MAST90050 Group Project
Sports Fixture Scheduling

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1 Introduction

1.1 Background

Sport events have always been the center of attention and reckoned as a fruitful business in modern economy. Professional sports leagues, in particular, have attracted millions of fans all over the world. Billions of dollars were invested each year to organize events and matches that not only brought up popularity of certain sports but also offered a leverage for numerous job opportunities, merchandising and advertisements.

It is therefore important for organizers, teams and leagues to find an schedule that can bring up matches of quality while also optimizing their uses of investments. The sports scheduling problem has been extensively studied by researchers from various backgrounds since 1970s (Anderson, 1997; Eatson, Nemhauser, and M. A. Trick, 2004; Rasmussen and M. A. Trick, 2008). The general problem is often described as a round-robin tournament scheduling, whose terminology was later unified by Rasmussen and M. A. Trick (2008), with proposed framework for tournaments designs and solution approaches.

The generalization was brought even further by a unified data format specified in the three field notation ($\alpha|\beta|\gamma$) (Van Bulck et al., 2020; Graham et al., 1979). It its base case, a sports scheduling problem can be specified by a competition format α, where matches are organized in a k round-robin (kRR) fashion, a k bipartite round-robin (kBRR) pattern or a non-round-robin (NRR) one; a constraints specification, β, describes hard constraints that must always be respected regardless of problem type and additional soft constraints like breaks, fairness and home-away patterns; and the objectives, γ, assigns a infeasibility value and a objective value to each timetable.

There also exists many variants and extensions to the original problem, where some have became problems of their own that yield numerous further extensions and applications. The traveling tournament problem (TTP), for example, is a popular application to the general problem. Inspired by the Major League Baseball format, the TTP was first introduced by Easton, Nemhauser, and M. A. Trick (2001) in 2001 and and believed to be one of the most popular variants among many others due to its complexity (Thielen and Westphal, 2011).

As tournament formats varies from games to games, there are many interpretation of the general problem with a predefined sets of basic constraints. For example, the sports league scheduling problem (SLSP) is the more recognized interpretation of the general problem in recent studies. It is a basic framework that allows one to consider different variants of the general sports scheduling and generate schedule respecting multiple objectives or constraints with parallelization design (Henz, 1999). In most cases, the SLSP deals with a kRR tournament respecting Home-Away Patterns (HAP), where tournaments are either temporally constrained or relaxed. Computational advances in both algorithmic and machine level enabled sports leagues like the National Football League (NFL), Australian Football League (AFL) and etc. to adapt specific formulations of SLSP and there have been many insightful studies that help organizers and teams to organize matches in practice (Dilkina and Havens, 2004; Barone et al., 2006).

1.2 Problem Interpretation

In a sports game, the preparation of the organizer is to make a sports schedule for each team, including when and where the match should be started. What's more, they need to arrange which two teams should meet in each round according to some organising authorities. It is called a sports Fixture scheduling problem.

There exist lots of factors that may influence the schedule, which is listed as follows:

- whether it is a home match or an away match. Usually, the team tends to have a high probability to win at home.
- travelling, which includes the traveling distance time and adjustment over difference in time zone.
- site allocation of the game, mainly over the stadium usage.
- expected payoff of the tickets. If the games between two teams are scheduled consecutively, then fewer and fewer people are willing to buy the tickets.

It is essential for sports leagues to have a fair and reasonable schedule in a season, above factors are all required to be evaluated. For example, in order to make the game fair for every team, the number of home games and travelling distance to the venue should be considered. They are the subjective functions that need to be optimized in a mathematical model. Therefore from the perspective of objectives, it is a multi-objective problem which is proposed by Barone et al. (2006). Furthermore, there does not exist an optimal solution that makes each criterion best. Since there exist conflict constraints to optimize the different objectives. It results in the trade-offs among those objectives.

In this report, we will consider the sports league fixturing problem generally as it is league agnostic, however for implementation purposes we will consider the Australian Football League (AFL).

1.3 AFL Overview

The AFL is the largest professional sports league in Australia, first founded in 1897 and now governed by the AFL Commission, tasked with the administration of the competition. The league consists of 18 teams located around Australia, primarily in Victoria however teams are also home to other states including South Australia, Western Australia, Queensland and Sydney.

The AFL has a season running each year that consists of a home-and-away series that every team participates in, as well as a finals series for the Top 8 teams. Finals games are scheduled between teams that finish in specific positions on the AFL ladder, which is the table that notes where teams are ranked throughout the season.

In order for the sport to run, there must be a fixture that specifies which team plays who, where and when. In a typical AFL season, each team plays 22 times - once against every team, and twice against five teams, determined by where the team finished on the ladder in the previous season. In the 2020 AFL season, all 18 teams played 1 game each due to COVID-19. It is much simpler to consider the fixture like this, where there is 17 rounds. For the rest of this report, we will consider the 17-game per club fixture. This is known as a round-robin fixture, where every team plays every other team once. Each week in the season holds a "round" of 9 games, with most games falling on the weekend. Typical timeslots for games are Thursday night, Friday night, Saturday afternoon, Saturday night and Sunday afternoon. Saturday's can have up to 5 games and Sunday's up to 4, whereas Thursday and Friday night will have maximum 1 game each.

As the game has gotten more popular, there are varying objectives to consider when a fixture for the season is created, most notably fairness and television ratings. For a teams schedule to be fair, they should alternate between home and away games each week, as teams can find it difficult playing multiple away games in a row and may have it easy if they play too many games at home consecutively.

To maximize television ratings, the AFL wants the best games with the best teams (based either on the teams following or how good they are) in the best timeslots. Typically, the best timeslots are Thursday night, Friday night and Saturday night games as they attract the largest viewer amounts.

In this project, we will aim to create feasible AFL schedules with objectives that seek to maximize fairness through home-away recurrence for teams, as well as maximize television ratings.

