

# DIMENSIONS OF TIME

Dimitry A. Demin  
*Program in MAS*

*Certified by*  
Prof. Dr. Ludger Hovestadt

*Advised by*  
Klaus Wassermann

December 6, 2012

# Dimensions of Time

Dimitry A. Demin  
MAS CAAD  
ETH Zurich

Submitted to the Program in Computer Aided Architectural Design, in partial fulfillment of the requirements for the degree of Master of Advanced Studies at the  
Swiss Federal Institute of Technology Zurich, 2011  
Eidgenössische Technische Hochschule Zürich, 2011

Copyright ETH Zurich  
All rights reserved

## Abstract

This thesis proposes a completely new understanding of time, emancipated from the myth of single dimensionality, by viewing time as a dimensional object. An extension mechanism of equilibrium and its relationship to entropy, based on the principle of time-based organization, is expanded upon in the body of the text. The notion of time as an independent mathematical object emerges as having a significant importance in the aesthetics of architectural design.

# Contents

<b>1</b>	<b>Of Time's Inception</b>	<b>9</b>
1.1	Physical Flow of Time . . . . .	16
1.2	Mathematical Flow of Time . . . . .	23
<b>2</b>	<b>Dimensional Time</b>	<b>31</b>
2.1	Parallelarity . . . . .	35
2.2	Dimensional Time Moment . . . . .	39
<b>3</b>	<b>Consequences</b>	<b>44</b>
<b>4</b>	<b>Appendix</b>	<b>46</b>

# Foreword

*“I am lying”* [Russell, 1910]

Sometimes one may have a strange, ambiguous feeling about time flow, that time elapses without any rules governing structure and organization. This may be said to have its origins in the XX century battlefield around the problem of the calculation of time, where the atomic clock(AC) and computers(C) become the prime heart of the *time revolution*. But subconsciously we do understand - the truth is far more complex and that crossed gap is only a small branch of the visible tree, the rest is the dark matter implicit in this hidden information.

The invention of the computer and atomic clock defined the time revolution, and the result of this becomes the dependence of almost every human activity on these devices: whether we move, stay still or even sleep, thousands of parallel processes are registered as having occurred. The structural heart of both these devices is a very similar mechanism that beats with a specific frequency - a second becomes a race for infinity. Understanding a continuous indirect dependence but combined usage between mathematics and physics, made this race very speedy. Better results were obtained - resulting in a rigorous interpretation of nature in which an axiomatic theory based on two natural numbers becomes the link in the thread. This in turn has produced a new form of global sovereignty, where companies such as Google, IBM or Microsoft become the most important technological players in a new empire. In other words, relativity and set theory is now one of the implicit and deterritorialized factors that are used to govern people world wide.

There is a quite pure definition of AC or C as spatial devices, yet they are at the heart of general relativity and homomorphism, determined through complex chains of nonlinear partial differential equations and laws of thought. Consistent with this heuristic, in this context it is necessary to separate time from the concept of space-time. There is no space between these virtual environments and information, rather they should be calculated in an imaginary dimensional manner. This is why time is clearly a touchstone, where space will meet its own entity.

With this invention of space-time we are postulating the inseparability of time from space. Later developments have involved increasing the dimensionality of space-time with every “theory update”, but this thesis is maybe a first thought pattern constructing *dimensional time*.

# Preface

*“~ . . . it would be best to be both loved and feared. But since the two rarely come together, anyone compelled to choose will find greater security in being feared than in being loved.” [Machiavelli, 1513]*

The idea to write an MAS thesis in architecture about time, even to propose a mathematical-physical prediction about this possible dimensionality, started with a project that combined the forecasting of human activities with this aesthetic representation. During that process some strange and special results occurred that awakened my interest to look at this concept in more detail, as for example what time could mean for the spatial aesthetic of architectural design. This interest caused me to delve deeper into the science of the 19th and 20th centuries. I found in mathematics, physics and philosophy some specific discrepancies, and suddenly one relatively simple idea filled my mind, the idea that time should be considered not as “an arrow of time”, but as an emergence in some particular cases as a dimensional manifold.

I found that it was important to create an aesthetic layer to describe this concept of time - and as an architect I understood that it should include not just architecture, but also all the disciplines of art and media. One invisible layer should be created to combine all these *aesthetic disciplines* into a new science - *an aesthetic science* - where it is not just necessary to create a simple link to computer science or engineering, but rather to produce an invisible, abstract informational way of thinking to create a bond between them. My thesis uses analogies with architecture, design and film to explore this concept; it also engages with propositions in mathematics and physics. It delineates a path leading towards an understanding of the huge set of data that today gives us millions of potential insights into user dimensionality, parallelarity, personal tastes, and indeed into human behavior in general.

The practical goal of this thesis is to create, on an aesthetic interdisciplinary level, an explanation of time from the perspective of discrete mathematics and to put forward a revolutionary idea concerning the *dimensionality of time*.

## Notation

Einstein said that if you are not able to explain an idea even to a child, then you are not familiar with it at all. This may seem to be a very strange pronouncement to hear from the man whose idea of relativity is totally unclear to most people.

This thesis has employed several modes of exposition - literary, mathematical, diagrammatical and pictorial, all of them in order to facilitate the reading process, to simplify complex ideas and make them more understandable for the general reader. I will draw parallels between these different modes and employ metaphors. Thus a hidden message of this thesis, inspired by the novels of Lewis Carroll, is to show the possibility of creating an interdisciplinary layer that will combine aesthetic and scientific disciplines that will make even children familiar with the idea of dimensionality, parallelarity and time breaks.

In order to make my prediction concerning dimensional time, I was using an old method: the thought experiment (deut: Gedankenexperiment), to provide a basis for rejecting the common idea of the “arrow of time”. Proof by contradiction is used in the part where a moment of time is considered as an infinite imaginary number.

The grammar of mathematics will be used to explain the concept of dimensional time. There are no differences in the spelling of mathematical variables which are numerical or scalar quantities as they will be familiar to anyone dealing with mathematics. They are written using Lyx math grammar.

The mathematics of today, in particular, references to the mathematics of nonlinear systems, has almost no general techniques and requires to be studied individually.

At the end of the thesis is one of the algorithms that supports the agenda in an aesthetic manner. This was programmed in python.

The bibliography can be found at the end of the thesis. Some of the books listed there are not quoted directly but are important sources informing the work of writers quoted in the text.

Diagrams and pictures generated using processing [processing.org] and rhinoceros [rhino3d.com] accompany the whole thesis, mostly in order to enable the reader to follow the trajectory of the mathematical reasoning. These diagrams and pictures are of an illustrative character and are inserted into the body of the text rather than appearing as appendices. Listed at the end of the thesis is the biographical content.

Inspired by the open source movement, I am making all the material that informs this thesis available for others to use, build and comment on.

## Overview

The form of this thesis follows the pattern of my work on this subject from my initial research, to prediction and then to mathematical calculation. The purpose of adopting this tripartite approach is to produce an account of the concept of dimensional time that is accessible for the general reader and is not difficult to understand. Accordingly, the thesis is divided into three sections.

### **Part I**

The first part provides a background of time understanding (of how time has been understood in) from different mathematical, physical and aesthetic disciplines. In particular the question of precision in the calculation of time in different disciplines will be dealt with. It begins by semantically describing in a metaphorical way the idea of dimensional time and the noumenal realm and my personal interest in it. This serves as a critique on the disinclination and inattentiveness of modern science to engage with aesthetics in general.

### **Part II**

This section provides mathematical, logical and physical proofs with the help of six thought experiments. Each system has its own approach and background. This part may be said to provide an Intermezzo between this background and the future of these aesthetics.

### **Part III**

The concluding part of the thesis is entitled “Consequences” rather than “Conclusion” in acknowledgment of the continuing effects of the changing/developing understanding of the nature of time and how this continues to impact upon aesthetics.

### **Appendix**

The IronPython code representing the Calabi-Yau Manifold



## Acknowledgements

I would like to express my thanks to everyone at ETH CAAD who has been involved in the emergence of this interdisciplinary study. In particular I would like to thank Professor Dr. Ludger Hovestadt and Klaus Wassermann for their unfailing help and support; also for their vision in creating the philosophic-mathematical background at ETH CAAD without which I would not have been able to explore and bring to fruition the ideas which inform this thesis. I am indebted also to Denis Petrov, a mathematician currently working in Geneva, for supporting me with his huge mathematical knowledge and discussing with me all the steps involved in the mathematical aspect of my work.

For their assistance in producing a readable English version of this thesis I must thank Frances Sander and Dmitri Berzon.

Last but not least, I owe a deep debt of gratitude to my mother for her financial support, and to Anne Müller and my daughter Charlotte for their understanding when, due to pressure of work, I had no time to devote to them.

# Chapter 1

## Of Time's Inception

“Time present and time past  
Are both perhaps present in time future,  
...” [Eliot, 1941]

A very great deal is known about the physical measurement of a moment<sup>1</sup> – that statistical mechanics formulates the idea of the *direction of time*, general relativity, *space-time* and philosophy, that time might be *tensed* or *tenseless* and *conventional* - but very little work from a dimensional perspective that might lead to a holomorphic explanation has emerged. Three dimensional objects processing different properties at different times - the encoding of motion and change in four dimensions, one of the biggest mysteries of the nature of time, its actual existence or unreality - sharpened this question about the reality of time and what might be known about it from a holomorphic perspective (*fig.1.1*).

Of the many possibilities for the explanation of time, there are two which have been implemented theoretically, each of which says something intuitive

---

<sup>1</sup>Terms as resonance, frequency, units of time, ect..., connected directly on time

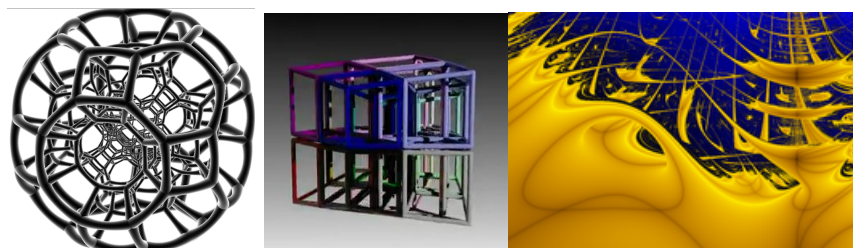


Figure 1.1: *left*: Omnitruncated tesseract; *middle*: stereographic; *right*: Lyapunov logistic fractal

about its matter. *Traditional time theory is tensed time*, it originates from the simplest theory of matter in motion where objects are perceived as being in time and space relative to constant acceleration (*fig. 1.2*) and following from this, the present is seen as a continuous dynamic process, the future is undetermined and unreal and the past is unchangeable. However - and most extraordinarily - by reversing the direction of time the same physical laws are satisfied. [McClure, 2005, Rundle, 2009]

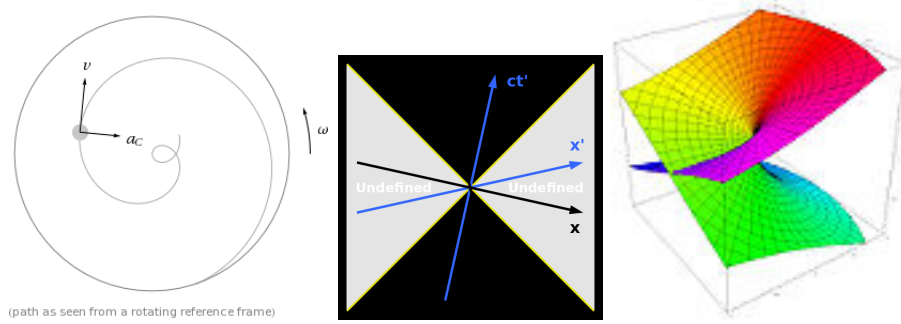


Figure 1.2: *left*: Coriolis effect; *middle*: space-time diagram; *right*: Riemann surface  $f(z) = z^{\frac{1}{2}}$

*Modern time theory is tenseless* - space and time are similarly represented - the past, present and future all exist but not at the same times. The origins of this theory may be traced back to Aristotle and Zeno's paradox of Achilles and the tortoise - the idea of measuring time by examining one process relative to another. The limitations of this idea were pointed out by rationalists, that if time should be measured by change, what happens if there is no change, does time stop? Galilean relativity answered that question - *the laws of physics are indifferent to the speed at which a body maintains straight uniform movement*. This was the idea that shaped Einstein's relativity, not least because of the legacy of Riemannian hyperbolic geometry<sup>2</sup> (*fig. 1.2; right*) and formed the essential framework for the general theory of relativity. [Penrose, 2004] The past, present, and the future are all considered to be equally real, even though they are not able to be separately observed - therefore unique time does not exist, and space and time are seen as not existing separately<sup>3</sup>.

<sup>2</sup>With M. Grossman's (1878 – 1936) help A. Einstein suited Riemann's work and the methods developed by G. Ricci-Curbastro (1853 – 1925) and T. Levi-Civita (1873 – 1941) to reframe the differential equations of classical laws of physics for variations in curvature and geometry – topology has been found applications to the large-scale structure of space and space-time. [O'Shea, 2007, Monastyrsky, 1999]

<sup>3</sup>At the end of XIX century M. Faraday (1791 – 1867), J.C. Maxwell (1831 – 1879), H.R. Hertz (1857 – 1894) proved that light spread in space in form of electromagnetic waves, through Michelson–Morley experiment that do not need any medium or aether.

