3 Stationarity

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Time Series

- Suatu *time series* atau runtun waktu adalah himpunan pengamatan yang berurut dalam waktu.
  - If the set is continuous,the time series is said to be *continuous*.
  - If the set is discrete, the time series is said to be *discrete*.
- Kuliah ini hanya membahas *time series* yang diskrit dengan pengamatan pada waktu t = 1, 2, … , N.

Discrete time series may arise in two ways:

1. **By *sampling*** a continuous time series. Mengambil pengamatan pada waktu-waktu tertentu
   - Nilai tukar rupiah bulanan, yg diamati tiap awal bulan
2. **By *accumulating*** a variable over a period of time. Mengakumulasikan pengamatan pada periode waktu tertentu.
   - rainfall, which is usually accumulated over a period such as a day or a month,
   - GDP, which is accumulated over the quarterly or annual.

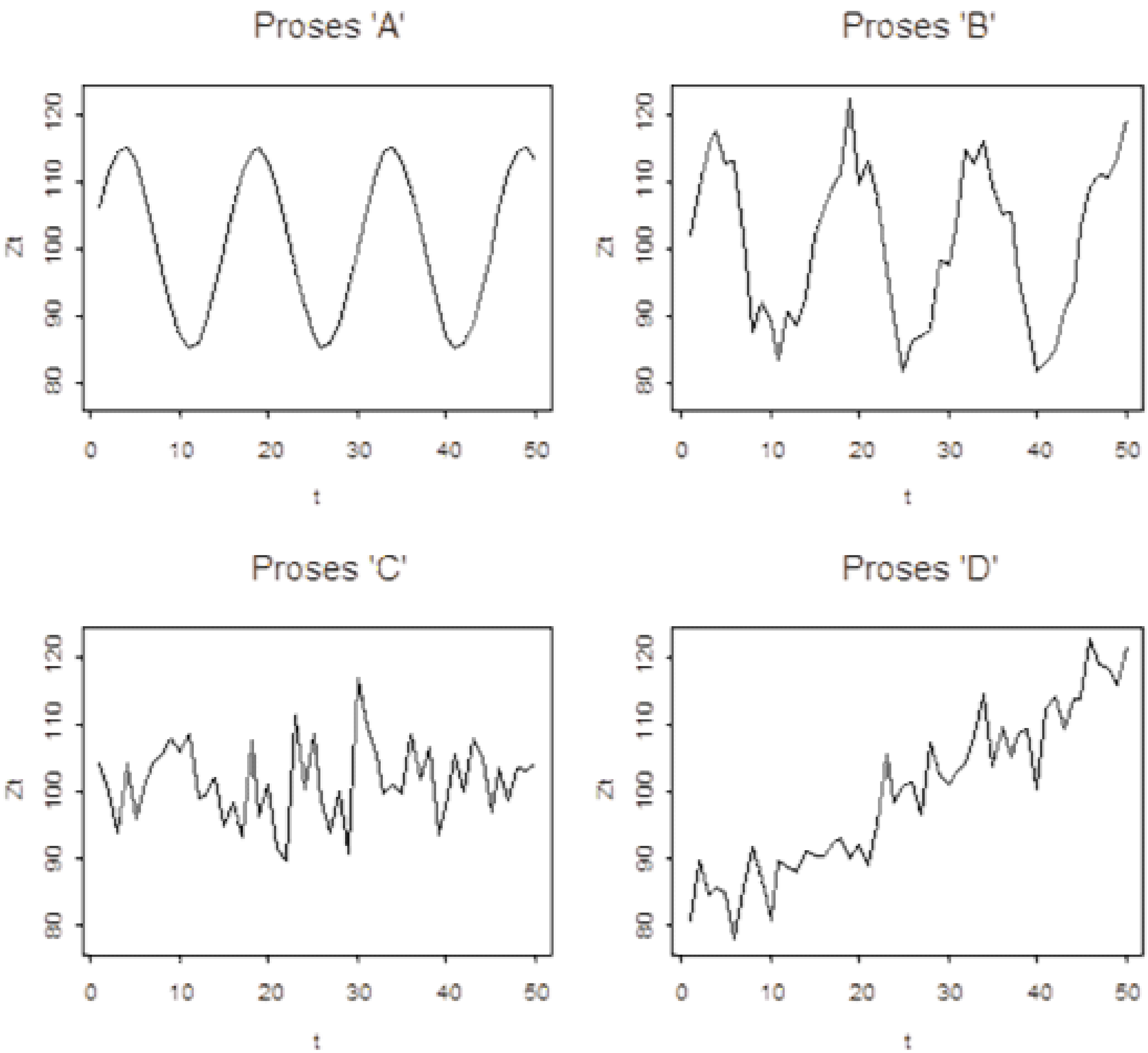
Deterministic and Statistical Time Series

- If future values of a time series are exactly determined by some mathematical function such as:

  $$z_t = \cos(2\pi f_t)$$

  the time series is said to be ***deterministic***.
- If future values can be described only in terms of a probability distribution, the time series is said to be nondeterministic or stochastic, or simply a ***statistical time series***.

  $z_t = \cos(2\pi f_t) + u_t$, *where* u_t *is probabilistic*


Proses 'A'
Proses 'B'
Proses 'C'
Proses 'D'
Zt
t
80
90
100
110
120
0
10
20
30
40
50

Stochastic Processes

- **Stochastic (Random) Process:** collection of random variables ordered in time.
  - **NOTATIONS:** Let **Y** a random variable, **Y(t)** if continuous (e.g. electrocardiogram), and $\mathbf{Y_t}$ if discrete (e.g. GDP, PDI, etc.).
  - Now, If we let **Y** represent GDP, then we can have Y_1, Y_2, Y_3, ... , Y_{20} where the subscript 1 denotes the 1st observation (i.e. GDP for the 1st quarter of 1st year) and the subscript 20 denotes the last observation (i.e. GDP for the 4th quarter of 5th year).

STATIONARITY

Stationary Stochastic Processes

- **Stationary Stochastic Processes:** A stochastic process is said to be stationary/ weakly /covariance/2nd-order stationary if:
  - Its mean and variance are constant over time, and
  - The value of the **covariance** between the two time periods depends only on the distance/lag between the two time periods and not the actual time at which the covariance is computed.
  - E.g. let's $\mathbf{Y_t}$ be a stochastic process, then;
  - Mean: $E(Y_t) = \mu$ .... (1)
  - Variance: $var(Y_t) = E(Y_t - \mu)^2 = \sigma^2$ .... (2)
  - Covariance: $\gamma_k = E[(Y_t - \mu)(Y_{t+k} - \mu)]$ .... (3)
    - Where γ_k, the covariance (or auto-covariance) at lag k,
    - If k = 0, we obtain γ_0, which is simply the variance of Y (= σ^2); if k = 1, γ_1 is the covariance between two adjacent values of Y

- A very special class of stochastic processes, called ***stationary processes***, is based on the assumption that the process is in a particular state of *statistical equilibrium*.
  - Strictly Stationarity
  - Weakly Stationarity
  - Covariance Stationarity
  - 2nd Order Stationarity

(REVIEW) Sifat proses stokastik data yang stasioner:

1. $f(Z_t, \ldots, Z_{t+k}) = f(Z_{t+m}, \ldots, Z_{t+k+m}) \;\; \forall$ m,t,k
2. $E(Z_t) = \mu_Z$ (tidak tergantung pada t)
3. $Var(Z_t) = \sigma^2_Z = E[(Z_t - \mu_Z)^2]$ (tidak tergantung pada t)
4. $\gamma_k = cov(Z_t, Z_{t+k})$; tidak tergantung pada t

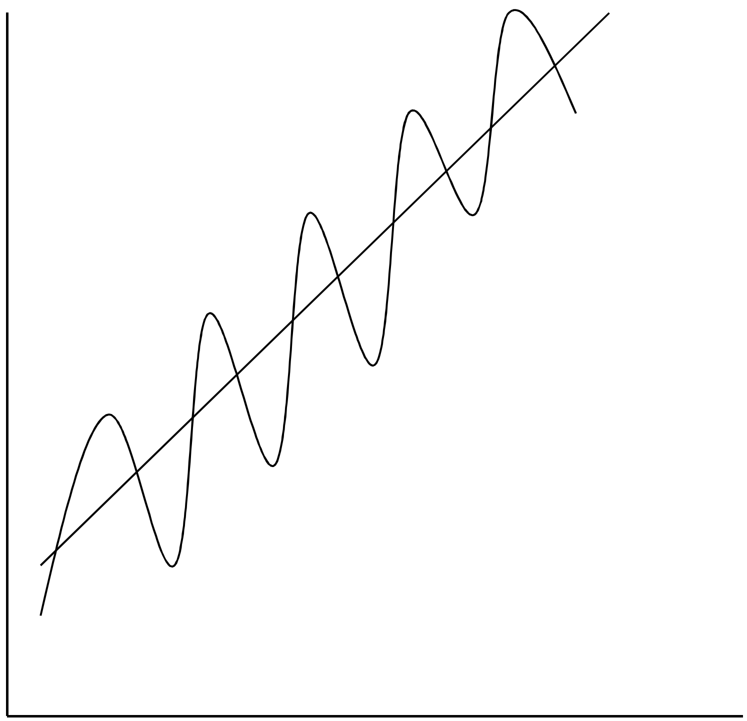
   $= cov(Z_{t+m}, Z_{t+m+k})$

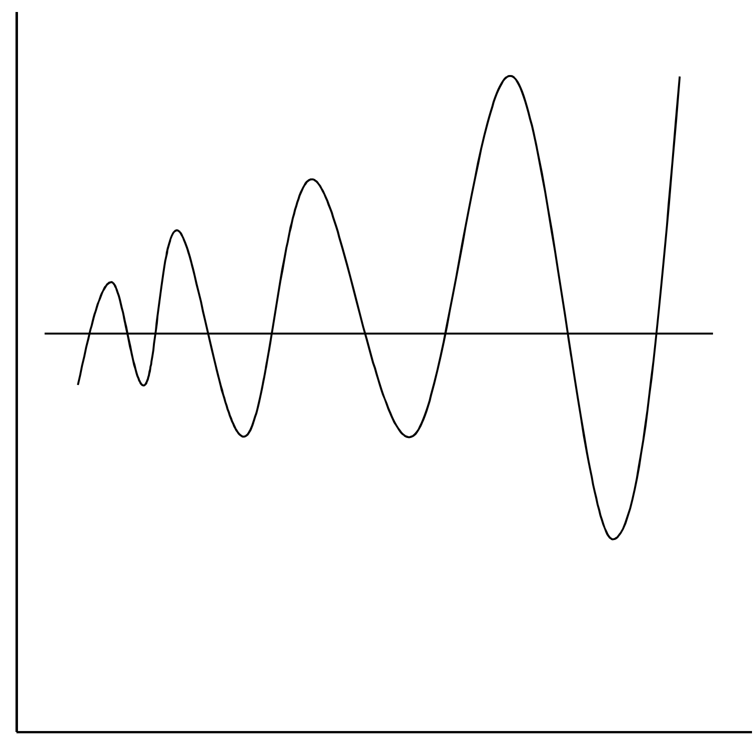
Sebagai catatan: Untuk lag nol, atau k=0, berlaku:

$\gamma_0 = cov(Z_t, Z_t) = var(Z_t) = \sigma^2_Z$

Strict Stationerity (Stationarity)

Suatu proses stokastik dinamakan stasioner (*strict stationerity*) jika *pdf* bersama $f(Z_{t1}, Z_{t2}, ..., Z_{tn})$ dan *pdf* bersama $f(Z_{t1+k}, Z_{t2+k}, ..., Z_{tn+k})$ adalah sama untuk sebarang bilangan bulat positif n dan sebarang pilihan $t1, t2, ..., tn$. Dalam hal ini struktur probabilistik dari proses tidak berubah dengan berubahnya waktu.