2 Literature Review

Throughout years of development, there has seen many different formulations of SLSP. In the basic round-robin format with break minimization as the main objective, the problem can be simply interpreted as graph coloring problem with colored edges as matches that obey HAP (de Werra, 1980). The graph model was further investigated in de Werra (1981) with so-called canonical 1-factorization. It was later shown that one n even teams can have $n-2$ breaks (de Werra, 1982; Werra, 1985); Froncek and Meszka (2005) later showed that scheduling odd teams with home-away-bye patterns and no breaks is unique, yielding a similar results for even teams if there are more than one round. A more recent study gained insights from network flow problem and proposed a balanced HAP pattern that also considers the differentiability of games (Knust and Von Thaden, 2006).

The integer programming (IP) formulation was first proposed in Mcaloon, Tretkoff, and Wetzel (1997) and solved the tournament scheduling problem with 12 teams, achieving same results as ones from a constraint programming (CP) solver with running time significantly reduced. Rasmussen and M. Trick (1978) adapted a similar formulation with benders decomposition and with improved running time. It was later discussed in Kendall et al. (2010) that IP schemes failed to improve further if more complex constraints are applied, and large instances results are rarely practical for real-world scenarios.

An improved CP based approach was proposed by Henz (1999) to first split the problem into three stages that can each be solved with CP formulation, which was especially important as it established a general constraint-based round-robin planning framework for later studies. A local search on solution based on tabu search (TS) was then proposed by J.-P. Hamiez and J.-K. Hao (2000) to reduce solution search space through restricted neighborhood J.-P. Hamiez and J.-K. Hao, 2000, where solution quality was shown no worse than early contributions but with remarkable short running time. The idea of global constraints was introduced later in Henz, Müller, and Thiel (2004) to develop propagation algorithm for refining search space. A 40 teams solution was found in a reasonable running time, which is enough for real world practice, albeit better results were found in later contributions with a reformulation done in J.-P. Hamiez and J.-K. Hao (2004). However, like the IP formulation, CP also relies heavily on other heuristics or metaheuristics in order to reduce complexity of solution space search (Kendall et al., 2010). Moreover, implementation of CP can get much more complex compared with an IP one if more complicated constraints are introduced, due to the fact that one must also identify connections between different variables to constrain a feasible solutions.

In addition to various formulations, objective functions are also different and play a heavy role in practical SLSP studies. As discussed earlier, TTP was one of the most popular sport scheduling problem whose objective is to minimize travel distance across different teams while respecting HAP. It is often addressed as an extension to or a subproblem of SLSP and has been studied extensively on heuristics design (Easton, Nemhauser, and M. A. Trick, 2001; Kendall et al., 2010), but seen major modifications in recent literature with insights gained from real world practice (Melo, Urrutia, and Ribeiro, 2009; Biajoli and Lorena, 2006). The solution approach to TTP was first proposed in Easton, Nemhauser, and M. Trick (2003) with a parallel branch-and-price model that uses IP scheme to find the master problem of tournament schedule, and a constraint programming (CP) for the pricing problem as well as lower bound for the master problem. Over the course of IP/CP development, there has not been many discussion on the formulation of TTP. Heuristics designs for TTP, on the other hand, has changed drastically over the past decades, where focus was shifted from hybrid heuristics (Costa, Urrutia, and Ribeiro, 2012) to novel designs with inspirations drawn from other fields (Alatas and Bingol, 2019; Fernandes et al., 2020).

3 Methodology

In this section, we introduce our proposed methods for the AFL 2020 season scheduling in a three-phased approach, starting with a simple problem and stepping it up to more sophisticated ones that assembles to a single problem after all problems are solved.

Specifically, we want to first generate a feasible schedule for the 2022 AFL season, that is, a basic 17-round-robin schedule with no other objectives involved. We will then be looking at feasible schedules that respect the home-away recurrence fairness for each team as much as possible. At this stage, however, there is still no numerical objectives associated with these schedules. We therefore introduce a television rating scheme that assigns a numerical value to each games, where their sums are collected as a objective value for later comparison.

3.1 Data Collection and Instances

Instances to our scheduling problem include the set of 18 teams in the AFL competition. Since sports fixture scheduling is inherently a timetabling problem, where the language is constrained in these 18 teams. Therefore, to distinguish different instances, we randomly shuffled rankings of teams, and each permutation is counted as a single instance. On top of the 18 teams, we also need the time where games are played, the location of each teams home ground, and the ranking of each team, used for television ratings.

Thus, in general, data we are interested are:

- The set of 18 teams in the AFL
- Each teams home ground
- Each teams ranking
- Time slots for each game per round

The data is accessible through the official AFL website or any website that houses AFL statistics.

Team	**Home Ground**	**Ranking**
Adelaide	Adelaide Oval	15
Brisbane	GABBA	4
Carlton	Marvel Stadium	13
Collingwood	MCG	17
Essendon	MCG	8
Fremantle	Optus Stadium	11
Geelong	GMHBA Stadium	3
Gold Coast	Metricon Stadium	16
GWS	GIANTS Stadium	7
Hawthorn	UTAS Stadium	14
Melbourne	MCG	1
North Melbourne	Blundstone Arena	18
Port Adelaide	Adelaide Oval	2
Richmond	MCG	12
St Kilda	Marvel Stadium	10
Sydney	SCG	6
West Coast	Optus Stadium	9
Western Bulldogs	Marvel Stadium	5

Table 1: AFL team ranking from 2021 AFL ladder

We've also find typical time slots for TV broadcasted games:

Time Slot
Thursday 7.50pm
Friday 7.50pm
Saturday 1.45pm
Saturday 2.10pm
Saturday 7.25pm
Saturday 7.40pm
Sunday 1.10pm
Sunday 3.20pm
Sunday 4.40pm

Table 2: Typical times slots for TV broadcasted games

3.2 Optimization Goal

As discussed earlier, our threefold process requires specific methods to find feasible solutions. As the solution is presented in a 17-round timetables, the general idea is to find the best feasible schedule that respects particular constraints while maximizing the overall return at each stage. Individual goals will be discussed in later sections.

