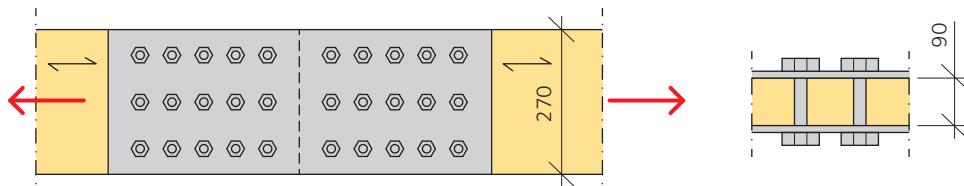


Figure 4.5

4.3.1 Problem description

A bolted connection in pure tension shall be designed. The connection could be the bottom chord in a truss or a purlin in tension. The number of bolts is unknown at the onset of design.

- design tensile force $N_{td} = 250 \text{ kN}$
- thickness of the steel plate $t = 8 \text{ mm}$
- load duration class M, service class 2
- bolt diameter $d = 16 \text{ mm}$
- tensile strength of bolt $f_u = 360 \text{ MPa}$
- glulam GL30c, $\rho_k = 390 \text{ kg/m}^3$.

4.3.2 Strength parameters

The yield moment of the bolt is calculated using *Volume 1*:
equation 4.6:

$$M_{y,Rk} = 0,3f_u d^{2,6} = 0,3 \cdot 360 \cdot 16^{2,6} = 146 \text{ Nm}$$

The embedding strength of wood is determined according to *Volume 1*:
equation 4.2:

$$f_{h,2,k} = 0,082(1 - 0,01 \cdot 16)390 = 26,9 \text{ MPa}$$

4.3.3 Resistance for a single bolt

The resistance for a single bolt is the sum of the shear and tensile resistances.

4.3.3.1 Shear capacity

The shear capacity is established through using the expressions in *Volume 1*: *table 4.7*, double shear steel-to-timber joints with $t = 0,5d$, failure mode j and k, that is pinned support for the bolt in relation to the steel plate:

$$F_{v,Rk} = \min \begin{cases} 0,5f_{h,2,k}t_2d = 0,5 \cdot 26,9 \cdot 10^6 \cdot 0,090 \cdot 0,016 = 19,3 \text{ kN} \\ 1,15\sqrt{2M_{y,Rk}f_{h,2,k}d} = 1,15\sqrt{2 \cdot 146 \cdot 26,9 \cdot 10^6 \cdot 0,016} = 12,9 \text{ kN} \end{cases}$$

Thus, the characteristic shear capacity is 12,9 kN/bolt.

4.3.3.2 Withdrawal capacity

The withdrawal capacity of a bolt is determined using *Volume 1: equation 4.18*. Since a washer is not used, but rather a continuous steel plate, the fictitious washer diameter is $\min(12 \cdot 8; 4 \cdot 16) = 64$ mm:

$$A_{\text{washer}} = \pi(64^2 - 16^2)/4 = 3016 \text{ mm}^2$$

$$F_{\text{ax,washer,Rk}} = 3 \cdot 2,5 \cdot 3016 = 22,6 \text{ kN/side}$$

4.3.3.3 Bolt resistance

The resistance of one bolt is determined:

$$F_{\text{Rk}} = F_{\text{v,Rk}} + 2 \cdot F_{\text{ax,washer,Rk}}/4 = 12,9 + 2 \cdot 22,6/4 = 24,2 \text{ kN/bolt}$$

Because a continuous steel plate exists, the whole withdrawal capacity is counted (rope effect). The design capacity is determined using $k_{\text{mod}} = 0,8$ and $\gamma_M = 1,3$:

$$F_{\text{Rd}} = \frac{0,8 \cdot 24,2}{1,3} = 14,9 \text{ kN/bolt}$$

4.3.4 Connection layout

The number of bolts will depend on the group effect and the number of rows in the connection. The group effect is in turn dependent on the number n of fasteners in a row, and therefore it is of interest to determine the number of rows that can be placed within the beam height 270 mm. The minimum distance between fasteners in a row is $4d$ according to *Volume 2: table 10.4*. The edge distance to an unloaded edge is $3d$. Thus, the maximum number of rows that can be placed perpendicular to grain is:

$$n_{\text{rows}} = \left(\frac{270 - 2 \cdot 3d}{4d} \right) + 1 = 3 \text{ rows}$$

4.3.5 Group effect

According to *Volume 1: equation 4.23*, bolted joints are affected by a group effect. The spacing for bolts parallel to the grain is chosen as $7d$, which is more than what the standard requires. Knowing that the boundary for n to result as n_{ef} is a spacing of $13d$, the determination of the number of fasteners in one row can be established as:

$$\begin{aligned} 3n_{\text{ef}} \cdot 14,9 &= 250 \\ n^{0,9} \sqrt[4]{\frac{7d}{13d}} &= \frac{250}{3 \cdot 14,9} \\ n &\geq 8,04 \approx 8 \text{ bolts/row} \end{aligned}$$

Without the group effect the connection could have been established using $250/14,9 = 17$ bolts, but now 24 bolts results. If more rows could be accommodated within the beam height, this would yield a lower number of bolts in total. Increasing the beam height can therefore be a good suggestion if not other parameters affect the selection of height.

For a connection of this type, a block shear check would have been necessary in practice, see *Volume 1: section 4.9.3*.

Example 4.4: Design of moment resisting column base

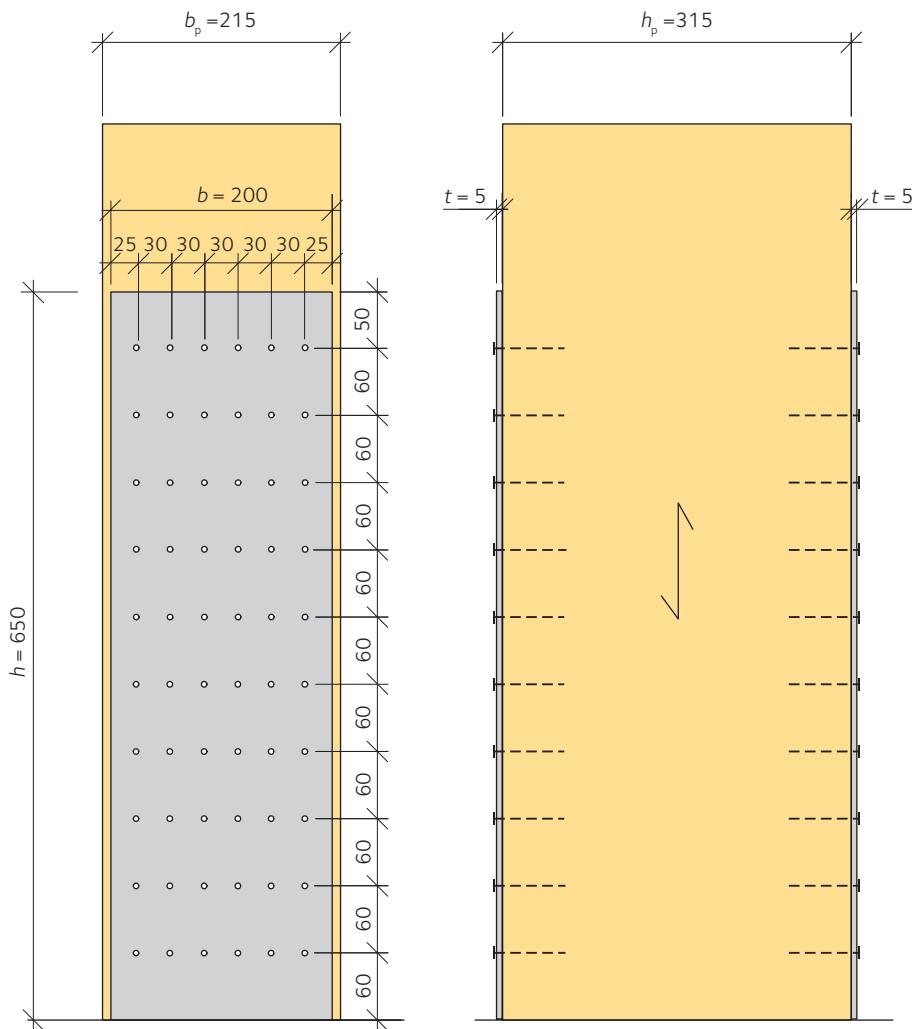


Figure 4.6

The base of a column in a storage building shall be designed as moment resisting. The timber column is fastened with nails to steel sheets cast into the concrete. The column is made of glulam GL30c with the dimensions $h_p = 315$ mm and $b_p = 215$ mm. The steel sheets are made of steel S235 with the dimension $h = 650$ mm, $b = 200$ mm and a thickness $t = 5$ mm. The number of annular ring shanked nails is 60 pieces per sheet, the dimension 60-4,0 and with a tensile strength of 600 MPa.

Check if the connection has enough capacity to withstand a design horizontal force $H = 25$ kN, a design moment $M = 25$ kNm and a design vertical force $V = 25$ kN.

1. Geometry

Column width	$b_p = 215 \text{ mm}$
Column depth	$h_p = 315 \text{ mm}$
Length steel sheet	$h = 650 \text{ mm}$
Width steel sheet	$b = 200 \text{ mm}$
Thickness steel sheet	$t = 5 \text{ mm}$
Diameter nail	$d = 4 \text{ mm}$
Diameter nail head	$d_h = 8 \text{ mm}$
Diameter hole drilled in the steel sheet	$d_{\text{hole}} = d + 1 \text{ mm} = 5 \text{ mm}$
Nail length	$\ell_s = 60 \text{ mm}$
Number of nails \perp grain	$n_h = 6$
Number of nails // grain	$n_v = 10$
Penetration depth, nail	$t_1 t_{\text{pen}} = \ell_s - t = 55 \text{ mm}$

2. Material properties

Shear strength glulam GL30c	$f_{v,k} = 3,5 \text{ MPa}$
Tension strength parallel to the grain, glulam GL30c	$f_{t,0,k} = 19,5 \text{ MPa}$
Compression strength perpendicular to the grain, glulam GL30c	$f_{c,90,k} = 2,5 \text{ MPa}$
Density, glulam GL30c	$\rho_k = 390 \text{ kg/m}^3$
Tension strength, nail	$f_u = 600 \text{ MPa}$
Yield strength, S235	$f_{yk} = 235 \text{ MPa}$
Ultimate strength, S235	$f_{uk} = 340 \text{ MPa}$

3. Modification factors

Material factor for glulam at ULS, GL30c	$\gamma_M = 1,25$
Material factor for glulam at ULS, connection	$\gamma_{M,\text{connection}} = 1,3$
Material factor for glulam at ULS, steel	$\gamma_{M2} = 1,2$
Factor for medium duration loading and service class 2	$k_{\text{mod}} = 0,8$

4. Design section forces

Horizontal force	$H = 25 \text{ kN}$
Vertical force	$V = 25 \text{ kN}$
Moment	$M = 25 \text{ kNm}$

The horizontal force is taken as contact pressure between the column and the steel sheet. The vertical force is taken as contact between the bottom of the column and the foundation. The moment is taken as a force couple in the steel sheets.

