Clase 6

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1 Representación de Fock

$$\hat{a}\,|0\rangle := 0$$

Base $|0\rangle\,,\ \hat{a}^\dagger\,|0\rangle\,,\ \cdots\,,\ \frac{1}{\sqrt{n!}}(\hat{a}^\dagger)^n\,|0\rangle$ donde $\frac{1}{\sqrt{n!}}(\hat{a}^\dagger)^n\,|0\rangle = |n\rangle$

Complementariamente

$$\hat{a}^\dagger\,|n\rangle = (n+1)^{\frac{1}{2}}\,|n+1\rangle$$
$$\hat{a}\,|n\rangle = n^{\frac{1}{2}}\,|n-1\rangle$$
$$\langle 0|\,\hat{N}\,|0\rangle = \langle 0|\,\hat{a}^\dagger\hat{a}\,|0\rangle = 0$$

La ultima ecuacion nos dice que en promedio la energia del vacio es cero pero eso no quiere decir que no haya fluctuaciones de energia en este.

$$\langle 0|\,\hat{H}\,|0\rangle = \langle 0|\,\omega\hat{N}\,|0\rangle = 0$$
$$\hat{H}\,|0\rangle = \omega\hat{N}\,|0\rangle = \omega\hat{a}^\dagger\hat{a}\,|0\rangle = 0$$

Pero recordemos que tenemos dos espacios de Hilbert $\mathcal{H}$, $\tilde{\mathcal{H}}$ con dos Hamiltonianos $\hat{H}$, $\hat{\tilde{H}}$

1.1 Modelo del Sistema Auxiliar

$$\hat{H} = \omega\hat{a}^\dagger\hat{a} \qquad \rightarrow \qquad \hat{\tilde{H}} = \omega\hat{\tilde{a}}^\dagger\hat{\tilde{a}}$$

Notemos que en el sistema auxiliar tambien utilizamos ω y no $\tilde{\omega}$ ya que el sistema es el mismo y lo medible va a ser igual en ambos subsistemas. En este subsistema tenemos nuevos operadores

$$\hat{\tilde{a}}^\dagger\hat{\tilde{a}} = \hat{\tilde{N}} \qquad\qquad \hat{\tilde{N}}\,|\tilde{n}\rangle = n\,|\tilde{n}\rangle$$

Reglas de conmutacion

$$\left[\hat{\tilde{a}},\hat{\tilde{a}}\right] = \left[\hat{\tilde{a}}^\dagger,\hat{\tilde{a}}^\dagger\right] = 0$$
$$\left[\hat{\tilde{a}},\hat{\tilde{a}}^\dagger\right] = 1$$

Se asume

$$\left[\hat{a},\hat{\tilde{a}}\right] = \left[\hat{a},\hat{\tilde{a}}^\dagger\right] = \left[\hat{a}^\dagger,\hat{\tilde{a}}^\dagger\right] = \left[\hat{a}^\dagger,\hat{\tilde{a}}\right] = 0$$

Representación de Fock

$$\hat{\tilde{a}}\left|\tilde{0}\right\rangle := 0$$

Base $\left|\tilde{0}\right\rangle, \hat{\tilde{a}}^\dagger, \cdots, \frac{1}{\sqrt{n!}}\left(\hat{\tilde{a}}^\dagger\right)^n\left|\tilde{0}\right\rangle$
Complementariamente

$$\hat{\tilde{a}}^\dagger\left|\tilde{n}\right\rangle = (n+1)^{\frac{1}{2}}\left|\tilde{n}+1\right\rangle$$
$$\hat{\tilde{a}}\left|\tilde{n}\right\rangle = n^{\frac{1}{2}}\left|\tilde{n}-1\right\rangle$$
$$\left|0,\tilde{0}\right\rangle \equiv \left|\bar{0}\right\rangle$$

2 Para el sistema total

$$\left|\bar{0}\right\rangle,\ \bar{a}^\dagger\left|\bar{0}\right\rangle = \left|1,\tilde{0}\right\rangle,\ \hat{\tilde{a}}^\dagger\left|\bar{0}\right\rangle = \left|0,\tilde{1}\right\rangle,\ \hat{a}^\dagger\hat{\tilde{a}}^\dagger\left|\bar{0}\right\rangle = \left|1,\tilde{1}\right\rangle, \cdots$$

3 Cálculo de $\left|0(\beta)\right\rangle$

$$\begin{aligned}
\left|0(\beta)\right\rangle &= Z^{-\frac{1}{2}}(\beta)\sum_n e^{-\beta\frac{E_n}{2}}\left|n,\tilde{n}\right\rangle \\
&= Z^{-\frac{1}{2}}(\beta)\sum_n e^{-\beta\frac{E_n}{2}}\left|n\right\rangle\otimes\left|\tilde{n}\right\rangle \\
&= Z^{-\frac{1}{2}}(\beta)\sum_n e^{-\beta\frac{E_n}{2}}\frac{1}{\sqrt{n!}}\left(\hat{a}^\dagger\right)^n\left|0\right\rangle\otimes\frac{1}{\sqrt{n!}}\left(\hat{\tilde{a}}^\dagger\right)^n\left|\tilde{0}\right\rangle \\
&= Z^{-\frac{1}{2}}(\beta)\sum_n e^{-\beta\frac{n\omega}{2}}\frac{1}{n!}\left(\hat{a}^\dagger\right)^n\left(\hat{\tilde{a}}^\dagger\right)^n\left|\bar{0}\right\rangle
\end{aligned}$$