3.3 Round Robin Scheduling

In our first solution approach, our goal is to simply create a feasible schedule. Here, we do not yet consider home-away fairness or optimizing for television ratings. A feasible schedule is one that satisfies the optimization goal above.

It turns out this is a well studied problem, known as the **Round Robin Tournament**. One of the most common solution approaches is the Circle Method. We have coded up the circle method in Python as `round_robin.py`.

Below we present the basic idea of such approach:

Algorithm 1 Circle method for Round Robin Schedule

Assign competitors a number and create two lists that create pairings in the first round by joining the lists
Set $nrounds \leftarrow nteams - 1$
$k = 1$
while $k < nrounds$ **do**
    Fix the first competitor in position and rotate every other team clockwise. Assign as schedule for round k
    $k = k + 1$
end

As shown in fig. 1, if there are 14 teams, for example, we want to schedule 13 rounds of matches, where at each round except for the last round, one of the teams is fixed while others start to rotate to the position of their next team in the next consecutive round. In particular, games are scheduled such that teams in the first row match with those in the second row. After a round of match finishes, teams, except for the fixed one, moves to

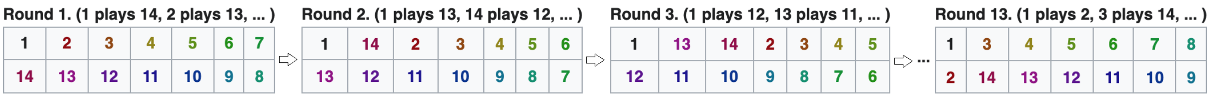


Figure 1: Visual representation of round-robin schedules for 14 teams solved by the circle method

their next position (to their right). If there are no further positions in the next column, the team goes to the next row of the same position; and teams go the next position of the previous row if there are no teams to their left.

While this method secures a feasible schedule, it is not a practical one that respect home-away fairness. For instance, if we consider the team North Melbourne, they play all their games at home (see table 8 in the Appendix), which is clearly unfair due to home advantage as discussed in section 1.3. For a national competition like the AFL, this is not acceptable and we must enforce fairness to a certain level.

3.4 Round Robin Scheduling with Home-Away Fairness

In order to address the issue with the first schedule we generated, we seek to create an algorithm that generates a feasible schedule that also respects fairness constraints. The idea is to ensure teams play home and away in a recurring fashion at each round for maximal fairness. For example, in the basic round-robin schedule (table 8), North Melbourne played every game at home, which as we noted, is simply not fair. We want this team and every other team to play at maximum 9 games either at home or away, ideally in a recurring pattern (e.g. home-away-home-away). We want to improve such a schedule by respecting the match order (i.e. first row being home, second one being away) in the circle method, which can be formalized and made possible by the following procedure

Algorithm 2 Home-Away Fair Schedule

Assign competitors a number and create two lists that create pairings in the first round by joining the lists
$nrounds \leftarrow nteams - 1$
$k = 1$
while $k < nrounds$ **do**
    Fix the first competitor in position and rotate every other team clockwise.
    Reverse the second list.
    If k is odd, flip the first match (home becomes away and vice versa)
    Assign as schedule for round k
    $k = k + 1$
end

While this schedule is enough scheduling a fair season of games, the AFL is a business and one may want to compare different schedules and explore other possibilities. In order to evaluate a schedule, we need to associate a numerical value that can explicitly used for comparison and represent solution quality.

3.5 Television Ratings Optimization

To evaluate different schedules, we must assign a value to each of them. Sport leagues like AFL gain their popularity mostly through broadcasting. Thus, we attempted to quantify this as the return to be maximized. In a simpler model, this can be measured and estimated through TV watching rate. It is expected that certain time slots are better than the others, during which matches often have higher TV ratings. We collected statistics from the past and found feasible time slots to distinguish ordering of these games played through weekend. In particular, each time slot is associated with a ranking that indicates their viewerships.

Time Slot	**Ranking**
Thursday 7.50pm	1
Friday 7.50pm	2
Saturday 1.45pm	6
Saturday 2.10pm	5
Saturday 7.25pm	3
Saturday 7.40pm	4
Sunday 1.10pm	8
Sunday 3.20pm	7
Sunday 4.40pm	9

Table 3: TV ratings ranked by viewership

Moreover, certain matches are more popular than the others and therefore, the TV rating can be further amplified by the team ranking.

Here, the goal is to assign top matches (by summing the ratings of the teams playing) to the best time slot and descend accordingly at each round. Specifically, this procedure can be formalized with the following pseudocode:

Algorithm 3 Optimize TV Ratings for a Round

Input: Schedule for a round with 9 games and team rankings
Create list *rankinglist* of each games ranking
for *each game in the round* **do**
| Add ranking of home team and ranking of away team and append to *rankinglist*
end
Sort *rankinglist*
Assign game to time slots based on *rankinglist*
Return TV-optimized round schedule

The above procedure is essentially a re-optimization of existing schedules, which can be either a basic round-robin schedule or a home-away fair one.

4 Results

4.1 Objective Measures

In order to demonstrate the performance of our scheduling heuristics, we need to first define an objective function. Moreover, for a more comprehensive study, we take the official AFL 2020 home-and-away season fixture, which is available in the table 11 of the appendix.

Moreover, we want to respect trades off between **tightness and tardiness**, as introduced in the lecture. In our case, a schedule can be fair but do not secure a high TV rating due to poor assignment; or it can be high in TV rating at the cost of game fairness.