Why are Stationary Time Series so Important?

- Jika time series **tidak stasioner**, maka hanya dapat mempelajari perilaku data untuk periode waktu under consideration, tidak bisa untuk forecasting (peramalan)
- Data time series yang tidak stasioner juga bisa menimbulkan **spurious regression** → jika dua variabel dibuat tren, regresi satu dan yang lainnya bisa memiliki R^2 tinggi tapi tidak berkorelasi (meaningless)

White Noise Processes *(stationary process)*

- We call a stochastic process(time series) purely random/white noise process if it has zero mean, constant variance $\boldsymbol{\sigma}^2$, and is serially uncorrelated i.e. [$u_t \sim IIDN(0, \sigma^2)$].
- **Note:** Here onward, in all equations the assumption of "white noise" will be applicable on u_t .

Random Walk Model (RWM)

Non Stationary Stochastic Processes

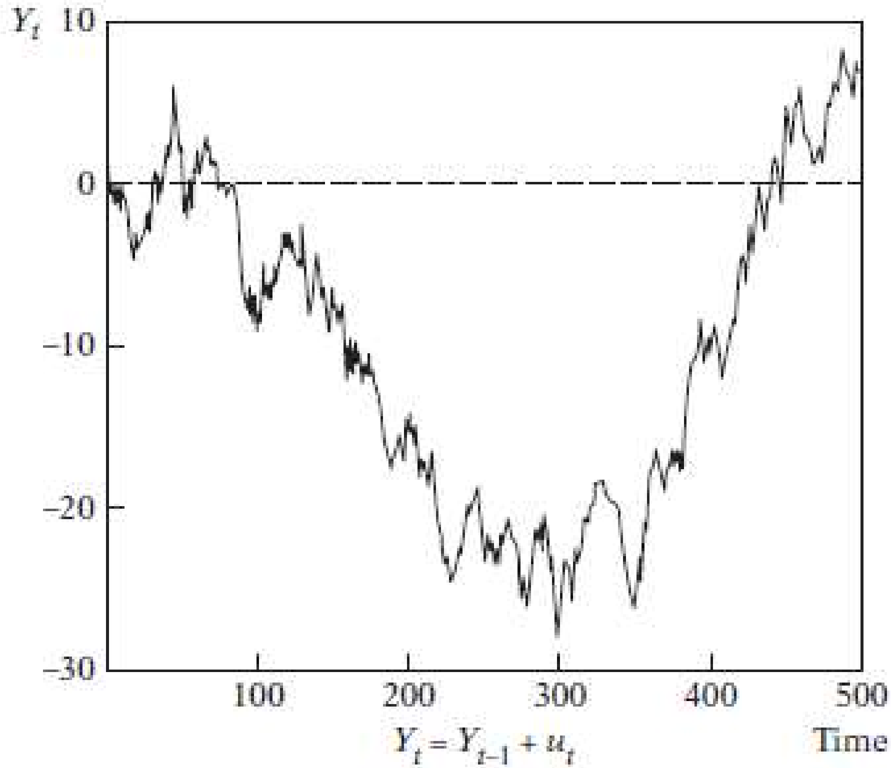
Random Walk merupakan model time series stokastik yang paling sederhana, dan merupakan contoh klasik dari model yang tidak stasioner.

1. ***Random walk without drift*** atau dikenal juga dengan pure random walk → tanpa intercept

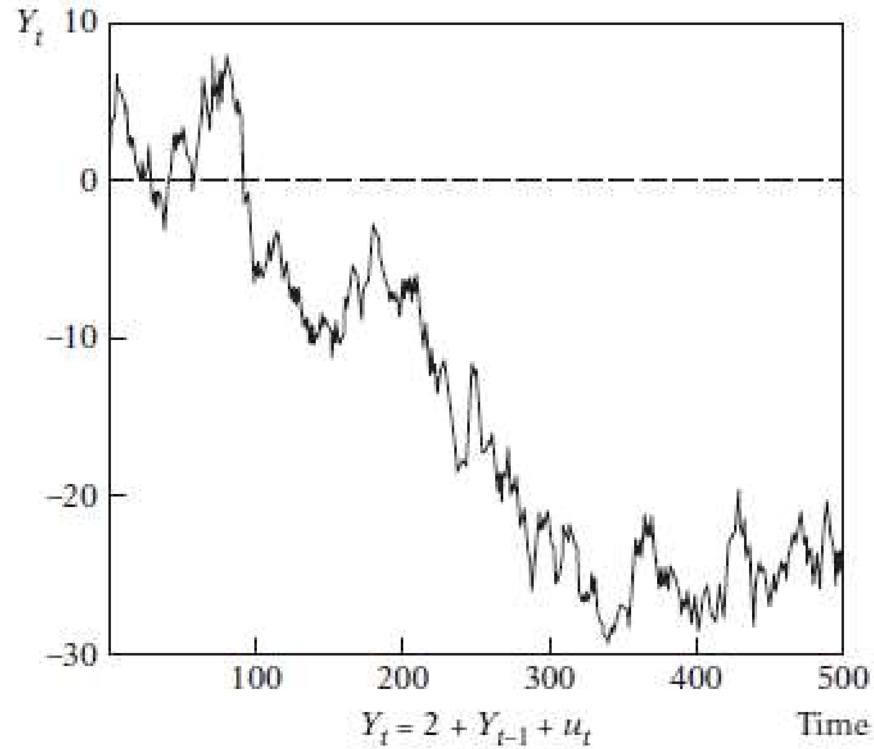
$$Y_t = Y_{t-1} + u_t$$

2. ***Random walk with drift*** → dengan intercept

$$Y_t = \delta + Y_{t-1} + u_t$$



A random walk without drift.



A random walk with drift.

a) Random Walk without Drift

Asumsi pada model ini adalah perubahan nilai Y_t berurutan berdasarkan suatu distribusi probabilitas dengan mean 0. Modelnya dapat dinyatakan dalam bentuk:

$$\mathbf{Y_t = Y_{t-1} + u_t; \quad \text{atau} \quad Y_t - Y_{t-1} = u_t}$$

$$\mathbf{E(u_t) = 0; \quad E(u_t u_s) = 0; \quad t \neq s}$$

Dimana: u_t adalah error yang "*white noise*" atau "*purely random*", dengan mean = 0 dan varian = σ^2.

Model tsb juga dapat diartikan bahwa nilai Y pada waktu ke-t sama dengan nilai Y pada waktu ke-t-1 ditambah random.

Bukti bahwa random walk tidak stasioner:

Model randam walk: $Y_t = Y_{t-1} + u_t$

dapat ditulis dengan:

$$Y_1 = Y_0 + u_1.$$

$$Y_2 = Y_1 + u_2 = Y_0 + u_1 + u_2.$$

$$Y_3 = Y_2 + u_3 = Y_0 + u_1 + u_2 + u_3.$$

Maka:

$$Y_t = Y_0 + \Sigma u_t.$$

Sehingga:

$$E(Y_t\ ) = E(Y_0 + \Sigma u_t) = E(Y_0\ ) + E(\Sigma u_t)$$

Y_0 adalah konstanta, sehingga nilai harapannya konstan, Y_0

u_t adalah "*white noise*", sehingga nilai harapannya = 0.

Jadi: $E(Y_t\ ) = E(Y_0 + \Sigma u_t) = E(Y_0\ ) + E(\Sigma u_t) = Y_0 + 0 = Y_0.$

Maka dapat disimpulkan bahwa rata-rata ***random walk tanpa intersep*** adalah konstan.

Sekarang kita lihat varian-nya, yaitu:

$$\mathbf{Var(Y_t) = Var(Y_0 + \Sigma u_t)}$$
$$\mathbf{= Var(Y_0) + Var(\Sigma u_t)}$$

Y_0 adalah konstanta, sehingga varian-nya = 0.

u_t adalah "white noise", sehingga variannya = σ^2.

Jadi:

$$\mathbf{Var(Y_t) = Var(Y_0 + \Sigma u_t)}$$
$$\mathbf{= Var(Y_0) + Var(\Sigma u_t)}$$
$$\mathbf{= 0 + \Sigma\, \sigma^2}$$
$$\mathbf{= t\, \sigma^2}$$

(tidak konstan, tergantung pada t)

Random Walk dengan Tren

Model: $\mathbf{Y_t = Y_{t-1} + \delta\, t + u_t}$

Bukti model ini tidak stasioner:

$$Y_1 = Y_0 + \delta + u_1$$

$$Y_2 = Y_1 + \delta + u_2 = Y_0 + \delta + \delta + u_1 + u_2$$

……….

$$Y_t = Y_0 + t\,\delta + \Sigma u_t$$

Maka:

$$E(Y_t) = E(\,Y_0 + t\,\delta + \Sigma u_t)$$

$$= Y_0 + t\,\delta$$ *(depend on t)*

$$Var(Y_t) = Var(Y_0 + t\,\delta + \Sigma u_t)$$

$$= t\,\sigma^2$$ *(depend on t)*

b) Random Walk with Drift

- Let's modify, $Y_t = Y_{t-1} + u_t$................(4) as follows:

$$Y_t = \delta + Y_{t-1} + u_t \text{....................}(9)$$

where δ is the drift parameter.

- The name drift comes from the fact that if we write the preceding equation as:

$$Y_t - Y_{t-1} = \Delta Y_t = \delta + u_t \quad \text{.............} \qquad (10)$$

- It shows that Y_t drifts upward/downward, depending on δ being positive/negative.

Further Explanation

- Note that model $Y_t = \delta + Y_{t-1} + u_t$ .... (9) is also an AR(1) model.
- Following the procedure discussed for Random Walk Without Drift, it can be shown that for the random walk with drift model (9),

$$E(Y_t) = Y_0 + t \cdot \boldsymbol{\delta} \text{ .... } (11)$$

$$\text{var } (Y_t) = t\boldsymbol{\sigma}^2 \text{ .... } (12)$$

- Here, again for RWM with drift the mean as well as the variance increases over time, again violating the conditions of stationarity.
- In short, RWM, with or without drift, is a non-stationary stochastic process.
- The random walk model is an example of what is known in the literature as a Unit Root Process.

