MAST90014 Optimization for Industry Group Assignment(GA)

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1 Introduction

In general, with the improvement of living standards and the aging of the population, people's demand for family services is increasing. People have different service needs, such as baby care, household cleaning, garden trimming, beauty, elderly care and carpentry. Home service is straightforward and convenient for people as service is provided in their own home since they only need to make an appointment by phone or online. According to Verified Market Research, the market for on-demand home services is expected to grow by USD 4730.31 billion between 2021-2025, of which 38% of the growth will come from the Asia-Pacific region. The market growth in the Asia-Pacific region will be faster than that in Europe and North America [3]. In addition, with the explosive growth of online booking services, some major companies (such as Google and Amazon) are involved in a market of about USD 400 billion, and Amazon Home Services covers more than 700 services [4]. In order to cope with the ever-increasing market and the pressure of competition among enterprises, service providers must reduce costs and improve efficiency.

For the purpose of this article, we consider that the company of 90014 Home Service (14HS) who hire service providers and provide specified types of services for their clients. For the benefit of the company, and on the basis of providing each client with vital home services, the company is looking to allocate service providers to clients with the minimal hiring cost possible. This problem can be commonly known as an alteration of the vehicle routing problem with time windows (VRPTW). Hence the relevant approach of a Mixed-Integer Program was implemented based on a location-allocation-routing (LAR) model, as well as Imperialist Competitive Algorithm (ICA) [2] to complement the computational complexity of the implementation, considering that is an NP-Hard problem. A feasible solution was achieved with our implementation with certain sample test inputs.

To model for a more realistic problem for 14HS, additional constraints were implemented. These constraints include hourly hire rates for the provider, allowing flexibility in the time window that the client demands for service, service providers having flexible finish times, having a variety of services available for clients to choose from, and for service providers to have special skills for the varied services respectively. The approach was again implemented as an MIP problem, and a feasible solution was achieved with a sample test input.

2 Literature Review

The problem 90014 Home Service (14HS) faces is a classical task assignment problem. The company currently has a numerous clients who demands home services from 14HS, and a numerous number of agents available for hire and placement. The clients demands a service at a specific time respectively, and each agent is only available within a specific time window. Each client must be assigned an agent that meets the demands of the client's services, within a common time window that matches both parties as well as other constraints and characteristics that satisfies both. In relevant literature, there are several approaches that are similar to the problem at hand. The Home Health Care Supply Chain Problem is an example of this, as well as other task assignment problems. All of which are well known as an alternation of the vehicle routing problem with time windows.

In the Home Health Care Supply Chain Problem [2], it attempts to solve an assignment problem with a set of patients, a set in a pharmacy and a set of nurses for home care services. The main decisions to be made are the location of pharmacies, the assignment of patients to established pharmacies, and the assignment of nurses to patients. The authors took the approach of a location-allocation-routing (LAR) model as the general modelling for assignment, as well as an Imperialist Competitive Algorithm (ICA) for the case of large sizes. Given that this is a case of a NP hard problem, the complexity of the model increases with increasing the number of clients, pharmacies and nurses included in the model.

Furthermore, the relevant problem mentioned can be extended with additional constraints for a more realistic problem [1]. Rather than starting at a common location first (known as "headquarters"), the extension of the home health care problem allows the nurses to start their workday from home, as well as taking into account the lunch breaks for the nurses. The approach to this extended problem not only involves a Mixed-Integer Program for the basic optimization, but a two-step heuristic was also implemented on top. The first stage is responsible for building multiple random and simultaneous tours for each nurse, complying with the defined constraints; and the purpose of the second stage was to improve the tours defined in the first stage.

3 Part 1: Basic Model

3.1 Problem Specification

Given the motivation and related literature discussed in earlier sections, in the following sections, we consider a home service company named 90014 Home Service (14HS) whose main business is to allocate the service providers from the its headquarters to serve the clients at home. It is assumed that, all service providers must go back to the headquarters after they have finish their work by the end of a normal work day. For now, we assume that the company only provides a single type of home service with several service providers, where each provider can have various available time and hiring costs. Meanwhile, each service requested by a client will have a start time that specifies client's demand, as well the associated duration to serve that client. To simplify our model, the location of the service is assumed to be the home of each clients; and each provider will spend different time moving from one location (client) to another location (client). To make the 14HS Company maximize its profit, the hiring cost of the service providers must be minimized, while all clients' requests should also be satisfied.

