Number Theory

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1.Single Prime Number efficient algorithm: this algorithm is 10^14 number.

```
int single_prime(long long n)
{
    if(n<2)return 0;
    if(n==2) return 1;
    if(n%2==0) return 0;
    for(int i=3;i<=sqrt(n);i+=2)
    if(n%i==0)
    return 0;
    return 1;
}</pre>
Example: n=1000;
If 1000 is prime number than yes . Others No .
```

18.anagrammatic_prime number total 22 ti . ai sob prime number gulo jevabe lekhi sob somoy prime hobe .

```
anagrammatic_prime[]={2,3,5,7,11,13,17,31,37,71,73,79,97,113,131, 199,311,337,373,733,919,991};
```

. this algorithm used to 10^18 number .this number used to single prime | This algorithm called by miller_rabin theory|

```
const int S=8;
long long mult_mod(long long a,long long b,long long c)
 a=a%c;
 b=b%c;
 long long ret=0,tmp=a;
 while(b)
 {
   if(b&1)
     ret+=tmp;
     if(ret>c)
     ret-=c;
   tmp<<=1;
   if(tmp>c)
   tmp-=c;
   b>>=1;
 }
 return ret;
```

```
long long pow_mod(long long a,long long n,long long mod)
  long long ret=1;
  long long temp=a%mod;
  while(n)
  {
    if(n&1)
    ret=mult_mod(ret,temp,mod);
    temp=mult_mod(temp,temp,mod);
    n>>=1;
  }
  return ret;
}
bool check(long long a,long long n,long long x,long long t)
{
  long long ret=pow_mod(a,x,n);
  long long III=ret;
  for(int i=1;i<=t;i++)
    ret=mult_mod(ret,ret,n);
    if(ret==1 and III!=1 and III!= n-1)
    return true;
    III=ret;
  }
  if(ret!=1)
  return true;
  else
  return false;
}
```

```
bool miller_rabin(long long n)
  if(n<2) return false;
  if(n==2)return true;
  if((n&1)==0) return false;
  long long x=n-1;
  long long t=0;
  while((x&1)==0)
  {
    x>>=1;
    t++;
  }
  srand(time(NULL));
  for(int i=0;i<S;i++)
    long long a=rand()%(n-1)+1;
    if(check(a,n,x,t))
    return false;
  }
  return true;
}
```

here , n number single number . if this number is prime than YES others $\ensuremath{\mathsf{NO}}$.

3. Seive prime algorithm: this algorithm used 1 to N prime number.

```
#define MAX 1002 //it is N
#define LMT 19 // it is squre(MAX) and LMT is odd number
#define i64 long long
long int flag[MAX+1], primes[MAX], total;
#define ifc(x) (flag[ x >> 6 ] & (1 << ((x >> 1) & 31)))
#define isc(x) (flag[ x >> 6] | =(1 << ((x >> 1) & 31)))
void sieve()
 int i, j, k;
 primes[0] = 2;
 total = 1;
 for(i = 3; i <=LMT; i += 2)
 {
    if(!ifc(i))
      primes[total++] = i;
      for(j = i*i; j <=MAX; j += 2*i)
      isc(j);
    }
 for(i = LMT+2; i <=MAX; i += 2) //LMT value must be odd number
    if(!ifc(i))
    {
      primes[total++] = i;
    }
 }
}
```

This algorithm used array primes[] , store to prime number 1 to N.

4.Range Primes Number :this algorithm previous store , max range difference.how many A from B difference prime number? And A and B 10^18 projanto hoy . tahole mone rakhte hobe . A and B difference 10^7 er besi hobe na . tai age 10^7 projnato prime store kore rakhte hobe .

```
#define N 1000006 // max difference number A and B
char x[N];
long long range_prime(long long a,long long b)
 long long t=0,n=0,d,i,k,che=0,j=0;
 if(a>b) return 0;
 if(a==b and b==1) return 0;
  CLR(x); //x array er sob position a 0 rakhte hobe
 if(a<3)
    cout<<"2";
    che=1;
    t=1;
    a=3;
 }
 if(a%2==0)
  a=a+1;
 if(b%2==0)
  b=b-1;
 d=sqrt(b);
 i=0;
  while(prime[i]<=d)
                                   //prime[] array te age max difference prime store kore rakhte hobe .
    n=ceil((a*1.0)/prime[i]);
    if(n==1)
```

```
n=2*prime[i];
   else
     n=n*prime[i];
   for(k=n;k<=b;k=k+prime[i])
   x[k-a]=1;
   i=i+1;
 }
 for(i=a;i<=b;i=i+2)
 {
   if(x[i-a]!=1)
   {
     if(che==1)
     cout<<" "<<i;
     else
     cout<<i;
     che=1;
     t=t+1;
   }
 }
 cout<<endl;
 return t; // total prime number range er modhe
}
If A =3 and B =10 hoy .tahole
Print korbe 3, 5,7
```