The objective function is defined by:

$$Z_{fixture} = \sum_{r \in R} \sum_{j \in J} \frac{35}{GR_{rj}} \frac{9}{TR_j} - \alpha |HAV|,$$

where:

- GR_{rj} is the game ranking for game $j \in J$ for round $r \in R$, which is the addition of the two teams ranking scores, found in section 3.1.
- TR_j is the time ranking for game $j \in J$ which we showed in section 3.5.
- $|HAV|$ is the number of home-away violations, where a violation is counted if a team plays 2 consecutive home or away games.

For simplicity of result interpretation, the above equation can be written as

$$Z_{fixture} = TVOPT - \alpha |HAV|.$$

Note the top team has a ranking of 1 and the bottom team ranking 18, we take $\frac{35}{GR_{rj}}$ so the top ranking game gets the highest score. In other words, the worst possible game will have a rank of 35, which is $18 + 17$ if the two worst teams play. For example, if teams 1 and 2 play, the relative contribution to the objective from that game will be $\frac{35}{3} = 11.7$, whereas if teams 17 and 18 play, the objective contribution will be $\frac{35}{35} = 1$.

Similarly, for ranking of time slots, as there are 9 possible ranks as shown in algorithm 3. Thus, taking $\frac{9}{TR_j}$ ensures the top ranked time-slot contributes the most to TV rating of a given game. Finally, $|HAV|$ is the number of home-away violations, counted if a team plays 2 home games in a row or 2 away games in a row.

Our objective function multiplies the game rank with the time slot rank, which will be highest when the best teams play in the best time slot. This ensures optimal TV ratings. We also want to minimize the number of home-away violations, and the relative importance of a home-away violation can be scaled by the a parameter α.

For the three scheduling methods we used, as well as the AFL 2020 schedule, we have included in the appendix the first four rounds of the schedules. Full schedules are available in attached .csv files.

4.2 Experiment setup

In the following experiments, we generated 100 random instances to test out efficiency of our algorithms. Moreover, to study the effect of Home-Away violation on solution qualities, we applied a set of scaling constant $\alpha = 0.5, 1.0, 2.0$ and collect statistics correspond to each α. The objective values of each algorithm is then compared with the actual AFL schedule of the 2020 season to enrich our exploration of home-away fairness.

4.3 Basic Round Robin

Schedules	**Objective values**		
	$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 2.0$
Basic round-robin$_{best}$	**1097.45**	**969.45**	**713.45**
Basic round-robin$_{average}$	867.14	739.14	483.14
Basic round-robin$_{worst}$	738.13	610.13	354.13
AFL 2020	889.12	822.12	688.12
Basic round-robin$_{stdev}$	102.02	102.02	102.02
Share of solutions better than AFL 2020	0.35	0.19	0.08

Table 4: Objective value statistics of basic round-robin schedules from 100 random instances, compared with 2020 AFL schedule (solutions in bold mark suggest an improvement to the actual schedule)

We can read from table 4 that even basic round-robin can generate better solution than the actual AFL schedule under the metrics of our choice. In fact, there are **35 instances** that yield better schedules than the AFL 2020 one when $\alpha = 0.5$; 19 instances better when $\alpha = 1.0$ and 8 instances for $\alpha = 2.0$. Scaling of home-away violations are significant and this implies that the basic round-robin schedules has more violations than the actual schedule, which amplifies the scaling effect of HAVs.

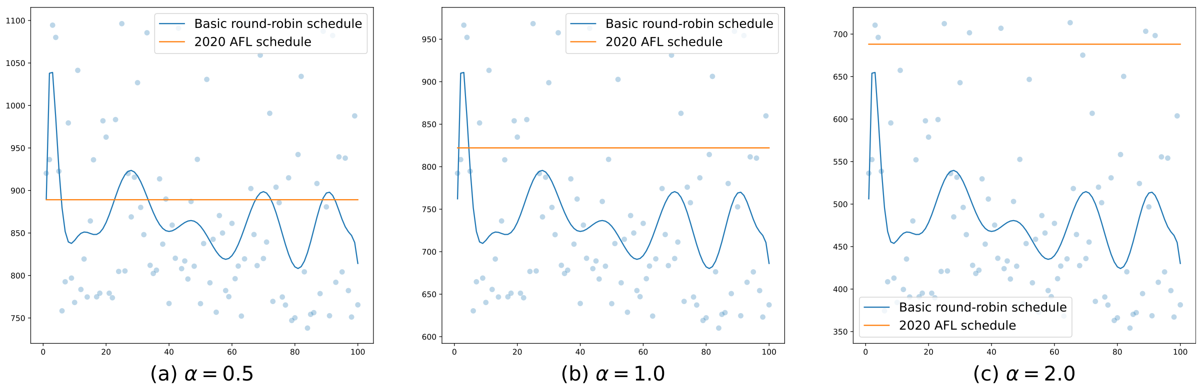


Figure 2: Objective values of basic round-robin schedules from 100 random instances, compared with 2020 AFL schedule and controlled by $\alpha = 0.5, 1.0, 2.0$

We can also use fig. 2 to observe that the scaling of α is clearly linear, identifying the fact that the basic round-robin has no respect over the fairness at all.

Moreover, the standard deviation for solutions acquired from the basic round-robin scheme are high across all α, which is also shown in fig. 2 as solutions are scattered across the entire domain where even a 10th polynomial fails to capture its pattern. We believe that the basic round-robin scheme performs nearly a randomized search that does not confine solution search space well

4.4 Home-Away Fairness

In the home-away fairness scheme, whose results are shown in table 5, we observe that almost most solutions achieved better performance than the actual schedule. Moreover, as we scale up α, which translates to a penalization of fairness violations, the actual schedule falls short in terms of desired objective values and solutions satisfying maximal home-away fairness only decreases linearly with the scale, as suggested in fig. 2.