Our approach, as discussed earlier, is to first formulate the problem as a vehicle routing problem with time windows (VRPTW) with slightly looser constraints, and with some influence drawn from the location-allocation-routing (LAR) model. The goal is, clearly, to minimize the hiring cost with the following assumptions:

- Time windows are assumed to be "hard" that allow earliness but not tardiness. That is, if a provider arrives at a client's home prior to the earliest start time, then it must wait until the beginning of the time window (i.e. when client demands a service) before it may start serving the client. On the other hand, lateness is, in any case, prohibited, so that the service provider cannot serve the client if they arrived late.
- The number of service providers, $|M|$, should be large enough to yield a feasible solution.

3.2 Mathematical Model

An Mixed Integer Programming model was created and implemented with the following data, variables and constraints.

Data

- o: single depot at 14HS headquarters which all service providers must start from and end at
- l: time of the end of the workday (when all service providers should be back at 14HS headquarters).
- N: set of clients.
  - s_i: service start time for client i.
  - d_i: service duration for client i.
- M: set of service providers.
  - f_k: hiring cost of service provider k.
  - w_k: time that service provider k is available at 14HS.
- t_{ij}: travel time between locations $i, j \in N \cup \{o\}$.

Variable

- x_{ijk}: binary variables to indicate whether provider k goes from client i to client j.
- c_{ik}: the time that the provider k visits client i.

Model

$$\begin{aligned}
\min & \sum_{j \in N} \sum_{k \in M} f_k x_{0jk} && && (1)\\
\text{s.t.} & \sum_{j \in N} x_{0jk} = 1, && k \in M && (2)\\
& \sum_{j \in N} x_{i0k} = 1, && k \in M && (3)\\
& \sum_{i \in N} x_{ihk} - \sum_{j \in N} x_{hjk} = 0, && i \in N && (4)\\
& \sum_{k \in M} \sum_{j \in N} x_{ijk} = 1, && i \in N && (5)\\
& c_{ik} + t_{ij} + d_i - \mathbf{M}(1 - x_{ijk}) \leq c_{jk}, && i, j \in N, k \in M && (6)\\
& c_{ik} = s_i, && i \in N, k \in M && (7)\\
& s_i \geq t_{0i}, && i \in N && (8)\\
& c_{ik} + d_i + t_{i0} \leq l, && j \in N, k \in M && (9)\\
& w_k + t_{0j} \cdot x_{0jk} \leq s_j, && j \in N, k \in M && (10)
\end{aligned}$$

Constraints (2) and (3) are imposed to require each service provider starts from and end at the headquarter. (4) is the classical flow conservation constraint that requires service provider to leave the client i after its service is done. Constraint (5) ensures each client is visited exactly once. The inequality (6) respects a complete service, in which case a service provider k cannot leave client i and head to the next client j before it finishes its service. Note that this can be done by placing a bounding constant $\mathbf{M}$ that ensures every time window is properly summed and provider k may not leave to j (i.e. $x_{ijk} = 1$), otherwise this constraint will be violated. Similarly, constraint (7) ensures service time window is matched between a client and a service provider by setting the time that provider k visits client i to be equal to the time that client i demands a service. Constraint (8) ensures that the service is delivered on time by restricting the service start time for client i to be always greater than the time it takes to travel from headquarters to client i. Constraint (9) states that each service provider should return to headquarter before the end of the day. The final constraint (10) states that the start time of the first service from provider k at client j should be larger than sum of the time till service provider k is available at the headquarters, and the travel time from the headquarters to that client.