Galileo who was *heuristic*, and Einstein (as Aristotle) nominally came to the idea of relativity, his general and relativity was created with almost no base number of experiments,

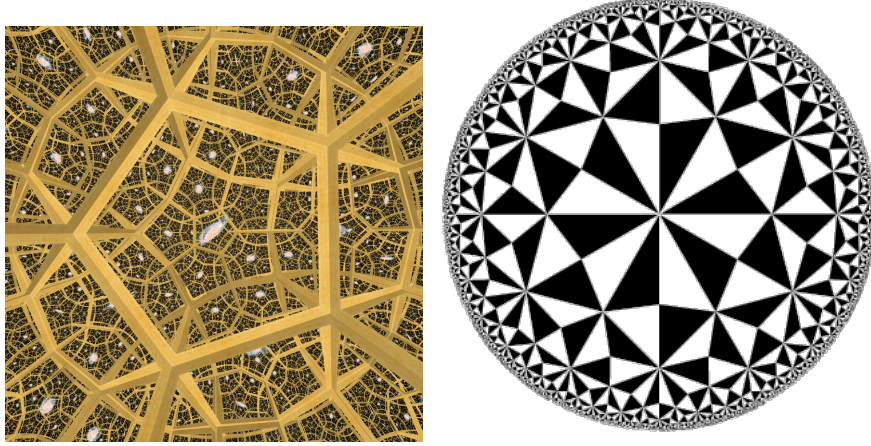


Figure 1.3: *left*: Curved Spaces (3.manifolds); *right*: Poincaré Hyperbolic Disk

Observer-dependency is a property of the march of time in special relativity. Within one particular observer's light cone (hypersurface in *Minkowski space*<sup>4</sup>, separated areas of past and future event-dependency), the order of events is definite. But another observer, moving relative to our first observer, will disagree with the first as to what events are simultaneous with event  $P$ . There is a plethora of times, one for every inertial frame of reference, and they are all equally legitimate - so-called proper time [Geroch, 1981]<sup>5</sup>.

In the 1930's, the physicists Paul Dirac(1902-1964) and Arthur Milne(1896-1950) shared the idea of a probable necessity<sup>6</sup> to introduce more than one time scale. This necessity was predicted by the current law of physics, that tells - there is no perfect clock at all. That was based on discrepancies of different timekeepers such as the *atomic clock or pendulum* - it could be that there are

---

data, great intuition and thought experiments. Until now all his predictions confirm with experiments.

Before relativity Euler, Laplace, J.L. Lagrange(1736-1813), Legendre, Gauss, Liouville, Ostrogradski, Poisson, Jacobi and W.R. Hamilton (1805-1865) and many others (besides) after Newtonian Laws created theory for interpretation of the universe.

<sup>4</sup>form a four-dimensional manifold for representing with Lorentz transformations:  $x = \frac{x' - vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$ ,  $t = \frac{t' - \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}}$ , where velocity  $v$  and  $t$  is time. Relationship between space and time coordinates represented by square of the proper length  $s^2 = c^2t^2 + x^2 + y^2 + z^2$ , where  $c$  is speed of light,  $t$  is time and  $x, y, z$  spacial coordinates.

<sup>5</sup>Dutch astronomer O. Roemer(1644-1710) was the first to measure with high precision the light-speed 214.000 km/s, using G.D.Cassini's measurements of the shadows of the moons of Jupiter . [Novikov, 1990]

<sup>6</sup>which is influencing of predecessors in the sequence, investigating the presence of consequences

two or more “times” – one for gravity, one for electromagnetism, etc., but if this is true then all physics is not unified. On the other hand it is assumed that mathematics concerns itself with the study of arbitrarily general abstract systems.

J.E. McTaggart (1866-1925), who was the teacher of Bertrand Russell, presented an argument that concludes that *time doesn't exist* - a cup of tea is hot at time  $t$  but not hot at time  $t^*$ , where  $t^*$  is later than  $t$ , that is logically incompatible. Later he concluded that time is unreal [Rochelle, 1998].<sup>7</sup>

D. C. Williams in his “The Myth on Passage” asks “How fast does time flow?” - the moment is always changing with respect to time (if now moves). But if NOW is also time, so does it move with respect to itself? Williams supposed that there is a second time, that being the time with respect to which the first time moves. Hence the question, “How fast does time move?” with “One second per second?”. C.D. Broad held the following view: that it was necessary to start positing *extra dimensions of time*.<sup>8</sup>

*The results described in Einstein's theory of special relativity show that what is the present for one observer is the future for another. Following from this, for the observer who is moving faster, the event is already in the past, and so is real and determinate. By removing these results from the physical context and formulating them in terms of philosophical argument, Hilary Putnam revealed the impossibility of the tensed theory, and showed it to be both unreal and indeterminate.*<sup>9</sup>

From the theory of general relativity is derived the concept that four-dimensional space-time can be curved. The object which is curved can be curved in the direction of a higher dimension, for example, the one dimensional curve becomes a two-dimensional surface. However, this does not mean that the four-dimensional space could be translated into a fifth. The evidence suggests that our universe is curved and that gravity is the result of this curvature. Also, light rays travel in straight lines and near the sun they can be observed to deflect slightly. Time too exhibits a slight delay.<sup>10</sup>

<sup>7</sup>Newton: “absolute, true and mathematical time, of itself, and from its own nature, flows equably without relation to anything external.” this idea is an ideal of objective knowledge

<sup>8</sup>An interesting addition is the philosophical argument concerning the puzzle of paradoxes of *Being* in the *flow of time* (called “projection”). Considered from the relationship between eternity and infinite time, reality is not controlled by time flow. [Reichenbach, 1957]

<sup>9</sup>The measurement of the time interval between events can be represented as a certain amount,  $T$ . Say that one object is moving past another, which is not accelerating or decelerating. The measurement of the time interval between A and B according to this object can be represented as amount  $T^*$ .  $T = \gamma T^*$ , where  $0 < \gamma < 1$ . The faster this object is moving the smaller will be  $\gamma$ , at the speed of light even  $= 0$ . This is the cause of the twin paradox.

<sup>10</sup>Since about five billion years existent of the Sun and the Earth time dilation grow about ten thousand years. Space and time are infinite rigid framework, that will be curvature under massive object. Therefore is more mass more will be it curved. One of the greatest ideas of A. Einstein was to connect time and space to the properties of matter, therefore prediction

Kurt Gödel (1906-1978) found a new solution to the equations of general relativity - each solution of the equation of general relativity describes a space-time by the laws of general relativity, and therefore is allowed by our laws of Nature: Gödel's solution is very strange - the universe is expanding without having a center; and from the view of an observer all the matter in the universe is rotating.

It is like dragging a paddle, when the water will continuously rotate. The future could be "tipped over" - that the light-cones are oriented in the same way. Imagine that the plane where all the light-cones are could be curved. In the universe the distance between particles is constant and there can be no simultaneous notation. But if the universe is spreading, Gödel's theory is incorrect<sup>11</sup>.

One of the amazing things about Gödel's theory of space-time is that through every single point in space-time there are physically possible paths that allow time-travel - and from each event it is possible to reach any other event. Gödel thought that the space-time he discovered reveals something important about the nature of time; namely, that it does not exist. Like McTaggart [Penrose, 2004], Gödel seemed to have in mind something like the tensed theory of time. The tensed theory, to recall, said that the non-relational present moves, turning the unreal future, real. The trouble that special relativity gives to understanding the nature of time has been noted earlier.

Putnam and others had argued that the relativity of simultaneity proved this view of time false. Gödel's theory of space-time posed further problems for special relativity. For example it allows paths for closed loops through every single point. Closed causal loops like this are incompatible with the tensed theory. If two points  $A$  &  $B$  are considered on the closed loop and suppose  $A$  is present and therefore Real. Then  $B$  is the Past and Future of  $A$  and therefore both REAL & NOT REAL, which is impossible. Why could the existence and nature of time not depend on the distribution of matter and energy, was the question asked by Putnam and others. Many important things do depend on it, for instance whether space and time are finite or infinite.

J. Richard Gott introduced cosmic strings - extremely thin filaments of pure energy, hypoionathetical relics of the Big Bang. If two fast cosmic strings pass by each other very closely the gravitational interaction between them will curve space-time sufficiently to allow paths back in time.

To consider another feature of time, the idea of "non-orientable" time, a

---

of Black-Holes and even his famous formula  $E = mc^2$  is related. [Novikov, 1990]

<sup>11</sup>From today's physical point of view the universe is both homogeneous and infinite, therefore the observation of this universe requires infinite time. Instead of an expanding three-sphere-universe, it is correct to imagine an expanding plane-universe where the distance between two points is constantly and homogeneously expanding..

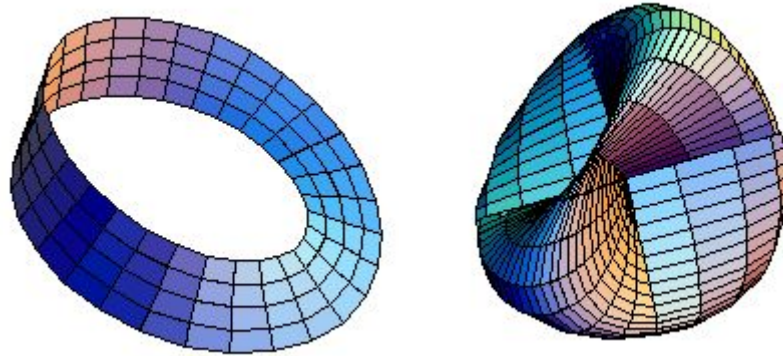


Figure 1.4: *left*: Möbius strip; *right*: Roman Surface;

Möbius strip may be imagined (*fig.1.4left*), that turns an object over without anything touching it. A Möbius strip in Time means that space-time might be non-orientable.

When considering the notion that time could *branch*, the idea is that space might divide into a minimum of two pieces, with time running separately on each. Does space “Run out”? Nowhere in your travels will you encounter an “edge” of space. It must therefore be assumed that space is extended without limit. Finite space without limit. None of Descartes inferences hold in general relativity, because space-time can be curved and space can be extended without limit and yet be finite.

Add to this the idea that the shape or geometry of space can change with time as it can in general relativity, then it is easy to contemplate the concept of space dividing . So space need not be a unity, and it is possible that time also can branch (*fig.1.4, right*).

“Topology of space” changes with time - which, for our purposes, happens when space divides and thus when time branches - then the space-time must also either have paths for time travel or be non-orientable in time. The laws of general relativity that permit something as weird as branching time must also permit something even weirder, time travel or time non-orientability! [Geroch, 1981]

Nietzsche’s Theory of Eternal Recurrence was proved by Roger Penrose and Stephen Hawking - that a general relativistic world such as ours must have had its matter and energy increasingly concentrated as we go into the past. The future is commonly taken to be evidence of a point, a so-called singularity - the BIG BANG, where matter and energy are so concentrated that the forces would be infinite. The relevance of the Big Bang is that it does not specify whether the singularity is pointing within the light-cone or pointing outside

the light-cone. If within, it could be associated with a temporal bridge to another phase of the universe much as wormholes connect bits of space-time. [Penrose, 2004, Hawking, 2008, Kaku, 2006, Lowith, 1997]

Another matter that requires to be considered is a simple philosophical objection to eternal recurrence. If each phase of the universe is exactly the same, infinitely recycled, then what possible evidence could there be for believing in an infinite number of phases rather than merely one? There are speculations about similar space-times, but ones wherein the repeated phases are not identical to one another. Therefore relative to the observer, time could be closed (cyclic) or open.

The temporal double standard means that both ends of the universe must be treated equally. The world that ends in a Big Crunch posits a world wherein the direction of time flips - one end of the universe is just as significant as another. Here "initial" and "final" have no objective meaning - the time reversal invariant that causes anti-thermodynamic behaviour - energy has to apply in the right place, at the right strengths and direction to enable reversibility. It is important to view this from a reversed temporal perspective - which is equally legitimate according to physics - that this sort of unlikely behaviour is occurring all the time around us.

In this unlikely state, the reversal of time's arrow assumes that past entropy<sup>12</sup> was low and in the future will be high. This does not allow Arthur Eddington's "time's arrow" flipping in the middle of the universe. If it is supposed that by reaching a point of high entropy in the future, entropy turns and heads for a final low state, again it can be observed that the direction of time turns around.

There is current scientific speculation concerning how entropy may start to increase in the single universe, creating a local arrow of time, while in some universes, time could actually run backwards. Following the writings of Hegel, Bergson and Heidegger, philosophy took a different point of view concerning the nature of time. Time came to be seen as our existential dimension, and for Bergson: time is "invention". But throughout all of these works, from the speculations of St Augustine until the XXI Century, there has emerged no clear statement about what time *is*.

Science has revealed more complex and interesting paradoxes of time and in the next chapters the implications of these paradoxes will be discussed.

---

<sup>12</sup>for further discussion of the emergence of entropy - see section below



## 1.1 Physical Flow of Time

Difficulties in the descriptions of time even from the classical view lead to unexpected complexity, that does not correspond to the historical time-oriented evolution. Time in mechanics is symmetric, turns backwards or forewords and vice versa, doesn't make a difference:

$$\begin{aligned}x_{n+1} &= f(x_n, t), \\x_n &= f^{-1}(x_{n+1}, -t).\end{aligned}$$

In this case the laws of physics state functional relationships and asserting certain physical quantity  $x_n$  with certain value, another quantity  $x_{n+1}$  has a determined value [Reichenbach, 1957]. Time is in the case of Newton's deterministic theory *reversal* invariable and appears as a *reversed or reversible* process.