Schedules	Objective values		
	$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 2.0$
Home-Away Fair$_{best}$	**1215.34**	**1207.34**	**1191.34**
Home-Away Fair$_{average}$	**974.97**	**966.97**	**950.97**
Home-Away Fair$_{worst}$	864.01	**856.01**	**840.01**
AFL 2020	889.12	822.12	688.12
Home-Away Fair$_{stdev}$	94.42	94.42	94.42
Share of solutions better than AFL 2020	0.84	1.0	1.0

Table 5: Objective value statistics of home-away fair schedules from 100 random instances, compared with 2020 AFL schedule (solutions in bold mark suggest an improvement to the actual schedule)

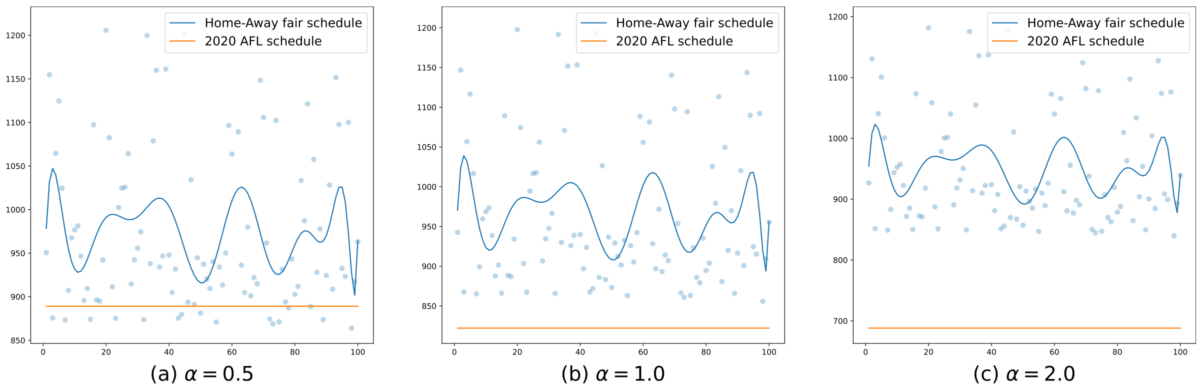


Figure 3: Objective values of round-robin schedules respecting home-away fairness from 100 random instances, compared with 2020 AFL schedule and controlled by $\alpha = 0.5, 1.0, 2.0$

The standard deviation for feasible solutions decreased as well, which implies a more confined solution space, which is indeed the case as this scheme tighten the constraints.

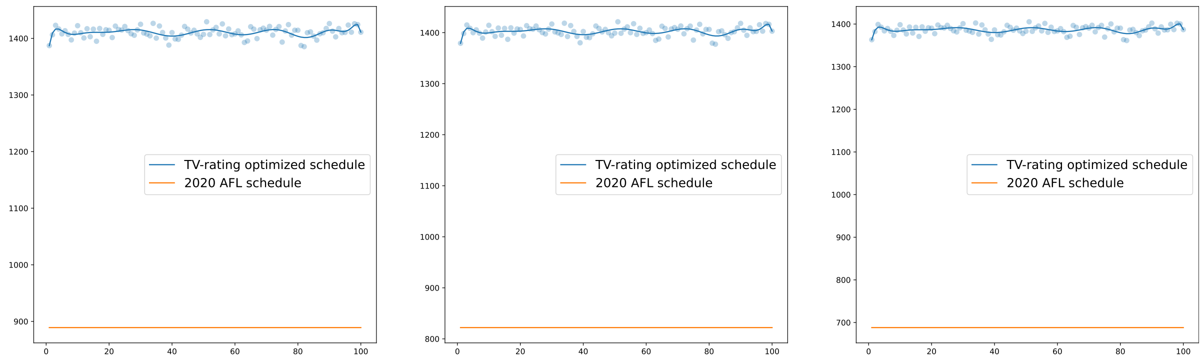
4.5 Home-Away Fairness and TV Rating Optimization

Finally, we present only the re-optimization with home-away fairness schedules due to the fact that they are the best schedules obtained so far and we are only interested on the improvement over existing solutions. Results for this solution scheme is therefore trivial in general.

Schedules	**Objective values**		
	$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 2.0$
TV-optimized$_{best}$	**1429.34**	**1421.34**	**1405.34**
TV-optimized$_{average}$	**1410.57**	**1402.57**	**1386.57**
TV-optimized$_{worst}$	**1385.41**	**1377.41**	**1361.41**
AFL 2020	889.12	822.12	688.12
TV-optimized$_{stdev}$	9.45	9.45	9.45

Table 6: Objective value statistics of round-robin schedules, re-optimized by TV ratings, respecting home-away fairness from 100 random instances, compared with 2020 AFL schedule (solutions in bold mark suggest an improvement to the actual schedule)

However, it is clear that that the standard deviation dropped by a significant amount, suggested by both table 6 and fig. 4. This new optimization scheme not only output solutions that produces solutions within the desired range of objective values, but also minimizes gaps between optimal and suboptimal solutions, yielding a more robust scheme that are less subjective to various instances.



(a) $\alpha = 0.5$ (b) $\alpha = 1.0$ (c) $\alpha = 2.0$

Figure 4: Objective values of round-robin schedules, re-optimized by TV ratings, respecting home-away fairness from 100 random instances, compared with 2020 AFL schedule and controlled by $\alpha = 0.5, 1.0, 2.0$

In fact, there are no constraint in place as discussed earlier. We use numerical values to confine solutions, where they are restricted to a given pattern of assignment, yielding a surprisingly low standard deviation.

4.6 Objective Function Analysis

A final round of comparison is drawn to show the overall improvement of the reoptimization scheme based on TV ratings. We apply the relative gaps, defined by

$$GAP = \frac{OPT_{TV-OPT} - OPT_{HAF}}{OPT_{HAF}}$$

to measure the overall improvement across all 100 sets of solutions.