3.3 Computational Result

Result Description

In running a computation[1] with 5 providers and 6 clients for a normal workday duration of 8 hours, we yield a minimized total hiring cost of 35 by hiring service provider 1, 3 and 4, whose routes are described as follows:

- Provider 1 is hired at a cost of 15
  - **0h**: it is available at headquarter, and leaves to the client 2.
  - **1h**: it takes 1 hour to arrive at client 2. (The service duration at client 2 is 1 hour.)
  - **4h**: it takes 2 hours to arrive at client 1. (The service duration at client 1 is 1 hour.)
  - **6h**: it take 1 hour to return to the headquarter.
- Provider 3 is hired at a cost of 8
  - **4h**: it is available at headquarter, and leaves to the client 3.
  - **6h**: it takes 1 hour to arrive at client 3, and wait for one hour to start the service. (The service duration at client 3 is 1 hour.)
  - **8h**: it takes 1 hour to return to the headquarter.
- Provider 4 is hired at a cost of 12
  - **0h**: it is available at headquarter, and leaves to the client 4.
  - **1h**: it takes 1 hour to arrive at client 4. (The service duration at client 4 is 1 hour.)
  - **3h**: it takes 1 hour to arrive at client 5. (The service duration at client 5 is 1 hour.)
  - **5h**: it takes 1 hour to arrive at client 6. (The service duration at client 3 is 2 hour.)
  - **8h**: it takes 1 hour to return to the headquarter.

As shown in figure 1, the feasible solution is marked by routes of each individual service provider. Looping routes at a specific node (that can be either a client or the depot) denotes the "stay" of corresponding service provider at that node for a given period of time, where, in particular, a dashed loop represents a waiting time that can either describe the

- time till the corresponding provider is available at 14 HS HQ; or
- time which the corresponding provider must wait before serving the client.

[1] All numerical experiments were run on the Gurobi optimizer with standard configuration

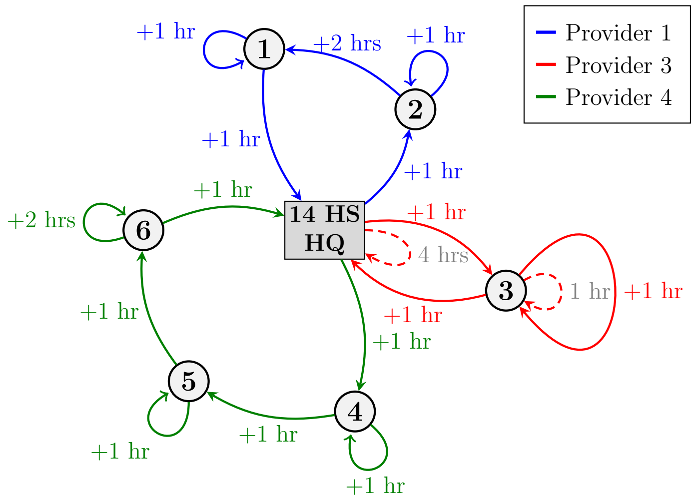

Figure 1: Illustration of feasible solution obtained from base model with 5 providers and 6 clients for a duration of 8 hours

4 Part 2: Improved Model

4.1 Problem Specification

In section 3 we solve our problem with a MIP formulation of VRPTW with extra constraints to successfully arranged optimal routes for service providers in the given scenario described in our dataset. However, such a model faces a great challenge dealing with real-world problems that is often far more complex and thus requires a more thorough inspection. In this section, we concern an improvement of previously defined model by feeding additional data that describe both client and service provider behaviours, as well as additional constraints that help refine the model to better suit real-world demands. In particular, we use a larger set of data to further inspect the efficiency of our model. However, due to the NP-hard nature of MIP, and given the computational challenge caused by the size of data as well as the complexity to illustrate a feasible solution, we again only consider a scenario where only 5 providers and 6 clients are in place. Now, for the improved model, we also consider the following:

1. 14HS offers a variety of services, hence each client service demand is one particular type of service. For the purpose of the implementation in this paper, 14HS offers 3 types of services: plumbing, gardening, and babysitting.

2. Each provider will have specialised skills in one or more types of services. Hence each type of client service demand will need a specialist to account for.

3. The hiring costs for providers will be hourly and calculated based on the hours they spent in the day.

4. Clients available for a given time period, where the service duration may fall within. For example, if a clients who submit a gardening request is available throughout an afternoon between 1pm to 5pm, the provider must start their service any time in within this time window as long as the service end time matches end time which the client is available.