Constructed on the basis of a causal indeterminism classical physics, quantum theory leads to similar strange reversible time properties. Particle systems are definable with single time coordinates, then a free particle has its own independent number of spatial coordinates. In considering the *Schrödinger equation* (1.1) that led to the development of the  $\psi$ -function<sup>13</sup>, with respect to the time parameter  $t$ , where  $q$  is the set of spatial coordinates<sup>14</sup>, when the state changes with time, the variable  $t$  enters as another argument into the function, which is then written in the form  $\psi(q, t)$ . For the time development of this function, Schrödinger's differential equation represents the fundamental *law of change* [Reichenbach, 1957]:

$$H_{op}(q, t) = c \frac{\partial \psi(q, t)}{\partial t}, \quad c = \frac{i\hbar}{2\pi} \quad (1.1)$$

$$H_{op}(q, -t) = -c \frac{\partial \psi(q, -t)}{\partial t}, \quad c = \frac{i\hbar}{2\pi}; \quad (1.2)$$

the fundamental quantum-mechanical law governing the time development of physical systems *does not* distinguish one time direction from an opposite one (1.2). The direction of time in quantum mechanics is presented in the same manner as in classical physics<sup>15</sup>. Additional strange time properties follow from Werner Heisenberg's relation of indeterminacy<sup>16</sup> that represents the indetermin-

<sup>13</sup> $\psi(q)$ -function, a mathematical device to collect all distribution functions. In mechanics a physical quantity  $u$  is characterized by its probability distribution (a function  $d(u)$ )

<sup>14</sup> $\psi(q)$  supplies a description of the physical state of a system, and that value is a complex number, but  $q$  values are real numbers.

<sup>15</sup>*statistical reversibility* is expressed in the form of *temporal symmetry in quantum mechanics* — a law determining the time development of *probability distributions*. [Reichenbach, 1957]

<sup>16</sup>This relation has its origin in the German word "Unbestimmtheit", and states the impossibility of simultaneous measurement of different quantities following from the impossibility of predicting a particle's future position when the value of its velocity, or momentum, at the present moment are unknown. [Reichenbach, 1957]

ism of quantum mechanics [Reichenbach, 1957].

Laws of classical and quantum mechanics<sup>17</sup> as well as special and general relativity permit the view that the laws of nature allow no distinction between past and future - time is simply a bookkeeping parameter without any direction (*fig.1.5*).

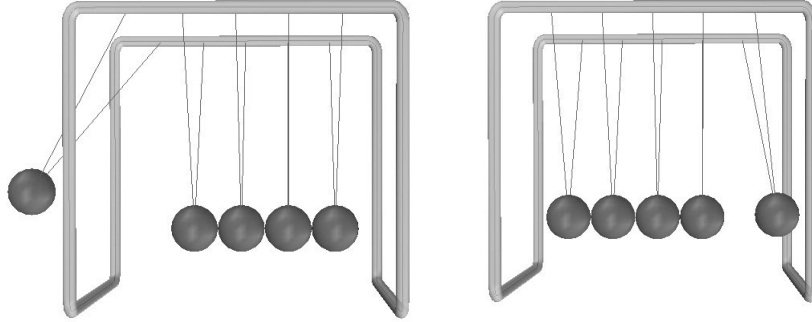


Figure 1.5: Newton's Cradle is an example of reversibility in classical systems. Particles in Newtonian physics are colliding in different directions, representing a model of *integrable systems*. [Craig, 2007]

Although time-reversibility or *T-symmetry*<sup>18</sup> properties are occurring, it is necessary also to look to the material contents of our universe and their relationship to the direction of time. There are sensitive processes to the arrow of time - the processes of Nature that behave in a temporally asymmetric manner - the *irreversible* processes: which may also be observed in so-called non-integrable systems<sup>19</sup>: it is impossible that, *once built, a building will ever spontaneously separate into its original materials*.

The explanation for why time flows in only one direction relates, in XIX Century, to the *Hamiltonian function*<sup>20</sup> - the geometrically encapsulated dynamical evolution of a system. Thermodynamics can certainly be derived from

<sup>17</sup>By formulating of quantum mechanics in the 1920s, Schrödinger and Heisenberg ignored Einstein's work and treated time in Newton's spirit, as an absolute that is ticking away in the background. Reason why these two theories might be so hard to reconcile and from time perspective absolutism was here to consider. Until today there is currently no working theory that unites quantum mechanics with general relativity [Novikov, 1990].

<sup>18</sup>*symmetry of equations which explain physical laws in relation to changing time from  $t$  to  $-t$ .*

<sup>19</sup>that is, of systems for which it is not possible to construct a unitary transformation, which eliminates the interactions.

<sup>20</sup>Even modern physics uses Hamilton or Lagrange in creating new theories because of the required consistency and invariance properties and implicity of "Newton's third law" [Penrose, 2004].

Hamiltonian [Prigogine, 2003], where causal laws<sup>21</sup> determining the path are the equations of motion:  $\frac{\partial H}{\partial p_i} = \frac{dq_i}{dt}$ ,  $\frac{\partial H}{\partial q_i} = \frac{dp_i}{dt}$ , that is, a kind of generalized form of Newton's fundamental laws of mechanics known as the *canonical equations*.

For a long time it was possible to eliminate the coordinates in the Hamiltonian. The continuing possibility for finding "privileged" canonical variables  $(q, p)$  [Prigogine, 2003], ceased to be a viable proposition following the works of Poincaré. Poincaré made a fundamental discovery where elimination was only possible for a class of dynamical systems - called "integrable systems".

The opposite happens in non-integrable systems which cannot construct a unitary transformation. When considering existing resonances such as a particle as a harmonic oscillator in a field (as in electromagnetism), a particle is dissolved in the continuum. The mechanism known as the Einstein and Bohr mechanism explains this effect. By extending the idea of unitary transformations because of marginal denominators, Poincaré showed how such resonances led to difficulties through the appearance of divergent terms. The vibrating atoms of crystals represent independent motions where it is possible to define the basic frequencies through these normal coordinates. If these crystals are considered as a point cloud rather than in a classical manner in terms of a number of points, employing the physics of distributions instead of physics of points; a statistical description is obtained and it becomes necessary to give up classical determinism and distributivity. An example of this is the Langevin stochastic differential equation that describes Brownian motion, the apparently random movement of a particle in a fluid due to collisions with the molecules of the fluid. Therefore the Langevin equation has a broken time symmetry. As a consequence, the description of the non-integrability rules of nature, the properties of which are: the appearance of new fluctuations resulting in the negation of determinism, and finally the appearance of a privileged direction of time - a nondistributivity which leads to new uncertainty relations.

The new transformed variables are random, leading to probability theory, which starts from the first science to handle asymmetry - thermodynamics, which is based on the theory of ensembles, and as a first science introduces irreversibility in a form of *the second law of thermodynamics* with the increasing of entropy. [Prigogine, 2003]

Thermodynamics itself starts with describing the spontaneous transfer of heat from a hotter body to a colder one - and when equilibrium is obtained the transfer stops. In gas expansion the gas expands through its "available volume" until equilibrium is achieved. An asymmetric phenomenon is observed - heat never goes from cold to warm, and gas spontaneously expands throughout the available volume. The second law of thermodynamics<sup>22</sup> says that: a closed sys-

<sup>21</sup>where events of the physical world are arranged in causal chains

<sup>22</sup>Hystory of the Second Law:

tem always increases with time. This claim is called entropy: heat change  $\Delta Q$  divided by temperature  $T$ :  $\Delta E = \frac{\Delta Q}{T}$ .

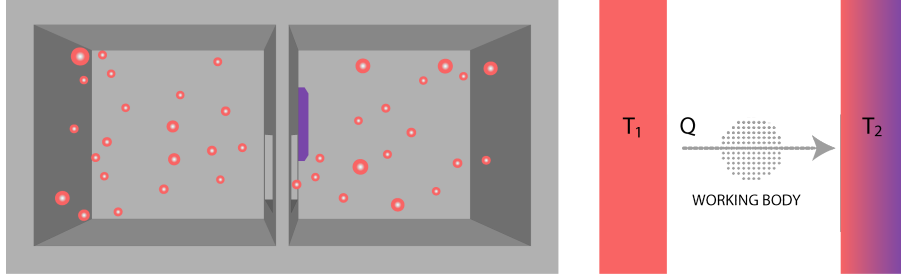


Figure 1.6: *left*: Maxwell's demon diagram; *right*: Entropy diagram. In addition to thermodynamics, entropy (the disorder of a system measurement) becomes an important term in different sciences

In terms of dynamical processes the second law of thermodynamics provides the distribution of the photon as an irreversible process leading to entropy production. Particles, without proper mass should, from the thermodynamical point of view, dissolve into a continuum. Probably the main event in the history of our universe, the Big Bang is this kind of differentiation. Certain time  $t$  or cross section is a state through the four-dimensional space-time world (if the universe is spatially finite). This it is possible to speak of the macroprobability, or of the entropy, of an instantaneous state of the universe. [Reichenbach, 1957]

"For some reason, the universe at one time had a very low entropy... that is the origin of all irreversibility..." [Feynman, 1963]

The Second Law of Thermodynamics rules out these reversed processes. But from the point of view that every body consists of Newtonian particles (neglecting the scale of quantum fields) the ruled out reverse processes are in fact possible, and therefore the Second Law of Thermodynamics is also an approximation:

$$pv = RT \frac{M}{m},$$

in considering the symmetrical functional relationships of physics in the laws of Boyle and Mariotte for perfect gases, quantities of pressure  $p$ , volume  $v$ , molecular weight  $m$ , mass  $M$ , and temperature  $T$  are in a certain functional

- 
- 1824 heat engine efficiency by S. Carnot(1796-1832)
  - 1850 formal statement of second law by R. Clausius
  - 1851 another statement of second law by W. Thomson/Lord Kelvin
  - 1865 entropy defined by R. Clausius(1822-88)
  - 1867 Maxwell demon thought experiment by J.C. Maxwell
  - 1872 H-theorem by L. Boltzmann

relationship, where a certain substance-independent constant,  $R$  is required [Reichenbach, 1957].

These two apparently irreconcilable views can be explained by statical mechanics, rules that were invented during XIX-th century and can be simply explained by the distribution rules of  $n$  number of balls between two boxes (*fig.1.6*). A process of gas mixing may be considered from two perspectives: thermodynamic and statistic. That thermodynamical interpretation submit the entropy becoming larger and in statistical interpretation - the process goes from an ordered macro-state to an unordered one, and thus from low to high probability [Reichenbach, 1957].

This, again, is asymmetry - and it is called statistical asymmetry. Boltzmann noticed that there are many more ways to spread the balls between the two boxes evenly or not evenly. Therefore the new Second Law states that reversal processes are most improbable and entropy here becomes a measure of how probable a state is. In other words in any system that evolves with time as a system, its entropy is *very likely* to increase rather than that its entropy *must* increase. This does not mean that entropy only increases towards the future, but that entropy increases toward the past also or does not increase at all, following the Gibbs paradox<sup>23</sup>.

This is known as the reversibility paradox (*fig.1.7*) of time's arrow, which predicts that entropy will increase in both directions rather than in one, called the future. This reversibility paradox has been explained on the basis of probability, why only one direction of time is observed in: the universe - like objects with 50-50 per cent distribution having a small probability that the distribution may change suddenly to 5-15% <sup>24</sup>. Such irreversible processes may only define a unidirectional causality and cannot supply causality for an asymmetrical relation[Reichenbach, 1957].

---

<sup>23</sup>Consider a system consisting of a thermally insulated container, divided into two equal parts by a thin rigid wall, on either side of which are two different ideal gases. If the baffle is opened the gases will mix. Since they are ideal, the process can be represented as an independent extension of two ideal gases in a vacuum. Expansion of the gas in a vacuum is an irreversible process which increases the entropy of the system. Thus, for each considered gas entropy increases, and because of its additive nature, respectively, increases the entropy of the system. The paradox arises if it is imagined that on both sides of the walls is the same gas at the same pressure and temperature. Opening the partition in this case does not affect the status of the system, it is just a state of equilibrium. This contradicts the fact that the entropy of the system will increase after the opening of the partition.

<sup>24</sup>The reversibility paradox was named after the teacher of Boltzmann, J.J. Loschmidt (1821-95). Boltzmann had the ambition to become the Darwin of physics.

Fundamental discoveries in thermodynamics were done by this Vienna group ...,in the second Law of Thermodynamics they found a small fluctuation from the 'order' universe, a function, which plays the role of entropy - which is true also for the existence of the Big Bang

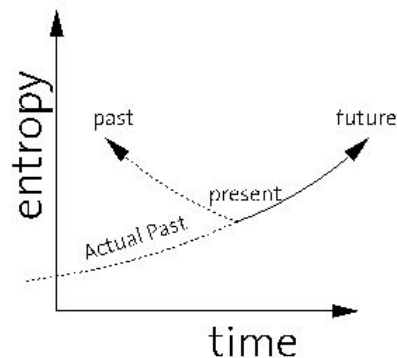


Figure 1.7: Penrose presentation of Loschmidt's reversibility paradox

By introducing temporally asymmetric boundary conditions (fig . . . ), it is assumed, that at the point of the universe called the past, the universe was in an incredibly *unlikely state* and that at some point in the distant future it will be in a *likely state*.

This monumental improbability that the expansion of the universe is the real reason that entropy increases (that the bigger the radius of the universe, the greater will be the entropy) should be a deeper explanation of the time arrow than merely the fact that the universe began in a very specific state (does entropy really increase?). Huw Price called this a temporal double standard. The smallness of the radius at the Big Bang is explained by the time reversal invariant where the low entropy begins and the increase of entropy is explained by the existence of a small radius constraint at the Big Bang and the lack of such a constant at the end of the universe.[Price, 1997]

The same field of inquiry was viewed from a scientific perspective by Ilya Prigogine who described dissipative structures, complex systems, and irreversibility. He introduced a view of Nature which goes beyond the mechanical timeless automaton which was inherited from classical physics, but also beyond a meaningless random game. In his works, evolution, emergence of structures and creativity are the keynotes of natural processes at all levels<sup>25</sup>.