Schedules	Improvement* (in %)		
	$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 2.0$
Home away fair	45.95	46.34	47.15

* relative gap measured in percentage

Table 7: Improvement of TV-optimization over the original schedule, measured by relative gaps from 100 random instances

As we read from table 7 that the improvement is remarkable and magnified by the scaling of violations. In other words, as we respect the home-away fairness more, the TV optimization scheme becomes the more important factor to objective value. We therefore believe that the objective function successfully captures the goal we aim to maximize, as in the return in terms of TV ratings and our solution scheme also corresponds to this objective function well in terms of maximization.

5 Discussion

The game scheduling is based on the round robin tournament. In order to make the game fair enough for each team, our algorithm aims to construct an equal number of the home match and away match in each round of the game. Furthermore, with the purpose of increase in the audience rate, television rating optimization is added, which is another objective in the mathematical model. It is achieved by arrange the popular teams to a more convenient time slot. So that the profit from the advertisement can be maximized. As mentioned above, this kind of the optimization problem is called multi-objective function. What's more, in our problem, the objectives are not conflicting with each other. Fairness and profit optimization can be reached at the same time.

In order to reduce the bias, Barone et al. (2006) suggest that a logical team name and round number can be used to replace the actual name. A team is randomly assigned to a number, so the team name is unknown during the organizing process. Therefore the interference of man-made factors is minimized, and accordingly increase the fairness of the game.

There are still some factors not included in the models that are influential to the the team performance, such as the travel and the ticket payoffs. In terms of the factor "travel", the travelling distance and different time zones between different venues. However, they are difficult to quantify using the mathematical model. In addition, TV is not the only way to watch the match nowadays. Young people prefer to watch videos online. So in the future research, view counts online should also be considered.

What's worse, when all factors are included, some constraints will be conflicted in an scheduling optimization problem. For example, to guarantee the fairness of the game, the travelling distance is possible to be increased. So the optimal result is hard to be reached. in such multi-objective evolutionary problem. Barone et al. (2006) propose an approach called "Pareto optimal". All results in this set are mutually non-dominated with each other and are at least as good as the other results. These results can be seen as the candidate results when different weights are assigned to different objectives. If the scheduling regularity prefers a fair game compared with all other objectives, then the mathematical model should put most of the weights on the objective "fairness". So that the outcome with the fair game will be provided even though the objective "distance" will be worsen. So the candidates can be selected with the different needs of the scheduling problem.

Our algorithm is based on the Round-Robin tournament, several other methods can also be applied to this problem. In the article written by J. P. Hamiez and J. K. Hao (2001), a heuristic method called Tabu search is applied to SLSP. It helps to get rid of the local optima and already visited solutions. What's more, a useful method called identification (stored the beat solution) and diversification(directly search the unvisited solutions) is also applied to increase the efficiency of the tabu search method. It is suitable when the number of teams is large (up to 40 teams). However, Tabu search contains lots of unknown parameters that may influence the quality of the result. Usually, Tabu search takes a long time to finish the algorithm Since the tabu search has to go over all of the neighbourhoods in each iteration, which is really time-consuming.

Another limitation of our method is that it only applies to AFL. The data used for our analysis is 2020 AFL. When it comes to other game with different scheduling rules, the mathematical model will have totally different constraints and objectives. However, the multi-objective evolutionary algorithm is a good choice for SLSP. More sports fixtures with different rules can be used to examine the feasibility and generality of this algorithm in the future.

6 Conclusion

Our project is about to investigate the optimization scheduling problem called SLSP. We focus on AFL to do the mathematical model and result analysis. It can be seen as a evolutionary-based multi-objective problem. We solve this kind of problem by the algorithm called multi-objective evolutionary algorithm. First problem is to create a feasible solution for AFL by Round-Robin tournament. Secondly, to maintain the fairness of the game, a new objective is included to the model. Thirdly, the TV ratings is also considered, which is another objective in the model. Then the optimal result is established. Our model is flexible enough for a user to see how much weights we want to put on each of the objectives.

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Appendix

For the purpose of results interpretation, we will only show the first 4 rounds of selected schedule from each method. Use attached .csv files to view the full schedule.

