This appendix is available online at https://github.com/alherm/TSO-DSO_coordination.

Appendix A. DSO Market Lower Level Problem

The DSO pre-qualification optimization problem has both constraints from the DA-market and the scenarios for the Real-time realization. Every DSO has its own separate problem such that Cost_e contains one value for every DSO e. The day ahead market is cleared for each distribution network separately, where the day ahead market has no nodal information. The real time realization is a stochastic SOCP problem.

$$\begin{aligned}
\max_{\Xi^{\mathrm{E}}} \quad & \mathcal{SW}_e = \sum_{d \in D_e^{\mathrm{D}}} \pi_d^{\mathrm{DA}} \widetilde{p}_d^{\mathrm{DA}} - \sum_{g \in G_e^{\mathrm{D}}} \pi_g^{\mathrm{DA}} \widetilde{p}_g^{\mathrm{DA}} \\
& - VOLL_e^{\mathrm{DA}} s_e^{\mathrm{DA}} - \sum_{r \in R_e^{\mathrm{D}}} \pi^R w_r^{\mathrm{DA}} - \pi_e^{\mathrm{PCC,DA}} p_e^{\mathrm{PCC,DA}} \\
& - \sum_{\omega} \phi_\omega \Bigg[\sum_{g \in G_e^{\mathrm{D}}} \Big(\pi_g^{\mathrm{DA}} (p_{g\omega}^{\mathrm{RT}} - \widetilde{p}_g^{\mathrm{DA}}) + \pi_g^{\uparrow} p_{g\omega}^{\uparrow} \\
& + \pi_g^{\downarrow} p_{g\omega}^{\downarrow} \Big) + \sum_{d \in D_e^{\mathrm{D}}} \Big(\pi_d^{\mathrm{DA}} (\widetilde{p}_d^{\mathrm{DA}} - p_{d\omega}^{\mathrm{RT}}) \\
& + \pi_d^{\uparrow} p_{d\omega}^{\uparrow} + \pi_d^{\downarrow} p_{d\omega}^{\downarrow} \Big) + \sum_{n \in N_e^{\mathrm{D}}} VOLL_n^{\mathrm{RT}} s_{n\omega}^{\mathrm{RT}} \\
& + \pi_e^{\mathrm{PCC,DA}} (p_{e\omega}^{\mathrm{PCC,RT}} - p_e^{\mathrm{PCC,DA}}) \\
& + \pi_e^{\uparrow PCC} p_{e\omega}^{\uparrow PCC} + \pi_e^{\downarrow PCC} p_{e\omega}^{\downarrow PCC} \\
& + \sum_{r \in R_e^{\mathrm{D}}} \Big(\pi^R (w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}}) + \pi^{\uparrow R} w_{r\omega}^{\uparrow} + \pi^{\downarrow R} w_{r\omega}^{\downarrow} \Big) \Bigg]
\end{aligned} \tag{A.1a}$$

subject to:

DA-level constraints:

$$\begin{aligned}
& \sum_{g \in G_e} \widetilde{p}_g^{\mathrm{DA}} - \sum_{d \in D_e^{\mathrm{D}}} \widetilde{p}_d^{\mathrm{DA}} + \sum_{r \in R_e^{\mathrm{D}}} w_r^{\mathrm{DA}} + s_e^{\mathrm{DA}} \\
& \quad + p_e^{\mathrm{PCC,DA}} = 0, \quad : (\lambda_e^{\mathrm{DA}})
\end{aligned} \tag{A.1b}$$

$$\underline{P}_g \le \widetilde{p}_g^{\mathrm{DA}} \le \overline{P}_g, \quad \forall g \in G_e^{\mathrm{D}} \quad : (\varsigma_g^{\mathrm{DA}-}, \varsigma_g^{\mathrm{DA}+}) \tag{A.1c}$$

$$\underline{P}_d \le \widetilde{p}_d^{\mathrm{DA}} \le \overline{P}_d, \quad \forall d \in D_e^{\mathrm{D}} \quad : (\varsigma_d^{\mathrm{DA}-}, \varsigma_d^{\mathrm{DA}+}) \tag{A.1d}$$

$$0 \le w_r^{\mathrm{DA}} \le W_r^{\mathrm{DA}}, \ \forall r \in R_e^{\mathrm{D}} \quad : (\iota_r^-, \iota_r^+) \tag{A.1e}$$

$$\underline{f}_e \le p_e^{\mathrm{PCC,DA}} \le \overline{f}_e, \quad : (\rho_e^{\mathrm{DA}-}, \rho_e^{\mathrm{DA}+}) \tag{A.1f}$$

$$0 \le s_e^{\mathrm{DA}} \le \sum_d p_d^{\mathrm{DA}}, \quad : (\Upsilon_e^{\mathrm{DA}-}, \Upsilon_e^{\mathrm{DA}+}) \tag{A.1g}$$

Real-time constraints:

$$p_{g\omega}^{\mathrm{RT}} = p_g^{\mathrm{DA}} + p_{g\omega}^{\uparrow} - p_{g\omega}^{\downarrow}, \quad \forall \omega, g \in G_e^{\mathrm{D}}, \quad : (\zeta_{g\omega}^p) \tag{A.1h}$$

$$p_{d\omega}^{\mathrm{RT}} = p_d^{\mathrm{DA}} - p_{d\omega}^{\uparrow} + p_{d\omega}^{\downarrow}, \quad \forall \omega, d \in D_e^{\mathrm{D}}, \quad : (\zeta_{d\omega}^p) \tag{A.1i}$$

$$w_{r\omega}^{\mathrm{RT}} = w_r^{\mathrm{DA}} + w_{r\omega}^{\uparrow} - w_{r\omega}^{\downarrow},\ \forall\omega, r \in R_e^{\mathrm{D}}, : (\zeta_{r\omega}^{p}) \tag{A.1j}$$

$$\sum_{g\in G_n} p_{g\omega}^{\mathrm{RT}} - \sum_{d\in D_n} p_{d\omega}^{\mathrm{RT}} + \sum_{r\in R_n} w_{r\omega}^{\mathrm{RT}} + p_{e\omega}^{\mathrm{PCC,RT}}|_{n=n_e^{\mathrm{LV}}}$$

$$+ s_{n\omega}^{\mathrm{RT}} = \sum_{l\in n\rightarrow} p_{l\omega}^{\mathrm{RT}} - \sum_{l\in\rightarrow n} p_{l\omega}^{\mathrm{RT}},\ \forall\omega, n \in N_e^{\mathrm{D}}\ : (\lambda_{n\omega}^{\mathrm{p,RT}}) \tag{A.1k}$$