And total 3 ta prime ache range er modhe .

5. a single number how many divisor and use to 10^14?

```
int divi[100000];
int number_of_divisor(int n)
{
  int i,s,p;
  s=0;
  for(i=1;i<=sqrt(n);i++)
    if(n%i==0)
      divi[s]=i;
      s++;
      p=n/i;
      if(p!=i)
         divi[s]=i;
         s++;
      }
    }
  }
  return s;
}
Example: n=10;
n=1,2,5,10 // this number is stor divi array.
```

S is count of divisor .

6.Seive divisor algorithm .this algorithm use to number of divisor , prime divisor, prime factor , sum of all divisor .range to 10^14 projnato ber kora jai . number of divisor , sum of all divisor .

```
long long str[1000];
i64 divisor(i64 n)
 i64 ret = 1,k,ssss=1,lll=1, i,j=0,cnt, rt = (LL)sqrt((double)n);
 for(i = 0; i < total && primes[i] <= rt; i++)
    if(n % primes[i] == 0) // before stor primes[] number sqrt(n) projanto
      n /= primes[i], cnt = 1;
      str[j]=primes[i];
      j++;
      while(n % primes[i] == 0)
         n/= primes[i], cnt++;
         str[j]=primes[i];
        j++;
      rt = (int) sqrt((double)n);
      ret*=(cnt+1);
      II*=(pow(primes[i],(cnt+1))-1);
      III*=(primes[i]-1);
      if(||%|||==0)
         II=II/III;
         III=1;
      }
    }
```

Number of divisor=(2+1)*(1+1)=3*2=6 //cnt is number of divisor

7. 1 from n projanto every number, number of divisor. 10^15 projanto ninnoy kora jai.

8. 1 from n projanto every number, sum of divisor .10^15 projanto ninnoy kora jabe.