The *tenseless* concept of time has been delivered as a central proof following from the theory of relativity, but also it may exist only as a single concept of time as a mathematical abstraction. If relativity is considered from the perspective of thermodynamics, entropy could reach the maximum, but that does not mean that time stops. Absolute and relative entropy is considered in the *paradox*

<sup>25</sup> An other remarkable entropy notion is uncovered in Prigogine's theorem, that states that the importation and dissipation of energy into chemical systems could reverse the maximization of entropy rule imposed by the second law of thermodynamics.

*of normalization* (also called Gibbs's paradox) which is inherent in the use of an absolute entropy for Maxwell-Boltzmann statistics. But it is a remarkable fact that still time is a one dimensional manifold in today's understanding of the space-time dimensional concept, and a moment of time is abstracted in the form of the real number  $\mathbb{R}$  that has an infinite spreading, whereas from the point of view of physics, time is dividable until it turns to quarks. Present day science is implicitly based on the consideration of time as a single dimensioned manifold, at some points bringing big surprises in the contents of the mathematical notion on time as a kind of flexible structure with a relationship to the outside fixed system.

## 1.2 Mathematical Flow of Time

On the peculiarities of modern mathematics lies in the fact that it studies artificially invented objects. In nature there is not such a thing as multidimensional space - there are no groups, rings and fields of the kind whose properties are extensively studied by mathematicians. And as the possibilities of modern technologies permit the building of more and more complex inventions so, at the same time, mathematics is inventing its counterparts for logical analysis.

Mathematics needs a clear structure. Where a ratio has a clear structure it should be interpretable as well, to be responsible in the deep sense. Time independence in mathematical relationships plays an interesting role - the most interesting fact being that there is no direct application of time in pure mathematics as it exists in physics.

Time in the mathematical sense could be both reversible and irreversible. Although there are some contradictions in mathematics preventing a return to the past, in physics there are none. Consider in mathematics the Euler method - the most basic explicit method for numerical integration of ordinary differential equations. According to this method, if a step with a negative value is taken, the result will be a different point (Euler method for numerical solving of differential equations [Crivelli, 2008]). An additional amazing thing arising from Euler's work is the so-called Eulerian nonsense summation  $1+2^2+2^4+2^6+2^8+\dots = -\frac{1}{3}$  of the convergence of power series [Penrose, 2004], which from a mathematical point of view makes the arrow of time meaningless.

Before the development of thermodynamics the time parameter is absent from mathematics. The notion that time could play a part in mathematics was derived from thermodynamics - from physics in mathematics it is well known as the theory of probability, and will be considered here.

Mixing processes: reference class  $A$ , attribute class  $B$ , frequency of observed events where  $P(A, B) = p$ . Interpretation of the  $p$  - the formula means that *probability exists*. It is sufficient that the sequence has a *practical limit*, thus cannot supply a convergence in the mathematical sense of the term "*limit*".

Relative frequency: for future observations, this will converge toward a limit; and  $p$  is measured by this limit of frequency:  $\lim_{n \rightarrow \infty} \frac{N^n(A, B)}{N^n(A)} = 0$ .

In the number of elements belonging to  $N^n()$ ,  $n$  is an element of the sequence.

There are further properties which a sequence must satisfy in order to be called a *random sequence*. This includes a chapter on a theory of order, which treats sequences of various types of order and defines special types.

An aftereffect probability is written by means of *phase superscripts*.

Probability lattice is the sequence of all the events participating in a process



$$\begin{array}{c}
x_{11} x_{12} \dots x_{1i} \dots \rightarrow p \\
x_{21} x_{22} \dots x_{2i} \dots \rightarrow p x_{2i} \dots \rightarrow p \\
x_{j1} x_{j2} \dots x_{ji} \dots \rightarrow p \\
\vdots \\
\check{p}_1 \check{p}_2 \dots \check{p}_i \dots \rightarrow p,
\end{array}$$

horizontal row represents the history of an event by short time intervals  $\Delta t$   
horizontal probability  $P(B^{ki})^i = p$ ,  
vertical probability  $P(B^{ki})^k = p_i$ ,

$$\begin{array}{ccc}
P(B^{ki})^i & < & P(B^{ki})^k, \\
\lim_{i \rightarrow \infty} P(B^{ki})^k & = & P(B^{ki})^i.
\end{array}$$

Horizontal and vertical probabilities are mutually independent mathematical quantities where the lattice satisfies two restrictive conditions: independence (trivial identity) and lattice invariance. This consideration shows a lattice of mixture and conditions allow it to be transformed from horizontal probabilities into vertical probabilities. The row is called a *time ensemble* (which refers to the temporal succession of states of one object), and the column a *space ensemble* (which refers to the simultaneous states of many objects). [Reichenbach, 1957]

There is the condition which makes possible an *inference from the time ensemble to the space ensemble*.

Random sequences do not restrict the general theory of probability, including the probability conditions of independence and lattice invariance which are a part of the theory of order. [Reichenbach, 1957]

A deterministic and probability interpretation of physical occurrences can be expressed by the strictly predictable causal chain in an attribute space, a *distribution function*  $\gamma(y_1, \dots, y_{n-1})$ .

Liouville's investigation of the condition of incompressibility of a liquid assumes that there is a domain  $A$  in the phase space. From every point of  $A$  the path of the phase point for a time interval  $\Delta t = t_{n+1} - t_n$ , can be constructed, in which the interval is the same for all points.

The fundamentals of the probability metric are based on this equality of "times of sojourn" which caused Boltzmann's *equations of motion*:  $\lim_{t \rightarrow \infty} \frac{\sum t_A}{t} = \text{const}$ , where total sojourn time  $\sum t_A$  in  $A$  maintains a certain ratio to  $t$ . Analogically to the probable frequency interpretation - as ratio limit of periods of time. This was formulated by Boltzmann (also P. and T. Ehrenfest) as ergodic hypothesis<sup>26</sup>, and solved in von Neumann and Birkhoff's theorem as derivation

---

<sup>26</sup>ergos = energy, odos = path

of a probability metric from causal laws alone<sup>27</sup>.

The laws of quantum physics are probability laws (not causal laws), thus the reversibility of elementary processes follows a symmetry relating to probability distributions [Reichenbach, 1957], where the distribution of molecules  $D$  is distinguished from an arrangement as a class of these. A distribution is a class of arrangements (of which the total number is  $N$ ), with a *probability metric* on which the computation is based (Boltzmann)  $\varphi(x_1, x_2, x_i) = \text{const}$ , where  $x_1, x_2, x_i$  are spacial coordinates. The geometrical meaning of this is to find where the special element is located in  $m$  (a kind of box-container, where the sum will look like this  $\sum_{i=1}^m n_i = n$ , which is also used as the description for every velocity distribution, and every arrangement).

Using Lagrange's method of infinitesimal variations with *multiplier*  $\alpha$ :

$\sum_{i=1}^m n_i [\delta(n, \log n_i) + a \cdot \delta(n_i)] = 0$ , means that all cells are occupied by equal numbers of molecules. If probability  $W$  reaches 1, that means that entropy  $S$  reached the maximum.  $S = \log W$

To express the dependence of  $S$  on the total energy  $E$ , with the fundamental relation of non-statistical thermodynamics, the amount of heat  $Q$  represents the energy  $E$ :  $\frac{dS}{dE} = k\beta = \frac{1}{T}$ , where  $\beta$  constant and  $T$  temperature.

[Reichenbach, 1957]

### Macrostatistics

Concerning cause and effect: the cause is the interaction at the lower end of the branch run through by an isolated system which displays order; and the state of order is the effect.

The significance of macrostatistics is that it supplies criteria of order which escape microstatistics, and may be used to discover causal chains in the macrocosm.

The shuffling mechanism consists of simple mechanical processes of an openly deterministic character

This process, in which order is transformed into disorder, is quasi-irreversible but ordered in an implicit way.

### Cause and Effect: Producing and Recording

States of high order can be preserved for a long time and can be observed conveniently thus supplying *records* and forward understanding of *causal explanation*.

<sup>27</sup>the ergodic theorem (distinguished from the ergodic hypothesis, where total space depends on the initial conditions) provides the explanation of the manner by which the phase point enters every volume element.

Von Neumann and Birkhoff's theorem states that the ergodic hypothesis exists for some subspace that is satisfied by the ergodic theorem. For not entered spaces  $\lim_{t \rightarrow \infty} \frac{\sum_t tA}{t} = 0$ . The phase space for the energy surface given by *ergodic density*  $\sigma$  function is constant (because neighbouring energy surfaces do not have a constant distance),  $\sigma$  is the reciprocal value of the *gradient of energy*. [Reichenbach, 1957]

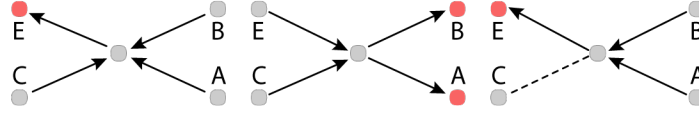


Figure 1.8: *left*: Double fork; *middle*: fork open toward the future; *right*: fork open toward the past

Causal explanation, both in its post-interaction and pre-interaction state, leads to finality instead of causality.

### The Principle of the Common Cause

The principle of the common cause can be stated in this form: if an improbable coincidence has occurred, there must exist a common cause.

(*fig.1.8, left*) a double-fork arrangement in which  $A$  and  $B$  represent two events, the simultaneous occurrence of which is improbable;  $C$  is their common cause;  $E$ , their common effect. If  $C$ , and not  $E$ , explains the simultaneous occurrence of  $A$  and  $B$  the upper part of the diagram can be omitted permitting the study of the single fork (*fig.1.8, middle*) in which merely the common cause  $C$  is indicated.

In order to explain the coincidence of  $A$  and  $B$ , which has a probability exceeding that of a chance coincidence, it is assumed that there exists a common cause  $C$  defining a *conjunctive fork*, which makes the conjunction of the two events  $A$  and  $B$  more frequent than it would be for independent events. The common cause  $C$  *explains* the *frequent coincidence*. A *statistical dependence* is here derived from an independence.

$$\begin{aligned} P(A) = a, \quad P(B) = b, \quad P(C) = c, \quad P(A, B) = w, \\ P(C, A) = u, \quad P(C, B) = v, \quad P(\bar{C}, A) = r, \quad P(\bar{C}, B) = s. \end{aligned}$$

The rule of elimination furnishes the following equations:

$$\begin{aligned} w &= cuv + (1 - c)rs, \\ a &= cu + (1 - c)r, \quad b = cv + (1 - c)s, \quad 0 < c < 1, \end{aligned}$$

$$\begin{aligned} u &> a > r, & v &> b > c, \\ uv &> w. \end{aligned}$$

Now the time order is known; the fork can also be drawn as in (*fig.1.8, right*), replacing the  $E$  point by  $C$ . It is not possible to use these conditions to determine the time direction of the fork directly, for that there is an indirect way.

Consequently, if there were no common cause  $C$ , the common effect would establish a statistical dependence between  $A$  and  $B$ ; and the explanation would be given in terms of a *final cause*<sup>28</sup>. [Reichenbach, 1957]

Principle of the common cause: If coincidences of two events  $A$  and  $B$  occur more frequently than would correspond to their independent occurrence there exists a common cause  $C$  for these events such that the fork  $ACB$  is conjunctive.

### Entropy and Information.

The second law of thermodynamics leads to the existence of records of the past, and records store information - thus there is a close relationship between *entropy* and *information*.

The term *information* refers to a *probability disjunction*, or *probability situation* of independent events  $B_1 \vee \dots \vee B_m$  for which a set of probabilities  $p_1 \dots p_m$  is known.  $p_1$  satisfies the condition:

$$\sum_{i=1}^m p_i = 1.$$

There are two kinds of information: first: *intentional information*<sup>29</sup> and second: *extensional information*<sup>30</sup>.

$p_1 > p_2 > p_3$  means inappropriate measurements for larger disjunctions, therefore it is necessary to permit a double prediction - and attach a higher weight  $w_1$  to the prediction if the first part comes true; and a lower weight  $w_2$  if the second part comes true.

$$p_1 w_1 + p_2 w_2 + p_3 w_3, p_1 > p_2 > p_3, w_1 > w_2 > w, \quad (1.3)$$

From information theory it has been shown that it is convenient to use the  $w$  in a form:

$$w_i = \log p_i, \quad (1.4)$$

as a measure of intentional information:

$$H(p) = \sum_{i=1}^m p_i \log p_i + \log m, \quad (1.5)$$

If disjunction  $B_1 \vee \dots \vee B_m$  is associated with the set of probabilities  $q_1 \dots q_m$  and controlled by  $p_1 \dots p_m$ , the following is deducible:

<sup>28</sup>is incompatible with the second law of thermodynamics and consider such forks impossible

<sup>29</sup>*information about the event* increases with the percentage of true predictions and thus expresses the degree of predictability associated with the disjunction.

<sup>30</sup>*Information from the event* measures increases with the number of false predictions. Extensional information is the inverse of intentional information.

$$H(p; q) = \sum_{i=1}^m p_i \log q_i + \log m, \quad (1.6)$$

$$H(p/q) = H_1 - H_2. \quad (1.7)$$

Therefore  $H$  is a negative entropy, being the negative of the specific entropy  $S$ .

$$H = -S. \quad (1.8)$$

Entropy can be regarded as extensional information.

If the  $p_i$  is very unequal and the entropy is low, the information is high and vice versa. Entropy increases with ignorance *about* the event predicted and is in inverse ratio to intentional information. Entropy therefore measures information *from* the event, or extensional information.

Not by the probability  $W$  of (1.6, 1.7), but by the frequencies with which molecules occur in the different cells of the space considered, that is, by the  $p$ , of (1.6, 1.7)<sup>31</sup>.