$$p_{e\omega}^{\mathrm{PCC,RT}} = p_e^{\mathrm{PCC,DA}} + p_{e\omega}^{\uparrow PCC} - p_{e\omega}^{\downarrow PCC},\ \forall\omega,\ : (\zeta_{e\omega}^{\mathrm{PCC}}) \tag{A.1l}$$

$$\sum_{g\in G_n} q_{g\omega}^{\mathrm{RT}} - \sum_{d\in D_n} q_{d\omega}^{\mathrm{RT}} + s_{n\omega}^{\mathrm{q,RT}} + q_{e\omega}^{\mathrm{PCC,RT}}|_{n=n_e^{\mathrm{LV}}}$$

$$= \sum_{l\in n\rightarrow} q_{l\omega}^{\mathrm{RT}} - \sum_{l\in\rightarrow n} q_{l\omega}^{\mathrm{RT}},\ \forall\omega, n \in N_e^{\mathrm{D}}\ : (\lambda_{n\omega}^{\mathrm{q,RT}}) \tag{A.1m}$$

$$p_{l\omega}^{(RT)2} + q_{l\omega}^{(RT)2} \le \varphi_{l\omega}^{\mathrm{RT}} v_{n\omega}^{\mathrm{RT}}, \quad \forall\omega, l \in L_e^{\mathrm{D}}\ : (\gamma_{l\omega}) \tag{A.1n}$$

$$p_{l\omega}^{\mathrm{RT}} + p_{l'\omega}^{\mathrm{RT}} = R_l \varphi_{l\omega}^{\mathrm{RT}}, \quad \forall\omega, l \in L_e^{\mathrm{D}}\ : (\mu_{l\omega}^{p}) \tag{A.1o}$$

$$q_{l\omega}^{\mathrm{RT}} + q_{l'\omega}^{\mathrm{RT}} = X_l \varphi_{l\omega}^{\mathrm{RT}}, \quad \forall\omega, l \in L_e^{\mathrm{D}}\ : (\mu_{l\omega}^{q}) \tag{A.1p}$$

$$p_{l\omega}^{(RT)2} + q_{l\omega}^{(RT)2} \le S_l, \quad \forall\omega, l \in L_e^{\mathrm{D}}\ : (\eta_{l\omega}) \tag{A.1q}$$

$$v_{m\omega}^{\mathrm{RT}} = v_{n\omega}^{\mathrm{RT}} - 2(R_l p_{l\omega}^{\mathrm{RT}} + X_l q_{l\omega}^{\mathrm{RT}}) + (R_l^2 + X_l^2)\varphi_{l\omega}^{\mathrm{RT}},$$

$$\forall\omega, l \in L_e^{\mathrm{D}} \quad : (\beta_{l\omega}) \tag{A.1r}$$

$$\underline{V}_n^2 \le v_{n\omega}^{\mathrm{RT}} \le \overline{V}_n^2, \quad \forall\omega, n \in N_e^{\mathrm{D}} \quad : (\sigma_{n\omega}^{-}, \sigma_{n\omega}^{+}) \tag{A.1s}$$

$$0 \le w_{r\omega}^{\mathrm{RT}} \le W_{r\omega}^{\mathrm{RT}}, \quad \forall\omega, n \in N_e \quad : (\nu_{n\omega}^{-}, \nu_{n\omega}^{+}) \tag{A.1t}$$

$$\underline{P}_g \le p_{g\omega}^{\mathrm{RT}} \le \overline{P}_g, \quad \forall\omega, g \in G_e \quad : (\varsigma_{g\omega}^{\mathrm{RT}-}, \varsigma_{g\omega}^{\mathrm{RT}+}) \tag{A.1u}$$

$$\underline{P}_d \le p_{d\omega}^{\mathrm{RT}} \le \overline{P}_d, \quad \forall\omega, d \in D_e \quad : (\varsigma_{d\omega}^{\mathrm{RT}-}, \varsigma_{d\omega}^{\mathrm{RT}+}) \tag{A.1v}$$

$$\underline{Q}_g \le q_{g\omega}^{\mathrm{RT}} \le \overline{Q}_g, \quad \forall\omega, g \in G_e \quad : (\kappa_{g\omega}^{\mathrm{RT}-}, \kappa_{g\omega}^{\mathrm{RT}+}) \tag{A.1w}$$

$$\underline{Q}_d \le q_{d\omega}^{\mathrm{RT}} \le \overline{Q}_d, \quad \forall\omega, d \in D_e \quad : (\kappa_{d\omega}^{\mathrm{RT}-}, \kappa_{d\omega}^{\mathrm{RT}+}) \tag{A.1x}$$

$$\underline{f}_e \le p_{e\omega}^{\mathrm{PCC,RT}} \le \overline{f}_e, \quad \forall\omega \quad : (\rho_{e\omega}^{\mathrm{RT}-}, \rho_{e\omega}^{\mathrm{RT}+}) \tag{A.1y}$$

$$p_{g\omega}^{\uparrow} \ge 0,\ \forall\omega, g\ : (\epsilon_{g\omega}^{\mathrm{p}\uparrow}), \quad p_{g\omega}^{\downarrow} \ge 0,\ \forall\omega, g\ : (\epsilon_{g\omega}^{\mathrm{p}\downarrow}) \tag{A.1z}$$

$$p_{d\omega}^{\uparrow} \ge 0,\ \forall d, \omega\ : (\varepsilon_{d\omega}^{\mathrm{p}\uparrow}), \quad p_{d\omega}^{\downarrow} \ge 0,\ \forall d, \omega\ : (\varepsilon_{d\omega}^{\mathrm{p}\downarrow}) \tag{A.1aa}$$

$$p_{e\omega}^{\uparrow PCC} \ge 0,\ \forall\omega,\ : (\epsilon_{e\omega}^{\uparrow PCC}),\ p_{e\omega}^{\downarrow PCC} \ge 0,\ \forall\omega,\ : (\epsilon_{e\omega}^{\downarrow PCC}) \tag{A.1ab}$$

$$0 \le s_{n\omega}^{\mathrm{RT}} \le \sum_{d\in D_n} p_{d\omega}^{\mathrm{RT}},\ \forall\omega, n \in N_e^{\mathrm{D}},\ : (\Upsilon_{n\omega}^{\mathrm{RT}-}, \Upsilon_{n\omega}^{\mathrm{RT}+}) \tag{A.1ac}$$

$$w_{r\omega}^{\uparrow} \ge 0,\ \forall\omega, w\ : (\epsilon_{r\omega}^{\mathrm{p}\uparrow}), \quad w_{r\omega}^{\downarrow} \ge 0,\ \forall\omega, w\ : (\epsilon_{r\omega}^{\mathrm{p}\downarrow}) \tag{A.1ad}$$