9.number of Divisor, number of prime factor, sum of all divisor. 10^18 projanto ninnoy kora jai.

```
typedef unsigned long long uint64;
typedef long double float80;
typedef long long II;
typedef long long int64;
typedef unsigned uint;
typedef unsigned char uint8;
static const uint64 PRIME_MAX = 1024;
static const uint64 THRESHOLD = PRIME_MAX * PRIME_MAX;
static const uint64 POLLARD_RHO_M = 250;
static const uint64 MOD = 100000000000000000001l;
static const float80 MOD_INV = float80(1) / MOD;
static const int primes[] = {
         2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97,101,103,107,109,113,127,131,137,139,149,151,157,163,16
7,173,179,181,191,193,197,199,211,223,227,229,233,239,241,251,257,263,269,271,277,281,283,293,307,311,313,317,331,337,347,349,353,3
59,367,373,379,383,389,397,401,409,419,421,431,433,439,443,449,457,461,463,467,479,487,491,499,503,509,521,523,541,547,557,563,569,
571,577,587,599,601,607,613,617,619,631,641,643,647,653,659,661,673,677,683,691,701,709,719,727,733,739,743,751,757,761,769,773
013,1019,1021,1031
};
inline uint xrand(void)
         static uint x = 123456789, y = 362436069, z = 521288629, w = 88675123;
         uint t = x ^ (x << 11); x = y; y = z; z = w;
         return w = (w ^ (w >> 19)) ^ (t ^ (t >> 8));
}
```

```
inline uint randrange(uint64)
{
           return (uint64(xrand()) * 0xFFFFFFFF) >> 32;
}
template <typename T>
T gcd(T a, T b)
{
           if(b > a)
  {
                      T tmp = a;
                      a = b;
                      b = tmp;
           }
           while(1)
  {
                      if(!b) return a;
                       a-=b; if(a>=b)\{a-=b; if(a>=b)\{a-=b; if(a>=b)\{a-=b; if(a>=b)\{a\%=b;\}\}\}\}
                      if(!a) return b;
                      b-=a; if(b>=a)\{b-=a; if(b>=a)\{b-=a; if(b>=a)\{b-=a; if(b>=a)\{b-=a; if(b>=a)\}\}\}
           }
}
inline uint square_add_mod(uint a, uint c, uint mod, float80)
{
           return (uint64(a) * a + c) % mod;
}
inline uint mul_mod(uint a, uint b, uint mod, float80)
{
           return uint64(a) * b % mod;
}
```

```
template <typename T>
T pow_mod(uint base, T exp, T mod, float80 modi)
{
          T ret = 1;
          T q = base;
          while(exp) {
                    if(exp & 1) {
                               ret = mul_mod(ret, q, mod, modi);
                    }
                    exp >>= 1;
                    q = mul_mod(q, q, mod, modi);
          }
          return ret;
}
template <typename T>
bool miller_rabin_pass(uint base, uint m, T exp, T mod, float80 modi)
{
          T n = pow_mod(base, exp, mod, modi);
          if(n == 1)
          return true;
          for(uint i = 0; i < m; ++i) {
                    if(n == mod - 1)
                               return true;
                    n = mul_mod(n, n, mod, modi);
          }
          return n == mod - 1;
}
inline uint ilog2(uint64 x)
{
```

```
union Data {
                     uint64 u64;
                      double d;
          } n;
           n.d = double(x) + 0.5;
           return (n.u64 >> 52) - 1023;
}
bool miller_rabin(uint64 n)
{
           static const uint BASES1[] = {2, 3};
           static const uint BASES2[] = {2, 299417};
           static const uint BASES3[] = {2, 7, 61};
           static const uint BASES4[] = {15, 176006322, 4221622697u};
           static const uint BASES5[] = {2, 2570940, 211991001, 3749873356u};
           static const uint BASES6[] = {2, 2570940, 880937, 610386380, 4130785767u};
           static\ const\ uint\ BASES7[] = \{2,\,325,\,9375,\,28178,\,450775,\,9780504,\,1795265022\};
           if(n <= 1)
          {
                     return false;
          }
           if(n <= 3) {
                     return true;
          }
           uint64 d = n - 1;
           uint s = ilog2(d \& -d);
           d >>= s;
           uint bases_size;
           const uint* bases;
           if(n < 1373653)
          {
```

```
bases_size = 2;
           bases = BASES1;
} else if(n < 19471033) {
           bases_size = 2;
           bases = BASES2;
} else if(n < 4759123141ull) {
           bases_size = 3;
           bases = BASES3;
} else if(n < 154639673381ull) {
           bases_size = 3;
           bases = BASES4;
} else if(n < 47636622961201ull) {
           bases_size = 4;
           bases = BASES5;
} else if(n < 3770579582154547ull) {
           bases_size = 5;
           bases = BASES6;
} else {
           bases_size = 7;
           bases = BASES7;
}
if(n < 0x10000000ull) {
           for(uint rep = 0; rep < bases_size; ++rep) {</pre>
