Clase 18

Manuel Garcia.

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1 Momento Angular

$$[J^\alpha, J^\beta] = i\hbar\epsilon^{\alpha\beta\gamma}J^\gamma \qquad\qquad [J^x, J^y] = i\hbar J^z \text{ etc.}$$
$$\vec{J}^2 = (J^x)^2 + (J^y)^2 + (J^z)^2$$
$$[J^z, \vec{J}^2] = 0$$

Notacion provisional:

$$\vec{J}^2 |a,b\rangle = a |a,b\rangle \qquad\qquad J^2 |a,b\rangle = b |a,b\rangle$$

Operador $J_\pm$

$$J_\pm = J_x \pm iJ_y$$
$$[\vec{J}^2, J_\pm] = 0$$

$$[J^z, J_\pm] = \pm\hbar J_\pm$$

Si tenemos el estado $J_\pm |a,b\rangle = |\beta_\pm\rangle$, qué propiedades tiene? Utilizando que $[\vec{J}^2, J_\pm] = 0$

$$\vec{J}^2 |\beta_\pm\rangle = a |\beta_\pm\rangle$$

Y con $[J^z, J_\pm] = \pm\hbar J_\pm$

$$J^z |\beta_\pm\rangle = (b \pm \hbar) |\beta_\pm\rangle$$

Entonces podemos escribir:

$$J^z J_+ |a,b\rangle = (J_+ J^z - [J^z, J_\pm]) |a,b\rangle$$
$$= (J_+ J_z + \hbar J_+) |a,b\rangle = (b+\hbar) \underbrace{J_+ |a,b\rangle}_{|\beta_+\rangle}$$

Dado a qué limites tiene b?

$$\vec{J}^2 - (J^z)^2 = (J^x)^2 + (J^y)^2$$
$$\langle a,b| \underbrace{\vec{J}^2 - (J^z)^2}_{=a-b^2} |a,b\rangle = \langle a,b| (J^x)^2 + (J^y)^2 |a,b\rangle \geq 0$$
$$a - b^2 \geq 0 \qquad \rightarrow \qquad a \geq b^2$$

Tenemos que :

$$J_+ |a, b_{max}\rangle = 0$$
$$J_- J_+ |a, b_{min}\rangle = 0$$

$$\begin{aligned} J_- J_+ &= (J^x - iJ^y)(J^x + iJ^y) \\ &= (J^x)^2 + (J^2)^2 + i[J^x, J^y] = \vec{J}^2 - (J^z)^2 - \hbar J^z - \hbar J^z \end{aligned}$$

Tenemos que:

$$b_{max} = \frac{m\hbar}{2} \qquad n = 0, 1, 2, 3, ...$$
$$b_{max} = \hbar J \quad \rightarrow \quad J = \frac{b_{max}}{\hbar} = \frac{m}{2}$$
$$a = \hbar^2 J(J+1)$$

$$J^z |J, m\rangle = \hbar m |J, m\rangle \qquad m : -J, -J+1, ..., +J$$
$$\vec{J}^2 |J, m\rangle = \hbar^2 J(J+1) |J, m\rangle$$

$$J = 0 \qquad \rightarrow \qquad |0, 0\rangle$$
$$J = \frac{1}{2} \quad \rightarrow \quad \left|\frac{1}{2}, -\frac{1}{2}\right\rangle ; \quad \left|\frac{1}{2}, +\frac{1}{2}\right\rangle$$
$$J = 1 \quad \rightarrow \quad |1, -1\rangle , \quad |1, 0\rangle , \quad |1, 1\rangle$$

Momento angular **Orbital**

$$\vec{L} = \vec{r} \times \vec{p} \quad \rightarrow \quad \langle \vec{r}| \vec{L} |\psi\rangle = -i\hbar \vec{r} \times \boldsymbol{\vec{\nabla}} \psi(\vec{r})$$