Where $\Xi^{E} = \{\widetilde{p}_g^{\mathrm{DA}}, \widetilde{p}_d^{\mathrm{DA}}, p_{g\omega}^{\mathrm{RT}}, p_{g\omega}^{\uparrow}, p_{g\omega}^{\downarrow}, p_{d\omega}^{\mathrm{RT}}, p_{d\omega}^{\uparrow}, p_{d\omega}^{\downarrow}, q_{g\omega}^{\mathrm{RT}}, q_{d\omega}^{\mathrm{RT}}, s_{n\omega}^{\mathrm{RT}}, s_e^{\mathrm{DA}}, w_{n\omega}^{\mathrm{RT}}, p_{l\omega}^{\mathrm{RT}}, q_{l\omega}^{\mathrm{RT}}, \varphi_{l\omega}^{\mathrm{RT}}, v_{n\omega}^{\mathrm{RT}}, w_e^{\mathrm{DA}}, p_e^{\mathrm{PCC,DA}}, p_e^{\mathrm{PCC,RT}}, p_e^{\uparrow PCC}, p_e^{\downarrow PCC}, s_{n\omega}^{\mathrm{q,RT}}\}$ is the variable-set of the DSO-level combined day-ahead and real-time market clearing.

The Lagrangian of above problem is:

$$\mathcal{L}_e = \sum_{d\in D_e^{\mathrm{D}}} \pi_d^{\mathrm{DA}} \widetilde{p}_d^{\mathrm{DA}} - \sum_{g\in G_e^{\mathrm{D}}} \pi_g^{\mathrm{DA}} \widetilde{p}_g^{\mathrm{DA}}$$

$$
\begin{aligned}
& - VOLL_e^{\mathrm{DA}} s_e^{\mathrm{DA}} - \sum_{r \in R_e^{\mathrm{D}}} \pi^R w_r^{\mathrm{DA}} - \pi_e^{\mathrm{PCC,DA}} p_e^{\mathrm{PCC,DA}} \\
& - \sum_{\omega} \phi_\omega \Bigg[\sum_{g \in G_e^{\mathrm{D}}} \Big(\pi_g^{\mathrm{DA}} (p_{g\omega}^{\mathrm{RT}} - p_g^{\mathrm{DA}}) + \pi_g^{\uparrow} p_{g\omega}^{\uparrow} + \pi_g^{\downarrow} p_{g\omega}^{\downarrow} \Big) \\
& + \sum_{d \in D_e^{\mathrm{D}}} \Big(\pi_d^{\mathrm{DA}} (p_d^{\mathrm{DA}} - p_{d\omega}^{\mathrm{RT}}) + \pi_d^{\uparrow} p_{d\omega}^{\uparrow} + \pi_d^{\downarrow} p_{d\omega}^{\downarrow} \Big) \\
& + \sum_{n \in N_e^{\mathrm{D}}} VOLL_n^{\mathrm{RT}} s_{n\omega}^{\mathrm{RT}} \\
& + \sum_{r \in R_e^{\mathrm{D}}} \Big(\pi^R (w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}}) + \pi^{\uparrow R} w_{r\omega}^{\uparrow} + \pi^{\downarrow R} w_{r\omega}^{\downarrow} \Big) \\
& + \pi_e^{\mathrm{PCC,DA}} (p_{e\omega}^{\mathrm{PCC,RT}} - p_e^{\mathrm{PCC,DA}}) \\
& + \pi_e^{\uparrow PCC} p_{e\omega}^{\uparrow PCC} + \pi_e^{\downarrow PCC} p_{e\omega}^{\downarrow PCC} \Bigg] \\
& - \lambda_e^{\mathrm{DA}} \Bigg[\sum_{g \in G_e} \widetilde{p}_g^{\mathrm{DA}} - \sum_{d \in D_e^{\mathrm{D}}} \widetilde{p}_d^{\mathrm{DA}} + \sum_{r \in R_e^{\mathrm{D}}} w_r^{\mathrm{DA}} \\
& \quad + s_e^{\mathrm{DA}} + p_e^{\mathrm{PCC,DA}} \Bigg] \\
& + \sum_{g \in G_e^{\mathrm{D}}} \left[\varsigma_g^{\mathrm{DA}-} \left(\underline{P}_g - \widetilde{p}_g^{\mathrm{DA}} \right) + \varsigma_g^{\mathrm{DA}+} \left(\widetilde{p}_g^{\mathrm{DA}} - \overline{P}_g \right) \right] \\
& - \sum_{r \in R_e^{\mathrm{D}}} \left[\iota_r^- w_r^{\mathrm{DA}} - \iota_r^+ (w_r^{\mathrm{DA}} - W_r^{\mathrm{DA}}) \right] \\
& + \sum_{d \in D_e^{\mathrm{D}}} \Big[\varsigma_d^{\mathrm{DA}-} \left(\underline{P}_d - \widetilde{p}_d^{\mathrm{DA}} \right) + \varsigma_d^{\mathrm{DA}+} \left(\widetilde{p}_d^{\mathrm{DA}} - \overline{P}_d \right) \Big] \\
& + \rho_e^{\mathrm{DA}-} (\underline{f}_e - p_e^{\mathrm{PCC,DA}}) + \rho_e^{\mathrm{DA}+} (p_e^{\mathrm{PCC,DA}} - \overline{f}_e) \\
& - \sum_{g \in G_e^{\mathrm{D}}} \zeta_{g\omega}^{\mathrm{p}} \Big(p_{g\omega}^{\mathrm{RT}} - p_g^{\mathrm{DA}} - p_{g\omega}^{\uparrow} + p_{g\omega}^{\downarrow} \Big) \\
& - \sum_{d \in D_e^{\mathrm{D}}} \zeta_{d\omega}^{\mathrm{p}} \Big(p_{d\omega}^{\mathrm{RT}} - p_d^{\mathrm{DA}} + p_{d\omega}^{\uparrow} - p_{d\omega}^{\downarrow} \Big) \\
& - \sum_{r \in R_e^{\mathrm{D}}} \zeta_{r\omega}^{p} \Big(w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}} - w_{r\omega}^{\uparrow} + w_{r\omega}^{\downarrow} \Big) \\
& - \sum_{n \in N_e^{\mathrm{D}}, \omega} \lambda_{n\omega}^{\mathrm{p,RT}} \Big(\sum_{g \in G_n} p_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} p_{d\omega}^{\mathrm{RT}} + \sum_{r \in R_n} w_{r\omega}^{\mathrm{RT}} \\
& + s_{n\omega}^{\mathrm{RT}} + p_{e\omega}^{\mathrm{PCC,RT}} |_{n = n_e^{\mathrm{LV}}} - \sum_{l \in n \rightarrow} p_{l\omega}^{\mathrm{RT}} + \sum_{l \in \rightarrow n} p_{l\omega}^{\mathrm{RT}} \Big) \\
& - \sum_{\omega} \zeta_{e\omega}^{\mathrm{PCC}} \Big(p_{e\omega}^{\mathrm{PCC,RT}} - p_e^{\mathrm{PCC,DA}} - p_{e\omega}^{\uparrow PCC} + p_{e\omega}^{\downarrow PCC} \Big)
\end{aligned}
$$