                      if(!miller_rabin_pass<uint>(bases[rep], s, d, n, 0))
                                 return false;
           }
} else {
           float80 modi = float80(1) / n;
           for(uint rep = 0; rep < bases_size; ++rep) {</pre>
                      if(!miller_rabin_pass<uint64>(bases[rep], s, d, n, modi))
                                 return false;
```

```
}
          }
          return true;
}
template <typename T>
T pollard_rho(T n)
{
          if(!(n & 1))
          {
                     return 2;
          }
          Ty = randrange(n - 1) + 1;
          T c = randrange(n - 1) + 1;
          T m = POLLARD_RHO_M;
          T g = 1, q = 1;
          T x, ys;
           uint64 r = 1;
           float80 n_inv = float80(1) / n;
           while(g == 1) {
                     x = y;
                     for(uint i = 0; i < r; ++i) {
                                y = square_add_mod(y, c, n, n_inv);
                     }
                     T k = 0;
                     while(k < r && g == 1) {
                                ys = y;
                                T end = (r - k < m ? r - k : m);
                                 for(uint i = 0; i < end; ++i) {
                                           y = square_add_mod(y, c, n, n_inv);
```

```
T dif = (x >= y ? x - y : y - x);
                                            q = mul_mod(q, dif, n, n_inv);
                                 }
                                 g = gcd(q, n);
                                 k += m;
                      }
                      r <<= 1;
           }
           if(g == n) {
                      while(1) {
                                 ys = square_add_mod(ys, c, n, n_inv);
                                 T dif = (x >= ys ? x - ys : ys - x);
                                 g = gcd(dif, n);
                                 if(g > 1)
                                 break;
                      }
           }
           return g;
}
uint64 ps[1000];
uint64 pwr[1000];
uint factors(uint64 n)
{
           if(n <= 1)
           return 0;
           uint pos = 0;
           uint v = sqrt(n);
           if(uint64(v) * v == n && miller_rabin(v))
```

```
{
          ps[pos] = v;
          pwr[pos] = 2;
          ++pos;
          return pos;
}
uint e = ilog2(n \& -n);
if(e > 0)
{
          n >>= e;
          ps[pos] = 2;pwr[pos++] = e;
          v = sqrt(n);
}
uint end = (n > THRESHOLD ? PRIME_MAX : v + 1);
uint p_idx = 1;
uint p = primes[p_idx++];
while(p < end) {
          if(n % p == 0) {
                     n /= p;
                     uint e = 1;
                     while(n % p == 0) {
                               n /= p;
                               ++e;
                     }
                     end = (n > THRESHOLD ? PRIME_MAX : sqrt(n) + 1);
                     ps[pos] = p;pwr[pos++] = e;
          }
          p = primes[p_idx++];
}
```

```
p = primes[p_idx-1];
uint64 cut_off = uint64(p) * p;
if(n > 1)
{
           if(cut_off > n || miller_rabin(n))
           {
                      ps[pos] = n;pwr[pos++] = 1;
                      return pos;
           }
           while(1)
           {
                      uint64 p;
                      if(n < 0x10000000ull)
                      {
                                 p = pollard_rho<uint>(n);
                      }
                      else
                      {
                                 p = pollard_rho<uint64>(n);
                      }
                      if(!miller_rabin(p))
                      continue;
                      n /= p;
                      uint e = 1;
                      while(n % p == 0) {
                                 n /= p;
                                 ++e;
                      }
                      ps[pos] = p;pwr[pos++] = e;
                      if(n <= cut_off | | miller_rabin(n)) {</pre>
                                 if(n > 1) {
```

```
ps[pos] = n;pwr[pos++] = 1;
                                           }
                                           break;
                                }
                     }
          }
          return pos;
}
pair<uint64,uint64>A[100];
void solve()
{
  uint64 n,temp;
  scanf("%llu",&n);
  int sz = factors(n);
  for(int i=0;i<sz;i++)
    A[i] = make_pair(ps[i],pwr[i]);
  }
  sort(A,A+sz);
  for(int i=0;i<sz;i++)
  {
    II X = A[i].first;
    Il upto = A[i].second;
    for(int j=0;j<upto;j++)
      printf("%llu ",X);
    }
  }
  printf("\n");
```

