Clase 20

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$$H = \frac{\vec{p}^2}{2m} - \frac{e^2}{r}$$
$$[\vec{L}, H] = 0$$
$$\rightarrow [L_z, H] = 0$$
$$[\vec{L}^2, H] = 0$$

Nos dividiremos el spin:

$$H |\psi\rangle = E |\psi\rangle$$
$$|\psi\rangle = |E; l, m_e\rangle$$
$$H |E; l, m_e\rangle = E |E; l, m_e\rangle$$
$$\vec{L}^2 |E; l, m_e\rangle = \hbar^2 e(e+1) |E; l, m_e\rangle$$
$$L_z |E; l, m_e\rangle = \hbar m_e |E; l, m_e\rangle$$

Queremos hacer la proyeccion:

$$\left\langle \underbrace{R, \theta, \phi}_{=\vec{r}} \middle| E; l, m_e \right\rangle = f(r, \theta, \phi)$$
$$L_{ZA} f(r, \theta, \phi) = ?$$
$$L_{ZA} = -i\hbar \frac{\partial}{\partial \phi}$$

Para esto debemos tener en cuenta el momentum angular orbital:

$$\langle \theta, \phi | l, m_e \rangle = Y_{l, m_e}(\theta, \phi)$$

Entonces:

$$L_{ZA} Y_{l, m_e}(\theta, \phi) = \hbar m_e Y_{l, m_e}(\theta, \phi)$$
$$\vec{L}_A^2 Y_{l, m_e}(\theta, \phi) = \hbar^2 e(e+1) Y_{l, m_e}(\theta, \phi)$$
$$\vec{L}_A f_{..l, m_e}(r, \theta, \phi) = \hbar^2 e(e+1) f_{..l, m_e}(r, \theta, \phi)$$
$$f_{..l, m_e}(r, \theta, \phi) = \underbrace{R(r)}_{\text{Func. radial}} Y_{l, m_e}(\theta, \phi)$$

2 Atomo de Hidrogeno

$$V(r) = -\frac{e^2}{r}$$

Su hamiltoniano:

$$H = \frac{\vec{p}_1^2}{2m_e} + \frac{\vec{p}_2}{2m_p} + V(r) \qquad \vec{r} = \vec{r}_1 - \vec{r}_2 \qquad r = |\vec{r}|$$

Tenemos $\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2$

$$\begin{aligned}
[p_1^\alpha, r_1^\beta] &= -i\hbar\delta^{\alpha\beta} \\
[p_2^\alpha, r_2^\beta] &= -i\hbar\delta^{\alpha\beta} \\
[p_1^\alpha, r_2^\beta] &= 0 \\
[p_2^\alpha, r_1^\beta] &= 0
\end{aligned}$$

Posicion y momentum

$$\vec{P} = \vec{p}_1 + \vec{p}_2 \qquad \vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M}$$

$$\begin{aligned}
[p^\alpha, r^\beta] &= -i\hbar\delta^{\alpha,\beta} \\
\vec{p} &= \frac{m_2}{M}\vec{p}_1 - \frac{m_1}{M}\vec{p}_2
\end{aligned}$$

$$\begin{aligned}
[P^\alpha, R^\beta] &= -i\hbar\delta^{\alpha\beta}\underbrace{\left(\frac{m_1}{M} + \frac{m_2}{M}\right)}_{=1} \\
[p^\alpha, r^\beta] &= -i\hbar\delta^{\alpha\beta} \\
[p^\alpha, R^\beta] &= 0 \\
[P^\alpha, r^\beta] &= 0
\end{aligned}$$

$$H = \frac{\vec{p}_1^2}{2m_1} + \frac{\vec{p}_2^2}{2m_2} + V(r)$$

$$\text{Reemplazamos:} \qquad \vec{p}_1 = \vec{p} + \frac{m_1}{M}\vec{P} \qquad \vec{p}_2 = -\vec{p} + \frac{m_2}{M}\vec{P}$$

$$H = \frac{\vec{P}^2}{2M} + \frac{\vec{p}^2}{2m_{rad}} + V(r) \qquad \text{con:} \qquad \frac{1}{m_{rad}} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\left\langle \vec{R} \middle| \vec{P} \right\rangle = \frac{1}{(2\pi\hbar)^{3/2}} e^{i\frac{\vec{P}\cdot\vec{R}}{\hbar}} = \psi_{\vec{P}}(\vec{R})$$

$$H\left|\vec{P}; E_I\right\rangle = \left(\frac{\vec{P}^2}{2M} + E_I\right)\left|\vec{P}; E_I\right\rangle$$