The resulting force due to the moment:

$$F_x = \frac{M}{h_p} = 79 \text{ kN}$$

This assumes that the force is so small so that the steel sheet does not buckle, which has to be checked. In this case *Eurocode 3: section 6.3* gives us:

$$\lambda = 60 / 5 \cdot \sqrt{12} / 93,9 = 0,443$$

$$\Phi = 0,5 \cdot (1 + 0,49 \cdot (\lambda - 0,2) + \lambda 2) = 0,657$$

$$X = 1 / (\Phi + \sqrt{(\Phi^2 - \lambda^2)}) = 0,875$$

The capacity of the steel sheet with respect to buckling is then $N_{b,Rd} = 0,875 \cdot 0,005 \cdot 0,2 \cdot 235 \cdot 103 / 1,1 = 187 \text{ kN}$. This means that in this case there is no risk of buckling, but that is so because the moment being moderate in the example and the bottom row of nails are placed as close to the lower end as possible, with respect to end distance ($15d$). If the steel sheet would buckle, the lower end of the glulam column would transfer the force to the concrete through contact pressure and we would get a much shorter lever to withstand the moment and therefore an increased force in the tensioned sheet.

Force taken as contact between the steel sheet and the column.

$$F_y = H = 25 \text{ kN}$$

5. Embedding strength

Embedding strength glulam GL30c
(Volume 1: equation 4.1)

$$f_{h,k} = 0,082 \rho_k d^{-0,3}$$

$$f_{h,k} = 21,1 \text{ MPa}$$

6. Yield moment, nail

Yield moment, nail
(Volume 1: equation 4.5)

$$M_{y,Rk} = 0,45 f_u d^{2,6}$$

$$M_{y,Rk} = 9\,925 \text{ Nmm}$$

Somewhat simplified the formula for smooth, grooved nails has here been applied. In practice this value is usually obtained from each nail manufacturer.

7. Axial withdrawal capacity

Axial withdrawal strength
(Volume 1: equation 4.14)

$$f_{ax,k} = 20 \cdot 10^{-6} \rho_k^2$$

$$f_{ax,k} = 3 \text{ MPa}$$

Pull through strength

The steel sheet prevents pull through of the nail head, therefore the pull through strength is not being calculated.

Withdrawal capacity
(Volume 1: equation 4.16)

$$F_{ax,Rk} = f_{ax,k} \cdot d \cdot t_{pen} = 669 \text{ N}$$

Withdrawal capacity/4

$$F_{ax,Rk} / 4 = 167 \text{ N}$$

8. Shear capacity of the nails

Thick steelplate

(Volume 2: section 10.3
and Volume 1: table 4.3)

$$t/d = 1,25$$

the condition ≥ 1 is satisfied

Failure mode c

(Volume 2: section 10.3
and Volume 1: table 4.3)

$$F_{v,Rk,c} = f_{h,k} t_1 d =$$

$$= 21,1 \cdot 55 \cdot 4 = 4,64 \text{ kN}$$

Failure mode d

(Volume 2: section 10.3
and Volume 1: table 4.3)

$$F_{v,Rk,d} = f_{h,k} t_1 d \left(\sqrt{2 + \frac{4M_{y,Rk}}{f_{h,k} dt_1^2}} - 1 \right)$$

$$F_{v,Rk,d} = 21,1 \cdot 55 \cdot 4 \left(\sqrt{2 + \frac{4 \cdot 9925}{21,1 \cdot 4 \cdot 55^2}} - 1 \right) = 2,17 \text{ kN}$$

Failure mode e

(Volume 2: section 10.3
and Volume 1: table 4.3)

$$F_{v,Rk,e} = 2,3 \sqrt{M_{y,Rk} f_{h,k} d}$$

$$F_{v,Rk,e} = 2,3 \sqrt{9925 \cdot 21,1 \cdot 4} = 2,11 \text{ kN}$$

Capacity for one nail

$$F_{v,Rk} = \min(F_{v,Rk,c}, F_{v,Rk,d}, F_{v,Rk,e})$$

Deciding failure mode

Failure mode e

Contribution from rope effect

(can here be fully counted)

$$F_{ax,Rk} / 4 = 0,167 \text{ kN}$$

Total design capacity per nail
and shear plane

$$F_{Rd} = \frac{(F_{v,Rk} + F_{ax,Rk} / 4) \cdot k_{mod}}{\gamma_M} =$$

$$\frac{(2,11 + 0,167) \cdot 0,8}{1,3} = 1,40 \text{ kN}$$

Least spacing // grain for nails to be
effective (reduction is not needed)
(Volume 2: table 10.1)

$$14d = 56 \text{ mm}$$

Real distance between nails

$$a_1 = 60 \text{ mm}$$

Total capacity for the connection

$$F_{Rd,tot} = n_v \cdot n_h \cdot F_{Rd} = 10 \cdot 6 \cdot 1,40 = 84 \text{ kN}$$

Control of the capacity

$$\frac{F_x}{F_{Rd,tot}} = \frac{79}{84} = 0,94$$

The capacity of the nails is enough.

9. Nail spacing

Chosen spacing parallel to the grain

$$a_1 = 60 \text{ mm}$$

Chosen spacing perpendicular to the grain

$$a_2 = 30 \text{ mm}$$

Chosen distance to loaded end

$$a_3 = 60 \text{ mm}$$

Angle between force direction and grain

$$\alpha = 0$$

Required spacing parallel to the grain

$$a_1 = (5 + 5 \cdot |\cos(0)|) \cdot d \cdot 0,7 = 28 \text{ mm}$$

Required spacing perpendicular to the grain

$$a_2 = 5d \cdot 0,7 = 14 \text{ mm}$$

Required distance to loaded end

$$a_{3,t} = (10 + 5 \cdot \cos(0)) \cdot d = 60 \text{ mm}$$

► The capacity of the nails is enough to withstand the forces.

To perform a complete check of the connection it is also necessary to check plug shear failure, the strength of the steel sheets, and if the steel sheet can withstand the horizontal force as well as to check the connection between the steel sheets and the foundation.

Composite timber elements

Example 5.1: Design of an OSB-webbed I-girder (ULS and SLS)

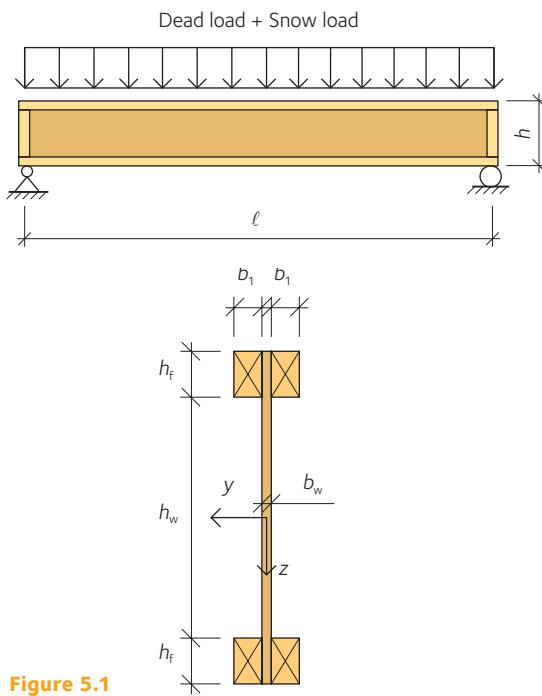


Figure 5.1

A composite beam with flanges in strength class C24 and a web panel of oriented strand board (OSB 3) is designed as shown below. Check all important resistances in the ultimate limit state (ULS), except crushing perpendicular to grain due to support reactions. Check also the maximum instantaneous and final deflections in the serviceability limit state (SLS). For the total load the requirements for the characteristic load combination are $w_{\text{inst}} \leq \ell/300$ and $w_{\text{fin}} \leq \ell/200$.

Note that the beam is braced sideways such that lateral torsional buckling cannot occur.

Note also that full composite action can be assumed between the flange timber and the web panel. Transverse web stiffeners are used at both supports.

The girder is located in a place where service class 2 can be assumed. The rest of the input data are given below:

User defined units:

Flange width for each side	$b_1 = 45 \text{ mm}$
Flange depth	$h_f = 70 \text{ mm}$
Web panel depth	$h_w = 500 \text{ mm}$
Web panel thickness	$b_w = 15,0 \text{ mm}$
Total span	$\ell = 7,0 \text{ m}$
Self weight of the roof, including the girder	$g_k = 1,65 \text{ kN/m}$
Snow zone 2,5, snow is a short term load in Sweden	$s_k = 3,0 \text{ kN/m}$ (Is obtained for example when one-sided slope of < 15°, without drifted snow and with spacing at 1 800 mm centres)
Factor for the quasi permanent $H > 1\ 000 \text{ m}$ over sea level	$\psi_2 = 0,2$

Material properties

Characteristic and design material properties of C24 timber (solid timber), see Volume 2, table 3.3.

$$\left\{ \begin{array}{l} \text{Bending strength} \\ \text{Tensile strength parallel to grain} \\ \text{Compressive strength parallel to grain} \\ \text{Shear strength} \end{array} \right\} = \begin{pmatrix} f_{m,k} \\ f_{t,0,k} \\ f_{c,0,k} \\ f_{v,k} \end{pmatrix} = \begin{pmatrix} 24 \\ 14,5 \\ 21 \\ 4,0 \end{pmatrix} \text{ MPa} \quad \begin{array}{l} \text{Modulus of elasticity} \\ E_{0,\text{mean}} = 11\,000 \text{ MPa} \end{array}$$

Partial factor $\gamma_M = 1,30$

Modification factor $k_{\text{mod}} = 0,90$
Short term load duration M and service class 2

Creep factor $k_{\text{def}} = 0,80$

The modification factor, k_h for size effects in bending and tension, is defined as:

$$k_{\text{hhh}}(h) = \begin{cases} k \leftarrow \min \left[\left(\frac{150 \text{ mm}}{h} \right)^{0,2}, 1,3 \right] \\ k \leftarrow 1,0 \quad \text{if } k < 1,0 \\ k \end{cases}$$