Fixture: Basic Round Robin

Round	Home Team	Away Team	Venue	Time
Round 1	North Melbourne	Brisbane	Marvel Stadium	Thursday 7.50pm
Round 1	Collingwood	Western Bulldogs	MCG	Friday 7.50pm
Round 1	Geelong	GWS	GMHBA Stadium	Saturday 1.45pm
Round 1	Gold Coast	Essendon	Metricon Stadium	Saturday 2.10pm
Round 1	West Coast	Melbourne	Optus Stadium	Saturday 7.25pm
Round 1	Hawthorn	Carlton	MCG	Saturday 7.40pm
Round 1	Sydney	Adelaide	SCG	Sunday 1.10pm
Round 1	St Kilda	Richmond	Marvel Stadium	Sunday 3.20pm
Round 1	Fremantle	Port Adelaide	Optus Stadium	Sunday 4.40pm
Round 2	North Melbourne	Western Bulldogs	Marvel Stadium	Thursday 7.50pm
Round 2	Brisbane	Melbourne	Gabba	Friday 7.50pm
Round 2	Gold Coast	Richmond	Metricon Stadium	Saturday 1.45pm
Round 2	Hawthorn	GWS	MCG	Saturday 2.10pm
Round 2	Collingwood	Carlton	MCG	Saturday 7.25pm
Round 2	West Coast	Essendon	Optus Stadium	Saturday 7.40pm
Round 2	St Kilda	Port Adelaide	Marvel Stadium	Sunday 1.10pm
Round 2	Geelong	Adelaide	GMHBA Stadium	Sunday 3.20pm
Round 2	Sydney	Fremantle	SCG	Sunday 4.40pm
Round 3	North Melbourne	Melbourne	Marvel Stadium	Thursday 7.50pm
Round 3	Western Bulldogs	Carlton	Marvel Stadium	Friday 7.50pm
Round 3	Hawthorn	Adelaide	MCG	Saturday 1.45pm
Round 3	West Coast	Richmond	Optus Stadium	Saturday 2.10pm
Round 3	Brisbane	Essendon	Gabba	Saturday 7.25pm
Round 3	Collingwood	GWS	MCG	Saturday 7.40pm
Round 3	Geelong	Fremantle	GMHBA Stadium	Sunday 1.10pm
Round 3	Gold Coast	Port Adelaide	Metricon Stadium	Sunday 3.20pm
Round 3	St Kilda	Sydney	Marvel Stadium	Sunday 4.40pm
Round 4	North Melbourne	Carlton	Marvel Stadium	Thursday 7.50pm
Round 4	Melbourne	Essendon	MCG	Friday 7.50pm
Round 4	West Coast	Port Adelaide	Optus Stadium	Saturday 1.45pm
Round 4	Collingwood	Adelaide	MCG	Saturday 2.10pm
Round 4	Western Bulldogs	GWS	Marvel Stadium	Saturday 7.25pm
Round 4	Brisbane	Richmond	Gabba	Saturday 7.40pm
Round 4	Gold Coast	Sydney	Metricon Stadium	Sunday 1.10pm
Round 4	Hawthorn	Fremantle	MCG	Sunday 3.20pm
Round 4	Geelong	St Kilda	GMHBA Stadium	Sunday 4.40pm

Table 8: Basic Round Robin schedule with objectives HAV = 256 and TVOPT = 883.9

Fixture: Home-Away Fair

Round	Home Team	Away Team	Venue	Time
Round 1	North Melbourne	Brisbane	Marvel Stadium	Thursday 7.50pm
Round 1	Collingwood	Western Bulldogs	MCG	Friday 7.50pm
Round 1	Geelong	GWS	GMHBA Stadium	Saturday 1.45pm
Round 1	Gold Coast	Essendon	Metricon Stadium	Saturday 2.10pm
Round 1	West Coast	Melbourne	Optus Stadium	Saturday 7.25pm
Round 1	Hawthorn	Carlton	MCG	Saturday 7.40pm
Round 1	Sydney	Adelaide	SCG	Sunday 1.10pm
Round 1	St Kilda	Richmond	Marvel Stadium	Sunday 3.20pm
Round 1	Fremantle	Port Adelaide	Optus Stadium	Sunday 4.40pm
Round 2	Brisbane	Port Adelaide	Gabba	Thursday 7.50pm
Round 2	Adelaide	Fremantle	Adelaide Oval	Friday 7.50pm
Round 2	Carlton	Gold Coast	MCG	Saturday 1.45pm
Round 2	Essendon	Geelong	MCG	Saturday 2.10pm
Round 2	Richmond	Sydney	MCG	Saturday 7.25pm
Round 2	GWS	St Kilda	GIANTS Stadium	Saturday 7.40pm
Round 2	Western Bulldogs	West Coast	Marvel Stadium	Sunday 1.10pm
Round 2	Melbourne	Hawthorn	MCG	Sunday 3.20pm
Round 2	North Melbourne	Collingwood	Marvel Stadium	Sunday 4.40pm
Round 3	Collingwood	Brisbane	MCG	Thursday 7.50pm
Round 3	West Coast	North Melbourne	Optus Stadium	Friday 7.50pm
Round 3	St Kilda	Essendon	Marvel Stadium	Saturday 1.45pm
Round 3	Geelong	Carlton	GMHBA Stadium	Saturday 2.10pm
Round 3	Hawthorn	Western Bulldogs	MCG	Saturday 7.25pm
Round 3	Gold Coast	Melbourne	Metricon Stadium	Saturday 7.40pm
Round 3	Fremantle	Richmond	Optus Stadium	Sunday 1.10pm
Round 3	Sydney	GWS	SCG	Sunday 3.20pm
Round 3	Port Adelaide	Adelaide	Adelaide Oval	Sunday 4.40pm
Round 4	Brisbane	Adelaide	Gabba	Thursday 7.50pm
Round 4	Richmond	Port Adelaide	MCG	Friday 7.50pm
Round 4	Melbourne	Geelong	MCG	Saturday 1.45pm
Round 4	Carlton	St Kilda	MCG	Saturday 2.10pm
Round 4	GWS	Fremantle	GIANTS Stadium	Saturday 7.25pm
Round 4	Essendon	Sydney	MCG	Saturday 7.40pm
Round 4	North Melbourne	Hawthorn	Marvel Stadium	Sunday 1.10pm
Round 4	Western Bulldogs	Gold Coast	Marvel Stadium	Sunday 3.20pm
Round 4	Collingwood	West Coast	MCG	Sunday 4.40pm