Tests of Stationarity

Tests of Stationarity

- In practice we face two important questions:
  - How do we find out if a given time series is stationary or not?
  - Is there a way that it can be made stationary?
- Prominently discussed tests in the literature are:
  - Graphical Analysis
  - Autocorrelation Function
  - The Unit Root Test

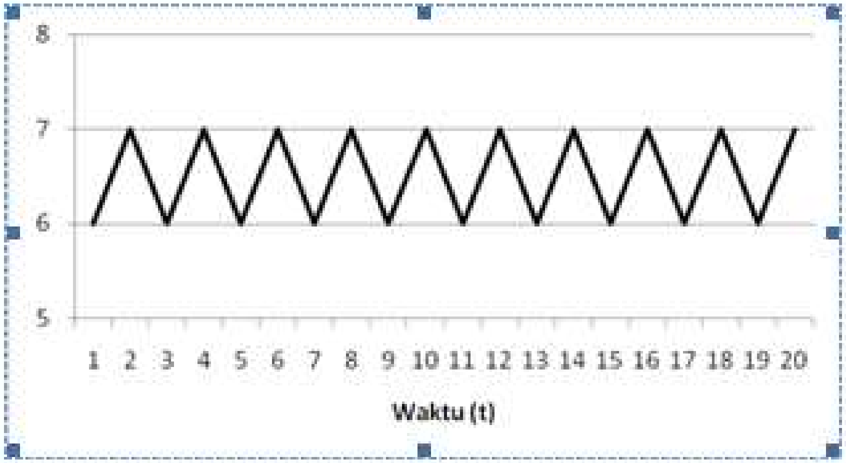
1. Graphical Analysis

- Before pursuing a formal test, it is always advisable to plot the time series under study
- E.g. take the GDP time series.
- You will see that over the period of study GDP has been increasing (i.e. showing an upward trend)
- This perhaps suggests that the GDP series is not stationary (also more or less true of the other economic time series).

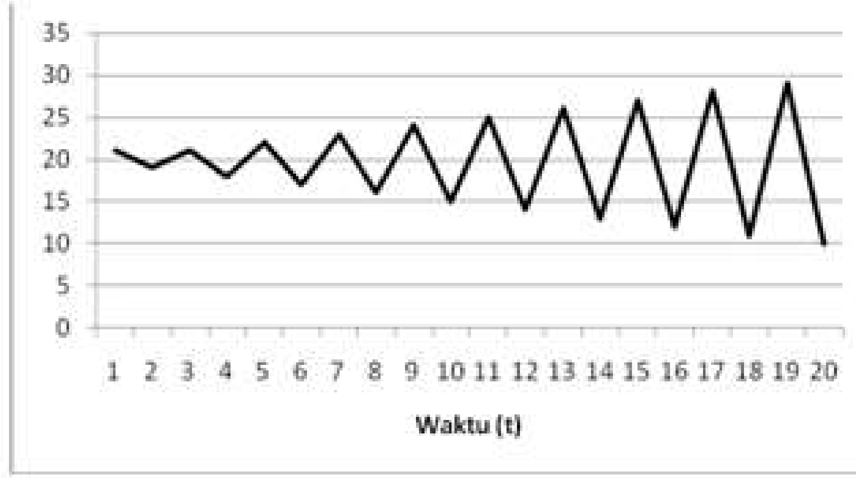
Analisis Grafik

Sebelum dilakukan formal test, lakukan dulu plot data time series. Plot tersebut dapat menjadi petunjuk awal mengenai perilaku data time series.

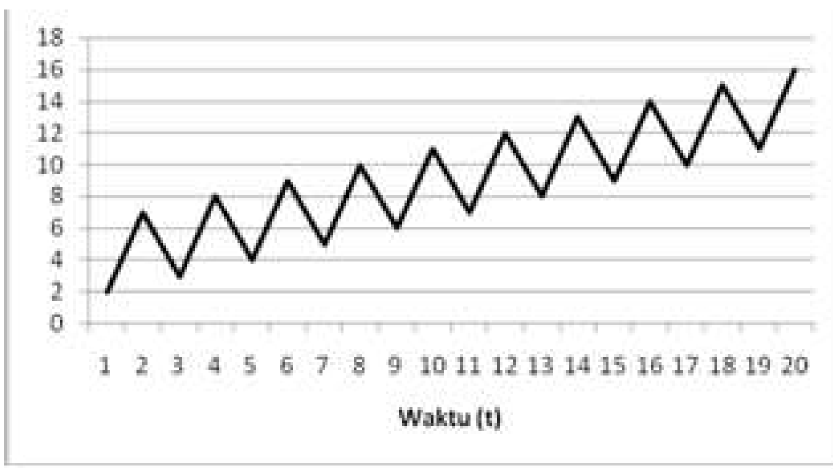
Pemeriksaaan Stasioneritas



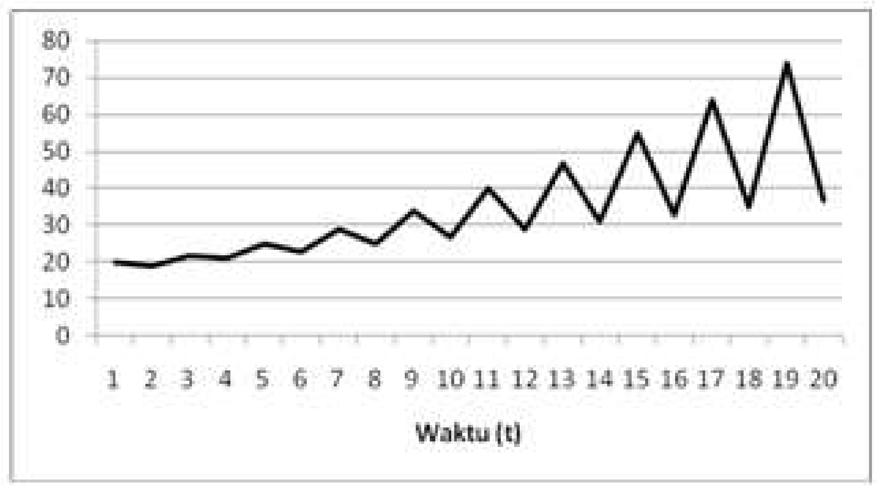
Gambar 2.1 Plot Data dalam Keadaan Stasioner Nilai Tengah dan Ragam.



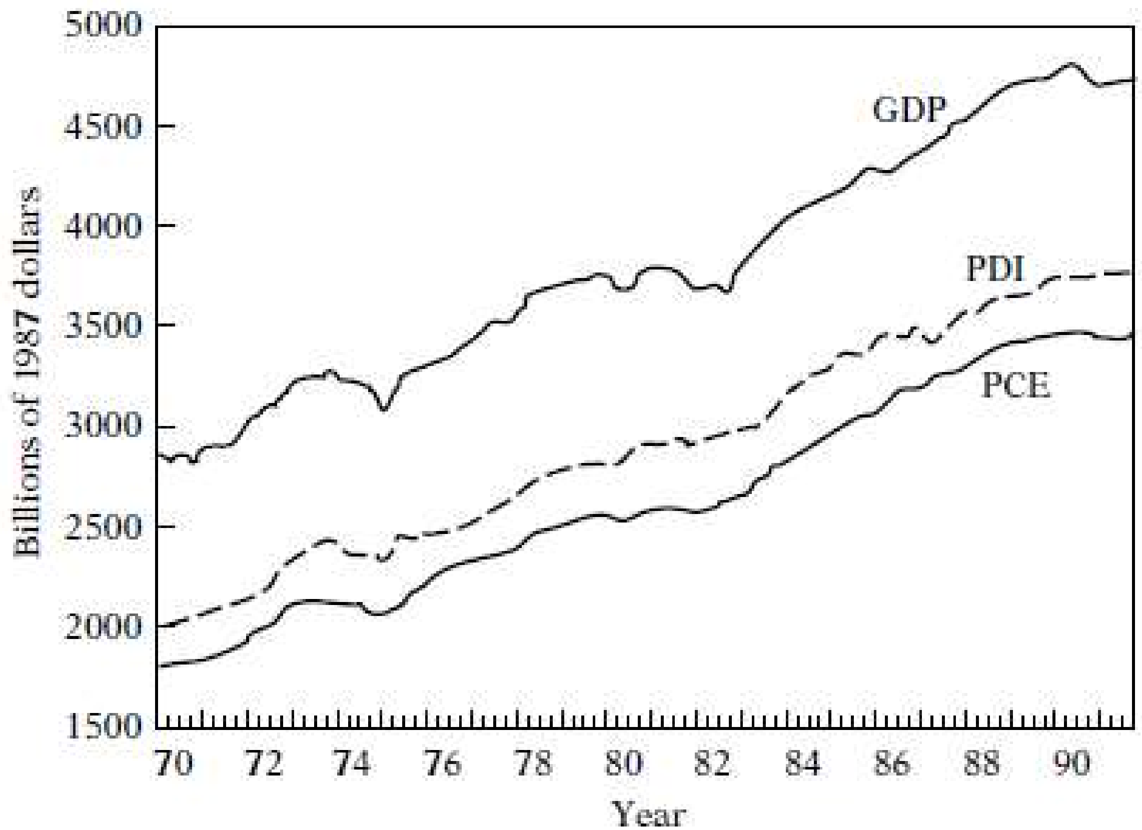
Gambar 2.3 Plot Data Stasioner pada Nilai Tengah, tapi Tidak Stasioner pada Ragam.



Gambar 2.2 Plot data yang tidak Stasioner pada Nilai Tengah, tapi Stasioner pada Ragam.



Gambar 2.4 Plot Data yang Tidak Stasioner pada Nilai Tengah maupun Ragamnya.



GDP, PDI, and PCE, United States, 1970–1991 (quarterly).

Dari Plot data time series di atas dapat dilihat GDP menunjukkan tren meningkat. Ini merupakan indikasi bahwa data GDP tidak stasioner