In the previous model specification, we assume that time windows are always "hard". By adding service time window for both providers and clients granting us some degree of flexibility in terms of time window. Note that this is not necessarily equivalent to the VRP with soft time windows as proposed in [6] as the time window is still considered "hard" and providers are expected to wait if they arrived earlier than the earliest starting time which clients demand.

4.2 Improvement of the Model

A similar Mixed Integer Programming model was created and implemented with as was shown in Section 3. To model for the additional constraints added to the problem, additional data and constraints was created based on the model in Section 3. These are as follows:

Additional data

- f_k: hiring cost per hour for provider k
- ms_k: time provider k can start service
- me_k: time provider k must end service
- ns_i: time client i can start getting the service
- ne_i: time client i must end service
- H: set of skills
  - ph_{kh}: Binary variable if provider k has skill h
  - nh_{ih}: Binary variable if client i requires skill h

Model

$$
\begin{aligned}
& \min \sum_{i \in N} \sum_{j \in N} \sum_{k \in M} f_k x_{ijk} d_j && && (11) \\
\text{or} \quad & \min \sum_{k \in M} f_k \left[\max_{j \in N, i \in N} (x_{ijk}(c_{jk} + d_j)) - c_{0k} \right] && && (12) \\
\text{s.t.} \quad & \text{Constraints } (2) - (6), (9) && && \\
& ms_k \leq c_{0k}, && k \in K && (13) \\
& c_{ik} + d_i + t_{i0} \leq me_k, && i \in N, i \in M && (14) \\
& \sum_{k \in M} \sum_{j \in N} x_{ijk} \cdot ph_{kh} \cdot nh_{ih} = 1, && i \in N, h \in H && (15)
\end{aligned}
$$

Similar to the model defined in earlier sections, the goal of this model is now to minimize the hourly hiring costs of service provides regarding various household services. There are two objective functions, (11) and (12) in place. For the computational result, we respect the first objective in (11), whereas (12) will also be respected, but with only results due to the complexity to illustrate a feasible solution. In particular, this model takes on constraints (2)-(10) with three additional constraints.

Here, constraint (13) simply states that service start time of provider k must be always greater than or equal to their respected available time. Constraint (14) ensures that service finish time is always later than the sum of the time it takes for a provider k to arrive at client i, total service duration and the waiting time if the client is currently unavailable. Constraint (15) ensures that each client's request would be served exactly once.

4.3 Computational Result

Result Description

In running a computation with our improved model given the same number of service providers clients as in part 1, we get a minimized hiring cost of 45 with service provider 2, 3, 4 and 5, whose routes are described as follows:

- Provider 2 is hired at a cost of 10
  - **0h**: it is available at headquarter, and leaves to the client 4.
  - **4h**: it takes 1 hour to arrive at client 4, and wait for 3 hours to start the service. (The service duration at client 4 is 1 hour.)
  - **6h**: it takes 1 hours to arrive at client 5. (The service duration at client 5 is 1 hour.)
  - **8h**: it take 1 hour to arrive at the headquarter.
- Provider 3 is hired at a cost of 8
  - **4h**: it is available at headquarter, and leaves to the client 3.
  - **6h**: it takes 1 hour to arrive at client 3, and wait for one hour to start the service. (The service duration at client 3 is 1 hour.)
  - **8h**: it takes 1 hour to arrive at the headquarter.
- Provider 4 is hired at a cost of 12
  - **0h**: it is available at headquarter, and leaves to the client 2.
  - **1h**: it takes 1 hour to arrive at client 2. (The service duration at client 2 is 1 hour.)
  - **4h**: it takes 2 hours to arrive at client 1. (The service duration at client 1 is 2 hours.)
  - **8h**: it takes 2 hours arrives at the headquarter.
- Provider 5 is hired at a cost of 15
  - **2h**: it is available at headquarter, and leaves to the client 6.
  - **4h**: it takes 2 hour to arrive at client 6. (The service duration at client 6 is 2 hours.)
  - **7h**: it takes 1 hour to arrive at the headquarter.