From 1.4 and 1.6:

$$W = N_D \cdot q_1^{n_1} \dots q_m^{n_m},$$

$$S = \frac{1}{n} \log W.$$

The links between information theory and statistical mechanics show that there is a connection concerning the inverse relation between entropy and information. The presentations of information theory include remarks that the entropy therein defined is closely related to the second law of thermodynamics derived from *recording processes* and the entropy of information theory does define a time direction.

### The Time Direction of Information and the Theory of Registering Instruments

Information processes indicate a direction of time.

For physical quantities of irregular variation, *prediction* is as yet technically impossible, whereas *postdiction* is easily achieved by the use of registering instruments. The difference can be explained in terms of the relations that hold for *conjunctive forks*.

The inference from the partial effect to the total cause is typical for all forms of recording processes.

If a fork is pointed towards the future and open towards the past, in order to predict the common effect  $E$  with a high probability, it is necessary to know the occurrence of both  $A$  and  $B$ . Such predictions usually require a large number of observational data. Generally there are multiple forks, in which a number of simultaneous events  $A_1 \dots A_i$  determine a common effect  $E$ .

---

<sup>31</sup>The corresponding probability disjunction, that is, the set of probabilities having this measure of information, is given not by the probability which measures the entropy

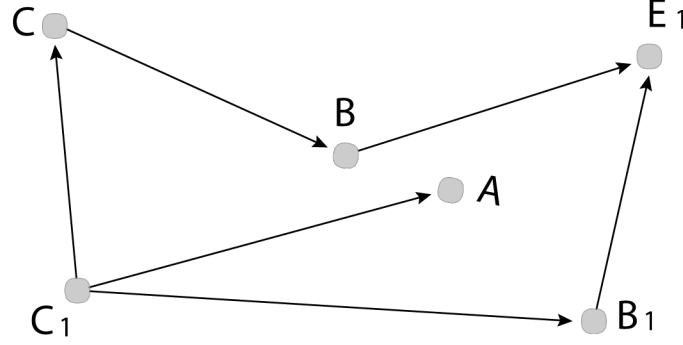


Figure 1.9: The observation of the record  $A$  of the event  $C$  makes possible the prediction of the later event  $E$ .

$A$  is the gathered value;  $C$ , the atmospheric pressure at the place of the instrument;  $B$  some other effects of this local pressure;  $C_1$  the atmospheric pressure over a wider area;  $B_1$  other effects of this pressure; and  $E$ , the rain. The fork  $ACB$  is open toward the future; but the fork  $CEB_1$  is closed on both sides, and therefore the partial cause  $C$  can be used for an inference toward the total effect  $E$ . Such an inference toward the future is made possible by a detour through the past, as it were.

intentional information

extensional information

C. Shannon, if the recording process is repeated  $n$  times, the individual amounts  $H$  may be added up:

$$H_n = n \cdot H, \quad (1.9)$$

where  $H_n$  total information,  $n$  successive acts of recording

The increase in information with growing numbers of recorded data thus finds a natural expression.

$$I = - \sum_{i=1}^N p_i \log_2 p_i,$$

where:

$I$  - quantity of information;

$N$  - quantity of possible events;

$p_i$  - possibility of particular ( $i$ ) events.

$$I = \log_2 N.$$

This quantitative aspect of recording information means that the larger the number of recorded items is, the lower is the probability that the items are a

product of chance, and the higher the probability that they represent a record of the past.

Growing order is an indication of positive time, contrary to the entropy rule, according to which positive time tends to produce disorder. But this apparent contradiction is easily resolved. The order of the registered items does not represent a succession of states in an isolated system, but results from the space ensemble of individual interaction states, each of which produces a specific recorded item. And order in the space ensemble indicates a time direction perpendicular to the ensemble. Thus there is a time sequence the order, or information content, of which grows with positive time, as expressed in 1.9. With increasing information, the manifestation of time direction becomes more and more pronounced.

## Chapter 2

# Dimensional Time

“Wir müssen wissen - wir werden wissen!” [Hilbert, 1930]

Following Hilbert’s speech [Hilbert, 1930], Kurt Gödel showed the impossibility of finding a system of logical axioms that would be able to establish every conceivable result in number theory and not lead to some sort of contradiction - there were limits to logic. To turn time as a geometrical event, time should be represented with set theory and calculated with probability, but where time is considered from a dynamic point of view, everything occurs in a predetermined way. From the thermodynamic point of view: everything goes to death, the so-called thermal death. Both are not able to describe time. When considering that light travels with finite speed some complications appear, hence looking far out is looking back in time [O’Shea, 2007].

Although the results obtained by classical or quantum mechanics or classical thermodynamics contain certainly a large part of truth, these descriptions are based on a too restricted form of dynamics. The illusion of irreversibility occurs in systems such as non-equilibrium structures that are large and where there is no exact knowledge of their time evolution.

As a result of Perelman’s work it is known that closed loops that cannot be shrunk to a point would show up as spikes in the *pair separation histogram*, a mathematical tool used to search for periodicity in data. Also, if there is only a finite number of *inequivalent closed loops* in the universe, then it must have a positive curvature. However, questions concerning the shape of the universe or the matter of time are still very much open.

Two events can never be said to be the same, or equivalent, when although they are the same in one sense, they are different in another. Their topological<sup>1</sup> or geometrical properties could belong to one domain in geometry, the domain

---

<sup>1</sup>Topological properties can be very different from geometric properties such as length and angle.



of *topology* (*homeomorphic* as same topologically), where points are *continuous*. What is not forbidden within the law of nature, should exist, even if only on a very small scale.

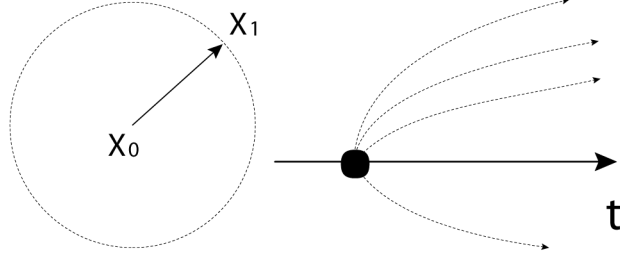
J.W.Cronin and V.L.Fitch made a fundamental discovery concerning the decay of an unstable particle [Novikov, 1990] that outlines the rule of irreversibility processes and can define a time direction and distribution. This discovery was a class of arrangements.

One of the mysterious questions is why the chain of time is one dimensional - opposite to space. The notion of a bundle is required by forming a geometrical sense for the gauge theories of particle interactions. The addition of extra "spatial" dimensions (or internal dimensions) is required, which move along in an "internal direction" that does not actually carry us away from the space-time point at which we are situated. In some modern theories, these notions are presented in a modified form, in relation to which space-time itself is thought of as acquiring extra dimensions.

Physical values are preserved with time because of space symmetry. In the absolute symmetric system they could appear as a random-based state of the system and from the view of Nature which goes beyond the mechanical timeless automaton which was inherited from classical physics, also beyond a meaningless random game. Evolution, emergence of structures and creativity are the keynotes of natural processes at all levels. Emmy Noether showed that energy persists because of the homogeneity of time. In addition, Einstein in his attempts to unite gravity and electricity introduced the "spin" of space-time to describe electromagnetic phenomena. The Kaluza-Klein theory unites Einstein's gravity and Maxwell's electromagnetic field on a geometric basis. Their proposal was that space-time is not 4th-dimensional but 5th. [Novikov, 1990].

The question then arises as to why time's extra dimension has no obvious appearances. From probability theory a point - the representation of a moment in time is from "now" to later - now is a probability vector (2.1, left).

Where  $x_0$  is a condition of a system in a moment of time  $x(t_0)$  here  $x$  represents a *time function*, and  $x_1$  is a next moment. In other words this is a vector with the length one:  $(\overrightarrow{x_0; x_1}) = 1$ . where zero is  $x_0$  and one is  $x_1$ . Following the Cantor set on the vector  $[0; 1)$  an infinite number of points may be ascribed to  $d$ , where  $d = 2^{\mathbb{N}}$ . At every point there is a probability of a change of trajectory in any possible direction, but still in the direction of the vector  $(\overrightarrow{x_0; x_1})$ . This means that the mathematical expectation is that movement of time will be along the trajectory of vector  $(\overrightarrow{x_0; x_1})$ . Following from this, there are no rules that actually could forbid the extension of time parameter  $t$  from Minkowski space-time  $t^2 - x^2 - y^2 - z^2 = 0$  and represent it as a bundle. This may be derived from the set of time series as a set of series of real numbers:

Figure 2.1: *left*: a time vector; *right*: forking possibilities

$$f : \{t, \{\mathbb{R}\}\} \mapsto \mathbb{RT} \mapsto \infty$$

and it increases towards infinity.

If first, as examined in the previous section, the number of points between zero and one is infinite, then any other vector could be parametrized as  $[a; b] \mapsto [0; 1]$ . As second, at every point there is an infinite number of movement directions: even on a two dimensional manifold it is possible to place every point - it is not a matter of one or two in that case. Even more, between two points on the “arrow of time” is it possible to place a point in-between - time is not interrupting the topological mechanism that lets time flow, independent of choice of coordinates or mechanism describing the rate of change.

In considering time separately as a dimensional matter, then it is has some similarities to curvature. At any point of an arrow of time, the time then should be given not by a single number but as the curvature as a mathematical object. *For example* the curvature tensor, it represents six different numbers to describe curvature on a three-manifold. As entropy increases towards the average value, in a similar manner as can be seen in Perelman’s proof of the Poincaré conjecture, that is called the heat equation<sup>2</sup>.

When time starts to deform and arrange itself - it could take any values, even become zero (*developing of singularities*) or negative, the values could go to infinity, or continuity could stop. This amazing complexity has its origins in the history of mathematics starting from Euclid’s fifth postulate<sup>3</sup> and ending in

<sup>2</sup>Perelman was using Hamilton’s equations for the Ricci flow  $\partial_t (g_{ij}) = -2R_{ij}$  a type of *heat equation*. This is a distant relative of the *Black-Scholes equation* and both are a type of differential equation, known as a partial differential equation. [Perelman, 2002][O’Shea, 2007]

<sup>3</sup>“If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side

the present with the proof of the Poincaré conjecture. In order to understand *dimensional matter*, knowledge of *dimensions* is required. This knowledge is important in order to provide a general explanation of an object and therefore to be able to continue following it. Leaving aside the matter of the arrow of time, it is now necessary to focus on the moment of time. In mathematical terms the infinite time interval  $[T, \infty]$  or finite time interval  $[T, T']$  is just a set with filled values, simply a moment of time. But the central question that remains to be answered concerns the meaning of 'dimensional' in this context. Growing/lamination/bundling (расслоение)<sup>4</sup> on dimensionals as a concept can be seen to have been introduced with the work of Euclid It is one interpretation of this Parallelarity<sup>5</sup> that is shown in the diagram (2.1, right). Handling dimensions requires a great deal of knowledge, and handling them near singularities requires great imagination as well. The next section examines this in greater detail.

---

on which the angles are less than the two right angles." [O'Shea, 2007]

<sup>4</sup>This word "расслоение", which exactly describes the forking possibilities in (2.1, right) *right* above, has no precise equivalent in English

<sup>5</sup>Parallelarity - State of being parallel, parallelism. 1804 -Mitford, Inquiry 85 The exactness of the parallelarity of its lines. OED 1971

## 2.1 Parallelarity

In the nineteenth century mathematicians were concerned with two dimensions, in the twentieth century with the third dimension, and in the beginning of the twentyfirst century with the work of Grigori Jakowlewitsch Perelman<sup>6</sup> scientists will be able to conquer the forth [Atiyah, 1990], [Gessen, 2009]. This has been the most explosive and productive era in research on dimensional matter in human history, where many concepts of dimensionality have been provided. Perelman resumed the long development of geometry, topology, algebra, and analysis which is so important to the development of generalizations concerning dimensionality.

These had their origins in Euclidean geometry, but from the eighteenth century with Carl Friedrich Gauß's (1777-1855) fundamental assumption<sup>7</sup> that the sum of the three angles of a triangle is less than  $\pi$ , the future of research into dimensionality was set [Göttingen, 2011]. Following this, with space as a posteriori concept, a product of the human mind, Nikolai Ivanovich Lobachevsky (1792-1856) launched his his theory of non-Euclidean geometry<sup>8</sup>, with the development of a hyperbolic geometry where the fifth Euclidean postulate was false.

Another substantial idea was established by Bernhard Riemann (1826-1866), who made a distinction between space and geometry<sup>9</sup>, and defined the manifold as being a type of space consisting of regions which may be named by collections of numbers:

- $\mathbb{R}$  - is a number line;
- $\mathbb{R}^2$  - is a plane;
- $\mathbb{R}^3$  - a three-space which consist from triples of real numbers.

If  $n$  is any positive integer, the set of all ordered  $n$ -tuples of real numbers is a space, which is called  $n$ -space, and designated  $\mathbb{R}^n$ . That points are real numbers,  $n$ -tuples and therefore  $n$ -dimensional, as particular continuous spaces may even be infinite-dimensional and  $n$ -tuple represented as real numbers - which could be even represented as time.

Following from the above, this summary of important terms and properties may be distinguished:

---

<sup>6</sup>In the first proof of the Poincaré conjecture, that states “*any compact three-manifold on which any closed path can be shrunk to a point, is the same topologically as (that is, homeomorphic to) the three-sphere.*” [O’Shea, 2007]

<sup>7</sup>which was made, not at least based on the works of Leonhard Euler (1707-1783).

<sup>8</sup>This theory is rooted in opposition to Kant’s transcendental idealism, where space and time are a priori factors, but was rejected by the Russian Academy of Sciences and went largely unnoticed.