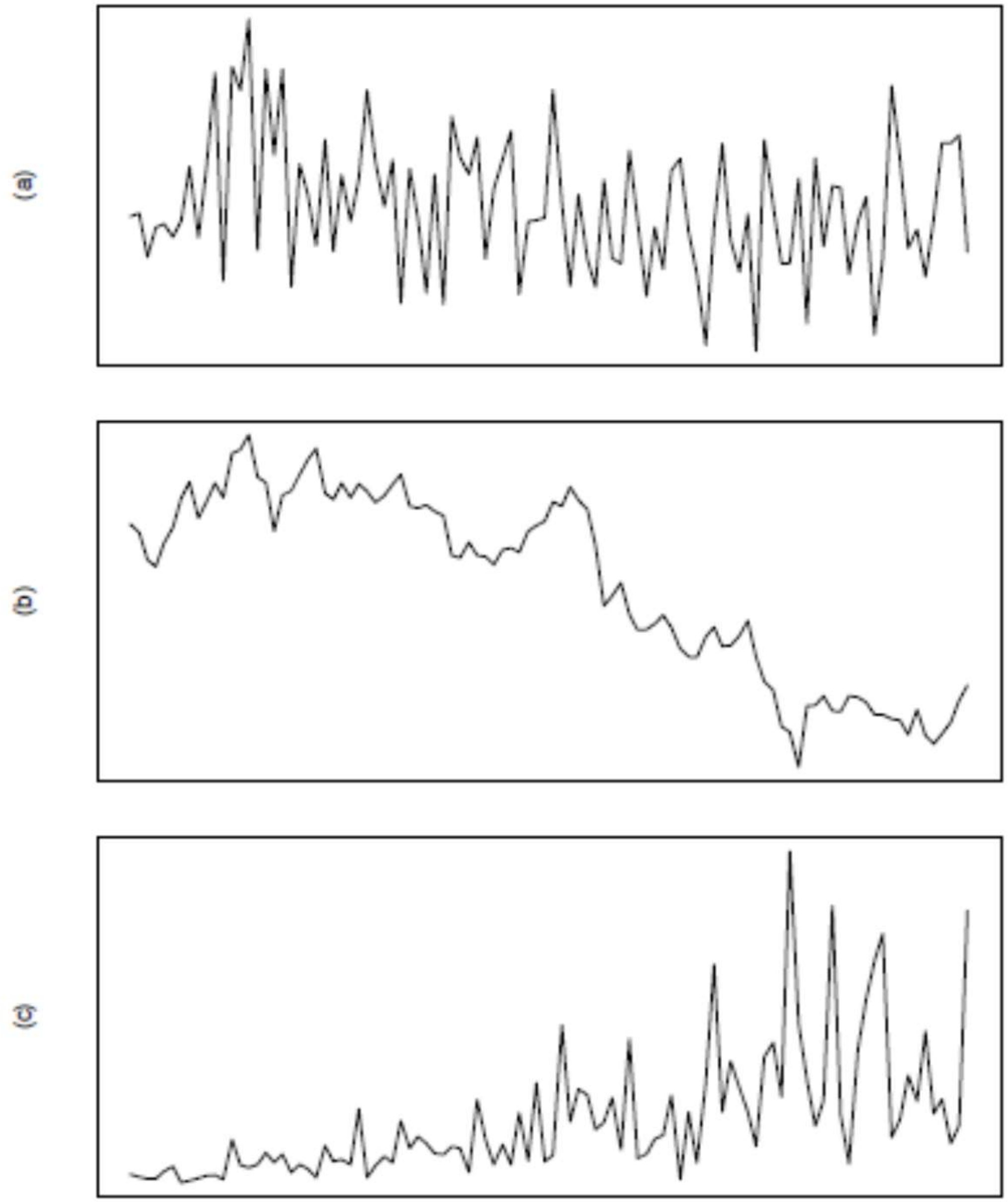


Figure 7-6: *Illustrations of time series data, showing (a) a series stationary in the mean; (b) a series non-stationary in the mean; and (c) a series non-stationary in the mean and variance. In each case, $n = 100$.*

2. Autocorrelation Function (ACF), Correlogram & Uji-ujinya

Autocorrelation Function (ACF)

ACF lag k dinotasikan ρ_k

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

$$= \frac{\text{covariance at lag } k}{\text{variance}}$$

Di Makridakis. Simbol r sebagai estimasi dari rho, korelasi Yt dg yt-1 sama dengan yt dg yt+1

Karena yang digunakan adalah sample, maka

$$\hat{\gamma}_k = \frac{\sum(Y_t - \bar{Y})(Y_{t+k} - \bar{Y})}{n}$$

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0}$$

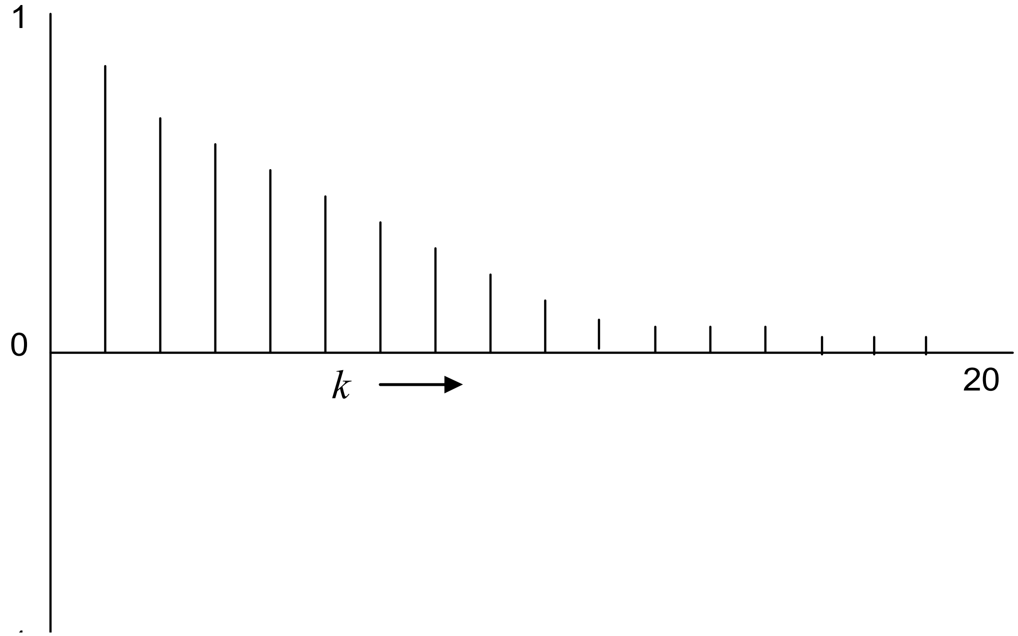
$$\hat{\gamma}_0 = \frac{\sum(Y_t - \bar{Y})^2}{n}$$

$$r_k = \frac{\sum_{t=k+1}^{n}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2}$$

Untuk suatu proses yang stasioner $\{Z_t\}$, autokorelasi pada lag k atau korelasi antara Z_t dan Z_{t-k}, didefinisikan sebagai

$$\rho_k = \frac{Cov(Z_t, Z_{t-k})}{\sqrt{Var(Z_t)}\sqrt{Var(Z_{t-k})}} = \frac{\gamma_k}{\gamma_0} .$$

Fungsi autokorelasi (*autocorrelation function*), yang disingkat ACF, dibentuk dengan himpunan $\{\rho_k;\ k = 0,1,2,...\}$ dengan $\rho_0 = 1$.



Contoh Fungsi autokorelasi (ACF) teoritik suatu data Zt

ACF

Dari suatu *time series* yang stasioner $Z_1, Z_2, \ldots, Z_n$, estimasi terhadap nilai mean μ, fungsi autokovarians $\{\gamma_k;\ k = 0,1,2,\ldots\}$, dan ACF $\{\rho_k;\ k = 0,1,2,\ldots\}$ dapat dilakukan dengan menggunakan statistik

$$\hat{\mu} = \overline{Z} = \frac{1}{n}\sum_{t=1}^{n} Z_t \quad ,$$

dan untuk $k = 0,1,2,\ldots$

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n} (Z_t - \mu)(Z_{t-k} - \mu)$$

Diperlukan n yang cukup besar untuk memperoleh nilai estimasi yang cukup baik, dan dalam praktek biasanya diperlukan $n \geq 50$. (Chatfield, 1996; Soejoeti, 1987). Dari formula diatas terlihat jelas bahwa nilai $\hat{\gamma}_k$ tidak dapat dihitung untuk $k > n-1$, dan dalam praktek biasanya tidak memerlukan $\hat{\gamma}_k$ untuk semua k, melainkan hanya kira-kira untuk $k \leq \frac{n}{4}$ saja. Nilai ACF ρ_k selanjutnya dapat diestimasi dengan

$$\hat{\rho}_k = r_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} = \frac{\sum_{t=1}^{n-k} (Z_t - \overline{Z})(Z_{t+k} - \overline{Z})}{\sum_{t=1}^{n} (Z_t - \overline{Z})^2}$$

Makridakis
pp 316

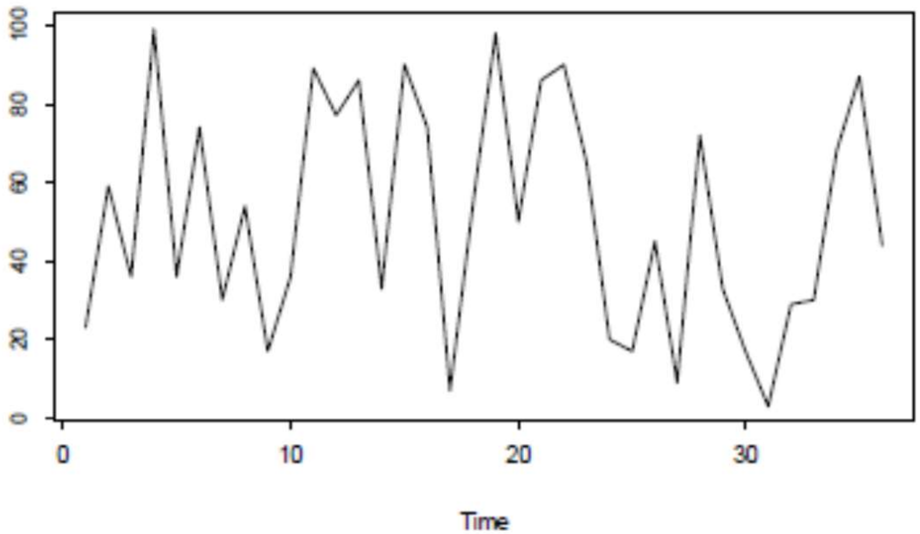


Figure 7-2: *Time plot of the data in Table 7-1.*

$r_1 = 0.103$
$r_2 = 0.099$
$r_3 = -0.043$
$r_4 = -0.031$
$r_5 = -0.183$
$r_6 = 0.025$
$r_7 = 0.275$
$r_8 = -0.004$
$r_9 = -0.011$
$r_{10} = -0.152$

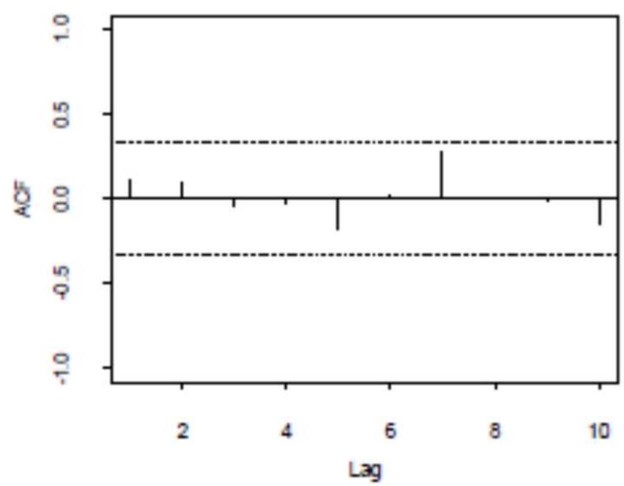


Figure 7-3: *Autocorrelations for the series in Table 7-1.*

Partial Autocorrelation Function (PACF)

- Makridakis pp 320/323

Partial autocorrelations are used to measure the degree of association between Y_t and Y_{t-k}, when the effects of other time lags—$1, 2, 3, \ldots, k-1$—are removed.

The value of this can be seen in the following simple example. Suppose there was a significant autocorrelation between Y_t and Y_{t-1}. Then there will also be a significant correlation between Y_{t-1} and Y_{t-2} since they are also one time unit apart. Consequently, there will be a correlation between Y_t and Y_{t-2} because both are related to Y_{t-1}. So to measure the real correlation between Y_t and Y_{t-2}, we need to *take out* the effect of the intervening value Y_{t-1}. This is what partial autocorrelation does.