For this flange we have:

$$k_h = k_{\text{hhh}}(h_f) = 1,16$$

$$\left\{ \begin{array}{l} \text{Bending strength} \\ \text{Tensile strength parallel to grain} \\ \text{Compressive strength parallel to grain} \\ \text{Shear strength} \end{array} \right\} = \frac{k_{\text{mod}}}{\gamma_M} \begin{pmatrix} f_{m,k} k_h \\ f_{t,0,k} k_h \\ f_{c,0,k} \\ f_{v,k} \end{pmatrix} = \begin{pmatrix} 1,93 \cdot 10^1 \\ 1,12 \cdot 10^1 \\ 1,45 \cdot 10^1 \\ 2,76 \cdot 10^1 \end{pmatrix} \text{ MPa}$$

Characteristic and design material properties of OSB/3, see Volume 2: table 3.10.

$$\left\{ \begin{array}{l} \text{In plane tensile strength} \\ \text{In plane compressive strength} \\ \text{Panel shear strength} \\ \text{Planar (rolling) shear strength} \end{array} \right\} = \begin{pmatrix} f_{t,w,k} \\ f_{c,w,k} \\ f_{v,w,k} \\ f_{r,w,k} \end{pmatrix} = \begin{pmatrix} 9,4 \\ 15,4 \\ 6,8 \\ 1,0 \end{pmatrix} \text{ MPa} \quad \begin{array}{l} \text{Modulus of elasticity} \\ E_{w,\text{mean}} = 3\,800 \text{ MPa} \end{array}$$

$$\quad \begin{array}{l} \text{Shear modulus} \\ G_{w,\text{mean}} = 1\,080 \text{ MPa} \end{array}$$

Partial factor for OSB/3 $\gamma_{M,OSB} = 1,2$

Modification factor for OSB/3 $k_{w,\text{mod}} = 0,70$
Short term load duration and service class 2

Creep factor for OSB/3 $k_{w,\text{def}} = 2,25$

Design values:

$$\left\{ \begin{array}{l} \text{In plane tensile strength} \\ \text{In plane compressive strength} \\ \text{Panel shear strength} \\ \text{Planar (rolling) shear strength} \end{array} \right\} \begin{pmatrix} f_{t,w,d} \\ f_{c,w,d} \\ f_{v,w,d} \\ f_{r,w,d} \end{pmatrix} = \frac{k_{w,\text{mod}}}{\gamma_{M,\text{OSB}}} \begin{pmatrix} f_{t,w,k} \\ f_{c,w,k} \\ f_{v,w,k} \\ f_{r,w,k} \end{pmatrix} = \begin{pmatrix} 5,48 \cdot 10^0 \\ 8,98 \cdot 10^0 \\ 3,97 \cdot 10^0 \\ 5,83 \cdot 10^{-1} \end{pmatrix} \text{ MPa}$$

Design loads in the ULS, according to Eurocode 0

Partial factors

$$\gamma_G = 1,35 \quad \gamma_Q = 1,5$$

ζ -coefficient for permanent loads

$$\zeta = 0,85$$

Total design load from combination, see Eurocode 0: equation 6.10b, which is the only relevant combination due to a fairly large difference between s_k and g_k .

Design load

$$q_{Ed} = \zeta \gamma_G g_k + \gamma_Q s_k = 6,39 \text{ kN/m}$$

Cross sectional properties

As we have a cross-section of two different materials it is convenient to go for a transformed section or fictitious section as explained in Volume 1: section 5.1.1.1. This cross-section will be different for the instantaneous, final SLS and final ULS conditions. Let the elastic modulus of the C24-flange be the reference material. The web thickness (parallel to the axis of bending) is then changed by the following factors:

$$\left\{ \begin{array}{l} \mu_{w,SLS,\text{inst}} = \frac{E_{w,\text{mean}}}{E_{0,\text{mean}}} = 0,345 \\ \mu_{w,ULS,\text{inst}} = \mu_{w,SLS,\text{inst}} = 0,345 \\ \mu_{w,SLS,\text{fin}} = \frac{E_{w,\text{mean}}}{1 + k_{w,\text{def}}} \frac{(1 + k_{\text{def}})}{E_{0,\text{mean}}} = 0,191 \\ \mu_{w,ULS,\text{fin}} = \frac{E_{w,\text{mean}}}{1 + \psi_2 k_{w,\text{def}}} \frac{(1 + \psi_2 k_{\text{def}})}{E_{0,\text{mean}}} = 0,276 \end{array} \right.$$

Effective width factors

Web thicknesses

$$\left\{ \begin{array}{l} b_{w,SLS,\text{inst}} = \mu_{w,SLS,\text{inst}} b_w = 5,18 \text{ mm} \\ b_{w,ULS,\text{inst}} = b_{w,SLS,\text{inst}} = 5,18 \text{ mm} \\ b_{w,SLS,\text{fin}} = \mu_{w,SLS,\text{fin}} b_w = 2,87 \text{ mm} \\ b_{w,ULS,\text{fin}} = \mu_{w,ULS,\text{fin}} b_w = 4,15 \text{ mm} \end{array} \right.$$

Total width of the timber flange

$$b = 2b_1 = 90,0 \text{ mm}$$

Total beam depth

$$h = 2h_f + h_w = 640 \text{ mm}$$

Function for the second area moment of inertia, which may be called for different web thicknesses

$$I_y(t_w) = \frac{(b + t_w)h^3 - b h_w^3}{12}$$

Second area moment of inertia of the transformed cross-sections

$$\begin{cases} I_{y,SLS,inst} = I_y(b_{w,SLS,inst}) = 1,14 \cdot 10^9 \text{ mm}^4 \\ I_{y,ULS,inst} = I_y(b_{w,ULS,inst}) = 1,14 \cdot 10^9 \text{ mm}^4 \\ I_{y,SLS,fin} = I_y(b_{w,SLS,fin}) = 1,09 \cdot 10^9 \text{ mm}^4 \\ I_{y,ULS,fin} = I_y(b_{w,ULS,fin}) = 1,12 \cdot 10^9 \text{ mm}^4 \end{cases}$$

Check the bending moment resistance, see Volume 1: section 5.1.1.2.

No system effect can be taken into account, so k_{sys} is not used. Also, no lateral torsional buckling can occur because we have sufficient bracing, that is $k_{crit} = 1,0$.

In this case the 3 most probable failure conditions are given to the right, which is realized by looking at the strength properties (Volume 1: equation 5.6)

$$\left. \begin{cases} \sigma_{f,t,d} = \frac{M_{y,Ed}}{I_{y,ULS,fin}} z_{flange,centre} \leq f_{t,0,d} \\ \sigma_{f,t,max,d} = \frac{M_{y,Ed}}{I_{y,ULS,fin}} z_{flange,edge} \leq f_{m,d} \\ \sigma_{w,t,max,d} = \mu_{w,ULS,inst} \frac{M_{y,Ed}}{I_{y,ULS,inst}} z_{flange,edge} \leq f_{t,w,d} \end{cases} \right\}$$

Just by looking at the numerical values of the parameters we understand that tension failure at the centre of the bottom flange will be the likely cause of failure. Note also that the flange is checked at final conditions, while the web edge is checked at instantaneous conditions. This is because the flanges have better creep properties than the web panel.

Maximum bending moment

$$M_{y,Ed} = \frac{q_{Ed} \ell^2}{8} = 39,1 \text{ kNm}$$

Stress in the C24-timber at the centre of the tension flange

$$\sigma_{f,t,d} = \frac{M_{y,Ed}}{I_{y,ULS,fin}} \frac{h - h_f}{2} = 9,97 \text{ MPa}$$

Compare!

$$f_{t,0,d} = 11,20 \text{ MPa}$$

Stress in the C24-timber at the flange edge

$$\sigma_{f,t,max,d} = \frac{M_{y,Ed}}{I_{y,ULS,fin}} \frac{h}{2} = 11,2 \text{ MPa}$$

$$f_{m,d} = 19,3 \text{ MPa}$$

Stress in the OSB web panel at the edge of the tension flange

$$\sigma_{w,t,max,d} = \mu_{w,ULS,inst} \frac{M_{y,Ed}}{I_{y,ULS,inst}} \frac{h}{2} = 3,79 \text{ MPa}$$

$$f_{t,w,d} = 5,48 \text{ MPa}$$

- We conclude that the tension flange is perfectly optimized with regard to bending failure of the timber flange as well as the web panel.

Check the shear force resistance at supports, see Volume 1: section 5.1.1.3.

We have a pure shear failure of the web panel if the free web depth $h_w = 500$ mm is smaller than $35 b_w = 525$ mm. The smallest possible value of this limit is $32 b_w = 480$ mm according to *Volume 1: table 5.2*. Shear buckling will have a negligible effect on the shear resistance, which can be determined by *Volume 1: equation 5.11* with $\chi_v = 1,0$.

Maximum design shear force

$$V_{z,Ed} = \frac{q_{Ed} \ell}{2} = 22,4 \text{ kN}$$

Shear force resistance without regarding shear buckling, noting that we have one web panel

$$V_{z,Rd} = b_w (h_w + h_f) f_{v,w,d} = 24,6 \text{ kN}$$

No fiddling around with fictitious cross sections is need, as the flanges are not involved in this failure. We conclude that also the web panel is well designed with regard to its shear resistance.

Check for failure in the glue line between the web panel and the flange timber, see Volume 1: section 5.1.1.4.

The failure criterion (*Volume 1: equation 5.20 or 5.21*) is:

$$\tau_{v,Ed} = \frac{V_{z,Ed} \Delta S}{I_y h_{gl}} \leq \begin{cases} f_{r,d} & \text{if } h_{gl} \leq \frac{4b_w}{n_{gl}} \\ f_{r,d} \left(\frac{4b_w}{n_{gl} h_{gl}} \right)^{0.8} & \text{if } h_{gl} > \frac{4b_w}{n_{gl}} \end{cases}$$

Here we have an I-beam for which $n_{gl} = 2$, that is 2 equal flange pieces are glued to both sides of a web panel. Note that b_w is the physical thickness and not a fictitious thickness. It is just that the shear stresses are more concentrated towards the inner corner in an I-beam than in a boxed beam that is taken into account through n_{gl} .

The depth of the glue line is:

$$h_{gl} = h_f = 70 \text{ mm, compare to } \frac{4b_w}{n_{gl}} = 30 \text{ mm}$$

We must, therefore, use a reduced planar (rolling) shear strength of the board material, which is lower than the shear strength of the timber.

ΔS is the first moment of area of the flange part belonging to each glue line, that is taken for the area that builds up the shear stress in the glue line, noting that the shear stress is zero at the free edge.

First area moment:

$$\Delta S_y = \frac{b_1}{2} h_f \frac{h - h_f}{2} = 4,49 \cdot 10^5 \text{ mm}^3$$

In this case the $I_{y,ULS,fin}$ will produce the largest shear stress. Note also that no $\mu_{w,ULS,fin}$ is needed because the stress is already smeared out along the real physical depth of the glue line.