$$
\begin{aligned}
& - \sum_{n \in N_e^{\mathrm{D}}, \omega} \lambda_{n\omega}^{\mathrm{q,RT}} \Bigg(\sum_{g \in G_n} q_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} q_{d\omega}^{\mathrm{RT}} + s_{n\omega}^{\mathrm{q,RT}} \\
& + q_{e\omega}^{\mathrm{PCC,RT}}\big|_{n=n_e^{\mathrm{LV}}} - \sum_{l \in n\rightarrow} q_{l\omega}^{\mathrm{RT}} + \sum_{l \in \rightarrow n} q_{l\omega}^{\mathrm{RT}} \Bigg) \\
& + \sum_{l \in L_e^{\mathrm{D}}, \omega} \gamma_{l\omega} \left[p_{l\omega}^{(RT)2} + q_{l\omega}^{(RT)2} - \varphi_{l\omega}^{\mathrm{RT}} v_{n\omega}^{\mathrm{RT}} \right] \\
& - \sum_{l \in L_e^{\mathrm{D}}, \omega} \Big[\mu_{l\omega}^{p} \left(p_{l\omega}^{\mathrm{RT}} + p_{l'\omega}^{\mathrm{RT}} - R_l \varphi_{l\omega}^{\mathrm{RT}} \right) \\
& \quad + \mu_{l\omega}^{q} \left(q_{l\omega}^{\mathrm{RT}} + q_{l'\omega}^{\mathrm{RT}} - X_l \varphi_{l\omega}^{\mathrm{RT}} \right) \Big] \\
& + \sum_{l \in L_e^{\mathrm{D}}, \omega} \left[\eta_{l\omega} \left(p_{l\omega}^{(RT)2} + q_{l\omega}^{(RT)2} - S_l \right) \right] \\
& - \sum_{l \in L_e^{\mathrm{D}}, \omega} \Big[\beta_{l\omega} \Big(v_{m\omega}^{\mathrm{RT}} - v_{n\omega}^{\mathrm{RT}} + 2 (R_l p_{l\omega}^{\mathrm{RT}} + X_l q_{l\omega}^{\mathrm{RT}}) \\
& \quad - (R_l^2 + X_l^2) \varphi_{l\omega}^{\mathrm{RT}} \Big) \Big] \\
& + \sum_{n \in N_e^{\mathrm{D}}, \omega} \left[\sigma_{n\omega}^{-} \left(\underline{V}_n^2 - v_{n\omega}^{\mathrm{RT}} \right) + \sigma_{n\omega}^{+} \left(v_{n\omega}^{\mathrm{RT}} - \overline{V}_n^2 \right) \right] \\
& - \sum_{r \in R_e^{\mathrm{D}}, \omega} \left[\nu_{r\omega}^{-} w_{r\omega}^{\mathrm{RT}} - \nu_{r\omega}^{+} \left(w_{r\omega}^{\mathrm{RT}} - W_{r\omega}^{\mathrm{RT}} \right) \right] \\
& + \sum_{g \in G_e^{\mathrm{D}}, \omega} \left[\varsigma_{g\omega}^{\mathrm{RT}-} \left(\underline{P}_g - p_{g\omega}^{\mathrm{RT}} \right) + \varsigma_{g\omega}^{\mathrm{RT}+} \left(p_{g\omega}^{\mathrm{RT}} - \overline{P}_g \right) \right] \\
& + \sum_{d \in D_e^{\mathrm{D}}, \omega} \left[\varsigma_{d\omega}^{\mathrm{RT}-} \left(\underline{P}_d - p_{d\omega}^{\mathrm{RT}} \right) + \varsigma_{d\omega}^{\mathrm{RT}+} \left(p_{d\omega}^{\mathrm{RT}} - \overline{P}_d \right) \right] \\
& + \sum_{g \in G_e^{\mathrm{D}}, \omega} \left[\kappa_{g\omega}^{\mathrm{RT}-} \left(\underline{Q}_g - q_{g\omega}^{\mathrm{RT}} \right) + \kappa_{g\omega}^{\mathrm{RT}+} \left(q_{g\omega}^{\mathrm{RT}} - \overline{Q}_g \right) \right] \\
& + \sum_{d \in D_e^{\mathrm{D}}, \omega} \left[\kappa_{d\omega}^{\mathrm{RT}-} \left(\underline{Q}_d - q_{d\omega}^{\mathrm{RT}} \right) + \kappa_{d\omega}^{\mathrm{RT}+} \left(q_{d\omega}^{\mathrm{RT}} - \overline{Q}_g \right) \right] \\
& + \sum_{g \in G_e^{\mathrm{D}}, \omega} \left[-p_{g\omega}^{\uparrow} \epsilon_{g\omega}^{\mathrm{p}\uparrow} - p_{g\omega}^{\downarrow} \epsilon_{g\omega}^{\mathrm{p}\downarrow} \right] \\
& + \sum_{r \in R_e^{\mathrm{D}}, \omega} \left[-w_{r\omega}^{\uparrow} \epsilon_{r\omega}^{\mathrm{p}\uparrow} - w_{r\omega}^{\downarrow} \epsilon_{r\omega}^{\mathrm{p}\downarrow} \right] \\
& + \sum_{d \in D_e^{\mathrm{D}}, \omega} \left[-p_{d\omega}^{\uparrow} \varepsilon_{d\omega}^{\mathrm{p}\uparrow} - p_{d\omega}^{\downarrow} \varepsilon_{d\omega}^{\mathrm{p}\downarrow} \right] \\
& + \sum_{\omega} \left[-p_{e\omega}^{\uparrow PCC} \epsilon_{e\omega}^{\uparrow PCC} - p_{e\omega}^{\downarrow PCC} \epsilon_{e\omega}^{\downarrow PCC} \right] \\
& + \sum_{\omega} \Bigg[\rho_{e\omega}^{\mathrm{RT}-} \left(\underline{f}_e - \sqrt{p_{e\omega}^{(PCC,RT)2} + q_{e\omega}^{(PCC,RT)2}} \right)
\end{aligned}
$$

