AL HABIB ULLAH

1

$$A_1 x = A_2 x + b$$

$$A_1 x^K = A_2 x^{(K-1)} + b$$

$$\Rightarrow \boxed{x^{(K)} = (A_1^{-1} A_2)\, x^{(K-1)} + A_1^{-1} b}$$

$$x = (A_1^{-1} A_2)\, x^{(0)} + A_1^{-1} b$$

$$\begin{aligned} \epsilon^{(K)} &= x - x^{(K)} \\ &= (A_1^{-1} A_2)\, x^{(0)} + A_1^{-1} b - (A_1^{-1} A_2)\, x^{(K-1)} - A_1^{-1} b \\ &= (A_1^{-1} A_2)\,(x^0 - x^{(K-1)}) \end{aligned}$$

$$\boxed{\epsilon^{(K)} = (A_1^{-1} A_2)\, \epsilon^{(0)}}$$

b)

$$A = \begin{bmatrix} -2 & 1 & \cdots & \cdots \\ 1 & -2 & 1 & \cdots \\ \cdot & \cdot & \cdot & \cdot \\ \cdots & \cdots & 1 & -2 \end{bmatrix}$$

$$x^{(k+1)} = (I + \alpha A)\,x^{(k)} - \alpha b$$

$$x^{(k+1)} - x^{(k)} = \left[(I+\alpha A)\,x^{(k)} - \alpha b\right] - \left[(I+\alpha A)\,x^{(k-1)} - \alpha b\right]$$

$$= (I+\alpha A)\,(x^{(k)} - x^{(k-1)})$$

$$= (I+\alpha A)^2\,(x^{(k)} - x^{(k-2)})$$

$$= (I+\alpha A)^k\,(x^{(k)} - x^{(0)})$$

$$= (I+\alpha A)^k\,x^{(k)}$$

we want x^{k+1} to be close to x^k when the norm of their difference is small fraction of the norm $x^{(k)}$.

$$\left\| x^{(k+1)} - x^{(k)} \right\| = \left\| (I - \alpha A)^{k} x^{(k)} \right\|$$

$$\leq \left\| 1 - 2\alpha + 2\alpha \cos\left(\frac{\pi j}{N}\right)^{k} \right\| \left\| x^{(k)} \right\|$$

$$\leq$$

??

2|

$$A_1 x^{(k+1)} = A_2 x^{(k)} + f$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Scheme-1

$$A_1 = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & -b \\ -c & 0 \end{bmatrix}$$

Scheme-2

$$A_1 = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 \\ -c & 0 \end{bmatrix}$$

$$A_1 x^{(k+1)} = A_2 x^{(k)} + f$$

$$\Rightarrow x^{(k+1)} = A_1^{-1} A_2 x^{(k)} + A_1^{-1} f$$

For convergence the spectral radius must be computed

Scheme -1

eigenvalues are

$$\lambda = \frac{b^{1/2} c^{1/2}}{a^{1/2} d^{1/2}}, \; -\frac{b^{1/2} c^{1/2}}{a^{1/2} d^{1/2}}$$

$|\lambda| < 1.$

Scheme - 2

$$\lambda = 0, \frac{bc}{ad}$$

$|\lambda| < 1$

$ad > bc.$