The shear stress is:

$$\tau_{\text{mean,d}} = \frac{V_{z,Ed} \Delta S_y}{I_{y,ULS,fin} h_{gl}} = 0,128 \text{ MPa}$$

Compare to the reduced planar shear strength:

$$f_{r,w,d} \left(\frac{4b_w}{n_{gl} h_{gl}} \right)^{0,8} = 0,296 \text{ MPa}$$

- We conclude that the glue line has more than sufficient strength.

Check the maximum beam deflection in the SLS

See Volume 1: section 5.1.1.5.

The deflection is to be calculated for the characteristic combination, that is as in Eurocode 0, Equation 6.14b.

$$\sum_{j \geq 1} G_{k,j} + Q_{k,l} + \sum_{i \geq 2} (\psi_{0,i} Q_{k,i})$$

Self weight $g_{ser} = g_k = 1,40 \text{ kN/m}$

Snow load $s_{ser} = s_k = 3,00 \text{ kN/m}$

Quasi permanent reduction factor for snow load $\psi_2 = 0,2$

Creep factors for service class 2 **Timber** **OSB**
 $k_{def} = 0,8$ $k_{w,def} = 2,25$

Second area moments in the SLS $I_{y,SLS,inst} = 1,14 \cdot 109 \text{ mm}^4$
 $I_{y,SLS,fin} = 1,09 \cdot 109 \text{ mm}^4$

Area of the web panel, its real physical area $A_w = b_w h = 9,60 \cdot 103 \text{ mm}^2$
 if we are a bit brave, otherwise use the distance between flange centres

Keep bending and shear deflections apart, because the web panels are much more prone to creep than the flanges acting in bending.
 The deflections calculated are based on Volume 1: Equations 5.85 and 5.86, but the “fca” index is replaced by “bend” for bending and “slip” is replaced by “shear”. The factor “2” is also removed.

Example 5.1: Design of an OSB-webbed I-girder (ULS and SLS)

Instantaneous bending deflection caused by the permanent load

* Note that this is the instantaneous value to be used for the final bending deflection.

Instantaneous shear deflection caused by the permanent load

Note that there is no difference between the two values as they are not based on a fictitious cross section.

* Note that this is the instantaneous value to be used for the final shear deflection.

Instantaneous bending deflection caused by the snow load

* Note that this is the instantaneous value to be used for the final bending deflection.

Instantaneous shear deflection caused by the snow load

Note that there is no difference between the two values as they are not based on a fictitious cross section.

* Note that this is the instantaneous value to be used for the final shear deflection.

Total instantaneous bending deflection

$$\begin{cases} w_{g,bend,SLS,inst} = \frac{5g_{ser} \ell^4}{384 E_{0,mean} I_{y,SLS,inst}} = 3,48 \text{ mm} \\ w_{g,bend,SLS,infin} = \frac{5g_{ser} \ell^4}{384 E_{0,mean} I_{y,SLS,fin}} = 3,65 \text{ mm}^* \end{cases}$$

$$\begin{cases} w_{g,shear,SLS,inst} = \frac{g_{ser} \ell^2}{8 G_{w,mean} A_w} = 0,828 \text{ mm} \\ w_{g,shear,SLS,infin} = w_{g,shear,SLS,inst} = 0,828 \text{ mm}^* \end{cases}$$

$$\begin{cases} w_{s,bend,SLS,inst} = \frac{5s_{ser} \ell^4}{384 E_{0,mean} I_{y,SLS,inst}} = 7,47 \text{ mm} \\ w_{s,bend,SLS,infin} = \frac{5s_{ser} \ell^4}{384 E_{0,mean} I_{y,SLS,fin}} = 7,82 \text{ mm}^* \end{cases}$$

$$\begin{cases} w_{s,shear,SLS,inst} = \frac{s_{ser} \ell^2}{8 G_{w,mean} A_w} = 1,78 \text{ mm} \\ w_{s,shear,SLS,infin} = w_{s,shear,SLS,inst} = 1,78 \text{ mm}^* \end{cases}$$

$$w_{bend,inst} = w_{g,bend,SLS,inst} + w_{s,bend,SLS,inst} = 11,0 \text{ mm}$$

$$w_{shear,inst} = w_{g,shear,SLS,inst} + w_{s,shear,SLS,inst} = 2,61 \text{ mm}$$

$$w_{inst} = w_{bend,inst} + w_{shear,inst} = 13,6 \text{ mm} \quad \text{compare to } \frac{\ell}{300} = 23,3 \text{ mm}$$

Final bending deflection from permanent loading

$$w_{g,bend,fin} = w_{g,bend,SLS,infin} (1 + k_{def}) = 6,57 \text{ mm}$$

Note the more severe creep factor

Final shear deflection from permanent loading

$$w_{g,shear,fin} = w_{g,shear,SLS,infin} (1 + k_{w,def}) = 2,69 \text{ mm}$$

Final bending deflection from snow loading

$$w_{s,bend,fin} = w_{s,bend,SLS,infin} (1 + \psi_2 k_{def}) = 9,9 \text{ mm}$$

Note the more severe creep factor

Final shear deflection from snow loading

$$w_{s,shear,fin} = w_{s,shear,SLS,infin} (1 + \psi_2 k_{w,def}) = 2,57 \text{ mm}$$

Total final bending deflection

$$w_{bend,fin} = w_{g,bend,fin} + w_{s,bend,fin} = 16,5 \text{ mm}$$

Total final shear deflection

$$w_{shear,fin} = w_{g,shear,fin} + w_{s,shear,fin} = 5,26 \text{ mm}$$

Total final deflection

$$w_{fin} = w_{bend,fin} + w_{shear,fin} = 21,8 \text{ mm} \quad \text{compare to } \frac{\ell}{200} = 35,0 \text{ mm}$$

Horizontal stabilization

Example 6.1: Design of bracing system for wind load on the gable of an industrial hall

For missing data see Volume 3: example 3.5 (this volume), page 14.

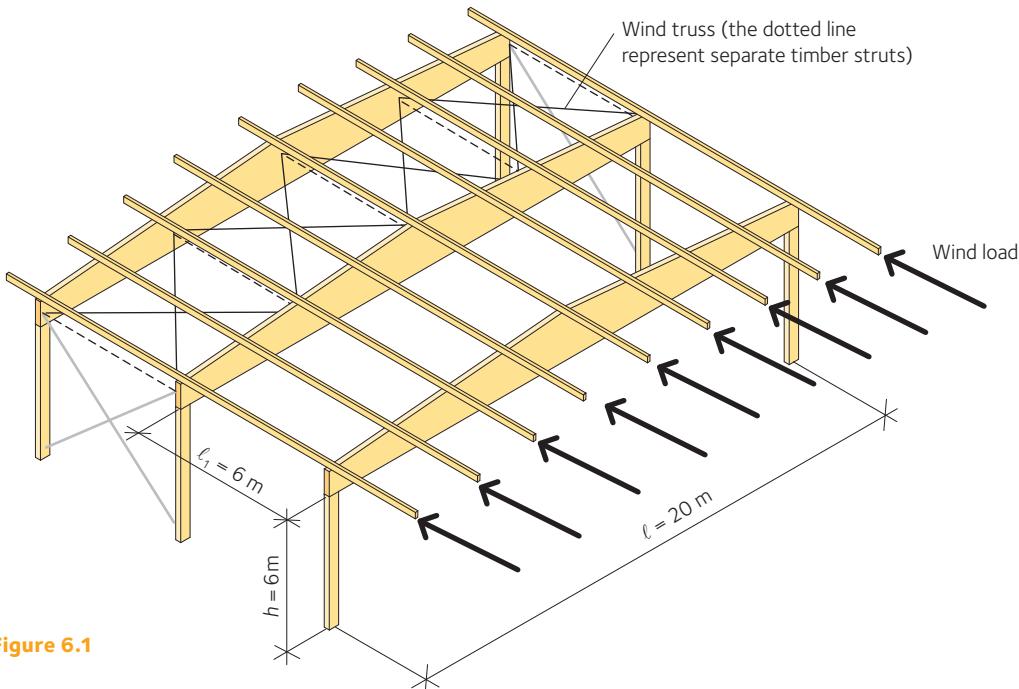


Figure 6.1

Geometry of the structure

The building with the measures $\ell \times \ell_{\text{tot}} 20 \times 36 \text{ m}$ is braced by means of a wind truss in the roof, consisting of diagonal steel bars and orthogonal compression struts of glulam. The wind truss is placed right below the roof plane. The horizontal loads are led by means of purlins to separate compression struts, which lay in the same plane as the diagonal steel bars. This to avoid eccentricities in the truss joints. The support reactions from the wind truss are led by means of a pair of diagonal steel bars in each longside wall to the foundation, but they are not designed in this example. The geometry of the structure is shown in figure 6.1.

Loads acting on the wind truss

The loads considered for the design of the roof wind truss are: wind as primary action and second order effects generated by beam self weight, roof dead load and snow. The characteristic value for wind

Example 6.1: Design of bracing system for wind load on the gable of an industrial hall

Table 6.1

Load type	Uniformly distributed load [kN/m ²]	Uniformly distributed load [kN/m]	Load combination factor ψ_0
Wind	$0,45 + 0,16 = 0,61$	$w_k = 0,61 \cdot 7,5 / 2 = 2,3$	0,6
Beam self weight	–	$g_{1k} = 1,10$	–
Roof dead load	0,6	$g_{2k} = 3,96$	–
Snow	1,5	$s = 8,77$	0,5

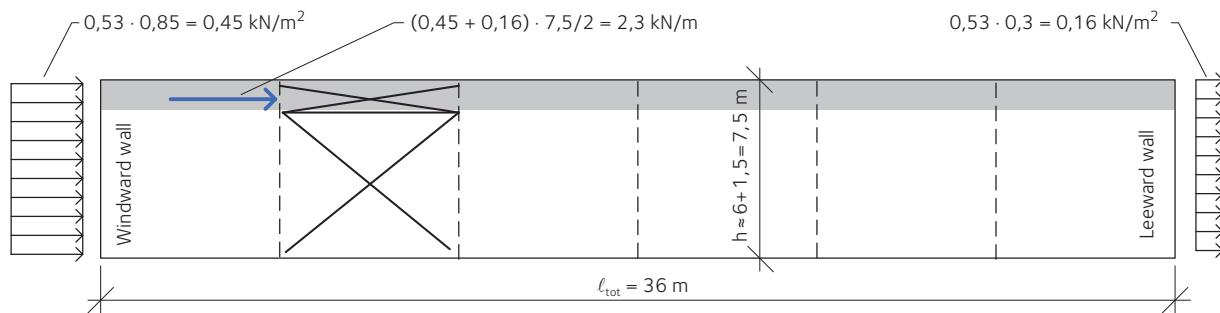


Figure 6.2

load is assumed 0,53 kN/m², which generates a pressure at the windward wall and suction at the leeward wall. The pressure coefficients are assumed 0,85 and 0,3 for the windward wall and the leeward wall respectively. The wind loads considered for the design of the roof wind truss are shown in figure 6.2.