Although figure 2 suggests a similar result compared to figure 1, we do observe some extra waiting time for certain providers, which implies that this improved model captures the "matching" between clients' requests and provider skills. However, as mentioned in earlier discussions, assuming an 8-hour cap may not be realistic as there often can be certain clients request service with late time windows. Since we are minimizing the hourly cost and given that providers are available after some waiting time, we put service providers to work 8 full hours once they are available, with another cap at 12 hours to not violate previous assumptions. Therefore,

in running a test with 16 clients and 10 service providers, we get a feasible solution as shown in figure 3. This result marks a few interesting differences compared with previous ones, as it presents a network-like structure that indicates the "skill matching" aspect in our formulation. In fact, running on a even larger instance with 30 clients and 20 providers yields a total cost of 567 with 17 providers hired and the following routes:

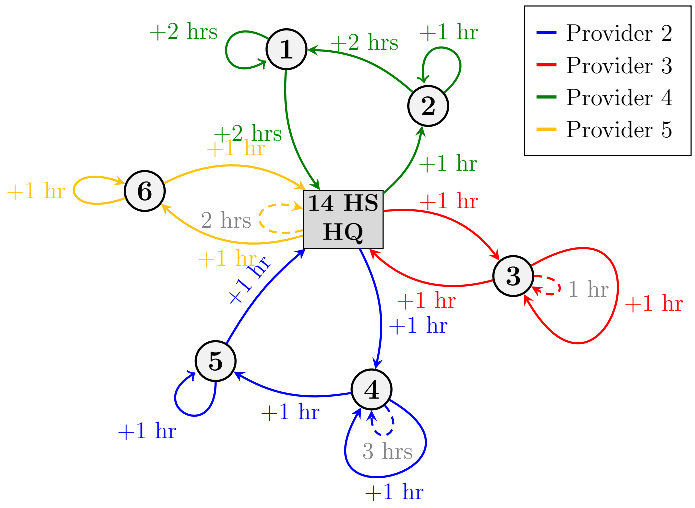

Figure 2: Illustration of feasible solution obtained from improved model with 5 providers and 6 clients

- **2-cycle**:
  - **Provider 2**: depot → 7 → depot
  - **Provider 4**: depot → 30 → depot
  - **Provider 11**: depot → 29 → depot
  - **Provider 15**: depot → 23 → depot
  - **Provider 19**: depot → 3 → depot
- **3-cycle**:
  - **Provider 1**: depot → 27 → 12 → depot
  - **Provider 3**: depot → 26 → 15 → depot
  - **Provider 5**: depot → 22 → 10 → depot
  - **Provider 8**: depot → 6 → 4 → depot
  - **Provider 13**: depot → 28 → 17 → depot
  - **Provider 14**: depot → 11 → 21 → depot
  - **Provider 16**: depot → 14 → 1 → depot
  - **Provider 18**: depot → 5 → 13 → depot
- **4-cycle**:
  - **Provider 6**: depot → 9 → 23 → 18 → depot
  - **Provider 9**: depot → 2 → 8 → 20 → depot
  - **Provider 12**: depot → 16 → 19 → 24 → depot

In conjunction with the original data, we observe that 2,3-cycle routes are the more dominant pattern in our routing network. We also notice that 4-cycle routes are assigned to providers that have less skills to offer but also cost less, while providers that cost more but can offer all types of skills are not hired at all.