PACF

Besaran statistik lain yang diperlukan dalam analisis time series adalah fungsi autokorelasi parsial (PACF), yang ditulis dengan notasi $\{\phi_{kk}; k = 1,2,\ldots\}$, yakni himpunan autokorelasi parsial untuk berbagai lag k. Autokorelasi parsial didefinisikan sebagai

$$\phi_{kk} = \frac{|P_k^*|}{|P_k|},$$

dimana P_k adalah matriks autokorelasi $k \times k$, dan P_k^* adalah P_k dengan kolom terakhir diganti dengan

$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_k \end{bmatrix}.$$

Sehingga diperoleh

$$\phi_{11} = \rho_1,$$

$$\phi_{22} = \frac{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & \rho_2 \end{vmatrix}}{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{vmatrix}} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2},$$

$$\begin{aligned}
\alpha_1 &= 0.103 \\
\alpha_2 &= 0.089 \\
\alpha_3 &= -0.062 \\
\alpha_4 &= -0.030 \\
\alpha_5 &= -0.171 \\
\alpha_6 &= 0.065 \\
\alpha_7 &= 0.315 \\
\alpha_8 &= -0.096 \\
\alpha_9 &= -0.092 \\
\alpha_{10} &= -0.168
\end{aligned}$$

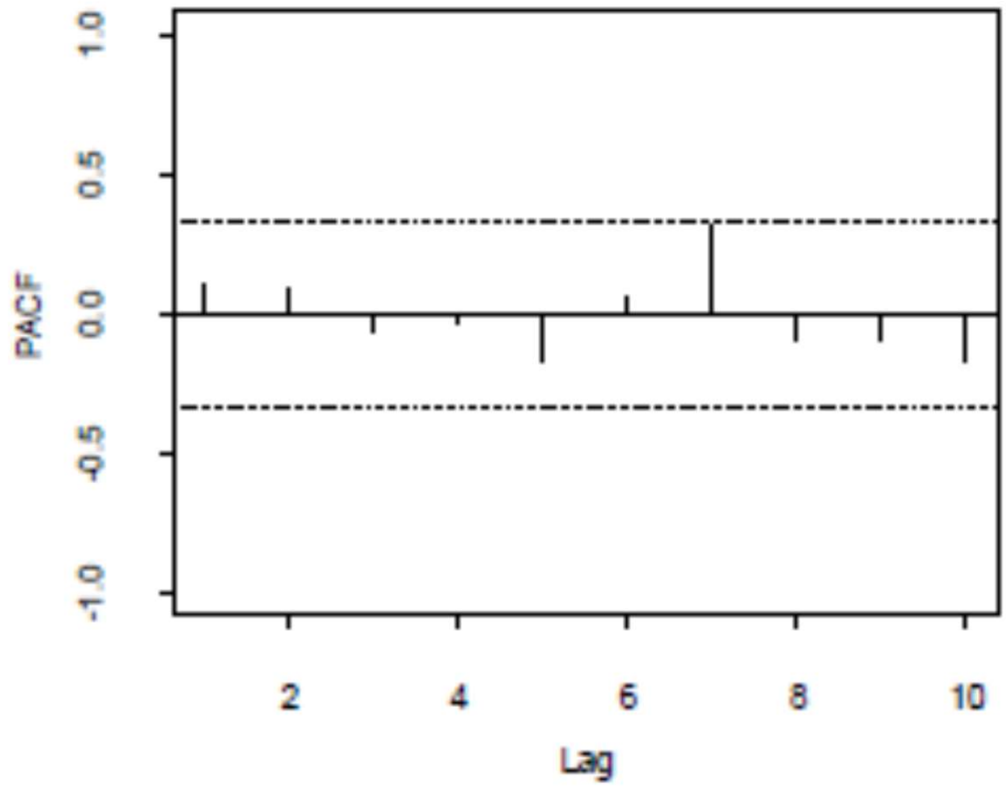


Figure 7-4: *Partial autocorrelations for the series in Table 7-1.*

Contoh 1.

Berikut ini adalah sepuluh nilai yang pertama dari suatu data *time series* yang panjang.

t	1	2	3	4	5	6	7	8	9	10
Z_t	13	8	15	4	4	12	11	7	14	12

Dari data ini, hitunglah nilai r_1, r_2, dan r_3.

<u>Jawaban</u> :

Berikut ini adalah perhitungan r_1, r_2, dan r_3.dari 10 data *time series* di atas.

t	Z_t	Z_{t+1}	Z_{t+2}	Z_{t+3}
1	13	8	15	4
2	8	15	4	4
3	15	4	4	12
4	4	4	12	11
5	4	12	11	7
6	12	11	7	14
7	11	7	14	12
8	7	14	12	-
9	14	12	-	-
10	12	-	-	-
Total	100	-	-	-

(a). $$r_1 = \frac{\sum_{t=1}^{10-1}(Z_t - \overline{Z})(Z_{t+1} - \overline{Z})}{\sum_{t=1}^{10}(Z_t - \overline{Z})^2}$$

$$= \frac{(13-10)(8-10) + (8-10)(15-10) + \ldots + (14-10)(12-10)}{(13-10)^2 + (8-10)^2 + \ldots + (12-10)^2}$$

$$= \frac{-27}{144} = -0{,}188$$

(b). $$r_2 = \frac{\sum_{t=1}^{10-2}(Z_t - \overline{Z})(Z_{t+2} - \overline{Z})}{\sum_{t=1}^{10}(Z_t - \overline{Z})^2}$$

$$= \frac{(13-10)(15-10) + (8-10)(4-10) + \ldots + (7-10)(12-10)}{(13-10)^2 + (8-10)^2 + \ldots + (12-10)^2}$$

$$= -0{,}201$$

(c). $$r_3 = \frac{\sum_{t=1}^{10-3}(Z_t - \overline{Z})(Z_{t+3} - \overline{Z})}{\sum_{t=1}^{10}(Z_t - \overline{Z})^2}$$

$$= \frac{(13-10)(4-10) + (8-10)(4-10) + \ldots + (11-10)(12-10)}{(13-10)^2 + (8-10)^2 + \ldots + (12-10)^2}$$

$$= 0{,}181$$

Dengan demikian dari data di atas diperoleh ACF r_k, $k = 1,2,3$ sebagai berikut.

k (lag)	1	2	3
r_k (ACF)	$-0,188$	$-0,201$	$0,181$

Contoh 2.

Berdasarkan data time series pada contoh 1 di atas, hitunglah nilai dari $\hat{\phi}_{11}, \hat{\phi}_{22}$ dan $\hat{\phi}_{33}$.

Jawaban :

Dengan menggunakan hasil dalam contoh 1, yaitu nilai-nilai dari perhitungan r_1, r_2, dan r_3, serta menerapkan rumus Durbin (1960) diperoleh

$$\hat{\phi}_{11} = r_1 = -0.188,$$

$$\hat{\phi}_{22} = \frac{r_2 - r_1^2}{1 - r_1^2} = \frac{-0.201 - (-0.188)^2}{1 - (-0.188)^2} = -0.245 \ .$$

Pemeriksaan Stasioneritas: **Korelogram**

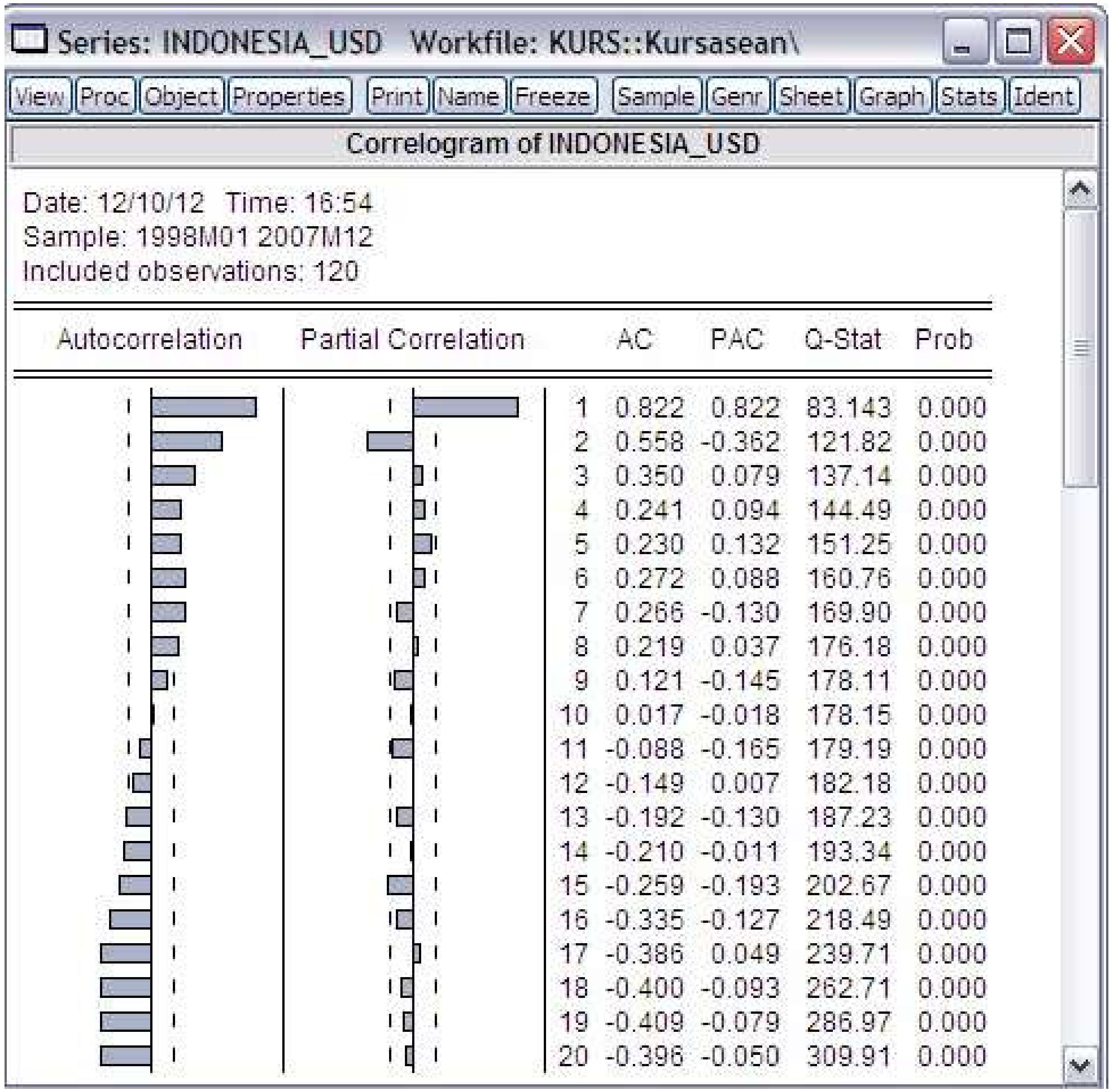
- Korelogram merupakan teknik identifikasi kestasioneran data time series melalui Fungsi Autokorelasi (ACF). Fungsi ini bermanfaat untuk menjelaskan suatu proses stokastik, tentang bagaimana korelasi antara data-data (Y_t) yang berdekatan.
- Korelogram diperoleh dengan membuat plot antara ρ_k dan k (lag). Plot antara ρ_k dan k ini disebut korelogram populasi. Dalam praktek, kita hanya dapat menghitung fungsi otokorelasi sampel (*Sample Autocorrelation Function*).

Data yang stasioner: korelogram menurun dengan cepat seiring dengan meningkatnya k.