In addition to wind, the load generated by the out-of-straightness of the main beams (second order effects) shall be taken into account. This load can be considered as uniformly distributed and its characteristic value can be calculated according to equation 6.28:

$$q_h = n \cdot \frac{M}{30h \cdot \ell} \cdot (1 - k_{\text{crit}})$$

where:

- $n = 6$ is the number of braced beams (note that the total number of beams is 7, but the beams at the two gables carry only one half of the vertical load).
- M is the bending moment in the beam due to vertical loads, considering the load combination "STR B", according to Eurocode 0. The load combination factor should be taken $\psi_0 = 0,6$ when wind is considered as principal load and $\psi_0 = 1,0$ when snow is considered as principal load:

$$\bullet \quad \ell = 20 \text{ m} \quad M = \frac{q\ell^2}{8} = \frac{[1,35 \cdot (g_{1,k} + g_{2,k}) + 1,5 \cdot \psi_0 \cdot s] \cdot \ell^2}{8}$$

- k_{crit} is the factor that takes into account the risk for lateral torsional buckling in an unbraced beam. The depth of the beam can be assumed $h_m = (h + h_{ap})/2 \approx 1200 \text{ mm}$ and $\ell_{\text{ef}} = 0,9 \cdot \ell + 2h$:

$$\lambda_{\text{rel},m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,\text{crit}}}} = \sqrt{f_{m,k} \cdot \frac{h \cdot \ell_{\text{ef}}}{0,78 \cdot b^2 \cdot E_{0,05}}}$$

- k_{crit} = see Volume 1: table 3.3

In this example the tilting load from the columns is not considered.

Table 6.2

Load combination	Bending moment in the beam M [kNm]	$\lambda_{\text{rel,m}}$	k_{crit}	q_h [kN/m]
a) Wind is the principal load	$\frac{[1,35 \cdot (1,1 + 3,96) + 0,5 \cdot 1,5 \cdot 8,77] \cdot 20^2}{8} = 670,6$	1,55	0,41	3,4
b) Snow is the principal load	$\frac{[1,35 \cdot (1,1 + 3,96) + 1,5 \cdot 8,77] \cdot 20^2}{8} = 1081$	1,55	0,41	4,7

Table 6.3

Service class	Load combinations [kN/m]	Load duration	k_{mod}
1	a) $q_{d,a} = 1 \cdot [1,5 \cdot w_k + 1,0 \cdot q_h] = 1,5 \cdot 2,3 + 3,4 = 6,9$	Short	0,9
1	b) $q_{d,b} = 1 \cdot [1,0 \cdot q_h + 1,5 \cdot \psi_{0,w} w_k] = 1,0 \cdot 3,4 + 1,5 \cdot 0,6 \cdot 2,3 = 5,5$	Medium	0,8

Load combinations

The beams are assumed to be indoors, in a heated environment. Therefore, they are characterized by an environment in which relative humidity very seldom, if ever, exceeds 65 %. Thus, the service class can be assumed to be 1. In this example only load combination in the ULS is considered. Only the load combination with wind as principal load is shown in this example. The governing load combination is thus a), that is wind as principal load.

Material

1. Timber elements

The material used for the struts is glulam GL30c ($\gamma_M = 1,25$, $k_{\text{mod}} = 0,9$). The adopted cross section for the strut is 190 × 180 mm. Upper chord and lower chord of the wind truss are two adjacent double tapered beams. The depth of these beams is varying along the span from 700 mm at support to 1 698 mm at mid-span. For the sake of the simplicity, however, it can be assumed that these beams have constant cross section 190 × 1 200 mm². The beams are also made of glulam GL30c. The relevant design values are shown below.

2. Steel diagonals

The diagonals are made of steel bars quality 4.8. The bars are threaded at their ends.

Table 6.4

Design values	
Compression parallel to grain	$f_{c,0,d} = 17,6$ MPa
Tension parallel to grain	$f_{t,0,d} = 14,0$ MPa
Modulus of elasticity	$E_{0,\text{mean}} = 13\,000$ MPa $E_{0,05} = 10\,800$ MPa

Table 6.5

Type of bar	Design yield stress f_{yb} [MPa]	Design ultimate stress f_{ub} [MPa]
Steel 4.8	320	400

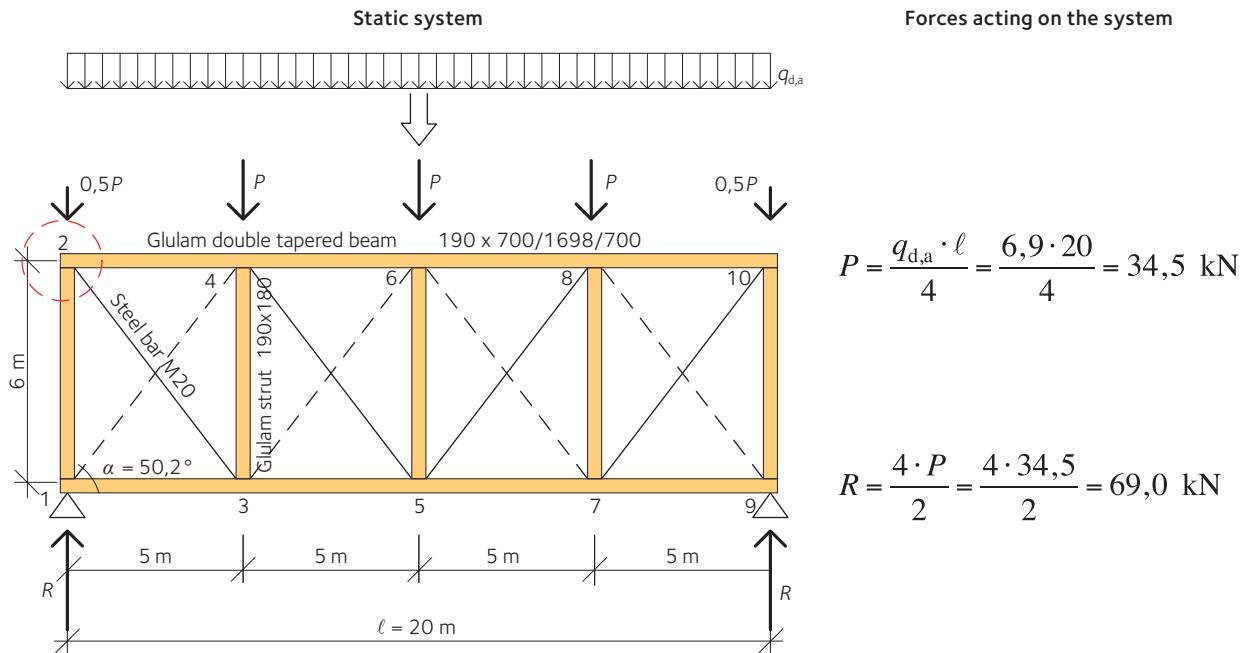


Figure 6.3

Design of the wind truss

The wind truss consists of a:

- lower chord and an upper chord of glulam
- orthogonal compression struts of glulam and
- diagonal bars of steel.

a. Compression struts

The most loaded struts are those at the supports (that is struts 1 – 2 and 9 – 10). The force acting in these struts is:

$$N_{sd} = N_{1-2} = N_{9-10} = R = 69,0 \text{ kN}$$

The same cross section, that is 190×180 mm is adopted for all the struts of the wind truss.

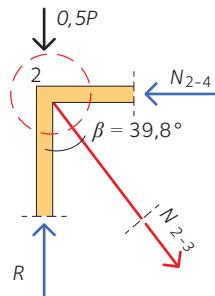
Table 6.6

Cross section	Relative slenderness $\lambda_{rel} = \frac{\lambda}{\pi} \sqrt{\frac{f_{ck}}{E_{0,05}}}$	Reduction factor k_c	Compression force in the strut N_{sd} [kN]	Compression strength of the strut N_{rd} [kN]	Utilization ratio
190×180	1,695	0,348	69,0	$k_c \cdot A \cdot f_{cd} = 0,348 \cdot 0,19 \cdot 0,18 \cdot 17,64 \cdot 10^3 = 210 \text{ kN}$	$69,0 / 210 = 0,33$

b. Diagonal steel bars

The most loaded steel diagonals are those closest to the supports (that is diagonals 2 – 3 and 7 – 10). The force acting in these diagonals is:

Graphical representation of the forces acting in node 2



Forces acting in the steel bar

$$N_{sd} = N_{2-3} = \left(R - \frac{P}{2} \right) \frac{1}{\cos \beta} = \\ = \left(69,0 - \frac{34,5}{2} \right) \cdot \frac{1}{\cos 39,8^\circ} = 67,4 \text{ kN}$$

Figure 6.4

The force acting in the diagonal steel bar is $N_{sd} = 67,4 \text{ kN}$. The design of the diagonal steel bars is performed as if they were bolts subjected to tensile force, since the end of the bars are threaded. The partial coefficient for bolts is $\gamma_{M2} = 1,2$.

Table 6.7

Steel quality	Size	Stress area $A_s [\text{mm}^2]$	Ultimate stress $f_{ub} [\text{MPa}]$	Tensile strength of the bar $F_{t,Rd} = 0,9 \cdot \frac{f_{ub} \cdot A_s}{\gamma_{M2}} [\text{kN}]$
4.8	M12	84	400	25,2
4.8	M16	157	400	47,1
4.8	M20	245	400	73,5
4.8	M24	353	400	105,9

Steel bar M20, 4.8 are chosen as diagonal members in the wind truss:

- $N_{sd} = 67,4 \text{ kN} < f_{t,Rd} = 73,5 \text{ kN} \rightarrow \text{OK}$

Design for serviceability

Example 7.1: Design of a straight timber joist with respect to deflection

A simply supported rectangular joist is subjected to characteristic permanent loading, $g_k = 0,5 \text{ kN/m}^2$ and characteristic medium duration variable load $q_k = 2,0 \text{ kN/m}^2$. The clear span ℓ is 4,5 m and the joists are spaced at 0,6 m centres. The cross section is 45 × 220 mm. Also see *example 3.1 in this Volume, page 5*.

The joist is part of an indoor floor in a residential building. Service class 1. Design the joist in structural timber of strength class C24.

Determine the deflection of the joist. Two different load cases will be discussed in the following to show different cases that might be of interest when checks of deflections are to be made.

Solution

First the deflection for characteristic load combination is determined, that is a deflection occurring quite seldom during the lifetime of the structure.