10.prime factor, a number how many prime factor and ki ki prime factor ache. 10^7 projanto ninnoy kora jabe.

```
vector<int>primefc[1001];
int check[1001];
int prime_factor()
 int i,j,k,nnn;
 for(i=2;i<=1000;i+=2)
    primefc[i].push_back(2);
    check[i]=1;
 for(i=3;i<=1000;i+=2)
    if(!check[i])
    {
      for(j=i;j<=1000;j+=i)
        primefc[j].push_back(i);
        check[j]=1;
      }
    }
 for(i=2;i<=1000;i++)
 reverse(primefc[i].begin(),primefc[i].end());
 return 0;
}
Whe N=20;
```

Tahole N er prime factor =2,5

11.Phi function ,relative_prime,co_prime , Euler Totient Function and gcd(1,n)=1.....gcd(n,n) =1. used to 10^7 projanto .

```
#define MAX 5000000
int etf[MAX + 1];
inline void coprime()
       register int i, j;
       for(etf[2] = 1, j = 4; j <= MAX; j+=2)
       etf[j] = j >> 1;
       for(i = 3; i <= MAX; i+=2)
                 if(!etf[i])
                          for(etf[i] = i-1, j = i<<1; j <= MAX; j+=i)
                                    if(!etf[j])
                                    etf[j] = j;
                                    etf[j] = etf[j] / i * (i-1);
                         }
                 }
}
Example: n=10;
Prime factor=2,5
Ans=10*(2-1)*)*(5-1)/(2*5);
\label{eq:discuss: 1/10,2/10,3/10,4/10,5/10,6/10,7/10,8/10,9/10,10/10; here \ vagfoler \ man <=1 \ tt=5 \ ; //ans+1 \ hobe \ .
Gcd(1,10) = 1, gcd(Gcd(2,10) = 2, Gcd(3,10) = 1, Gcd(4,10) = 2, Gcd(5,10) = 5, Gcd(6,10) = 2, Gcd(7,10) = 1, Gcd(8,10) = 1, Gcd(9,10) = 3, Gcd(10,10) = 10, G
Here gcd(a,b)=1 horche ans =4;
```

12.Phi function,erular.coprime.reletive.prime.phi_funtion 10^12 projanto ninnoy kora jai .

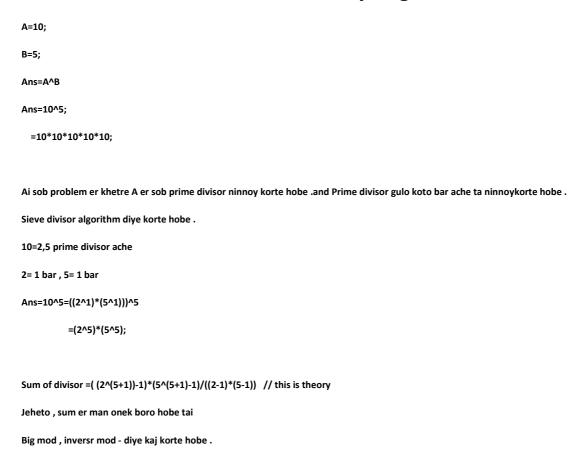
```
long long l,r,k,tests;
int pr[1<<20];
vector<int> vec[1<<20];
long long get_phi(long long val,vector<int>&primes)
           long long res=val;
           for (int i=0;i<primes.size();i++)
           {
                     while (val%primes[i]==0)
                     val/=primes[i];
                     res=res/primes[i]*(primes[i]-1);
          }
           if (val>1)
           res=res/val*(val-1);
           return res;
}
//www. hackerearth. com/code-monk-number-theory-iii/algorithm/monk-and-etf/description/\\
int main(){
           pr[1]=1;
           for (int i=2;i<=1000000;i++)
                     if (pr[i]==0)
                                for (int j=i*2;j<=1000000;j+=i)
                                           pr[j]=1;
           cin>>tests;
           for (;tests;--tests)
```

```
cin>>l>>r;
                     for (long long i=l;i<=r;i++)
                     vec[i-l].clear();
                     for (int i=2;i<=1000000;i++)
                     {
                                 if (pr[i])
                                 continue;
                                 long long bnd=l/i*i;
                                 if (l%i>0)
                                bnd+=i;
                                for(long long j=bnd;j<=r;j+=i)
                                vec[j-l].push_back(i);
                     }
                     ans=0;
                      cout<<l<" from "<<r<" Euler Totient Function value is : ";
                     for (long long i=l;i<=r;i++)
                     {
                                 long long Q=get_phi(i,vec[i-l]);
                                 cout<<Q<<" ";
                     }
                     cout<<endl;
          }
           return 0;
}
When I=2 and r=5;
Ans =1,2,2,4;
```

{

Ager problem ta dekhle hobe . agertar are tar modhe difference holo ager sdudhu 10^7 projanto ninnoy kora jabe . kintu atar L and R er difference 10^6 hole hobe . L and R er man 10^12 projnat ninnoy kora jai .