Data yang tidak stasioner: korelogram cenderung tidak menuju nol (tidak mengecil) meskipun k membesar

Date: 09/10/04 Time: 10:50
Sample: 1/01/2002 12/31/2002
Included observations: 164

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.956	0.956	152.75	0.000
		2	0.921	0.071	295.16	0.000
		3	0.878	-0.097	425.38	0.000
		4	0.839	0.016	545.13	0.000
		5	0.805	0.051	656.13	0.000
		6	0.773	0.004	759.11	0.000
		7	0.741	-0.024	854.30	0.000
		8	0.711	0.013	942.58	0.000
		9	0.676	-0.074	1022.8	0.000
		10	0.647	0.042	1096.7	0.000
		11	0.617	0.001	1164.6	0.000
		12	0.581	-0.111	1225.0	0.000
		13	0.546	-0.019	1278.9	0.000
		14	0.516	0.055	1327.2	0.000
		15	0.488	0.012	1370.8	0.000
		16	0.468	0.051	1411.1	0.000
		17	0.446	-0.010	1448.0	0.000
		18	0.423	-0.048	1481.4	0.000
		19	0.399	-0.017	1511.3	0.000
		20	0.370	-0.054	1537.2	0.000


Series: INDONESIA_USD Workfile: KURS::Kursasean\

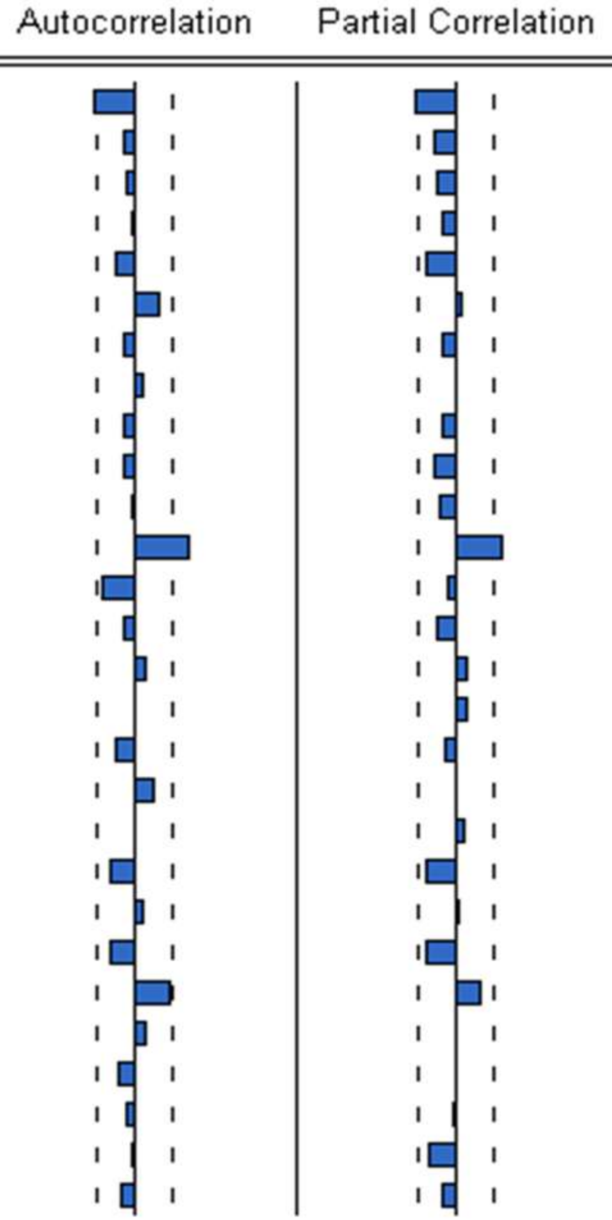
View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats | Ident

Correlogram of INDONESIA_USD

Date: 12/10/12 Time: 16:54
Sample: 1998M01 2007M12
Included observations: 120

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.822	0.822	83.143	0.000
		2	0.558	-0.362	121.82	0.000
		3	0.350	0.079	137.14	0.000
		4	0.241	0.094	144.49	0.000
		5	0.230	0.132	151.25	0.000
		6	0.272	0.088	160.76	0.000
		7	0.266	-0.130	169.90	0.000
		8	0.219	0.037	176.18	0.000
		9	0.121	-0.145	178.11	0.000
		10	0.017	-0.018	178.15	0.000
		11	-0.088	-0.165	179.19	0.000
		12	-0.149	0.007	182.18	0.000
		13	-0.192	-0.130	187.23	0.000
		14	-0.210	-0.011	193.34	0.000
		15	-0.259	-0.193	202.67	0.000
		16	-0.335	-0.127	218.49	0.000
		17	-0.386	0.049	239.71	0.000
		18	-0.400	-0.093	262.71	0.000
		19	-0.409	-0.079	286.97	0.000
		20	-0.396	-0.050	309.91	0.000

Date: 09/19/04 Time: 16:32
Sample: 1998:01 2003:05
Included observations: 64



	AC	PAC	Q-Stat	Prob
1	-0.255	-0.255	4.3580	0.037
2	-0.072	-0.147	4.7128	0.095
3	-0.056	-0.125	4.9267	0.177
4	-0.024	-0.095	4.9662	0.291
5	-0.117	-0.192	5.9424	0.312
6	0.150	0.039	7.5731	0.271
7	-0.080	-0.083	8.0448	0.329
8	0.050	0.002	8.2360	0.411
9	-0.079	-0.095	8.7154	0.464
10	-0.063	-0.145	9.0283	0.529
11	-0.019	-0.104	9.0568	0.617
12	0.350	0.292	18.987	0.089
13	-0.200	-0.042	22.302	0.051
14	-0.067	-0.122	22.678	0.066
15	0.078	0.059	23.207	0.080
16	0.000	0.060	23.207	0.108
17	-0.120	-0.071	24.510	0.106
18	0.128	-0.007	26.019	0.099
19	-0.008	0.053	26.025	0.130
20	-0.149	-0.195	28.156	0.106
21	0.057	0.010	28.475	0.127
22	-0.157	-0.195	30.957	0.097
23	0.235	0.157	36.634	0.035
24	0.076	-0.001	37.251	0.041
25	-0.111	-0.008	38.575	0.041
26	-0.043	-0.013	38.778	0.051
27	-0.023	-0.177	38.838	0.066
28	-0.090	-0.087	39.797	0.069

Uji Bartlett

- dilakukan untuk melihat signikansi r_k satu per satu. Barlet menunjukkan bahwa jika suatu time series dibentuk melalui proses *white noise*, maka sampel otokorelasi-nya akan berdistribusi normal dengan mean 0 dan standar deviasi $1/\sqrt{n}$, dimana n banyaknya pengamatan, atau dinotasikan dengan $r_k \sim N(0, 1/\sqrt{n})$. Bila n = 100, maka $r_k \sim N(0, 0.1)$.
- Oleh karena itu, bila ada $r_k > 0.2$ (dua kali standar deviasi), maka kita yakin dengan kepercayaan 95% bahwa $\rho \neq 0$ dan berarti time series yang sedang kita analis bukan berasal dari proses *white noise*. Atau secara matematis dituliskan dengan:

  $r_k \pm Z_{\alpha/2}$ s.e ; dimana s.e adalah standar error

- Signifikan atau tidaknya nilai autokorelasi melalui pengujian standar error (Se).

Hipotesis yang digunakan:

H_0: $\rho_k = 0$ (Stasioner)

H_1: $\rho_k \neq 0$

- Jika interval rk tidak mengandung nilai 0, maka H_0 tidak dapat ditolak, tetapi jika interval tidak mengandung nilai 0, maka H_0 dapat ditolak.
- Pada Korelogram uji ini digambarkan dengan: garis putus-putus
- Kelemahan: Kadang timbul keraguan dalam memutuskan stasioner atau tidak.

==➔ Perlu uji formal

Pemeriksaan Stasioneritas: ***Pormanteau Test***

Box-Pierce Q Statistic

- **Box-Pierce Q Statistic:**

$$Q = n \sum_{k=1}^{m} \hat{\rho}_k^{\,2}$$

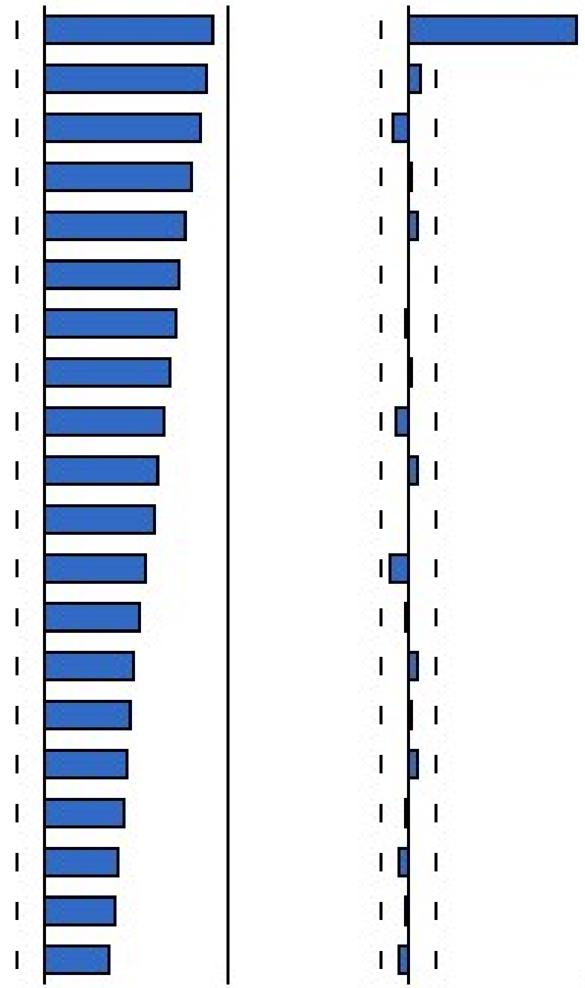
dimana: n = banyak sampel, m=panjang lag

- Jika statistik Q < $\chi^2_{(\alpha)}$, H_0 diterima, berarti data deret waktu adalah stasioner

Date: 09/19/04 Time: 16:32
Sample: 1998:01 2003:05
Included observations: 64

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.255	-0.255	4.3580	0.037
		2	-0.072	-0.147	4.7128	0.095
		3	-0.056	-0.125	4.9267	0.177
		4	-0.024	-0.095	4.9662	0.291
		5	-0.117	-0.192	5.9424	0.312
		6	0.150	0.039	7.5731	0.271
		7	-0.080	-0.083	8.0448	0.329
		8	0.050	0.002	8.2360	0.411
		9	-0.079	-0.095	8.7154	0.464
		10	-0.063	-0.145	9.0283	0.529
		11	-0.019	-0.104	9.0568	0.617
		12	0.350	0.292	18.987	0.089
		13	-0.200	-0.042	22.302	0.051
		14	-0.067	-0.122	22.678	0.066
		15	0.078	0.059	23.207	0.080
		16	0.000	0.060	23.207	0.108
		17	-0.120	-0.071	24.510	0.106
		18	0.128	-0.007	26.019	0.099
		19	-0.008	0.053	26.025	0.130
		20	-0.149	-0.195	28.156	0.106
		21	0.057	0.010	28.475	0.127
		22	-0.157	-0.195	30.957	0.097
		23	0.235	0.157	36.634	0.035
		24	0.076	-0.001	37.251	0.041
		25	-0.111	-0.008	38.575	0.041
		26	-0.043	-0.013	38.778	0.051
		27	-0.023	-0.177	38.838	0.066
		28	-0.090	-0.087	39.797	0.069