The moment of inertia

$$I = 39,93 \cdot 10^{-6} \text{ m}^4$$

Elastic modulus to be used in serviceability limit state

$$E_{0,\text{mean}} = 11\,000 \text{ MPa}$$

Characteristic load combination:

$$q_d = g_k + q_{k,l} + \sum \psi_{0,i} q_{k,i}$$

In this case $g_k = 0,5 \text{ kN/m}^2$, variable load is $q_{k,1} = 2,0 \text{ kN/m}^2$ and there is no other variable load, that is:

$$\sum \psi_{0,i} q_{k,i} = 0$$

The deflection for a simple supported joist is:

$$w = \frac{5q\ell^4}{384EI}$$

The instantaneous deflection for the self-weight is:

$$w_{\text{inst},G} = \frac{5g\ell^4}{384EI} = \frac{5 \cdot 0,5 \cdot 10^3 \cdot 0,6 \cdot 4,5^4}{384 \cdot 11000 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}} \cdot 10^3 = 3,6 \text{ mm}$$

The instantaneous deflection for the variable load is:

$$w_{\text{inst},Q} = \frac{5q\ell^4}{384EI} = \frac{5 \cdot 2,0 \cdot 10^3 \cdot 0,6 \cdot 4,5^4}{384 \cdot 11000 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}} \cdot 10^3 = 14,6 \text{ mm}$$

In total the instantaneous (short term) deflection corresponds to $\approx \ell/250$ ($4\ 500/18,2$).

When the long term deflections are to be estimated it is more relevant to refer to the quasi-permanent load case:

$$q_d = g_k + \sum \psi_{2,i} q_{k,i}$$

For residential building loads the value of ψ_2 is 0,3, see *Volume 2: table 2.2*.

The final deflection from the self-weight can either be determined by using the value:

$$E_{\text{mean,fin}} = \frac{E_{\text{mean}}}{1 + k_{\text{def}}}$$

in:

$$w = \frac{5q\ell^4}{384EI}$$

$k_{\text{def}} = 0,6$ is given by *Volume 2: table 9.1*.

This gives:

$$E_{\text{mean,fin}} = \frac{11000}{1 + 0,6} = 6875 \text{ MPa}$$

$$w_{\text{fin,G}} = \frac{5g\ell^4}{384E_{\text{mean,fin}}I} = \frac{5 \cdot 0,5 \cdot 10^3 \cdot 0,6 \cdot 4,5^4}{384 \cdot 6875 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}} \cdot 10^3 = 5,8 \text{ mm}$$

Alternatively this deflection can be given by:

$$w_{\text{fin,G}} = w_{\text{inst,G}} (1 + k_{\text{def}})$$

$$w_{\text{fin,G}} = 3,6 \cdot (1 + 0,6) = 5,8 \text{ mm}$$

The creep deflection for the variable load is determined from $\psi_2 q_k$. Corresponding to the self-weight the deflection for the variable load can be calculated as:

$$w_{\text{fin,Q}} = w_{\text{inst,Q}} (1 + \psi_2 k_{\text{def}})$$

$$w_{\text{fin,Q}} = 14,6 \cdot (1 + 0,3 \cdot 0,6) = 17 \text{ mm}$$

In total the final long term deflection (including the short term effects) corresponds to $\approx \ell/200$ ($4\ 500/22,8$).

The acceptance for a certain limit has to be determined based upon the use of the structural element. In some cases the value of $\ell/200$ can be a problem but in some cases it may be acceptable. It must be noticed that the short term effects of the characteristic load is included in $\ell/200$.

Example 7.2: Serviceability check of floor with timber joists – vibration

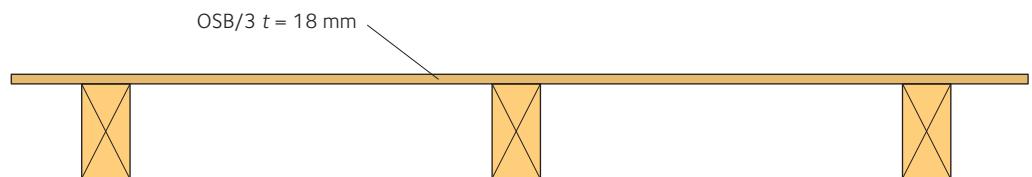


Figure 7.1

A timber floor in a residential building has a clear span of 3,7 m between supports. The structure comprises 45 × 220 mm timber joists spaced at 0,4 m centres, strength class C18 in service class 1.

The flooring consists of 18 mm thick OSB/3 panel and it is nailed to the joists. In this example the OSB panel is oriented with the main direction of the strands parallel to the timber joists.

The floor width is 4,4 m and the floor mass, based on permanent loading only, is 35 kg/m².

Carry out a design check to confirm that the vibration behaviour of the floor will be acceptable, see *Volume 2: section 9.4 or Eurocode 5: section 7.3*.

Moment of inertia I:

$$I_{\text{joist}} = \frac{bh^3}{12} = \frac{0,045 \cdot 0,22^3}{12} = 39,93 \cdot 10^{-6} \text{ m}^4$$

Second moment of area of flooring/meter width I_f :

$$I_f = \frac{b t^3}{12} = \frac{1 \cdot 0,018^3}{12} = 4,86 \cdot 10^{-7} \text{ m}^4$$

Width of floor structure, $B = 4,4 \text{ m}$

Strength class C18

Mean modulus of elasticity of the joists parallel to the grain,
 $E_{0,\text{mean}} = 9\,000 \text{ MPa}$

OSB/3

Mean modulus of elasticity of flooring parallel to the grain,
 $E_m = 4\,930 \text{ MPa}$

Mass of the floor (kg/m²), $m = 35 \text{ kg/m}^2$

a. Check the fundamental frequency of the floor (see Volume 1: equation 7.25):

$$f_1 = \frac{\pi}{2\ell^2} \sqrt{\frac{(EI)_\ell}{m}}$$

$$f_1 = \frac{\pi}{2 \cdot 3,7^2} \sqrt{\frac{\left(\frac{9000 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}}{0,4} \right)}{35}} = 18,4 \text{ Hz}$$

The fundamental frequency of the floor is acceptable as it is greater than 8 Hz and the equations given in Eurocode 5 can be used for a vibration check.

b. Check the static deflection of the floor according to the point load criterion in Volume 1: equation 7.19:

Maximum allowable deflection under a 1 kN point load is set to $a = 1,5 \text{ mm}$.

$$w = \frac{P\ell^3}{48EI}$$

Check one single joist:

$$P = 1 \text{ 000 N}$$

$$w = \frac{1000 \cdot 3,7^3}{48 \cdot 9000 \cdot 10^6 \cdot 39,93 \cdot 10^{-6}} \cdot 10^3 = 2,9 \text{ mm}$$

Since this is not good enough a check is made with regard to the effect of the rigidity in both floor directions (see Volume 1: equation 7.20–7.22):

$$w = \kappa \frac{P\ell^3}{48EI}$$

Where the load distribution factor κ is determined by:

$$\kappa = \begin{cases} -4,7\beta^2 + 2,9\beta + 0,4 & \text{when } 0 \leq \beta < 0,3 \\ 0,8 + 0,2\beta & \text{when } 0,3 \leq \beta \leq 1,0 \end{cases}$$

with:

$$\beta = \frac{(EI)_\ell}{(EI)_B} \left(\frac{s}{\ell} \right)^4$$

$$\beta = \frac{\left(\frac{9000 \cdot 39,93}{0,4} \right)}{\left(\frac{4930 \cdot 0,486}{3700} \right)} \left(\frac{400}{3700} \right)^4 = 0,051$$

$$\kappa = -4,7 \cdot 0,051^2 + 2,9 \cdot 0,051 + 0,4 = 0,53$$

We then get, after load distribution by means of the flooring, a deflection under the point load:

$$w = 0,53 \cdot 2,9 \text{ mm} = 1,6 \text{ mm}$$

This means that the static check not is totally fulfilled but all the same we go further with the control. We could also choose to change the timber quality.

The following should also be valid:

$$v \leq b^{(f_1 \zeta - 1)}$$

where v is the unit impulse velocity response ($\text{m}/(\text{Ns}^2)$). ζ is the modal damping ratio which normally can be set to 0,01. The constant b for the control of unit impulse response can be set to 100 (see *Volume 2: section 9.4*):

$$v \leq b^{(f_1 \zeta - 1)} = 100^{(18,4 \cdot 0,01 - 1)} = 0,023$$

The unit impulse velocity response is given by:

$$v = \frac{4(0,4 + 0,6n_{40})}{mB\ell + 200}$$

where n_{40} is the number of first order modes with natural frequencies up to 40 Hz and b is the floor width. The value of n_{40} may be calculated from:

$$n_{40} = \left\{ \left(\left(\frac{40}{f_1} \right)^2 - 1 \right) \left(\frac{B}{\ell} \right)^4 \frac{(EI)_\ell}{(EI)_B} \right\}^{0,25}$$

$$n_{40} = \left\{ \left(\left(\frac{40}{18,4} \right)^2 - 1 \right) \left(\frac{4400}{3700} \right)^4 \frac{\left(\frac{9900 \cdot 39,93}{0,4} \right)}{\left(\frac{4930 \cdot 0,486}{0,4} \right)} \right\}^{0,25} = 7,5$$

The unit impulse velocity response is then:

$$v = \frac{4(0,4 + 0,6 \cdot 7,5)}{35 \cdot 4,4 \cdot 3,7 + 200} = 0,025$$

- This value is higher than the limit value $0,023 \text{ m}/(\text{Ns}^2)$. The chosen floor design should therefore be adjusted in order to avoid problems with vibration.

Example 7.3: Serviceability check of floor with glulam elements – vibration

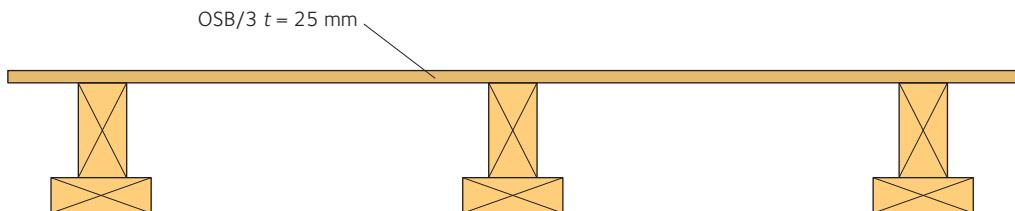


Figure 7.2

A timber floor in a residential multi-storey building is made up of a OSB/3 panel as upper flange and with glulam beams class GL28cs as webs and bottom flanges. The OSB panel has a thickness of 25 mm and is oriented with the main direction of the strands parallel to the glulam beams in this example. The web has a dimension of 42 × 315 mm and a lower flange with dimension 225 × 42 mm. The beams are spaced at 0,6 m centres.