$$
\begin{aligned}
&+ \rho_{e\omega}^{\text{RT}+}\left(\sqrt{p_{e\omega}^{(PCC,RT)2} + q_{e\omega}^{(PCC,RT)2}} - \overline{f}_e\right)\Big]\\
&+ \sum_{n\in N_e^{\text{D}},\omega}\left[-\Upsilon_{n\omega}^{\text{RT}-}s_{n\omega}^{\text{RT}} + \Upsilon_{n\omega}^{\text{RT}+}\left(s_{n\omega}^{\text{RT}} - \sum_{d\in D_n} p_{d\omega}^{\text{RT}}\right)\right]\\
&- \Upsilon_e^{\text{DA}-}s_e^{\text{DA}} + \Upsilon_e^{\text{DA}+}\left(s_e^{\text{DA}} - \sum_d p_d^{\text{DA}}\right) \qquad \text{(A.2)}
\end{aligned}
$$

The KKT conditions of above problem are (excluding the primal constraints of A.1):

$$
\begin{aligned}
&(\widetilde{p}_g^{\text{DA}}):\ -\pi_g^{\text{DA}} + \sum_\omega \phi_\omega \pi_g^{\text{DA}} - \lambda_e^{\text{DA}} - \varsigma_g^{\text{DA}-}\\
&\quad + \varsigma_g^{\text{DA}+} + \sum_\omega \zeta_{g\omega}^p = 0, \quad \forall g \in G_e^{\text{D}} \qquad \text{(A.3a)}\\
&(\widetilde{p}_d^{\text{DA}}): \sum_\omega \left(\zeta_{d\omega}^p - \phi_\omega \pi_d^{\text{DA}}\right) + \pi_d^{\text{DA}} + \lambda_e^{\text{DA}}\\
&\quad - \varsigma_d^{\text{DA}-} + \varsigma_d^{\text{DA}+} - \Upsilon_e^{\text{DA}+} = 0, \quad \forall d \in D_e^{\text{D}} \qquad \text{(A.3b)}\\
&(p_{g\omega}^{\uparrow}):\ -\phi_\omega \pi_g^{\uparrow} + \zeta_{g\omega}^p - \epsilon_{g\omega}^{\text{p}\uparrow} = 0, \quad \forall \omega, g \in G_e^{\text{D}} \qquad \text{(A.3c)}\\
&(p_{g\omega}^{\downarrow}):\ -\phi_\omega \pi_g^{\downarrow} - \zeta_{g\omega}^p - \epsilon_{g\omega}^{\text{p}\downarrow} = 0, \quad \forall \omega, g \in G_e^{\text{D}} \qquad \text{(A.3d)}\\
&(p_{d\omega}^{\uparrow}):\ -\phi_\omega \pi_d^{\uparrow} - \zeta_{d\omega}^p - \varepsilon_{i\omega}^{\text{p}\uparrow} = 0, \quad \forall \omega, d \in D_e^{\text{D}} \qquad \text{(A.3e)}\\
&(p_{d\omega}^{\downarrow}):\ -\phi_\omega \pi_d^{\downarrow} + \zeta_{d\omega}^p - \varepsilon_{d\omega}^{\text{p}\downarrow} = 0, \quad \forall \omega, d \in D_e^{\text{D}} \qquad \text{(A.3f)}\\
&(w_{r\omega}^{\uparrow}):\ -\phi_\omega \pi^{\uparrow R} + \zeta_{r\omega}^p - \epsilon_{r\omega}^{\text{p}\uparrow} = 0, \quad \forall \omega, r \in R_e^{\text{D}} \qquad \text{(A.3g)}\\
&(w_{r\omega}^{\downarrow}):\ -\phi_\omega \pi^{\downarrow R} - \zeta_{r\omega}^p - \epsilon_{r\omega}^{\text{p}\downarrow} = 0, \quad \forall \omega, r \in R_e^{\text{D}} \qquad \text{(A.3h)}\\
&(s_e^{\text{DA}}): -\text{VOLL}_e - \lambda_e^{\text{DA}} - \Upsilon_e^{\text{DA}-} + \Upsilon_e^{\text{DA}+} = 0 \qquad \text{(A.3i)}\\
&(s_{n\omega}^{\text{RT}}): -\text{VOLL}_n - \lambda_{n\omega}^{\text{p,RT}} - \Upsilon_{n\omega}^{\text{RT}-}\\
&\quad + \Upsilon_{n\omega}^{\text{RT}+} = 0,\ \forall \omega, n \in N_e^{\text{D}} \qquad \text{(A.3j)}\\
&(w_{r\omega}^{\text{RT}}): -\phi_\omega \pi^R - \zeta_{r\omega}^p - \left[\lambda_{n\omega}^{\text{p,RT}}\right]_{n_r} + \nu_{r\omega}^+\\
&\quad - \nu_{r\omega}^- = 0, \quad \forall \omega, r \in R_e^{\text{D}} \qquad \text{(A.3k)}\\
&(w_r^{\text{DA}}): -\pi^R + \sum_\omega \phi_\omega \pi^R - \lambda_e^{\text{DA}} - \iota_r^-\\
&\quad + \iota_r^+ + \sum_\omega \zeta_{r\omega}^p = 0, \forall r \in R_e^{\text{D}} \qquad \text{(A.3l)}\\
&(p_{g\omega}^{\text{RT}}): -\phi_\omega \pi_g^{\text{DA}} - \zeta_{g\omega}^p - \varsigma_{g\omega}^{\text{RT}-} + \varsigma_{g\omega}^{\text{RT}+}\\
&\quad - \left[\lambda_{n\omega}^{\text{p,RT}}\right]_{n_g} = 0, \quad \forall \omega, g \in G_e^{\text{D}} \qquad \text{(A.3m)}\\
&(q_{g\omega}^{\text{RT}}): -\kappa_{g\omega}^{\text{RT}-} + \kappa_{g\omega}^{\text{RT}+} - \left[\lambda_{n\omega}^{\text{q,RT}}\right]_{n_g} = 0, \quad \forall \omega, g \in G_e^{\text{D}} \qquad \text{(A.3n)}\\
&(p_{d\omega}^{\text{RT}}): \phi_\omega \pi_d^{\text{DA}} - \zeta_{d\omega}^p - \varsigma_{d\omega}^{\text{RT}-} + \varsigma_{d\omega}^{\text{RT}+}\\
&\quad + \left[\lambda_{n\omega}^{\text{p,RT}} - \Upsilon_{n\omega}^{\text{RT}+}\right]_{n_d} = 0, \quad \forall \omega, d \in D_e^{\text{D}} \qquad \text{(A.3o)}
\end{aligned}
$$

