ALHABIB ULLAH

1.

$$\frac{\partial \phi}{\partial t} = \alpha \frac{\partial^2 \phi}{\partial x^2}$$

$$\phi = e^{-\alpha k^2 t} e^{ikx}$$

$$= G(K$$

$$\frac{\phi_j^{n+1} - \phi_j^n}{\Delta t} = \alpha \frac{-\phi_{j+3}^n + 4\phi_{j+2}^n - 5\phi_{j+1}^n + 2\phi_j^n}{\Delta x^2}$$

$$\Rightarrow \phi_j^{n+1} = \phi_j^n + \underbrace{\frac{\alpha \Delta t}{\Delta x^2}}_{r} \left(-\phi_{j+3}^n + 4\phi_{j+2}^n - 5\phi_{j+1}^n + 2\phi_j^n\right)$$

$$\Rightarrow \phi_j^{n+1} = e^{ik\Delta x j} + r\left[-e^{ik\Delta x(j+3)} + 4e^{ik\Delta x(j+2)} - 5e^{ik\Delta x(j+1)} + 2e^{ik\Delta x j}\right]$$

$$= \left[1 + r\left(-e^{ik3\Delta x} + 4e^{ik2\Delta x} - 5e^{ik\Delta x} + 2\right)\right] e^{ik\Delta x j}$$

$$= G(K) e^{ik\Delta x j}$$

it will be stable if $G(K) \leq 1$

which is not possible conditions

2.

$$\frac{\partial u}{\partial t} = c\,\frac{\partial u}{\partial x}$$

$$\Rightarrow \frac{1}{\Delta t}\left[u_j^{n+1} - \frac{1}{2}\left(u_{j+1}^{n} + u_{j-1}^{n}\right)\right] = \frac{c}{2\Delta x}\left[u_{j+1}^{n} - u_{j-1}^{n}\right]$$

$$\Rightarrow \quad u_j^{n+1} = \frac{c\,\Delta t}{2\Delta x}\left[u_{j+1}^{n} - u_{j-1}^{n}\right] + \frac{1}{2}\left(u_{j+1}^{n} - u_{j-1}^{n}\right)$$

$$= \left(\frac{c\,\Delta t}{2\Delta x} + \frac{1}{2}\right)\left(u_{j+1}^{n} - u_{j-1}^{n}\right)$$

$$= \left(\frac{c\,\Delta t}{\Delta x} + 1\right)\left(\frac{u_{j+1}^{n} - u_{j-1}^{n}}{2}\right)$$

$$\boxed{\frac{c\,\Delta t}{\Delta x} < 0}$$