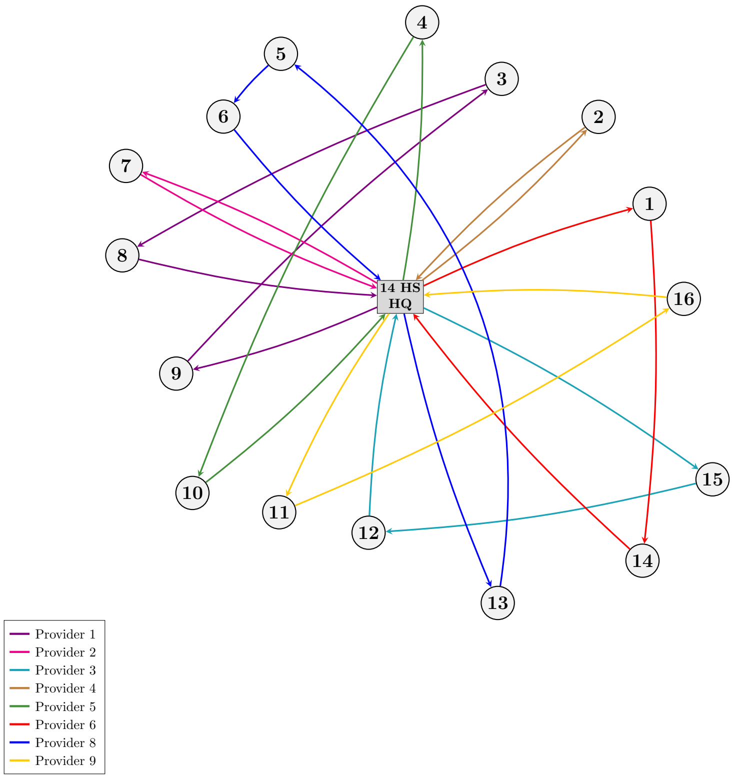

Figure 3: Illustration of feasible solution obtained from improved model with 10 providers and 16 clients; service and waiting time omitted, node distances are not drawn to scale

4.4 Result comparison

For feasible solutions, both models demonstrate promising results in terms of routing allocation. However, several experiments ran on a same Gurobi configuration with Intel Core i7-9700 CPU 16GB RAM, have suggested that runtime for the basic model is significantly larger than the one for the improved model. In particular, Gurobi callbacks read a 8,592,049-node traversal with 215,097,394 simplex iterations, costing a total of 1432.02 seconds to yield a feasible solution for a instance of 10 clients and 9 providers. On the other hand, when feeding an even larger instance to the improved model, Gurobi solver explored 1794 nodes with 124,288 simplex iterations to compute a feasible solution in 13.81 seconds, noting a remarkable difference in node efficiency as well as tightness of constraints. This significant improvement in runtime prove the efficiency of our revised formulation and additional constraints added do help regularize the model to reduce the computational cost.

5 Limitations and Heuristics

As mentioned in section 3, a relatively simple data set was carried out throughout this experiment to illustrate the modelling implementation. Although it is possible to find a globally optimal and feasible solution from a MIP, the computational complexity increases at an exponential rate as the size of the input data grows. Due to the NP-hard nature of this program, it can be a challenge to run the MIP implementation as the number of client and available providers grows to incorporate a more realistic problem faced by 14HS.

Some heuristics could be looked upon. Metaheuristic algorithms can be explored to take care of the computational complexities with a feasible optimal solution. One example is to undertake a greedy approach of sorting the data before implementing a MIP to reduce some iterations of what the optimiser explores [5]. First, sorting the providers' hiring costs from cheapest to highest, as well as the provider available hours and the client starting times. The initial assignment between providers and clients can be done iteratively by assigning clients to the cheapest and as early starting providers as possible, taking into account the time that each provider has spent on travels and clients they've served beforehand. With this approach, we can make sure to find a feasible solution that allocates each client to each provider, as well as reducing computational times relative to MIP because it won't explore as many combinatorial assignments. A limitation to this greedy approach is that we can achieve a locally optimal solution, which might not be as optimal as a MIP solution can achieve.

6 Conclusion and Discussion

In this report, we present a home service problem with a particular interest in the routing allocation of service providers to find the best optimal routes that yield least hiring costs. In particular, the model is developed in a two stage approach by first identifying the problem with basic formulation and then putting further constraints. More specifically, we adapt the classical VRPTW with hard time windows assumption in part 1, followed by a reformulation in part 2 that allows time window flexibility with extra data. Moreover, when testing on large instances, the basic model took significant computation time to yield a feasible solution and were often unbound or infeasible if data were not tailor-made for a specific scenario; whereas solutions from the improved model were feasible, and, most importantly, can be translated to interpretatble allocation decisions rather than variables simply satisfying constraints.

Although results from both models proved efficiency of the given formulation, there are clearly more questions to ask and details to put in the model. For example, optimal routes between the two are often relatively similar to each other for small instances. We argue that both models are inherently the same and in order to derive a different routes planning, a soft time window should be respected.

In addition, further studies can be explored on metaheuristic approaches to account for the computational complexity on a MIP implementation. By comparing a trade-off between the best objective value found and the algorithm runtimes, the most desirable approach can vary based on the use cases of what is valued most by the company.

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