The floor system can be assumed to have full interaction. The floor has a length of 5,5 m (in the direction of the webs) and a width of 8,0 m. The floor is simply supported.

The mass of the floor is 45 kg/m².

Carry out a design check to confirm that the vibration behaviour of the floor will be acceptable, see Volume 2: section 9.4 or Eurocode 5: section 7.3.

First transform the cross section to a fictive section consisting of glulam, GL28cs.

The effective width b_{eff} of the OSB can be determined to $0,15\ell$ according to *Volume 1: table 5.1* or *Eurocode 5: table 9.1*. Since $\ell = 5,5 \text{ m}$, $b_{\text{eff}} = 0,825 \text{ m}$ which is larger than 0,6 m. This means that the whole width of the OSB flange can be used in the calculation of the moment of inertia I .

Elastic modulus for GL28cs $E_{0,\text{mean}} = 12\,500 \text{ MPa}$

Elastic modulus for OSB $E_m = 4\,930 \text{ MPa}$

The fictitious width of the flange is then given by:

$$b_{f_u,\text{fic}} = \frac{E_m}{E_{0,\text{mean}}} b_u = \frac{4930}{12500} \cdot 600 = 237 \text{ mm}$$

Area of the cross-section:

$$\begin{aligned} A_c &= b_{f_i} \cdot h_\ell + b_w \cdot h_w + b_{f_u,\text{fic}} \cdot h_u = \\ &= 225 \cdot 42 + 42 \cdot 315 + 237 \cdot 25 = 28596 \text{ mm}^2 \end{aligned}$$

Example 7.3: Serviceability check of floor with glulam elements – vibration

Centre of gravity (from the lower edge):

$$x_{tp} = \frac{b_{f_l} \cdot h_{\ell} \cdot \frac{h_{\ell}}{2} + b_w \cdot h_w \cdot \left(h_{\ell} + \frac{h_w}{2} \right) + b_{fu,fic} \cdot h_u \cdot \left(h_{\ell} + h_w + \frac{h_u}{2} \right)}{A_c} =$$

$$\frac{225 \cdot 42 \cdot \frac{42}{2} + 42 \cdot 315 \cdot \left(42 + \frac{315}{2} \right) + 237 \cdot 25 \cdot \left(42 + 315 + \frac{25}{2} \right)}{28596} = 176 \text{ mm}$$

Second moment of area of the lower flange:

$$I_{f_l} = \frac{b_{f_l} h_{\ell}^3}{12} + b_{f_l} h_{\ell} \cdot \left[x_{tp} - \left(\frac{h_{f_l}}{2} \right) \right]^2 =$$

$$\frac{0,225 \cdot 0,042^3}{12} + 0,225 \cdot 0,042 \cdot \left[0,176 - \left(\frac{0,042}{2} \right) \right]^2 = 227 \cdot 10^{-6} \text{ m}^4$$

Second moment of area of the upper flange:

$$I_{f_u} = \frac{b_{f_u} h_u^3}{12} + b_{f_u} h_u \cdot \left[x_{tp} - \left(h_{f_l} + h_w + \frac{h_{f_u}}{2} \right) \right]^2 =$$

$$\frac{0,237 \cdot 0,025^3}{12} + 0,237 \cdot 0,025 \cdot \left[0,176 - \left(0,042 + 0,315 + \frac{0,025}{2} \right) \right]^2 = 222 \cdot 10^{-6} \text{ m}^4$$

Second moment of area of the web:

$$I_w = \frac{b_w h_w^3}{12} + b_w h_w \cdot \left[x_{tp} - \left(h_{f_l} + \frac{h_w}{2} \right) \right]^2 =$$

$$\frac{0,042 \cdot 0,315^3}{12} + 0,042 \cdot 0,315 \cdot \left[-0,176 + 0,042 + \frac{0,315}{2} \right]^2 = 117 \cdot 10^{-6} \text{ m}^4$$

Second moment of area:

$$I = I_{f_l} + I_{f_u} + I_w = 227 \cdot 10^{-6} + 222 \cdot 10^{-6} + 117 \cdot 10^{-6} = 566 \cdot 10^{-6} \text{ m}^4$$

Second moment of area of flooring/meter width:

$$I_f = \frac{b_t h_{f_u}^3}{12} = \frac{1 \cdot 0,025^3}{12} = 1,3 \cdot 10^{-6} \text{ m}^4$$

Width of floor structure, $B = 8 \text{ m}$

Mass of the floor (kg/m^2), $m = 45 \text{ kg}/\text{m}^2$

a. Check fundamental frequency of the floor:

$$f_1 = \frac{\pi}{2\ell^2} \sqrt{\frac{(EI)_{\ell}}{m}}$$

$$f_1 = \frac{\pi}{2 \cdot 5,5^2} \sqrt{\frac{\left(\frac{12500 \cdot 10^6 \cdot 566 \cdot 10^{-6}}{0,6} \right)}{45}} = 26,6$$

The fundamental frequency of the floor is acceptable as it is greater than 8 Hz and the equations given in *Volume 1: section 7.3.3.1* can be used for a vibration check.

b. Check the static deflection of the floor:

Maximum allowable deflection under a 1 kN point load is $a = 1,5$ mm:

$$w = \frac{P\ell^3}{48EI}$$

Check one single beam:

$$P = 1\,000 \text{ N}$$

$$w = \frac{1000 \cdot 5,5^3}{48 \cdot 12,5 \cdot 10^9 \cdot 566 \cdot 10^{-6}} \cdot 1000 = 0,5 \text{ mm}$$

The deflection of the floor is less than the allowable value.

The following should also be valid:

$$v \leq b^{(f_i \zeta - 1)}$$

where v is the unit impulse velocity response ($\text{m}/(\text{Ns}^2)$). ζ is the modal damping ratio which normally can be set to 0,01. The constant b for the control of unit impulse response can be set to 100 (see *Volume 2: section 9.4*):

$$v \leq b^{(f_i \zeta - 1)} = 100^{(26,6 \cdot 0,01 - 1)} = 0,034$$

The unit impulse velocity response is given by:

$$v = \frac{4(0,4 + 0,6n_{40})}{mB\ell + 200}$$

where n_{40} is the number of first order modes with natural frequencies up to 40 Hz and B is the floor width. The value of n_{40} may be calculated from:

$$n_{40} = \left\{ \left(\left(\frac{40}{f_1} \right)^2 - 1 \right) \left(\frac{B}{\ell} \right)^4 \frac{(EI)_\ell}{(EI)_B} \right\}^{0,25}$$

$$n_{40} = \left\{ \left(\left(\frac{40}{26,6} \right)^2 - 1 \right) \left(\frac{8000}{5500} \right)^4 \frac{\left(\frac{12500 \cdot 566}{0,6} \right)}{\left(\frac{4930 \cdot 1,3}{(4930 \cdot 1,3)} \right)} \right\}^{0,25} = 10,1$$

The unit impulse velocity response is then:

$$v = \frac{4(0,4 + 0,6 \cdot 10,1)}{45 \cdot 8 \cdot 5,5 + 200} = 0,012$$

- As the unit impulse velocity response of the floor is less than the allowable value it is acceptable. The floor has complied with the vibration requirements in Eurocode 5.

Symbols

Symbols used in EN 1995-1-1.

Symbol	Explanation
Latin upper case letters	
A	Cross-sectional area
A_{ef}	Effective area of the total contact surface between a punched metal plate fastener and the timber; effective total contact surface perpendicular to the grain
A_f	Cross-sectional area of flange
$A_{\text{net},t}$	Net cross-sectional area perpendicular to the grain
$A_{\text{net},v}$	Net shear area parallel to the grain
C	Spring stiffness
$E_{0,05}$	Fifth percentile value of modulus of elasticity
E_d	Design value of modulus of elasticity
E_{mean}	Mean value of modulus of elasticity
$E_{\text{mean,fin}}$	Final mean value of modulus of elasticity
F	Force
$F_{A,\text{Ed}}$	Design force acting on a punched metal plate fastener at the centroid of the effective area
$F_{A,\text{min,d}}$	Minimum design force acting on a punched metal plate fastener at the centroid of the effective area
$F_{ax,\text{Ed}}$	Design axial force on fastener
$F_{ax,\text{Rd}}$	Design value of axial withdrawal capacity of the fastener
$F_{ax,\text{Rk}}$	Characteristic axial withdrawal capacity of the fastener
F_c	Compressive force
F_d	Design force
$F_{d,\text{ser}}$	Design force at the serviceability limit state
$F_{f,\text{Rd}}$	Design load-carrying capacity per fastener in wall diaphragm
$F_{i,c,\text{Ed}}$	Design compressive reaction force at end of shear wall
$F_{i,t,\text{Ed}}$	Design tensile reaction force at end of shear wall
$F_{i,\text{vert},\text{Ed}}$	Vertical load on wall
$F_{i,v,\text{Rd}}$	Design racking resistance of panel i or wall i
F_{la}	Lateral load
$F_{M,\text{Ed}}$	Design force from a design moment
F_t	Tensile force
$F_{t,\text{Rk}}$	Characteristic tensile capacity of a connection
$F_{v,0,\text{Rk}}$	Characteristic load-carrying capacity of a connector along the grain
$F_{v,\text{Ed}}$	Design shear force per shear plane of fastener; Horizontal design effect on wall diaphragm
$F_{v,\text{Rd}}$	Design load-carrying capacity per shear plane per fastener; Design racking load capacity
$F_{v,\text{Rk}}$	Characteristic load-carrying capacity per shear plane per fastener
$F_{v,w,\text{Ed}}$	Design shear force acting on web
$F_{x,\text{Ed}}$	Design value of a force in x -direction
$F_{y,\text{Ed}}$	Design value of a force in y -direction
$F_{x,\text{Rd}}$	Design value of plate capacity in x -direction
$F_{y,\text{Rd}}$	Design value of plate capacity in y -direction
$F_{x,\text{Rk}}$	Characteristic plate capacity in x -direction
$F_{y,\text{Rk}}$	Characteristic plate capacity in y -direction
$G_{0,05}$	Fifth percentile value of shear modulus
G_d	Design value of shear modulus
G_{mean}	Mean value of shear modulus
H	Overall rise of a truss
I_f	Second moment of area of flange
I_{tor}	Torsional moment of inertia
I_z	Second moment of area about the weak axis
K_{ser}	Slip modulus
$K_{\text{ser,fin}}$	Final slip modulus
K_u	Instantaneous slip modulus for ultimate limit states
$L_{\text{net},t}$	Net width of the cross-section perpendicular to the grain
$L_{\text{net},v}$	Net length of the fracture area in shear
$M_{A,\text{Ed}}$	Design moment acting on a punched metal plate fastener
$M_{\text{ap},d}$	Design moment at apex zone
M_d	Design moment
$M_{y,\text{Rk}}$	Characteristic yield moment of fastener
N	Axial force
$R_{90,d}$	Design splitting capacity
$R_{90,k}$	Characteristic splitting capacity
$R_{ax,d}$	Design load-carrying capacity of an axially loaded connection
$R_{ax,k}$	Characteristic load-carrying capacity of an axially loaded connection
$R_{ax,\alpha,k}$	Characteristic load-carrying capacity at an angle α to grain
R_d	Design value of a load-carrying capacity
$R_{\text{ef},k}$	Effective characteristic load-carrying capacity of a connection
$R_{iv,d}$	Design racking capacity of a wall
R_k	Characteristic load-carrying capacity
$R_{sp,k}$	Characteristic splitting capacity