13.a^b sum of divisor .A and B very large number;



14.BIG MOD diye onek kaj kora jai.

```
II mod_pow(II a,II b,II m) // a^b and m is mod
{
          II r=1;
          while(b)
         {
                    if(b%2==1)
                    r=r*a%m;
                    a=(a*a)%m;
                    b/=2;
         }
          return r;
}
This algorithm a^b . here a and b is very large .
BIG NUMBER code diye inverse mod kaj kora jai . mod of couse Prime number hote hobe
If h=((a^b)%mod)/(c%mod)
Ans=(a^b)%mod; // ai kaj ti big mod diye korte hobe bigmod(a,b,mod)
Ans1=c%mod // ai kaj tuku bigmod(c-1,mod-2,mod)
Ans=ans*ans1; // ans holo ans .
```

15. Extended Euclidean Algo , INVERSE MOD this algorithm very important

```
void EE(II a, II b, II *x, II *y)
{
          if(a==0)
          {
                     *x=0;
                     *y=1;
                    return;
          }
          II temp_x,temp_y;
          EE(b%a, a, &temp_x, &temp_y);
          *y=temp_x;
          *x=temp_y - (b/a)*temp_x;
}
Il inverse_mod(Il a,Il m) //gcd(a,m)=1 hote hobe
{
          II x,y;
          EE(a,m,&x,&y);
          while(x<0)
          x+=m; // Importantly make it positive
          return x;
}
```

This algorithm a is hoy 1/a and m is mod , mod must be gcd(a,m)=1 hote hobe.

16.BIG mod (a^p)%m this problem is a,b,c is 10^18 projanto kora jai .

```
typedef long long vlong;
vlong bigmul ( vlong a, vlong b, vlong c )
 if ( b == 0 ) return 0;
 if(b&1){
    return ( a + bigmul ( a, b - 1, c ) ) % c;
 }
 else {
    return ( 2 * bigmul ( a, b / 2, c ) ) % c;
 }
}
vlong bigmod ( vlong a, vlong p, vlong m )
{
 vlong res = 1, x = a % m;
 while (p){
    if ( p & 1 ) res = bigmul ( res, x, m );
    x = bigmul(x, x, m);
    p >>= 1;
 }
 return res;
}
```

This problem 10^18 *10^18 gun korle rakha jai na . tai ata kea i algorithm diye korte hobe .

17. Base Conversion any base to any convert .

```
long any_base_to_decimal_base(long int n,long int m)
 long int i=0;
 long int ar[1000];
 while(n>0)
    ar[i]=n%10;
    i++;
    n=n/10;
 ar[i]=0;
 long int ans=0,j;
 for(j=0;j<i;j++)
    ans+=ar[j]*pow(m,j);
 }
 return ans;
char letters(int r)
 if(r<=35)
    for(int i=10;i<=35;i++)
     if(i==r)
     return (i-10+'A');
    }
 }
```

else

```
{
    for(int i=36;i<=61;i++)
    {
      if(i==r)
      return (i-35+'a');
    }
  }
}
void decimal_base_to_any_base(long int N, long int b)
{
  if (N == 0)
  return;
  long int x = N % b;
  N /= b;
  if(x < 0)
  N+=1;
  decimal_base_to_any_base(N, b);
  if(x>9)
  cout<<letters(x);
  else
  cout<< x < 0 ? x + (b * -1) : x;
  return;
}
```

19.single GCD ,LCM check algorithm :

```
LL gcd(LL a,LL b)
  while(b>0)
   a=a%b;
    a=a^b;
    b=b^a;
   a=a^b;
  return a;
}
LL lcm(LL a,LL b)
 LL x=(a*b)/gcd(a,b);
return x;
}
If a=10,b=12;
A=2*5
B=2*2*3
Gcd=2;
Lcm=2*2*3*5;
```

20.NcR algorithm is :-

```
LL ncr1(LL n ,LL r)
{
    if(r>(n/2))
    r=n-r;
    LL s=1,i;
    for(i=0;i<r;i++)
    {
        s=s*(n-i);
        s=s/(1+i);
    }
    return 0;
}</pre>
```

21.npr algorithm:

```
LL npr(LL n,LL r)
{
    LL s=1,i;
    for(i=1;i<=r;i++)
    s*=(n-i+1);
    return s;
}
N=5
R=2;
Npr=5!/(2!);
```