Clase 16

Manuel Garcia.

October 11, 2023

1 Ec. de Cauchy-Riemann

Cauchy-Riemann

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

En polares:

$$u_r = \frac{1}{r} v_\theta \qquad v_r = -\frac{1}{r} u_\theta$$

Ejercicio: $f(z) = \frac{iz^2+3}{z^3+i}, \quad z = re^{i\theta}$.

$$\begin{aligned}
f(re^{i\theta}) &= \frac{ir^2e^{2i\theta}+3}{r^3e^{3i\theta}+i} = \frac{ir^2(\cos 2\theta + i\sin 2\theta)+3}{r^3(\cos 3\theta + \sin 3\theta)+i} \\
&= \frac{[3 - r^2\sin 2\theta + ir^2\cos 2\theta](r^3\cos 3\theta - i(1+r^3\sin 3\theta))}{(r^3 i\cos 3\theta)^2 + (1+r^3\sin 3\theta)^2} \\
&= \frac{(3-r^2\sin 2\theta)r^5\cos 3\theta - i1 + r^3\sin 3\theta(3-r^2\sin 2\theta) + ir^2\cos 2\theta r^3\cos 3\theta + (1+r^3\sin 3\theta)r^2\cos 2\theta}{(r^3\cos 2\theta)^2 + (1+r^3\sin 3\theta)^2}
\end{aligned}$$

Derivada en polares: tenemos que $f'(z) = u_x + iv_x$

$$r^2 = x^2 + y^2 \qquad \theta = \tan^{-1}\frac{y}{x}$$

$$\begin{aligned}
f'(z) &= \left[\frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}\right] + i\left[\frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial v}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}\right] \\
&= \left[u_r\cos\theta - u_\theta\frac{\sin\theta}{r}\right] + i\left[v_r\cos\theta - v_\theta\frac{\sin\theta}{r}\right]
\end{aligned}$$

derivada en polares

$$f' = \left[u_r\cos\theta - u_\theta\frac{\sin\theta}{r}\right] + i\left[v_r\cos\theta - v_\theta\frac{\sin\theta}{r}\right]$$

Definicion

$$\epsilon(x) = -f'(x) + \frac{f(x) - f(x_0)}{x - x_0}$$

Esta será consecuente con que $\epsilon(x) \to 0$cuando $x_0 \to x$.

$$f(x) = f(x_0) + \epsilon(x)(x - x_0) + f'(x)(x - x_0)$$
$$f(x) = f(x_0) + \iota_1(x)\,|x - x_0| + f'(x)(x - x_0)$$
$$\epsilon_1(x) = \frac{e(x)(x - x_0)}{|x - x_0|}$$

2 Condiciones de suficiencia de las ecuaciones de Cauchy-Riemann

Teorema

Sea u y v funciones reales definidas en la vecindad del punto $(x_0, y_0) \in \mathbb{R}^2$
Si:

- $u_x(x_0, y_0)$ y $u_y(x_0, y_0)$ existen
- $u_x(x_0, y_0)$ y $u_x(x_0, y_0)$ son continuas en (x_0, y_0).

Entonces u es diferenciables en (x_0, y_0).