Date: 09/10/04 Time: 10:50
Sample: 1/01/2002 12/31/2002
Included observations: 164



Lag	AC	PAC	Q-Stat	Prob
1	0.956	0.956	152.75	0.000
2	0.921	0.071	295.16	0.000
3	0.878	-0.097	425.38	0.000
4	0.839	0.016	545.13	0.000
5	0.805	0.051	656.13	0.000
6	0.773	0.004	759.11	0.000
7	0.741	-0.024	854.30	0.000
8	0.711	0.013	942.58	0.000
9	0.676	-0.074	1022.8	0.000
10	0.647	0.042	1096.7	0.000
11	0.617	0.001	1164.6	0.000
12	0.581	-0.111	1225.0	0.000
13	0.546	-0.019	1278.9	0.000
14	0.516	0.055	1327.2	0.000
15	0.488	0.012	1370.8	0.000
16	0.468	0.051	1411.1	0.000
17	0.446	-0.010	1448.0	0.000
18	0.423	-0.048	1481.4	0.000
19	0.399	-0.017	1511.3	0.000
20	0.370	-0.054	1537.2	0.000

Pemeriksaan Stasioneritas: ***Pormanteau Test***

Ljung-Box (LB) Statistic

- Ljung-Box Statistic:

$$LB = n(n+2)\sum_{k=1}^{m}\left(\frac{\hat{\rho}_k^{\,2}}{n-1}\right) \sim \chi^2_m$$

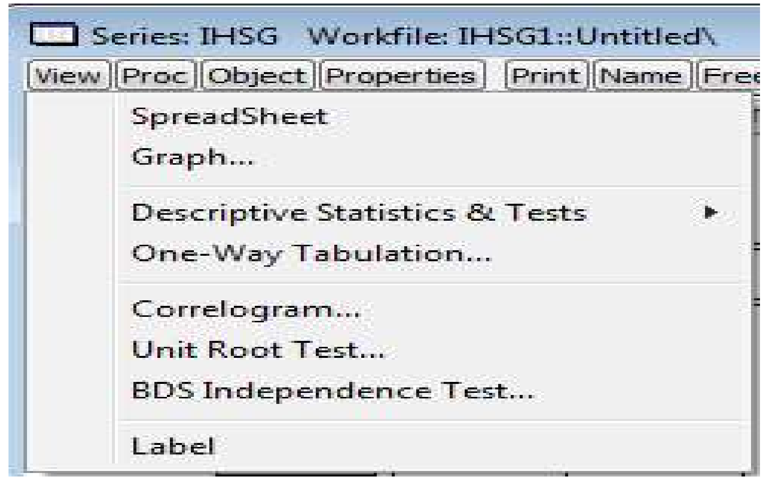
dimana: n = banyak sampel, m=panjang lag

- **Jika statistik LB lebih kecil dari nilai kritis statistik tabel Chi-Square dengan taraf nyata α maka data stasioner.**
- Lebih 'powerfull', Cocok untuk sampel kecil

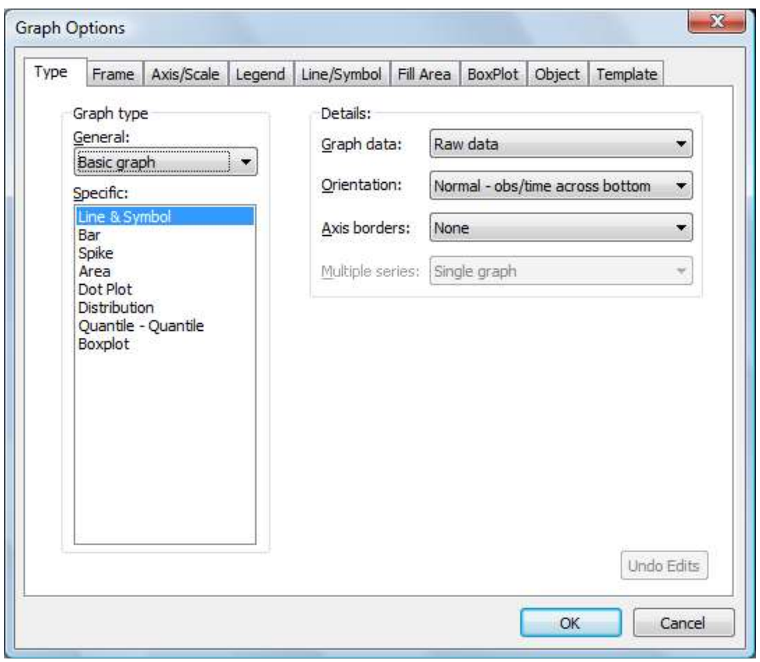
Ringkasan Prosedur Eviews
untuk Pemeriksaan Stasioneritas

Trend Data

- Dari *workfile* klik *View.*



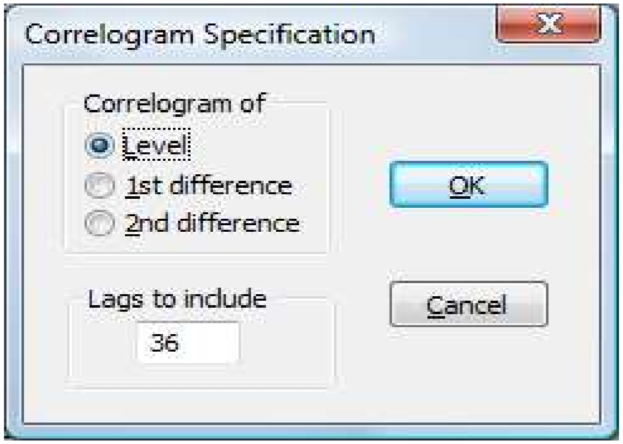
- Selanjutnya klik *Graph:*



- Lakukan semua pilihan seperti gambar disamping.
- Klik OK, akan diperoleh grafik data

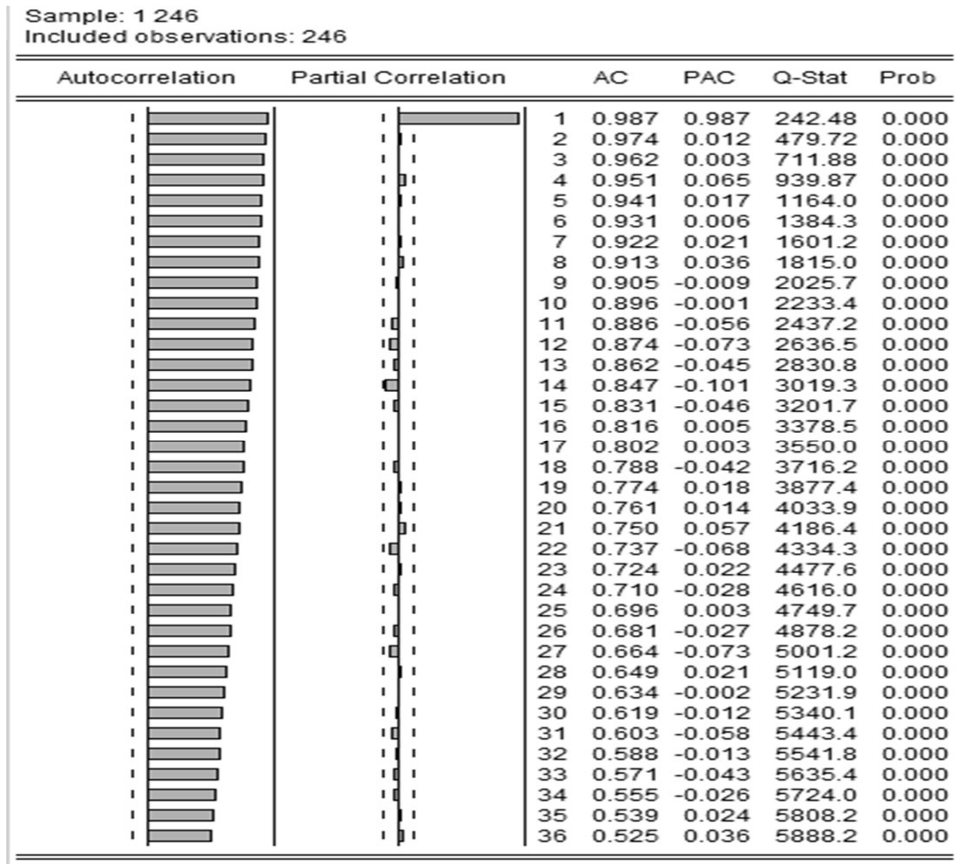
Autokorelasi dan Korelogram

- Dari menu *View*, klik *Correlogram.* Muncul tampilan berikut:



- Gunakan terlebih dahulu pilihan *level* pada *correlogram of*. Pilihan *Lag to include* = 36 (*by default*).
- Klik *OK*. Akan muncul output korelogram

Contoh output korelogram



Sample: 1 246
Included observations: 246

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.987	0.987	242.48	0.000
		2	0.974	0.012	479.72	0.000
		3	0.962	0.003	711.88	0.000
		4	0.951	0.065	939.87	0.000
		5	0.941	0.017	1164.0	0.000
		6	0.931	0.006	1384.3	0.000
		7	0.922	0.021	1601.2	0.000
		8	0.913	0.036	1815.0	0.000
		9	0.905	-0.009	2025.7	0.000
		10	0.896	-0.001	2233.4	0.000
		11	0.886	-0.056	2437.2	0.000
		12	0.874	-0.073	2636.5	0.000
		13	0.862	-0.045	2830.8	0.000
		14	0.847	-0.101	3019.3	0.000
		15	0.831	-0.046	3201.7	0.000
		16	0.816	0.005	3378.5	0.000
		17	0.802	0.003	3550.0	0.000
		18	0.788	-0.042	3716.2	0.000
		19	0.774	0.018	3877.4	0.000
		20	0.761	0.014	4033.9	0.000
		21	0.750	0.057	4186.4	0.000
		22	0.737	-0.068	4334.3	0.000
		23	0.724	0.022	4477.6	0.000
		24	0.710	-0.028	4616.0	0.000
		25	0.696	0.003	4749.7	0.000
		26	0.681	-0.027	4878.2	0.000
		27	0.664	-0.073	5001.2	0.000
		28	0.649	0.021	5119.0	0.000
		29	0.634	-0.002	5231.9	0.000
		30	0.619	-0.012	5340.1	0.000
		31	0.603	-0.058	5443.4	0.000
		32	0.588	-0.013	5541.8	0.000
		33	0.571	-0.043	5635.4	0.000
		34	0.555	-0.026	5724.0	0.000
		35	0.539	0.024	5808.2	0.000
		36	0.525	0.036	5888.2	0.000

3- The Unit Root Test

Unit Root Stochastic Process

- Let's write the RWM $Y_t = Y_{t-1} + u_t$..... (4) as:

  $Y_t = \rho Y_{t-1} + u_{t-1}$ $\quad -1 \leq \rho \leq 1$ .... (13)

- If $\rho = 1$, (13) becomes a RWM (without drift).
- If ρ is in fact 1, we face what is known as the unit root problem (non-stationarity); as the variance of Y_t is not stationary.
- The name unit root is due to the fact that $\boldsymbol{\rho} = 1$.
- Thus the terms non-stationarity, random walk, and unit root can be treated as synonymous.
- If, however, $|\rho| \leq 1$, then the time series Y_t is stationary in the sense we have defined it.
- **Note:** Unit Root Stochastic Process will be further explained in Unit Root Test of Stationarity.