Table 9: Homw-Away Fair schedule with objetives HAV = 16 and TVOPT = 1078.8

Fixture: TV Ratings Optimized

Round	Home Team	Away Team	Venue	Time
Round 1	West Coast	Melbourne	Optus Stadium	Thursday 7.50pm
Round 1	Geelong	GWS	GMHBA Stadium	Friday 7.50pm
Round 1	Collingwood	Western Bulldogs	MCG	Saturday 1.45pm
Round 1	North Melbourne	Brisbane	Marvel Stadium	Saturday 2.10pm
Round 1	Fremantle	Port Adelaide	Optus Stadium	Saturday 7.25pm
Round 1	Sydney	Adelaide	SCG	Saturday 7.40pm
Round 1	Gold Coast	Essendon	Metricon Stadium	Sunday 1.10pm
Round 1	St Kilda	Richmond	Marvel Stadium	Sunday 3.20pm
Round 1	Hawthorn	Carlton	MCG	Sunday 4.40pm
Round 2	Brisbane	Port Adelaide	Gabba	Thursday 7.50pm
Round 2	Essendon	Geelong	MCG	Friday 7.50pm
Round 2	Richmond	Sydney	MCG	Saturday 1.45pm
Round 2	GWS	St Kilda	GIANTS Stadium	Saturday 2.10pm
Round 2	Western Bulldogs	West Coast	Marvel Stadium	Saturday 7.25pm
Round 2	Melbourne	Hawthorn	MCG	Saturday 7.40pm
Round 2	Carlton	Gold Coast	MCG	Sunday 1.10pm
Round 2	Adelaide	Fremantle	Adelaide Oval	Sunday 3.20pm
Round 2	North Melbourne	Collingwood	Marvel Stadium	Sunday 4.40pm
Round 3	Sydney	GWS	SCG	Thursday 7.50pm
Round 3	Geelong	Carlton	GMHBA Stadium	Friday 7.50pm
Round 3	Hawthorn	Western Bulldogs	MCG	Saturday 1.45pm
Round 3	St Kilda	Essendon	Marvel Stadium	Saturday 2.10pm
Round 3	Gold Coast	Melbourne	Metricon Stadium	Saturday 7.25pm
Round 3	Port Adelaide	Adelaide	Adelaide Oval	Saturday 7.40pm
Round 3	Fremantle	Richmond	Optus Stadium	Sunday 1.10pm
Round 3	Collingwood	Brisbane	MCG	Sunday 3.20pm
Round 3	West Coast	North Melbourne	Optus Stadium	Sunday 4.40pm
Round 4	Melbourne	Geelong	MCG	Thursday 7.50pm
Round 4	Richmond	Port Adelaide	MCG	Friday 7.50pm
Round 4	Western Bulldogs	Gold Coast	Marvel Stadium	Saturday 1.45pm
Round 4	Brisbane	Adelaide	Gabba	Saturday 2.10pm
Round 4	Essendon	Sydney	MCG	Saturday 7.25pm
Round 4	GWS	Fremantle	GIANTS Stadium	Saturday 7.40pm
Round 4	Collingwood	West Coast	MCG	Sunday 1.10pm
Round 4	Carlton	St Kilda	MCG	Sunday 3.20pm
Round 4	North Melbourne	Hawthorn	Marvel Stadium	Sunday 4.40pm

Table 10: Home-Away scheduled re-optimized based on TV-rating with objectives HAV = 16 and TVOPT = 1419.53

Fixture: AFL 2020

Round	Home Team	Away Team	Venue	Time
1	Richmond	Carlton	MCG	Thursday 7.50pm
1	Western Bulldogs	Collingwood	Marvel Stadium	Friday 7.50pm
1	Essendon	Fremantle	Marvel Stadium	Saturday 1.45pm
1	Adelaide Crows	Sydney Swans	Adelaide Oval	Saturday 2.10pm
1	GWS Giants	Geelong Cats	GIANTS Stadium	Saturday 7.25pm
1	Gold Coast Suns	Port Adelaide	Metricon Stadium	Saturday 7.40pm
1	North Melbourne	St Kilda	Marvel Stadium	Sunday 1.10pm
1	Hawthorn	Brisbane Lions	MCG	Sunday 3.20pm
1	West Coast Eagles	Melbourne	Optus Stadium	Sunday 4.40pm
2	Collingwood	Richmond	MCG	Thursday 7.50pm
2	Geelong Cats	Hawthorn	GMHBA Stadium	Friday 7.50pm
2	Brisbane Lions	Fremantle	Gabba	Saturday 1.45pm
2	Carlton	Melbourne	Marvel Stadium	Saturday 2.10pm
2	Gold Coast Suns	West Coast Eagles	Metricon Stadium	Saturday 7.25pm
2	Port Adelaide	Adelaide Crows	Adelaide Oval	Saturday 7.40pm
2	GWS Giants	North Melbourne	GIANTS Stadium	Sunday 1.10pm
2	Sydney Swans	Essendon	SCG	Sunday 3.20pm
2	St Kilda	Western Bulldogs	Marvel Stadium	Sunday 4.40pm
3	Richmond	Hawthorn	MCG	Thursday 7.50pm
3	Western Bulldogs	GWS Giants	Marvel Stadium	Friday 7.50pm
3	North Melbourne	Sydney Swans	Marvel Stadium	Saturday 1.45pm
3	Collingwood	St Kilda	MCG	Saturday 2.10pm
3	Brisbane Lions	West Coast Eagles	Gabba	Saturday 7.25pm
3	Geelong Cats	Carlton	GMHBA Stadium	Saturday 7.40pm
3	Gold Coast Suns	Adelaide Crows	Metricon Stadium	Sunday 1.10pm
3	Fremantle	Port Adelaide	Metricon Stadium	Sunday 3.20pm
4	Sydney Swans	Western Bulldogs	SCG	Thursday 7.50pm
4	GWS Giants	Collingwood	GIANTS Stadium	Friday 7.50pm
4	Port Adelaide	West Coast Eagles	Metricon Stadium	Saturday 1.45pm
4	St Kilda	Richmond	Marvel Stadium	Saturday 2.10pm
4	Essendon	Carlton	MCG	Saturday 7.25pm
4	Gold Coast Suns	Fremantle	Metricon Stadium	Saturday 7.40pm
4	Brisbane Lions	Adelaide Crows	Gabba	Sunday 1.10pm
4	Melbourne	Geelong Cats	MCG	Sunday 3.20pm
4	Hawthorn	North Melbourne	Marvel Stadium	Sunday 4.40pm

Table 11: AFL 2020 season schedule with objectives HAV = 134 and TVOPT = 956.12