<sup>9</sup>Parallel to the work of Riemann, the concept of multidimensionality was proposed by a Swiss geometer, Ludwig Schläfli’s (1814-1895) in his work “*Theorie der vielfachen Kontinuität*” and by a British mathematician, Arthur Cayley (1821-1895). [Dimensions, 2011]

- *velocity vectors* are based at each point of an  $n$ -dimensional manifold are in direct correspondence with  $\mathbb{R}^n$ ;
- *tangent space* to the manifold at the given point - a set of all possible velocities, or derivatives, of any curve through the point;
- *metric* - a rule for measuring tool between two points;
- *geodesic lines* - shortest distance between points;
- *triangles* defining the *curvature*;
- *Riemann curvature tensor* is a mathematical tool for keeping track of the different curvatures in different directions;
- *homeomorphism* - appropriate notions of equivalence between objects and structures from a topological point of view, where *isometry* is the notion of equivalence for manifolds that carry geometries;
- Possibility of defining of straight lines and measuring angles, curvature measures the deviation from triangles having 180 degrees.
- our universe differs in Euclidean three-space. [Monastyrsky, 1999]

The extension of Riemann's work on investigating functions that were invariant under groups of motions of the complex plane was done by Felix Klein (1849-1925), who was the teacher of David Hilbert (1862-1943), who on his part restructured Euclidean geometry on a more rigorous ground [Hilbert, 1923]. Multidimensional geometries became central mathematical interests.

More Riemann-like than Riemann himself was the French mathematician Henri Poincaré (1854-1912), who did not invent topology, "but that he gave it wings" [O'Shea, 2007]. Poincaré proposed this deep concept about time matter: if one second of today is the same as one second of tomorrow, whether two events in different places happen at the same time. Before Einstein, Poincaré had combined space with time, calling this space-time, which he identified as a mathematical object. [Galison, 2004] From the topological point of view, Poincaré had formulated a fundamental but simple question that emerged in propositions about the shape and nature of the universe and it was this which caused such a development of interest in mathematical circles - starting from the Poincaré conjecture and ending with the Perelman proof.

Based on Richard Hamilton's proposal on the key notion of entropy using the Ricci flow equation<sup>10</sup>, Perelman suggested that the Ricci flow is a geometric object in its own right and allows the connection of manifolds with different topologies at different scales, and besides that analytical matter for obtaining geometric information. In other words the Ricci flow has been a mechanism that

---

<sup>10</sup>In implicit analogy with thermodynamic properties, how heat flows from warmer to cooler areas, the equation shows curvature flow from more curved areas to less curved areas.

processed the manifold, stretching and shaping it, cutting off pieces with homogeneous geometries. Is also important to underline that in Perelman's paper there are no assumptions on curvature or dimensionality applications. Therefore Perelman's work potentially provides new conceptual tools for thinking about space and time and had influence on dimensional time matter. [Perelman, 2002], [O'Shea, 2007]

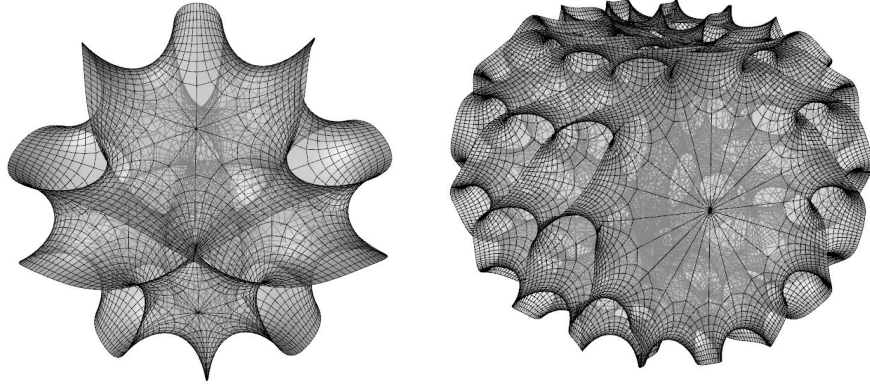


Figure 2.2: *left*: five-dimensional and *right*: ten-dimensional Calabi-Yau Manifolds

The Riemannian geometry lead to so called complex manifolds, as Riemann surfaces (*fig.1.2, right*) or Calabi-Yau manifold (*fig.2.2*) [Kodaira, 2008]. In Calabi-Yau's manifold (*fig.2.2*) when extra dimensions of space-time are conjectured these also take the form of a Calabi-Yau manifold. This geometrical notion of mirror symmetry<sup>11</sup>, was one of the causes of the emergence of string theory [Greene, 1997].

There are many concepts of dimensionality with different intrinsic properties which handle continuity and tearing or continuity, but they all have the same origins in Euclidean and, by extension, of Riemannian geometry.

The Russian mathematicians Nikolai Nikolaevich Luzin (1883-1950) and his students Pavel Aleksandrov (1896-1982) and Andrey Kolmogorov (1903-87) provided the last important input to this field. They discovered a new *set of algebraic structures* associated with manifolds and other topological spaces. These structures, called *cohomology* rings, which consider measures on an arbitrary space as functions, were a kind of mirror image of the homology groups that Poincaré had defined, but they carried two algebraic operations instead of the single one carried by the homology groups. [O'Shea, 2007] The development of

<sup>11</sup>Calabi-Yau manifolds provide a relation of an existing a symmetry between two manifolds, which may have different geometrically shape.

this investigation of topology had its final expression in the concept of finite probability space  $(\Omega, \mathbf{P}, \mathcal{F})$ <sup>12</sup>, formulated by Kolmogorov, which is sufficient to describe all of the significant features of probability theory. The term, finite probability space, is significant as its adoption denoted the establishment of probability theory as a rigorous branch of mathematics.[Kolmogorov, 1974, Kolmogorov, 1987] Another mathematician who made a significant contribution to this field of enquiry was Andrey Markov<sup>13</sup> (1856-1922). Markov chains and Markov processes were the primary results of his career and in his work the topological properties of a graph transition may be implicitly connected to the properties of probability space. These methods of probability will be investigated in this and the following sections to make a basis for a hypothetical prediction on time dimensionality.

---

<sup>12</sup>Where  $\Omega$  be a set,  $\mathcal{F}$  be a set of subsets which are called events of  $\Omega$ .  
 $\mathcal{F}$  is  $\sigma$ -algebra of sets;  $\Omega \in \mathcal{F}$ ;  $\mathbf{P}$  is a function from  $\mathcal{F}$  to  $[0; 1]$ ;  $\mathbf{P}(\Omega) = 1$

<sup>13</sup>It is interesting to note that his student was Georgy Voronoy, after whom the Voronoi diagram that is so popular nowadays in architectural practice is named.

## 2.2 Dimensional Time Moment

It is not an observational fact that time possess deterministic quality, but extrapolation in combination with probability by given parameters, or initial conditions could permit causality which can be extended to an unconditional form. This underlines the unconditional form of causality expressing determinism, which is a limit statement, introducing the convergence of probabilities resting on evidence and inductive inference. This in turn reveals a divergence of probabilities of time intervals. [Reichenbach, 1957, Heidegger, 1967]

If this is translated into mathematics the formulated divergence will lead to a double process of convergence and divergence. This is described with the following schema lattice in connection to the introduced mathematical model in the section 1.2:

$$\begin{array}{c}
 D_1^{(1)} D_1^{(2)} D_1^{(3)} \dots D_1^{(i)} \dots \\
 p_2^{(1)} p_2^{(2)} p_2^{(3)} \dots p_2^{(i)} \dots \rightarrow 1 \\
 p_3^{(1)} p_3^{(2)} p_3^{(3)} \dots p_3^{(i)} \dots \rightarrow 1 \\
 \dots \\
 p_k^{(1)} p_k^{(2)} p_k^{(3)} \dots p_k^{(i)} \dots \rightarrow 1 \\
 p_k^{(1)} p_k^{(2)} p_k^{(3)} \dots p_k^{(i)} \dots \rightarrow 1 \\
 \dots \\
 \check{p} \check{p} \check{p} \dots \check{p} \dots,
 \end{array} \tag{2.1}$$

where:  $D^{(0)}$  are boundary conditions,  $t$  is time,  $E$  predictable future state of initial conditions ( $v$ ). In human language this means that a later state at a later time will be predictable with a lower probability, but one that can be improved by using a more precise description of boundary conditions at the initial time.

The probabilities of all horizontal rows converge to 1<sup>14</sup> and this means that by making any probability statement at all about the occurrence of  $E$  after a long time, the knowledge of the probability of  $E$  must be used in general, and cannot be based on the initial predictions. Therefore the meaning of the divergence of probabilities lies not in the irrelevance of current information, but in the importance of average probability resulting from general statistics. [Reichenbach, 1957]

The defined divergence, refers to boundary conditions. The model of the schema resulting for a certain volume and the conditions of the boundaries,

<sup>14</sup>that calls the *antecedent probability*  $p$  of the state  $E$  in the vertical



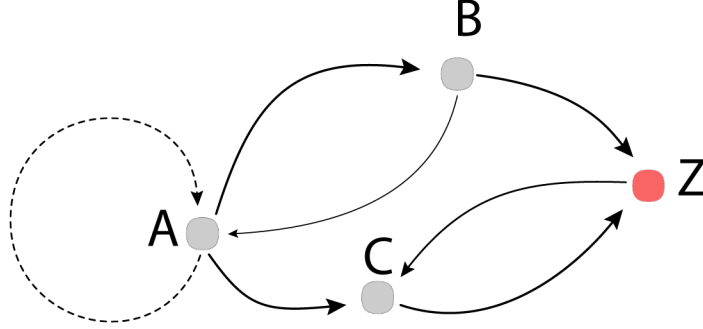


Figure 2.3: Markov chain

includes a description of the conditions at the boundaries of volume during  $[t_1, t_k]$ . A set of individual descriptions, each of which belongs to  $[t_k, t_m]$  according to the given conditions and output probabilities  $p_k^{(i)}$ ,  $p_m^{(i)}$ , characterizes the upper schema as a lattice of nonuniform convergence. In other words, an ambiguity duality is observed, which is familiar from the theory of limits, where a double transition to a limit often leads to differing results<sup>15</sup>. This in turn casts doubt concerning the predictive value of the schema 2.1, because it cannot be regarded as inductive evidence for the existence of nonuniform convergence. [Reichenbach, 1957]

A probability metric can be derived from not rigorous physical laws, by dealing with high probability to low probability laws. Following from this is the notion that there are no strict rules governing the universe and therefore time properties. If there are no strict rules governing a proper probability chain then there can be no determinism. According to this interpretation, the definition of an arrow of time - the order of time by means of causal laws in ordered causal chains, the entropy law shows that the reverse of a thermodynamical process is possible but very improbable, as was considered in the first chapter.

Considering entropy as a time factor in a stochastic process which could be characterized by the joint probability distribution which is time invariant  $t = t_n$ , it has a reciprocal input-output implicit connection<sup>16</sup> to the Markov chain process where the event outcome is at time  $t_{n+1}$  [Desurvire, 2009, Propp, 1985].

The usual Markov chain is a random process and is characterized as being memory-less and therefore having an irreversible mathematical character, but

<sup>15</sup>in the case of the results arising from the performance of an order

<sup>16</sup>which is based on Kolmogorov's forward equation

reversibility is also common and depends on probability distribution.

A Markov chain may be represented as an  $n$  number of vertexes  $A, B, C, Z$  and  $m$  number of edges  $AA, AB, BA, BZ, ZC, CZ, AC$ . A matrix below will be filled in the time interval:

$$\begin{array}{c} A \quad B \quad C \quad Z \\ \begin{array}{c} A \\ B \\ C \\ Z \end{array} \left[ \begin{array}{cccc} 0.1 & 0.6 & 0.3 & 0 \\ 0.2 & 0 & 0 & 0.8 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right] \end{array}$$

where the probability transition from stage  $A$  into stage  $B$  is  $P(A \rightarrow B) = 0.6$ . The main transition property of the matrix is from the formula:

$$\forall i \sum_j P(i \rightarrow j) = \sum_j P(j|i) = 1.$$

The calculation starts when the system is situated in the condition  $A$ , and a zero value appears in the row  $A$  in a case where there is no reversibility. At this stage  $A$  will generate per random  $RND(0, 1)$  to the next stage which in the example is equal to 0.75 and so on. After all the transitions are generated, a chain will be generated  $ABZ$ . By growing the complexity of the graph there may arise positive recurrent Markov chains for which, when in a stationary state, yield the same Markov chain in distribution if time is reversed [Sigman, 2009, Levin, 2012]. In other words, there is appearing a possibility of roaming about the graph space backwards and forwards on time at the same time.

This notion was laid down as the basis of the “Time Machine” project [Demin, 2010] on the calculation of social processes using a Markov chain and forwarding the building time calculation dependent upon multiple data.. The “Time Machine” project showed that backwards and forwards traveling in time could be happening simultaneously because of the parallel calculation of time-dependent data from the community considered by this system. In this instance time was considered as a moment of probable value where future and past values before were equally real. This was a paradoxical calculation technique of the moment of time, which led to a translation from discrete occasion/case time as a finite distribution of the Markov chain and could be also represented as the Chapman-Kolmogorov equation, using mathematical instruments from thermodynamics until quantum physics.

Although proceeding from different knowledge theories, this understanding of a time moment is anyway always a point on the arrow of time. Logically, this model does not contradict from the quantum mechanics point of view<sup>17</sup> - a

<sup>17</sup>which uses the Einstein-Smoluchowski relation instead of a mathematical model in the form of the Chapman-Kolmogorov equation.

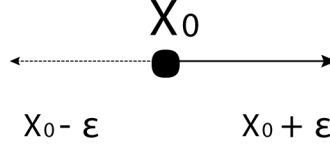


Figure 2.4: Diagram of infinitesimally small reversibility emergence

point, just like Planck's indivisible time, is a basic event. It is indivisible; to it is assigned a probability measure.