The Unit Root Test

$Y_t = \rho Y_{t-1} + u_t \quad -1 \leq \rho \leq 1$

Jika $\rho = 1$ → *random walk model without drift* → artinya non stationary stochastic process

Manipulasi Y_t

$Y_t - Y_{t-1} = \rho Y_{t-1} - Y_{t-1} + u_t$

$\Delta Y_t = (\rho - 1) Y_{t-1} + u_t$

$\Delta Y_t = \delta Y_{t-1} + u_t$

Jika $\delta = 0$, maka $\rho = 1$, artinya memiliki unit root sehingga tidak stasioner. Tetapi…

$$\Delta Y_t = u_t$$

$$Y_t - Y_{t-1} = u_t$$

Karena u_t adalah *white noise error term* → stasioner
Jadi data time series yang tidak stasioner tadi (random walk) menjadi stasioner setelah difference pertama

The Unit Root Test (cont.)

- We have to take the first differences of Y_t and regress them on Y_{t-1} and see if the estimated slope co-efficient in this regression (δ) is zero or not.
- If it is zero, we conclude that Y_t is non-stationary.
- But if it is negative, we conclude that Y_t is stationary.
- The only question is which test we use to find out if the estimated co-efficient of Y_{t-1} in (4.2) is zero or not?
- Unfortunately, under the null hypothesis that $\delta = 0$ (i.e., $\rho = 1$), the t value of the estimated coefficient of Y_{t-1} does not follow the t distribution even in large samples; i.e. it does not have an asymptotic normal distribution.

Unit Root Test: Dickey-Fuller (DF) Test

- Dickey and Fuller have shown that under the null hypothesis that $\delta = 0$, the estimated t-value of the coefficient of Y_{t-1} in (4.2) follows the **τ** (tau) statistic.
- These authors have computed the critical values of the **tau statistic** on the basis of Monte Carlo simulations.
- Interestingly, if the hypothesis that $\boldsymbol{\delta} = 0$ is rejected (i.e. the time series is stationary), we can use the usual t test.

DF-Test

- **Dickey-Fuller:** diuji dengan uji τ (dibaca: tau) atau yang dikenal dengan Dickey Fuller Test (DF). Statistik ini selanjutnya dikembangkan oleh Mc. Kinnon
- Pengujian Dickey–Fuller (DF) dgn nilai τ-statistik:

$$\boxed{\tau = \frac{\hat{\rho}}{Se(\hat{\rho})}}$$

- **Hipotesis:**

  Ho : δ **= 0** atau $\rho = 1$ **(Tidak Stasioner)**

  H1 : δ **< 0** atau $\rho < 1$

- Nilai τ-statistik dibandingkan τ-*McKinnon Critical Values*.
- **Tolak Ho berarti data stasioner**. Jika kita **tidak** menolak hipotesis $\delta = 0$, maka $\rho = 1$. Artinya data memiliki unit root, maka data time series Y_t tidak stasioner.

DF diestimasi untuk 3 bentuk random walk yang berbeda, yaitu

1. Y_t random walk

   $\Delta Y_t = \delta Y_{t-1} + u_t$

2. Y_t random walk with drift

   $\Delta Y_t = \beta_1 + \delta Y_{t-1} + u_t$

3. Y_t random walk with drift around a stochastic trend

   $\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + u_t$

Jika Ho ditolak, memiliki arti yang berbeda untuk masing bentuk

1. Y_t stasioner dengan rata-rata nol
2. Y_t stasioner dengan rata-rata tidak nol yaitu $\beta_1/(1- \rho)$
3. Y_t stasioner disekitar tren deterministik

Unit Root Test:
Augmented Dickey-Fuller (ADF) Test

- Model-model sebelumnya mengasumsikan u_t tidak berkorelasi → Hampir tidak mungkin.
- Korelasi serial antara residual dgn ΔYt, dapat dinyatakan dalam bentuk umum proses autoregressive.

- Jika error term memiliki korelasi, maka dilakukan ***Augmented Dicky Fuller test (ADF)*** dengan meregresikan

$$\boxed{\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \alpha_1 \Delta Y_{t-1} + \alpha_2 \Delta Y_{t-2} + \ldots \alpha_1 \Delta Y_{t-2} + u_t}$$

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-i} + \varepsilon_t$$

- Hipotesisnya sama dengan DF test

Berdasarkan model tersebut kita dapat memilih tiga model yang akan digunakan untuk melakukan Uji ADF, yaitu:

1. Model dengan intersep (β_1) dan trend (β_2), sebagaimana model diatas.
2. Model yang hanya intersep saja (β_1), yaitu:

$$\Delta Y_t = \beta_1 + \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-1} + \varepsilon_t$$

3. Model tanpa intersep dan trend (slop), yaitu:

$$\Delta Y_t = \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-1} + \varepsilon_t$$

Penghitungan manual cukup sulit → EViews

Unit Root Test:
Phillip Peron (PP) Test

- Asumsi penting dalam DF test adalah error term i.i.d. Sementara ADF menyesuaikan DF test untuk mengatasi masalah korelasi pada error term dengan menambahkan *lagged difference terms* terhadap regresi
- Phillip Peron menggunakan metode non parametrik untuk mengatasi masalah korelasi pada error term tanpa menambahkan *lagged difference terms.*

Pemeriksaaan Stasioneritas:
Unit Root Test / Uji Akar Unit (PP-Test)

- Nilai τ-statistik dari uji PP (Philip-Perron) dapat dihitung sbb:

$$\boxed{\tau = \sqrt{\frac{r_0}{h_0} t_0} - \frac{h_0 - r_0}{2 h_0 \sigma} \sigma_\theta}$$

dimana:

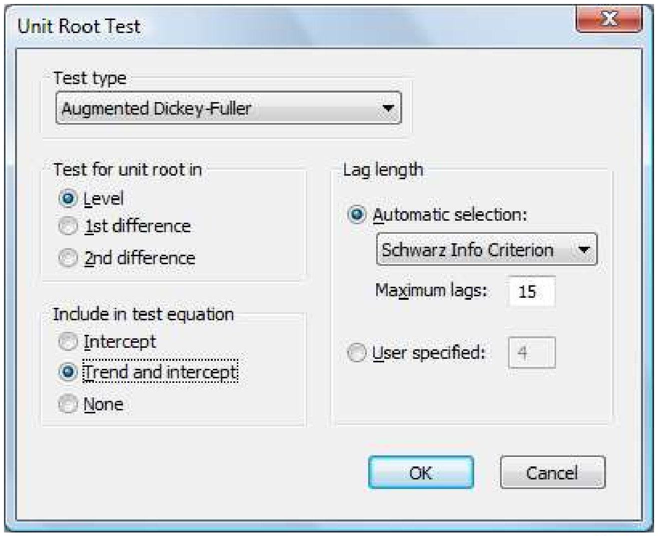
$$\boxed{h_0 = r_0 + 2 \sum_{r=1}^{M} \left(1 - \frac{j}{T}\right) r_j}$$

adalah spektrum dari ΔYt pada frekuensi nol, r_j adalah fungsi autokorelasi pada lag *j*, t_0 adalah τ-statistik pada θ, *adalah standar error dari* θ , dan σ adalah *standar error* uji regresi.

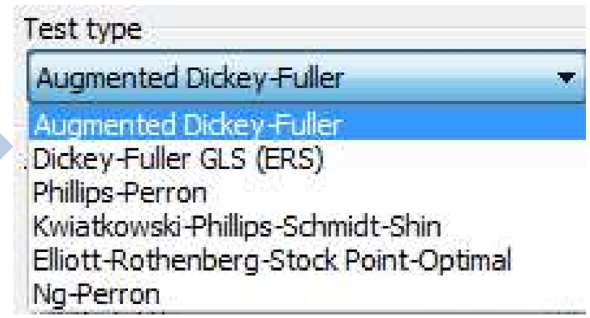
- Prosedur uji PP dapat diaplikasikan melalui cara yg sama dengan uji DF.

Uji Akar Unit

- Dari menu *View*, klik *Unit Root Test*. Muncul tampilan berikut:



Tentukan jenis uji. Pilihannya lihat gambar samping kanan



Contoh Output Uji Akar Unit (ADF test)

Null Hypothesis: IHSG has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic based on SIC, MAXLAG=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.454606	0.3506
Test critical values:	1% level	-3.995956	
	5% level	-3.428273	
	10% level	-3.137529	

*MacKinnon (1996) one-sided p-values.

EViews - [Series: CLOSING Workfile: ARIMA::Arima\]

File Edit Object View Proc Quick Options Add-ins Window Help

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats | Ident

Null Hypothesis: CLOSING has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.785722	0.9613
Test critical values:	1% level	-4.107947	
	5% level	-3.481595	
	10% level	-3.168695	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CLOSING)
Method: Least Squares
Date: 04/15/15 Time: 09:45
Sample (adjusted): 1/04/2005 4/01/2005
Included observations: 64 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CLOSING(-1)	-0.034075	0.043368	-0.785722	0.4351
C	7.179712	9.221236	0.778606	0.4392
@TREND(1/03/2005)	0.066212	0.043416	1.525052	0.1324

R-squared	0.112162	Mean dependent var	1.034844
Adjusted R-squared	0.083052	S.D. dependent var	1.945823
S.E. of regression	1.863269	Akaike info criterion	4.128282
Sum squared resid	211.7780	Schwarz criterion	4.229480
Log likelihood	-129.1050	Hannan-Quinn criter.	4.168149
F-statistic	3.853107	Durbin-Watson stat	1.554050
Prob(F-statistic)	0.026557		

Series: INDONESIA_USD Workfile: KURS::Kursasean\

View | Proc | Object | Properties | Print | Name | Freeze | Sample | Genr | Sheet | Graph | Stats | Ident

Augmented Dickey-Fuller Unit Root Test on INDONESIA_USD

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.693663	0.0002
Test critical values:	1% level	-3.486551	
	5% level	-2.886074	
	10% level	-2.579931	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INDONESIA_USD)
Method: Least Squares
Date: 12/10/12 Time: 17:16
Sample (adjusted): 1998M03 2007M12
Included observations: 118 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INDONESIA_USD(-1)	-0.240527	0.051245	-4.693663	0.0000
D(INDONESIA_USD(-1))	0.370500	0.086097	4.303283	0.0000
C	2201.267	471.0388	4.673217	0.0000

R-squared	0.214143	Mean dependent var	3.444712
Adjusted R-squared	0.200476	S.D. dependent var	641.0354
S.E. of regression	573.1889	Akaike info criterion	15.56540
Sum squared resid	37782731	Schwarz criterion	15.63584
Log likelihood	-915.3588	Hannan-Quinn criter.	15.59400
F-statistic	15.66854	Durbin-Watson stat	1.873386
Prob(F-statistic)	0.000001		

Assigment #03

- **Uji stasioneritas data** yang telah ditugaskan sebelumnya, dengan metode:
  - Grafik
  - Korelogram
  - Unit Root Test