Integrantes: Manuel Garcia, Carlos Llanos, Angel Almonacid.

1 ejercicio 5.13

$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta \qquad \theta = \arctan\left(\frac{y}{x}\right)$$
$$\partial_x r = \frac{x}{r} \qquad \partial_y r = \frac{y}{r}$$
$$\partial_r x = c_\theta \qquad \partial_\theta x = -rs_\theta \qquad \partial_x\theta = \frac{\frac{-y}{x^2}}{1+\frac{y^2}{x^2}} = \frac{-y}{x^2+y^2}$$
$$\partial_r y = s_\theta \qquad \partial_\theta y = rc_\theta$$
$$\partial_y\theta = \frac{x}{x^2+y^2}$$
$$dx\wedge dy = (c_\theta dr + -rs_\theta d\theta)\wedge(s_\theta dr + rc_\theta d\theta) = rc_\theta^2 dr\wedge d\theta + rs_\theta^2 dr\wedge d\theta = rdr\wedge d\theta$$

2 Ejercicio 5.15

$$\xi \in \Omega^q(M) \qquad \omega\in\Omega^r$$
$$\xi\wedge\omega = \xi_{i_1,\cdots,i_q}\omega_{j_1,\cdots,j_r}dx^{i_1}\wedge\cdots\wedge dx^{i_q}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_r}$$
$$d(\xi\wedge\omega) = \frac{1}{(q+r)!}\frac{\partial(\xi_{i_1,\cdots,i_q}\omega_{j_i,\cdots,J_r})}{x^v}dx^v\wedge dx^{i_1}\wedge\cdots\wedge dx^{i_q}\wedge dx^{j_1}\wedge\cdots\wedge dx^{j_r}$$
$$= \frac{1}{q!}\frac{\partial\xi_{i_1,\cdots,i_q}}{\partial x^v}dx^v\wedge dx^{i_1}\wedge\cdots\wedge dx^{i_q}\wedge\omega_{j_1,\cdots j_r}dx^{j_1}\wedge\cdots\wedge dx^{i_r}$$

$$\xi = fdx_{i_1}\wedge\cdots\wedge dx_{i_q} = fdx_I \qquad \omega = gdx_J$$

$$d(\xi\wedge\omega) = d(fdx_I\wedge gdx_J) = d\xi\wedge\omega + \xi\wedge d\omega$$

$$X[\omega(Y)] - Y[\omega(X)] - \omega([X,Y]) = \frac{\partial\omega_\mu}{\partial x^v}(X^vY^\mu - X^\mu Y^v)$$

$$\omega([X,Y]) = \omega_v(X^\mu\partial_\mu Y^v - Y^\mu\partial_\mu X^v)$$
$$X[\omega(Y)] = X^\mu\partial_\mu\omega_v Y^v + Y^\mu\omega_v\partial_\mu X^v$$
$$Y[\omega(X)] = Y^\mu\partial_\mu\omega_v X^v + Y^\mu\omega_v\partial_\mu X^v$$

$$X[\omega(Y)] - Y[\omega(X)] - \omega([X,Y]) = X^\mu\partial_\mu\omega_v Y^v - Y^\mu\partial_\mu\omega_v X^v = \partial_\mu\omega_v(X^\mu Y^v - Y^\mu X^v)$$
$$= \partial_\mu\omega_v(X^\mu Y^v - Y^\mu X^v)$$

$$d\omega(X_1,\cdots X_{p+1}) = \sum_{i=1}^{r}(-1)^{i+1}X_i\omega(X_1,\cdots\hat{X}_i\cdots X_{i+1}) + \sum_{i<j}(-1)^{i+1}\omega([x_i,X_j],X_1,\cdots\hat{X}_i,\cdots\hat{X}_j\cdots X_{i+1})$$

r-forma $\omega\in\Omega^r(M)$

$$\omega = \frac{1}{r!}\omega_{\mu_1\cdots\mu_r}dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_r}$$
$$d\omega = \frac{1}{r!}\left(\frac{\partial}{\partial x^v}\omega_{\mu_1\cdots\mu_r}\right)dx^v\wedge dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_r}$$

3 Ejercicio 5.19

(a) $\mathbb{R}^+ = \{x \in \mathbb{R} | x > 0\}$ $\qquad \frac{\partial}{\partial x} x^{-1} = -x^{-2} \neq 0$

(b)

$$\partial_x z = \partial_x (x+y) = 1$$
$$\partial_x (x^{-1}) = \partial_x (-x) = -1$$

(c)

$$(a,b) + (x,y) = (a+x, b+y) \qquad Dg = Dg(x) = \begin{bmatrix} a & \\ & b \end{bmatrix}$$
$$(x,y)^{-1} = (-x,-y) \qquad D(x,y)^{-1} = \begin{bmatrix} -1 & \\ & -1 \end{bmatrix}$$

Grupo de Lorentz

$$O(1,3) = \{M \in GL(4,\mathbb{R}) | M\eta M^T = \eta\} \qquad \eta = \mathrm{diag}(-1,1,1,1)$$