$$(q_{d\omega}^{\mathrm{RT}}): -\kappa_{d\omega}^{\mathrm{RT}-} + \kappa_{d\omega}^{\mathrm{RT}+} + \left[\lambda_{n\omega}^{\mathrm{q,RT}}\right]_{n_d} = 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.3p}$$

$$\begin{aligned}(p_{l\omega}^{\mathrm{RT}}): &\lambda_{n\omega}^{\mathrm{p,RT}} - \lambda_{m\omega}^{\mathrm{p,RT}} + 2\gamma_{l\omega} p_{l\omega}^{\mathrm{RT}} - \mu_{l\omega}^{p} - \mu_{l'\omega}^{p} + 2\eta_{l\omega} p_{l\omega}^{\mathrm{RT}} \\ &- 2\beta_{l\omega} R_l = 0, \quad \forall \omega, l = (n,m) \in L_e^{\mathrm{D}}\end{aligned} \tag{A.3q}$$

$$\begin{aligned}(q_{l\omega}^{\mathrm{RT}}): &\lambda_{n\omega}^{\mathrm{q,RT}} - \lambda_{m\omega}^{\mathrm{q,RT}} + 2\gamma_{l\omega} q_{l\omega}^{\mathrm{RT}} - \mu_{l\omega}^{q} - \mu_{l'\omega}^{q} + 2\eta_{l\omega} q_{l\omega}^{\mathrm{RT}} \\ &- 2\beta_{l\omega} X_l = 0, \quad \forall \omega, l = (n,m) \in L_e^{\mathrm{D}}\end{aligned} \tag{A.3r}$$

$$\begin{aligned}(\varphi_{l\omega}^{\mathrm{RT}}): &-\gamma_{l\omega} v_{n\omega}^{\mathrm{RT}} + \mu_{l\omega}^{p} R_l + \mu_{l\omega}^{q} X_l + \beta_{l\omega}(R_l^2 + X_l^2) \\ &= 0, \quad \forall \omega, l = (n,m) \in L_e^{\mathrm{D}}\end{aligned} \tag{A.3s}$$

$$\begin{aligned}(v_{n\omega}^{\mathrm{RT}}): &-\gamma_{l\omega} \varphi_{l\omega}^{\mathrm{RT}} - \beta_{l'\omega} + \beta_{l\omega} - \sigma_{n\omega}^{-} + \sigma_{n\omega}^{+} \\ &= 0, \quad \forall \omega, l = (n,m) \in L_e^{\mathrm{D}}\end{aligned} \tag{A.3t}$$

$$\begin{aligned}(p_e^{\mathrm{PCC,DA}}): &-\pi_e^{\mathrm{PCC,DA}} + \sum_{\omega} \left(\phi_\omega \pi_e^{\mathrm{PCC,DA}} + \zeta_{e\omega}^{\mathrm{PCC}}\right) \\ &- \lambda_e^{\mathrm{DA}} - \rho_e^{\mathrm{DA}-} + \rho_e^{\mathrm{DA}+} = 0\end{aligned} \tag{A.3u}$$

$$\begin{aligned}(p_{e\omega}^{\mathrm{PCC,RT}}): &-\phi_\omega \pi_e^{\mathrm{PCC,DA}} - \left[\lambda_{n\omega}^{\mathrm{p,RT}}\right]_{n_e^{\mathrm{LV}}} - \zeta_{e\omega}^{\mathrm{PCC}} \\ &- \rho_{e\omega}^{\mathrm{RT}-} + \rho_{e\omega}^{\mathrm{RT}+} = 0, \quad \forall \omega\end{aligned} \tag{A.3v}$$

$$(q_{e\omega}^{\mathrm{PCC,RT}}): -\left[\lambda_{n\omega}^{\mathrm{q,RT}}\right]_{n_e^{\mathrm{LV}}} = 0, \quad \forall \omega \tag{A.3w}$$

$$(p_{e\omega}^{\uparrow PCC}): -\phi_\omega \pi_e^{\uparrow PCC} + \zeta_{e\omega}^{\mathrm{PCC}} - \epsilon_{e\omega}^{\uparrow PCC} = 0, \quad \forall \omega \tag{A.3x}$$

$$(p_{e\omega}^{\downarrow PCC}): -\phi_\omega \pi_e^{\downarrow PCC} - \zeta_{e\omega}^{\mathrm{PCC}} - \epsilon_{e\omega}^{\downarrow PCC} = 0, \quad \forall \omega \tag{A.3y}$$

The complimentarity constraints are as follows:

$$0 \le \varsigma_g^{\mathrm{DA}+} \perp \overline{P}_g - \widetilde{p}_g^{\mathrm{DA}} \ge 0, \quad \forall g \in G_e^{\mathrm{D}} \tag{A.4a}$$

$$0 \le \varsigma_g^{\mathrm{DA}-} \perp \widetilde{p}_g^{\mathrm{DA}} - \underline{P}_g \ge 0, \quad \forall g \in G_e^{\mathrm{D}} \tag{A.4b}$$

$$0 \le \varsigma_d^{\mathrm{DA}+} \perp \overline{P}_d - \widetilde{p}_d^{\mathrm{DA}} \ge 0, \quad \forall d \in D_e^{\mathrm{D}} \tag{A.4c}$$

$$0 \le \varsigma_d^{\mathrm{DA}-} \perp \widetilde{p}_d^{\mathrm{DA}} - \underline{P}_d \ge 0, \quad \forall d \in D_e^{\mathrm{D}} \tag{A.4d}$$

$$0 \le \iota_r^{-} \perp w_r^{\mathrm{DA}} \ge 0, \quad \forall r \in R_e^{\mathrm{D}} \tag{A.4e}$$

$$0 \le \iota_r^{+} \perp W_r^{\mathrm{DA}} - w_r^{\mathrm{DA}} \ge 0, \quad \forall r \in R_e^{\mathrm{D}} \tag{A.4f}$$

