1.a) $\omega = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}$

$$= \left(\frac{\partial v}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial v}{\partial \eta} \cdot \frac{\partial \eta}{\partial x}\right) - \left(\frac{\partial u}{\partial \xi} \cdot \frac{\partial \xi}{\partial y} + \frac{\partial u}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}\right)$$

$$= \left(\frac{\partial v}{\partial \xi} \xi_x + \frac{\partial v}{\partial \eta} \cdot \eta_x\right) - \left(\frac{\partial u}{\partial \xi} \xi_y + \frac{\partial u}{\partial \eta} \eta_y\right)$$

b) $\dfrac{\partial T}{\partial t} = \dfrac{\partial}{\partial x}\left[k \dfrac{\partial T}{\partial x}\right] + \dfrac{\partial}{\partial y}\left[k \dfrac{\partial T}{\partial y}\right]$

$$= \frac{\partial}{\partial \xi}\left[k\left(\frac{\partial T}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial T}{\partial \eta} \cdot \frac{\partial \eta}{\partial x}\right)\right] \frac{\partial \xi}{\partial x} +$$

$$\frac{\partial}{\partial \eta}\left[k\left(\frac{\partial T}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial T}{\partial \eta} \cdot \frac{\partial \eta}{\partial x}\right)\right] \frac{\partial \eta}{\partial x}$$

$$+ \frac{\partial}{\partial \xi}\left[k\left(\frac{\partial T}{\partial \xi} \cdot \frac{\partial \xi}{\partial y} + \frac{\partial T}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}\right)\right] \frac{\partial \xi}{\partial y}$$

$$+ \frac{\partial}{\partial \eta}\left[k\left(\frac{\partial T}{\partial \xi} \cdot \frac{\partial \xi}{\partial y} + \frac{\partial T}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}\right)\right] \frac{\partial \eta}{\partial y}$$

$$\Rightarrow \frac{\partial T}{\partial t} = \frac{\partial}{\partial \xi}\left[k\left(\frac{\partial T}{\partial \xi}\cdot \xi_x + \frac{\partial T}{\partial \eta}\cdot \eta_x\right)\right]\xi_x$$

$$+ \frac{\partial}{\partial \eta}\left[k\left(\frac{\partial T}{\partial \xi}\cdot \xi_x + \frac{\partial T}{\partial \eta}\, \eta_{\cancel{x}}\right)\right]\eta_x$$

$$+ \frac{\partial}{\partial \xi}\left[k\left(\frac{\partial T}{\partial \xi}\, \xi_y + \frac{\partial T}{\partial \eta}\, \eta_y\right)\right]\xi_y$$

$$+ \frac{\partial}{\partial \eta}\left[k\left(\frac{\partial T}{\partial \xi}\, \xi_y + \frac{\partial T}{\partial \eta}\, \eta_y\right)\right]\eta_y.$$

$t+dt$ x

Laa Freidrich scheme

2)
$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + c\,\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} = 0$$

$$\Rightarrow u_i^{n+1} = u_i^n - \frac{c\Delta t}{2\Delta x}\left(u_{i+1}^n - u_{i-1}^n\right)$$

Lets assume $u(x,t) = \underbrace{e^{-ck^i t}}_{\text{growth factor}} e^{ikx}$

$$\therefore u_i^{n+1} = e^{(ik\Delta x)i} + P\left[e^{(ik\Delta x)(i+1)} - e^{(ik\Delta x)(i-1)}\right.$$

$\hookrightarrow P = -\frac{c\Delta t}{2\Delta x}$

$$= \underbrace{\left[1 + P\left(e^{(ik\Delta x)} - e^{-(ik\Delta x)}\right)\right]}\, e^{(ik\Delta x)i}$$

$$= \left[1 + 2P\,\sin(k\Delta x)\right] e^{(ik\Delta x)i}$$

$1 + 2P\sin(k\Delta x) \leq 1$

$\Rightarrow k\Delta x \leq 0$

$\Rightarrow \Delta x \leq 0$

Mac cormack scheme

$$u_i^{n+1} = \frac{1}{2}\left(\overline{u_i^{n+1}} + \overline{\overline{u_i^{n+1}}}\right)$$

$$= \frac{1}{2}\left[u_i^n - \frac{\Delta t}{\Delta x}\left(f_{i+1}^n - f_i^n\right) + u_i^n - \frac{\Delta t}{\Delta x}\left(\overline{f_{i+1}^n} + \overline{f_i^n}\right)\right]$$

$$u(x,t) = e^{-ckt}\, e^{ikx}$$

$$u_i^{n+1} = \frac{1}{2}\left[e^{(ik\Delta x)i} - \frac{c\Delta t}{\Delta x}\left(e^{(ik\Delta x)(i+1)} - e^{(ik\Delta x)i}\right) + e^{(ik\Delta x)i} - \frac{\Delta t}{\Delta x}\left(e^{-(ik\Delta x)(i+1)} - e^{-(ik\Delta x)i}\right)\right]$$

$$= \frac{1}{2}\left[2 - \frac{c\Delta t}{\Delta x}\left(e^{(ik\Delta x)} - 1\right) - \frac{\Delta t}{\Delta x}\left(e^{-ik\Delta x} - e^{-2(ik\Delta x)}\right)\right] e^{ik\Delta x}$$

$$\frac{1}{2}\left[2 - \frac{c\Delta t}{\Delta x}\left(e^{ik\Delta x} - 1\right) - \frac{\Delta t}{\Delta x}\left(e^{-ik\Delta x} - e^{-2(ik\Delta x)}\right)\right] \leq 1$$

??

$\Rightarrow$

3.

$$T_{steady}(x) = x^2 e^{-x}$$

$$S(x) = -(x^2 - 4x + 2)\, e^{-x}$$

$$\frac{d^2T}{dx^2} = \frac{d^2}{dx^2}\left(x^2 e^{-x}\right)$$

$$= \frac{d}{dx}\left[-x^2 e^{-x} + 2x e^{-x}\right]$$

$$= -(-x^2 e^{-x} + 2x e^{-x}) + 2e^{-x} - 2x e^{-x}$$

$$= x^2 e^{-x} - 4x e^{-x} + 2e^{-x}$$

$$= (x^2 - 4x + 2)\, e^{-x}$$

$$\therefore \frac{d^2T}{dx^2} + S_x = (x^2 - 4x + 2)\, e^{-x} - (x^2 - 4x + 2)\, e^{-x}$$

$$= 0$$

$$= \frac{\partial T}{\partial t} = 0$$

$\therefore$ T_{steady} is indeed a exact solution.