In recent years there is also a theory that appears from the Russian side. This says that trajectories are allowed to change the sign of time where loops (as time-space trajectories) are emerging and a particle can change the sign of the time. The elementary event is one path, which extends to the fact that the path is allowed to change - to run backwards with a movement opposite to the time moment. The formula is  $P \sim \cos S$  where the probability of the patch movement, because of the cos, allows the emergence of a space-time loop and therefore in one moment of time the value of time could be negative or positive. At this moment the appearance of time dimensionality can be observed<sup>18</sup>. [Kozyrev, 1991]

By using the methods of predicate logic at point  $x_0$  a dimensional manifold can be propose, with an infinitesimally small number  $\varepsilon$  as a geodesic curve as a shortest distance of  $[t_0, t_1]$ . This is possible if time may be measured as a moment of time, and associated with a time interval. Time can be considered as a system in such a case and can be seen to be given an impulse of oscillation. Furthermore the state of the system in this condition could be translated during an infinitesimally small time moment into stage/condition  $x_0$ :

$$\begin{aligned} \forall x_0 \exists \varepsilon : \{x_k\}_{k=1}^{\infty} : x_k \xrightarrow{K} x_0 \\ x_k \xrightarrow{K} x_0 : \forall \delta > 0 : |x_k - x_K| < \delta \Rightarrow \\ \exists \varepsilon > 0 : |x_k - x_0| < \varepsilon; \end{aligned}$$

and also could be represented in set theory grammar:

<sup>18</sup>This method may explain some paradoxes such as for example the arrow paradox of Zeno, or Schrödinger's cat. In this theory there are oscillations caused by the "loops" of time that it introduces in its turn

$$\forall \varepsilon > 0 \exists K < \infty$$

$$k > K \mid x_k - x_K \mid < \varepsilon$$

where  $\varepsilon = 0.00000001$ ,

$K$  finit time,

$k$  is any number,

Topology allows characterization of manifolds where the curvature is allowed to be zero [Perelman, 1994], where the arrow of time may be understood as multidimensional space, and where only one direction-manifold is explicitly represented and others are tending towards zero. Therefore time will be considered as an abstract vector space-bundle, which has an implicit relationship to aesthetic emergence.

## Chapter 3

# Consequences

“Es ist aber mit der Philosophie in der ihrer Anwendung auf Kunst wie mit der auf Naturlehre angewandten Mathematik;...”  
[Semper, 1852]

Understanding the historical condition of time by seeing through the illusion of the storm that perennially pushes along the infinite rail of linear time, leads to the notion of time generally accepted at the present day, as a single dimensional abstract object that succeeds in transforming the inability of humans to exit traditional deterministic concepts. This thesis puts forward the argument for a different view of time, by using probability theory and the notion of entropy. Following from this, a completely new dimensional object is proposed, which is called the dimensional time moment. This moment or interval perennially gears access to the sum of knowledge capable of reconciling the order between past and future. A property of this dimensional time moment is the loop which emerges at a point in the space-time manifold, as a path that leaves the point and returns. More generally, it is a continuous interval into the manifold so that both endpoints map to the distinguished point. This interval is deemed equivalent if it can be deformed in a continuous manner and can be realized as the set of a chain. It is by this means that a new understanding of time can be gained and time can be seen to have emancipated itself from myth.

This notion of the spatial appearance of additional time dimensions beyond physical and mathematical determination has implicit correspondence to aesthetics: the introduction of time-spatial properties causes shapes to emerge by which organized compositions are produced in natural systems. Ideas and associations can be discovered concerning the aesthetic experience of spatial time matter intersecting with other properties of time - simultaneity, bidirectionally, causation of equilibrium through entropy. Time's topology as a manifold for composition leads to the significance forwarded by phenomenology to architectural surfaces. These ideas and associations can influence the development of

architectural design. Instead of being merely a shape or a number of drawings represented as a set of planes and sections, architectural design viewed from this perspective can be seen to have more in common with film - a set of frames which are similar to the introduced time “coordinates”, being continually suspended between past and future.

Changes are introduced that each time they occur recover the significance of the original measurement in the progressive space. The presentation of an object for simultaneous collective reaction is characterized by the direct, intimate fusion of visual and emotional employment. By introducing the principle of time parallelarity, the key understanding of potential architectural embodiment can be based on the principle of cognitive *organization*, where time is a parameter of architecture which cannot be prescribed and should be distinguished from object time. Cognitive organization provides an underlying rhythm which has the form of a probable matrix where the order may increase. This matrix, by having reversible or irreversible matter, has found an adaptation in symmetrical and asymmetrical emergence (also in the sense of eurythmic symmetry), where behavior might be adaptive by responding to a stable equilibrium. Time dimensionality is a kind of extension mechanism which is needed for the continuous organization of activity that maintains and develops *equilibrium*.

It has been shown that equilibrium has a direct relationship to *entropy*. This concept has its origins in the decoding of the heat phenomenon in thermodynamics that has been expanded upon and has influenced complex areas of science. This concept of entropy has fundamental significance for life today where such things as computation or measurement of the quantity of information or energy is required. Consequently, the measurement of entropy can be seen to be of fundamental importance in the aesthetics of architectural design.

## Chapter 4

# Appendix

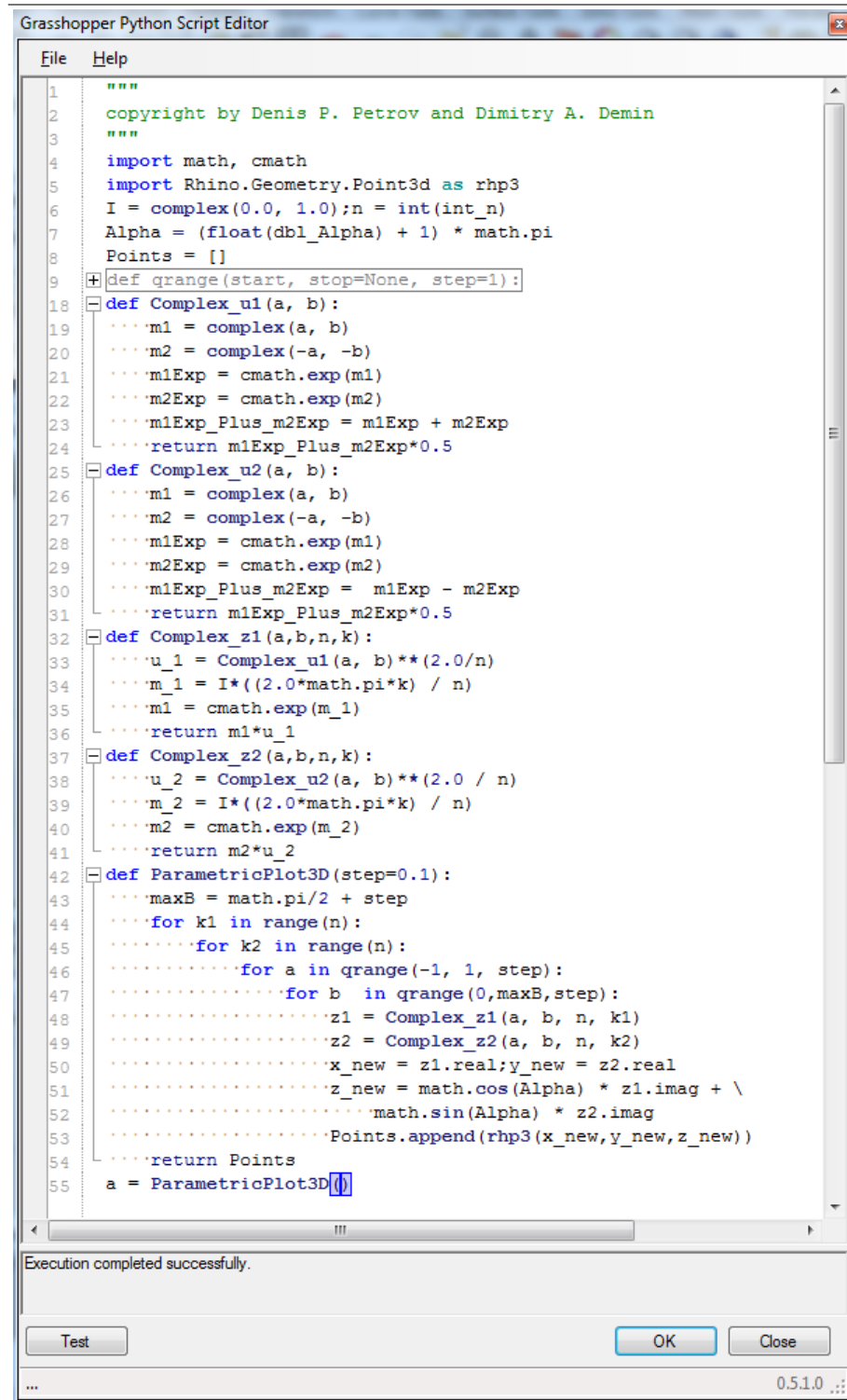
### Calabi-Yau Manifold

The following table contains an algorithm written in IronPython which represents the Calabi-Yau Manifold in Euclidean space. It could be used as a grasshopper patch, for results see [Demin, 2011]. The patch was developed with the assistance of Denis Petrov<sup>1</sup>.

---

<sup>1</sup>petrovdenis [at] gmail [dot] com

**Algorithm 4.1** This Rhinoceros based IronPython cod represents the Calabi-Yau Manifold in Euclidean space.



```

1  """
2  copyright by Denis P. Petrov and Dimitry A. Demin
3  """
4  import math, cmath
5  import Rhino.Geometry.Point3d as rhp3
6  I = complex(0.0, 1.0); n = int(int_n)
7  Alpha = (float(dbl_Alpha) + 1) * math.pi
8  Points = []
9  def qrangle(start, stop=None, step=1):
18 def Complex_u1(a, b):
19     m1 = complex(a, b)
20     m2 = complex(-a, -b)
21     m1Exp = cmath.exp(m1)
22     m2Exp = cmath.exp(m2)
23     m1Exp_Plus_m2Exp = m1Exp + m2Exp
24     return m1Exp_Plus_m2Exp*0.5
25 def Complex_u2(a, b):
26     m1 = complex(a, b)
27     m2 = complex(-a, -b)
28     m1Exp = cmath.exp(m1)
29     m2Exp = cmath.exp(m2)
30     m1Exp_Plus_m2Exp = m1Exp - m2Exp
31     return m1Exp_Plus_m2Exp*0.5
32 def Complex_z1(a,b,n,k):
33     u_1 = Complex_u1(a, b)**(2.0/n)
34     m_1 = I*((2.0*math.pi*k) / n)
35     m1 = cmath.exp(m_1)
36     return m1*u_1
37 def Complex_z2(a,b,n,k):
38     u_2 = Complex_u2(a, b)**(2.0 / n)
39     m_2 = I*((2.0*math.pi*k) / n)
40     m2 = cmath.exp(m_2)
41     return m2*u_2
42 def ParametricPlot3D(step=0.1):
43     maxB = math.pi/2 + step
44     for k1 in range(n):
45         for k2 in range(n):
46             for a in qrangle(-1, 1, step):
47                 for b in qrangle(0,maxB,step):
48                     z1 = Complex_z1(a, b, n, k1)
49                     z2 = Complex_z2(a, b, n, k2)
50                     x_new = z1.real;y_new = z2.real
51                     z_new = math.cos(Alpha) * z1.imag + \
52                         math.sin(Alpha) * z2.imag
53                     Points.append(rhp3(x_new,y_new,z_new))
54     return Points
55 a = ParametricPlot3D()

```

Execution completed successfully.

Test OK Close

0.5.1.0



# Figures

- 1.1 (*left and right*): [wikipedia.org](http://wikipedia.org), accessed April 2011
- 1.1 (*middle*): [\[processing.org\]](http://processing.org), accessed April 2011
- 1.2 (*all*): [wikipedia.org](http://wikipedia.org), accessed April 2011
- 1.3 (*left*): © Jeff Weeks, [geometrygames.org](http://geometrygames.org), accessed May 2012
- 1.3 (*right*): © Wolfram Mathworld™, [mathworld.wolfram.com/ PoincaréHyperbolicDisk.html](http://mathworld.wolfram.com/PoincaréHyperbolicDisk.html), accessed April 2011
- 1.4 (*both*): © Wolfram Mathworld™, accessed April 2011
- 1.5 (*both*): © Daniel Piker, [spacesymmetrystructure. wordpress.com](http://spacesymmetrystructure.wordpress.com), accessed December 2011

# Declaration

I hereby declare that this thesis is my own work and that it has not been previously submitted for any award.

Where other sources of information have been used, they have been acknowledged in the text.