$$0 \le \rho_e^{\mathrm{DA}-} \perp p_e^{\mathrm{PCC,DA}} - \underline{f}_e \ge 0 \tag{A.4g}$$

$$0 \le \rho_e^{\mathrm{DA}+} \perp \overline{f}_e - p_e^{\mathrm{PCC,DA}} \ge 0 \tag{A.4h}$$

$$0 \le \gamma_{l\omega} \perp \varphi_{l\omega}^{\mathrm{RT}} v_{n\omega}^{\mathrm{RT}} - (p_{l\omega}^{(RT)2} + q_{l\omega}^{(RT)2}) \ge 0,\ \forall \omega, l \in L_e^{\mathrm{D}} \tag{A.4i}$$

$$0 \le \eta_{l\omega} \perp S_l - p_{l\omega}^{(RT)2} - q_{l\omega}^{(RT)2} \ge 0,\ \forall \omega, l \in L_e^{\mathrm{D}} \tag{A.4j}$$

$$0 \le \sigma_{n\omega}^{-} \perp v_{n\omega}^{\mathrm{RT}} - \underline{V}_n^2 \ge 0, \quad \forall \omega, n \in N_e^{\mathrm{D}} \tag{A.4k}$$

$$0 \le \sigma_{n\omega}^{+} \perp \overline{V}_n^2 - v_{n\omega}^{\mathrm{RT}} \ge 0, \quad \forall \omega, n \in N_e^{\mathrm{D}} \tag{A.4l}$$

$$0 \le \nu_{r\omega}^{-} \perp w_{r\omega}^{\mathrm{RT}} \ge 0, \quad \forall \omega, r \in R_e^{\mathrm{D}} \tag{A.4m}$$

$$0 \le \nu_{r\omega}^{+} \perp W_{r\omega}^{\mathrm{RT}} - w_{r\omega}^{\mathrm{RT}} \ge 0, \quad \forall \omega, r \in R_e^{\mathrm{D}} \tag{A.4n}$$

$$0 \le \varsigma_{g\omega}^{\mathrm{RT}-} \perp p_{g\omega}^{\mathrm{RT}} - \underline{P}_g \ge 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.4o}$$

$$0 \le \varsigma_{g\omega}^{\mathrm{RT}+} \perp \overline{P}_g - p_{g\omega}^{\mathrm{RT}} \ge 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.4p}$$

$$0 \leq \varsigma^{\mathrm{RT-}}_{d\omega} \perp p^{\mathrm{RT}}_{d\omega} - \underline{P}_d \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_e \tag{A.4q}$$
$$0 \leq \varsigma^{\mathrm{RT+}}_{d\omega} \perp \overline{P}_d - p^{\mathrm{RT}}_{d\omega} \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_e \tag{A.4r}$$
$$0 \leq \kappa^{\mathrm{RT+}}_{g\omega} \perp q^{\mathrm{RT}}_{g\omega} - \underline{Q}_g \geq 0, \quad \forall \omega, g \in G^{\mathrm{D}}_e \tag{A.4s}$$
$$0 \leq \kappa^{\mathrm{RT-}}_{g\omega} \perp \overline{Q}_g - q^{\mathrm{RT}}_{g\omega} \geq 0, \quad \forall \omega, g \in G^{\mathrm{D}}_e \tag{A.4t}$$
$$0 \leq \kappa^{\mathrm{RT+}}_{d\omega} \perp q^{\mathrm{RT}}_{d\omega} - \underline{Q}_d \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_e \tag{A.4u}$$
$$0 \leq \kappa^{\mathrm{RT-}}_{d\omega} \perp \overline{Q}_d - q^{\mathrm{RT}}_{d\omega} \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_e \tag{A.4v}$$
$$0 \leq \rho^{\mathrm{RT-}}_{e\omega} \perp p^{\mathrm{PCC,RT}}_{e\omega} - \underline{f}_e \geq 0, \quad \forall \omega \tag{A.4w}$$
$$0 \leq \rho^{\mathrm{RT+}}_{e\omega} \perp \overline{f}_e - p^{\mathrm{PCC,RT}}_{e\omega} \geq 0, \quad \forall \omega \tag{A.4x}$$
$$0 \leq \epsilon^{\mathrm{p}\uparrow}_{g\omega} \perp p^{\uparrow}_{g\omega} \geq 0, \quad 0 \leq \epsilon^{\mathrm{p}\downarrow}_{g\omega} \perp p^{\downarrow}_{g\omega} \geq 0, \quad \forall \omega, g \in G^{\mathrm{D}}_e \tag{A.4y}$$
$$0 \leq \epsilon^{\mathrm{p}\uparrow}_{r\omega} \perp w^{\uparrow}_{r\omega} \geq 0,\ 0 \leq \epsilon^{\mathrm{p}\downarrow}_{r\omega} \perp w^{\downarrow}_{r\omega} \geq 0,\ \forall \omega, r \in R^{\mathrm{D}}_e \tag{A.4z}$$
$$0 \leq \varepsilon^{\mathrm{p}\uparrow}_{d\omega} \perp p^{\uparrow}_{d\omega} \geq 0, \quad 0 \leq \varepsilon^{\mathrm{p}\downarrow}_{d\omega} \perp p^{\downarrow}_{d\omega} \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_e \tag{A.4aa}$$
$$0 \leq \epsilon^{\uparrow PCC}_{e\omega} \perp p^{\uparrow PCC}_{e\omega} \geq 0, \quad 0 \leq \epsilon^{\downarrow PCC}_{e\omega} \perp p^{\downarrow PCC}_{e\omega} \geq 0, \quad \forall \omega \tag{A.4ab}$$
$$0 \leq \Upsilon^{\mathrm{RT-}}_{n\omega} \perp s^{\mathrm{RT}}_{n\omega} \geq 0, \forall n, \omega \tag{A.4ac}$$
$$0 \leq \Upsilon^{\mathrm{RT+}}_{n\omega} \perp \sum_{d \in D_n} p^{\mathrm{RT}}_{d\omega} - s^{\mathrm{RT}}_{n\omega} \geq 0,\ \forall n, \omega \tag{A.4ad}$$
$$0 \leq \Upsilon^{\mathrm{DA-}}_{e} \perp s^{\mathrm{DA}}_{e} \geq 0 \tag{A.4ae}$$
$$0 \leq \Upsilon^{\mathrm{DA+}}_{e} \perp \sum_{d} p^{\mathrm{DA}}_{d} - s^{\mathrm{DA}}_{e} \geq 0 \tag{A.4af}$$