Source: EN 1995-1-1:2004, 1.6

$R_{\text{to},k}$	Characteristic load-carrying capacity of a toothed plate connector	$f_{c,90,k}$	Characteristic compressive strength perpendicular to grain
$R_{v,d}$	Design racking capacity of a wall diaphragm	$f_{f,t,d}$	Design tensile strength of flange
V	Shear force; volume	$f_{h,k}$	Characteristic embedment strength
V_u, V_ℓ	Shear forces in upper and lower part of beam with a hole	$f_{\text{head},k}$	Characteristic pull-through parameter for nails
W_y	Section modulus about axis y	f_1	Fundamental frequency
X_d	Design value of a strength property	$f_{m,k}$	Characteristic bending strength
X_k	Characteristic value of a strength property	$f_{m,y,d}$	Design bending strength about the principal y -axis
Latin lower case letters		$f_{m,z,d}$	Design bending strength about the principal z -axis
a	Distance	$f_{m,\alpha,d}$	Design bending strength at an angle α to the grain
a_1	Spacing, parallel to grain, of fasteners within one row	$f_{t,0,d}$	Design tensile strength along the grain
$a_{1,CG}$	Minimum end distance to the centre of gravity of the screw in each timber member	$f_{t,0,k}$	Characteristic tensile strength along the grain
a_2	Spacing, perpendicular to grain, between rows of fasteners	$f_{t,90,d}$	Design tensile strength perpendicular to the grain
$a_{2,CG}$	Minimum edge distance to the centre of gravity of the screw in each timber member	$f_{t,w,d}$	Design tensile strength of the web
$a_{3,c}$	Distance between fastener and unloaded end	$f_{u,k}$	Characteristic tensile strength of bolts
$a_{3,t}$	Distance between fastener and loaded end	$f_{v,0,d}$	Design panel shear strength
$a_{4,c}$	Distance between fastener and unloaded edge	$f_{v,ax,\alpha,k}$	Characteristic withdrawal strength at an angle α to grain
$a_{4,t}$	Distance between fastener and loaded edge	$f_{v,ax,90,k}$	Characteristic withdrawal strength perpendicular to grain
a_{bow}	Maximum bow of truss member	$f_{v,d}$	Design shear strength
$a_{\text{bow,perm}}$	Maximum permitted bow of truss member	h	Depth; height of wall
a_{dev}	Maximum deviation of truss	h_{ap}	Depth of the apex zone
$a_{\text{dev,perm}}$	Maximum permitted deviation of truss	h_d	Hole depth
b	Width	h_e	Embedment depth; loaded edge distance
b_i	Width of panel i or wall i	h_{ef}	Effective depth
b_{net}	Clear distance between studs	$h_{f,c}$	Depth of compression flange
b_w	Web width	$h_{f,t}$	Depth of tension flange
d	Diameter; outer thread diameter	h_{rl}	Distance from lower edge of hole to bottom of member
d_1	Diameter of centre hole of connector; inner thread diameter	h_{ru}	Distance from upper edge of hole to top of member
d_c	Connector diameter	h_w	Web depth
d_{ef}	Effective diameter	i	Notch inclination
d_h	Head diameter of connector	$k_{c,y}, k_{c,z}$	Instability factor
$f_{h,i,k}$	Characteristic embedment strength of timber member i	k_{cr}	Crack factor for shear resistance
$f_{a,0,0}$	Characteristic anchorage capacity per unit area for $\alpha = 0^\circ$ and $\beta = 0^\circ$	k_{crit}	Factor used for lateral buckling
$f_{a,90,90}$	Characteristic anchorage capacity per unit area for $\alpha = 90^\circ$ and $\beta = 90^\circ$	k_d	Dimension factor for panel
$f_{a,u,\beta,k}$	Characteristic anchorage strength	k_{def}	Deformation factor
$f_{ax,k}$	Characteristic withdrawal parameter for nails	k_{dis}	Factor taking into account the distribution of stresses in an apex zone
$f_{c,0,d}$	Design compressive strength along the grain	$k_{f,1}, k_{f,2}, k_{f,3}$	Modification factors for bracing resistance
$f_{c,w,d}$	Design compressive strength of web	k_h	Depth factor
$f_{f,c,d}$	Design compressive strength of flange	$k_{i,q}$	Uniformly distributed load factor

Source: EN 1995-1-1:2004, 1.6

Symbols

k_m	Factor considering re-distribution of bending stresses in a cross-section	$u_{inst,Q,i}$	Instantaneous deformation for accompanying variable actions Q_i
k_{mod}	Modification factor for duration of load and moisture content	w_c	Precamber
k_n	Sheathing material factor	w_{creep}	Creep deflection
k_r	Reduction factor	w_{fin}	Final deflection
$k_{R,red}$	Reduction factor for load-carrying capacity	w_{inst}	Instantaneous deflection
k_s	Fastener spacing factor; modification factor for spring stiffness	$w_{net,fin}$	Net final deflection
$k_{s,red}$	Reduction factor for spacing	v	Unit impulse velocity response
k_{shape}	Factor depending on the shape of the cross-section	Greek lower case letters	
k_{sys}	System strength factor	α	Angle between the x -direction and the force for a punched metal plate; Angle between the direction of the load and the loaded edge (or end)
k_v	Reduction factor for notched beams	β	Angle between the grain direction and the force for a punched metal plate
k_{vol}	Volume factor	β_c	Straightness factor
k_y eller k_z	Instability factor	γ	Angle between the x -direction and the timber connection line for a punched metal plate
$l_{a,min}$	Minimum anchorage length for a glued-in rod	γ_M	Partial factor for material properties, also accounting for model uncertainties and dimensional variations
l	Span; contact length	λ_y	Slenderness ratio corresponding to bending about the y -axis
l_A	Distance from a hole to the centre of the member support	λ_z	Slenderness ratio corresponding to bending about the z -axis
l_{ef}	Effective length; Effective length of distribution	$\lambda_{rel,y}$	Relative slenderness ratio corresponding to bending about the y -axis
l_v	Distance from a hole to the end of the member	$\lambda_{rel,z}$	Relative slenderness ratio corresponding to bending about the z -axis
l_z	Spacing between holes	ρ_k	Characteristic density
m	Mass per unit area	ρ_m	Mean density
n_{40}	Number of frequencies below 40 Hz	$\sigma_{c,0,d}$	Design compressive stress along the grain
n_{ef}	Effective number of fasteners	$\sigma_{c,a,d}$	Design compressive stress at an angle α to the grain
p_d	Distributed load	$\sigma_{f,c,d}$	Mean design compressive stress of flange
q_i	Equivalent uniformly distributed load	$\sigma_{f,c,max,d}$	Design compressive stress of extreme fibres of flange
r	Radius of curvature	$\sigma_{f,t,d}$	Mean design tensile stress of flange
s	Spacing	$\sigma_{f,t,max,d}$	Design tensile stress of extreme fibres of flange
s_0	Basic fastener spacing	$\sigma_{m,crit}$	Critical bending stress
r_{in}	Inner radius	$\sigma_{m,y,d}$	Design bending stress about the principal y -axis
t	Thickness	$\sigma_{m,z,d}$	Design bending stress about the principal z -axis
t_{pen}	Penetration depth	$\sigma_{m,a,d}$	Design bending stress at an angle α to the grain
u_{creep}	Creep deformation	σ_N	Axial stress
u_{fin}	Final deformation	$\sigma_{t,0,d}$	Design tensile stress along the grain
$u_{fin,G}$	Final deformation for a permanent action G	$\sigma_{t,90,d}$	Design tensile stress perpendicular to the grain
$u_{fin,Q,1}$	Final deformation for the leading variable action Q_1		
$u_{fin,Q,i}$	Final deformation for accompanying variable actions Q_i		
u_{inst}	Instantaneous deformation		
$u_{inst,G}$	Instantaneous deformation for a permanent action G		
$u_{inst,Q,1}$	Instantaneous deformation for the leading variable action Q_1		

Source: EN 1995-1-1:2004, 1.6

$\sigma_{w,c,d}$	Design compressive stress of web
$\sigma_{w,t,d}$	Design tensile stress of web
τ_d	Design shear stress
$\tau_{f,d}$	Design anchorage stress from axial force
$\tau_{M,d}$	Design anchorage stress from moment
$\tau_{tor,d}$	Design shear stress from torsion
ψ_0	Factor for combination value of a variable action
ψ_1	Factor for frequent value of a variable action
ψ_2	Factor for quasi-permanent value of a variable action
ζ	Modal damping ratio

Source: EN 1995-1-1:2004, 1.6

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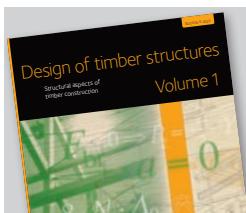
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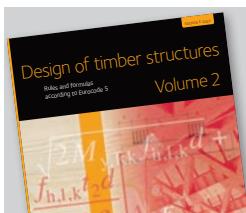
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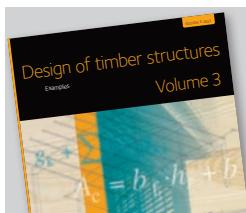
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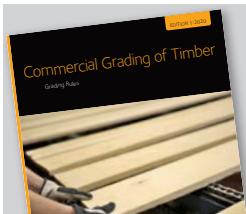
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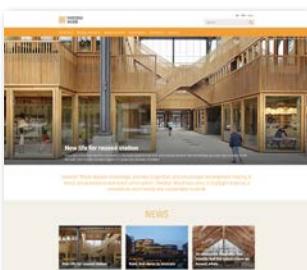
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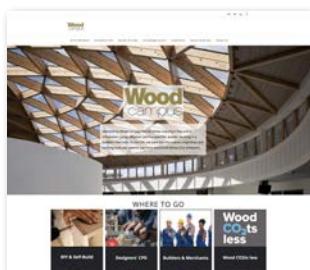
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