# Bibliography

- [Atiyah, 1990] Atiyah, M., The geometry and physics of knots, Cambridge university press, UK, 1990
- [Aurzada, 2011] Aurzada, F., Döring, H., Ortgiese, M., Scheutzow, M.; Moments of recurrence times for Markov chains, <http://arxiv.org/pdf/1104.1884v1.pdf> , 2011
- [Balmond, 2006] Balmond, C., A+U Special Issue: Cecil Balmond, Japan, 2006
- [Berners-Lee, 2005] Berners-Lee, T., Spinning the Semantic Web: Bringing the World Wide Web to Its Full Potential, The MIT Press, 2005
- [Bolzano, 1851] Bolzano, B, Paradoxien des Unendlichen, Germany, (1975)[1851]
- [Cantor, 1955] Cantor, G, ed.: Jourdain, E.B.P., Contributions to the Founding of the Theory of Transfinite Numbers, New York, (1955) [1915]
- [Carroll, 1865] Carroll, L., Alice's Adventures in Wonderland, UK, 1865
- [Cazeaux, 2011] Cazeaux, C., The Continental Aesthetics Reader, 2011
- [Conway, 1996] Conway, J. H. and Guy, R. K. The Book of Numbers. New York: Springer-Verlag, 1996
- [Craig, 2007] Craig, W., Hamiltonian Dynamical Systems and Applications, Springer, 2007
- [Crivelli, 2008] Crivelli, F., The Störmer-Verlet method, 2008, (<http://www.math.ethz.ch/education/bachelor/seminars/fs2008/nas/crivelli.pdf>)
- [Davies, 2009] Davies, S., Higgins, K., M., Hopkins, R., Stecker, R., Cooper D. E., A Companion to Aesthetics, Wiley-Blackwell, 2009, (<http://www.scribd.com/doc/54781196/A-Companion-to-Aesthetics>)
- [Demin, 2010] Demin, D., Kadivar, M., Time Machine, <http://vimeo.com/11826766>, 2010

- [Demin, 2011] Demin, D., Petrov, D., Calabi–Yau Manifold in Grasshopper, <http://vimeo.com/32417376>, 2011
- [Desurvire, 2009] Desurvire, E., Classical and Quantum Information Theory: An Introduction for the Telecom Scientist, Cambridge University Press, 2009
- [Dimensions, 2011] Leys, J., Ghys, É., Alvarez, A., Dimensions (film series), [www.dimensions-math.org](http://www.dimensions-math.org), 2011
- [Eliot, 1941] Eliot T.S. , Burnt Norton, Faber and Faber, UK, 1941
- [Feynman, 1963] Feynman, R., Six Easy Pieces, California Institute of Technology, 1963
- [Fraser, 2007] Fraser, J.T., Time and Time Again. Reports from a Boundary of the Universe, BRILL, 2007
- [Galison, 2004] Galison, P., Einstein’s Clocks, Poincare’s Maps: Empires of Time, W W Norton & Co, 2004
- [Gamow, 1988] Gamow, G., One Two Three... Infinity Facts and Speculations of Science, 1988
- [Geroch, 1981] Geroch, R., General Relativity from A to B, 1981
- [Gessen, 2009] Gessen M., Perfect Rigor: A Genius and the Mathematical Breakthrough of the Century, Harcourt Brace & Co, USA, 2009
- [Giedion, 1976] Giedion, S., Raum, Zeit, Architektur: Die Entstehung einer neuen Tradition, Birkhäuser, (1976)[1996]
- [Glasichev, 2002] Glasichev, V.,L., Architecture, Moskva AST, Russia, 2002, (rus)
- [Göttingen, 2011] Kurzer Führer durch das Universitätsarchiv Göttingen, <http://wwwuser.gwdg.de/~uniarch/>, accessed July 2011
- [Gray, 2009] Gray, R.,M., Entropy and Information Theory, Information Systems Laboratory Electrical Engineering Department Stanford University, 2009 (©1990 by Springer Verlag. Revised 2009 by R. M. Gray)
- [Greene, 1997] Greene, B., R., String Theory on Calabi-Yau Manifolds, [arxiv.org/pdf/hep-th/9702155.pdf](http://arxiv.org/pdf/hep-th/9702155.pdf), 1997
- [Gromov, 2011] Gromov, M., Structures, Learning and Ergosystems, <http://www.ihes.fr/~gromov/PDF/ergobrain.pdf>, 2011
- [Hardt, 2001] Hardt, M., Negri, A., Empire, Harvard University Press, USA, 2001

- [Hawking, 2008] Hawking, S., Mlodinow, L., A Briefer History of Time, Bantam, Reprint edition ,2008
- [Heidegger, 1967] Heidegger, M., Sein und Zeit, Max Niemeyer Verlag, Tübingen (Germany), 1967
- [Hilbert, 1923] Hilbert, D., Die Grundlagen der Geometrie, Leipzig, 1903
- [Hilbert, 1930] David Hilbert's Radio Broadcast, <http://topo.math.upsud.fr/~lcs/Hilbert/HContest.htm>, accessed April 2011
- [Hodgkin, 2005] Hodgkin, L., A History of Mathematics: From Mesopotamia to Modernity, Oxford University Press, USA, 2005
- [Hoflander, 1979] Hoflander, D.R., Gödel, Escher, Bach: An Eternal Golden Braid, Basic Books, USA, 1979
- [Hovestadt, 2010] Hovestadt, L., Beyond the Grid - Architecture and Information Technology: Applications of a Digital Architectonic, Birkhäuser, 2010
- [Ilachinski, 2001] Ilachinski A., Cellular automata: a discrete universe, UK, 2001
- [Kahneman, 2011] Kahneman, D., Thinking, Fast and Slow, Penguin books, 2011
- [Kaku, 2006] Kaku, M., Parallel Worlds: A Journey Through Creation, Higher Dimensions, and the Future of the Cosmos, Anchor Books, USA, 2006
- [Kodaira, 2008] Kodaira, K., Complex Manifolds and Deformation of Complex Structures, Springer Berlin Heidelberg, 2008
- [Kolmogorov, 1987] Kolmogorov, A.N., Information theory and Theory of computation, Moskow, 1987, (rus)
- [Kolmogorov, 1974] Kolmogorov, A.N., Introducing Probability Theory, Moskow, sec.ed. 1974, (rus)
- [Kozyrev, 1991] Kozyrev, N., A., Selected Works, Leningrad State Univ., Russia, 1991, (rus)
- [Kwinter, 2008] Kwinter, S., Davidson, C., Far from Equilibrium: Essays on Technology and Design Culture, Actar, 2008
- [Landes, 1983] Landes, D. S., Revolution in Time, 1983
- [Lemw, 1980] Lemw, S., Summa technologiae, Volk und Welt, 1980

- [Levin, 2012] Levin, D., A., Peres, Y., Wilmer, E., L., Markov Chains and Mixing Times, [pages.uoregon.edu/dlevin/MARKOV/markovmixing.pdf](http://pages.uoregon.edu/dlevin/MARKOV/markovmixing.pdf), accessed October 2012
- [Li, 2011] Li, T., M., Cellular Automata, Nova Science Publishers, Inc. New York, US, 2011
- [Loos, 2010] Loos, A., Opel A., (pub.), Adolf Loos Gesammelte Schriften, Lesethek, Germany, 2010
- [Lowith, 1997] Lowith, K., Nietzsche's Philosophy of the Eternal Recurrence of the Same, University of California Press, 1997
- [Machiavelli, 1513] Machiavelli, N., The Prince (Italian: Il Principe), (1513)[1992]
- [Manin, 1990] Manin, Yu., Panchishkin, A., Number theory, Introduction, VINITI, Moscow, 1990
- [Martzloff, 1988] Martzloff, J.-C., Histoire des Mathématiques chinoises, Masson, 1988
- [mathworld, 2011] <http://mathworld.wolfram.com/>
- [McClure, 2005] McClure, R., The Philosophy of Time: Time before Times, 2005
- [McMullen, 2010] McMullen, C. T., The evolution of geometric structures on 3-manifolds, <http://www.math.harvard.edu/~ctm/papers/home/text/papers/evo/evo.pdf>, 2010
- [Mersini, 2011] Mersini-Houghton, L., Is Eternal Inflation Eternal ?, <http://arxiv.org>, 2011
- [Monastyrsky, 1999] Monastyrsky, M., I., Riemann, Topology, and Physics, Birkhäuser Boston, 2nd edition, 1999
- [Novikov, 1990] Novikov I.D. , The River of Time, Moscow, 1990
- [O'Shea, 2007] O'Shea, D., The Poincaré conjecture. In Search of the Shape of the Universe, Walker & Company, New York, 2007
- [Pavlov, 2004] Pavlov, D., G., Chronometry of the three-dimensional time, <http://hypercomplex.xpsweb.com/articles/151/en/pdf/01-03-e.pdf>, 2004
- [Penrose, 2004] Penrose, R., The road to reality. A complete Guide to the Laws of the Universe, Jonathan Cape, London, 2004

- [Perelman, 1994] Perelman, G., "Proof of the Soul Conjecture of Cheeger and Gromoll," *Journal of Differential Geometry*, 40, p.:299-305, 1994
- [Perelman, 2002] Perelman, G., The Entropy Formula for the Ricci Flow and Its Geometric Applications, <http://arxiv.org/abs/math.DG/0211159>, 11 November 2002, Ricci Flow with Surgery on Three-Manifolds, <http://arxiv.org/abs/math.DG/0303109>, 10 March 2003, Finite Extinction Time for the Solutions to the Ricci Flow on Certain Three-Manifolds, <http://arxiv.org/abs/math/0307245>, 17 July 2003
- [Peterson, 1998] Peterson, I., *Mathematische Expeditionen*, Spektrum Verlag, 1998
- [Poincaré, 2012] Archives Henri Poincaré, <http://Poincaré.univ-nancy2.fr/>, accessed May 2012
- [Prigogine, 2003] Prigogine, I., *Is future given?*, World Scientific Publishing Co. Pte. Ltd., 2003
- [Price, 1997] Price, H., *Time's Arrow and Archimedes' Point: New Directions for the Physics of Time*, 1997
- [processing.org] <http://www.processing.org/>, May, 2011
- [Propp, 1985] Propp, M., B., *The Thermodynamic Properties of Markov Processes*, MIT, 1985
- [Rakov, 1996] Rakov, D., Morphological Synthesis Method of the Search for Promising Technical Systems, *IEEE Aerospace and Electronic Systems magazine*, december 1996, p.3-8
- [Reichenbach, 1957] Reichenbach, H., *The Philosophy of Space and Time*, 1957
- [Reichenbach, web] <http://www.iep.utm.edu/reichenb/>, Hans Reichenbach (1891—1953), September, 2012
- [rhino3d.com] <http://www.rhino3d.com/>, May, 2011
- [Rochelle, 1998] Rochelle, G., *Behind Time: The incoherence of time and McTaggart's atemporal replacement*, 1998
- [Rundle, 2009] Rundle, B., *Time, Space, and Metaphysics*, Oxford University Press, USA, 2009
- [Russell, 1910] Whitehead, North A., Russell, B., *Principia Mathematica*, 3 vols, Cambridge University Press, 1910, 1912, and 1913. Second edition, 1925 (Vol. 1)

- [Semper, 1852] Semper, G., Die vier Elemente der Baukunst, Nabu Press, (2010)[1852]
- [Semper, 1860-1863] Semper, G., Der Stil in den technischen und tektonischen Künsten oder Praktische Ästhetik, Nabu Press, (2010)[1860-1863]
- [Shakuntala, 2001] Shakuntala D., The Book of Numbers: Everything you always wanted to know about numbers but was difficult to understand, Orienat Paperbacks, Delhi, India, 2001.
- [Sigman, 2009] Sigman, K., Time-reversible Markov chains, [www.columbia.edu/~ks20/stochastic-I/stochastic-I-Time-Reversibility.pdf](http://www.columbia.edu/~ks20/stochastic-I/stochastic-I-Time-Reversibility.pdf), 2009
- [Sinclair, 2006] Sinclair, N., Pimm, D., Higginson W., (Eds.), Mathematics and the Aesthetic: New Approaches to an Ancient Affinity, Springer Science+Business Media, LLC, NY, USA, 2006
- [Sorabji, 2006] Sorabji, R., Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages, 2006
- [Stanford(time), 2010] Time Travel and Modern Physics, <http://plato.stanford.edu/entries/time-travel-phys/>, 2010
- [Stanford(axm.math), 2010] Philosophy of Mathematics, <http://plato.stanford.edu/entries/philosophy-mathematics/>, 2010
- [Tao, 2006] Tao, T., Perelman's proof of the Poincaré conjecture: a nonlinear PDE perspective, <http://arxiv.org/abs/math/0610903v1>, 2006
- [Thompson, 1942] Thompson, D.W., On Growth and Form, Cambridge University Press, Cambridge, UK, (1942)[ 1917]
- [Turner, 2009] Turner, A., A Simple Model of the Belousov-Zhabotinsky Reaction from First Principles, Bartlett School of Graduate Studies, University College London, London, 2009
- [Venturi, 1977] Venturi, R., Complexity and Contradiction in Architecture, Harry N. Abrams, 2nd ed., 1977
- [Volkenstein, 2009] Volkenstein, M., V., Entropy and Information, Birkhäuser, 2009 (1986)
- [Vrobel, 2011] Vrobel, S., Fractal Time: Why a Watched Kettle Never Boils (Studies of Nonlinear Phenomena in Life Science), 2011
- [von Borries, 2007] von Borries, F., Walz, S.P., Böttger, M., Space Time Play: Synergies Between Computer Games, Architecture and Urbanism, Birkhäuser Architektur, 2007



- [Wardrip-Fruin, 2003] Wardrip-Fruin, N., Montfort N., The New Media Reader, The MIT Press, UK, 2003
- [Watson, 2011] Watson (computer), <http://www-03.ibm.com/innovation/us/watson/>, May, 2011
- [Wechsler, 1988] Wechsler, J., On Aesthetics in Science, Birkhäuser, 1 edition, 1988
- [Whitford, 1999] Whitford, F., Kandinsky Aquarelle, Royal Academy of Arts, London, UK, 1999
- [Witten, 1987] Witten, E., Green, M., B., Schwarz, J., H., Superstring Theory: Vol. 1, Introduction, Cambridge University Press, 1987
- [Wolfram, 2002] Wolfram, S., A New Kind of Science, Wolfram Media, USA, 2002
- [Worringer, 1976(1907)] Worringer, W., Abstraktion und Einfühlung, München, Germany, (1976)[ 1907]
- [Zesdin, 1984] Zesdin, F.J., Inexhaustibility of the Universe, Moskow, 1984, (rus)