Appendix B. KKTs of DA market

For convenience problem (3) is repeated here, with dual varaibles for every constraint added.

$$\max_{\Xi^{\mathrm{DA}}} \mathcal{SW}^{\mathrm{DA}} = \sum_{d \in D} \pi^{\mathrm{DA}}_d \widehat{p}^{\mathrm{DA}}_d - \sum_{g \in G} \pi^{\mathrm{DA}}_g \widehat{p}^{\mathrm{DA}}_g$$
$$- VOLL^{\mathrm{DA}} s^{\mathrm{DA}} - \pi^R \sum_r w^{\mathrm{DA}}_r \tag{B.1a}$$

subject to:

$$\sum_{g \in G} \widehat{p}^{\mathrm{DA}}_g - \sum_{d \in D} \widehat{p}^{\mathrm{DA}}_d + \sum_r w^{\mathrm{DA}}_r + s^{\mathrm{DA}} = 0,\ :(\lambda^{\mathrm{T,DA}}) \tag{B.1b}$$
$$\underline{P}_g \leq \widehat{p}^{\mathrm{DA}}_g \leq \widetilde{p}^{\mathrm{DA}}_g,\ \forall g \in G^{\mathrm{D}}_e,\ \forall e \in E\ :(\varsigma^{\mathrm{T,DA-}}_{ge}, \varsigma^{\mathrm{T,DA+}}_{ge}) \tag{B.1c}$$
$$\underline{P}_g \leq \widehat{p}^{\mathrm{DA}}_g \leq \overline{P}_g,\ \forall g \in G^{\mathrm{T}}\ :(\sigma^{\mathrm{T,DA-}}_g, \sigma^{\mathrm{T,DA+}}_g) \tag{B.1d}$$
$$\underline{P}_d \leq \widehat{p}^{\mathrm{DA}}_d \leq \widetilde{p}^{\mathrm{DA}}_d,\ \forall d \in D^{\mathrm{D}}_e,\ \forall e \in E\ :(\varsigma^{\mathrm{T,DA-}}_{de}, \varsigma^{\mathrm{T,DA+}}_{de}) \tag{B.1e}$$
$$\underline{P}_d \leq \widehat{p}^{\mathrm{DA}}_d \leq \overline{P}_d,\ \forall d \in D^{\mathrm{T}}\ :(\sigma^{\mathrm{T,DA-}}_d, \sigma^{\mathrm{T,DA+}}_d) \tag{B.1f}$$
$$0 \leq w^{\mathrm{DA}}_r \leq W^{\mathrm{DA}}_r,\ \forall r \in R\ :(\nu^{\mathrm{T,DA-}}_r, \nu^{\mathrm{T,DA+}}_r) \tag{B.1g}$$
$$0 \leq s^{\mathrm{l,DA}} \leq \sum_d \widehat{p}^{\mathrm{DA}}_d,\ :(\rho^{\mathrm{T,DA-}}, \rho^{\mathrm{T,DA+}}) \tag{B.1h}$$

$\widehat{p}_g^{\mathrm{DA}}$ and $\widehat{p}_d^{\mathrm{DA}}$ is the day ahead dispatch from problem A.1.

The lagrangian of the TSO day-ahead market problem is as follows:

$$
\begin{aligned}
\mathcal{L}^{\mathrm{DA}} = & \sum_{d\in D} \pi_d^{\mathrm{DA}} \widehat{p}_d^{\mathrm{DA}} - \sum_{g\in G} \pi_g^{\mathrm{DA}} \widehat{p}_g^{\mathrm{DA}} \\
& - VOLL^{\mathrm{DA}} s^{\mathrm{DA}} - \pi^R \sum_r w_r^{\mathrm{DA}} \\
& - \lambda^{\mathrm{T,DA}} \left[\sum_{g\in G} \widehat{p}_g^{\mathrm{DA}} - \sum_{d\in D} \widehat{p}_d^{\mathrm{DA}} + \sum_{r\in R} w_r^{\mathrm{DA}} + s^{\mathrm{DA}} \right] \\
& + \sum_{g\in G_e^{\mathrm{D}}, e} \left[\varsigma_{ge}^{\mathrm{T,DA}-} \left(\underline{P}_g - \widehat{p}_g^{\mathrm{DA}} \right) + \varsigma_{ge}^{\mathrm{T,DA}+} \left(\widehat{p}_g^{\mathrm{DA}} - \widetilde{p}_g^{\mathrm{DA}} \right) \right] \\
& + \sum_{g\in G^{\mathrm{T}}} \left[\sigma_g^{\mathrm{T,DA}-} \left(\underline{P}_g - \widehat{p}_g^{\mathrm{DA}} \right) + \sigma_g^{\mathrm{T,DA}+} \left(\widehat{p}_g^{\mathrm{DA}} - \overline{P}_g \right) \right] \\
& + \sum_{d\in D_e^{\mathrm{D}}, e} \left[\varsigma_{de}^{\mathrm{T,DA}-} \left(\underline{P}_d - \widehat{p}_d^{\mathrm{DA}} \right) + \varsigma_{de}^{\mathrm{T,DA}+} \left(\widehat{p}_d^{\mathrm{DA}} - \widetilde{p}_d^{\mathrm{DA}} \right) \right] \\
& + \sum_{d\in D^{\mathrm{T}}} \left[\sigma_d^{\mathrm{T,DA}-} \left(\underline{P}_d - \widehat{p}_d^{\mathrm{DA}} \right) + \sigma_d^{\mathrm{T,DA}+} \left(\widehat{p}_d^{\mathrm{DA}} - \overline{P}_d \right) \right] \\
& - \sum_{r\in R} \left[\nu_r^{\mathrm{T,DA}-} w_r^{\mathrm{DA}} - \nu_r^{\mathrm{T,DA}+} \left(w_r^{\mathrm{DA}} - W_r^{\mathrm{DA}} \right) \right] \\
& - \rho^{\mathrm{T,DA}-} s^{\mathrm{DA}} + \rho^{\mathrm{T,DA}+} (s^{\mathrm{DA}} - \sum_d \widehat{p}_d^{\mathrm{DA}})
\end{aligned} \tag{B.2}
$$

The KKTs of the TSO day-ahead market (excluding primal constraints) are:

$$
\begin{aligned}
(\widehat{p}_g^{\mathrm{DA}}): \quad & -\pi_g^{\mathrm{DA}} - \left[\varsigma_{ge}^{\mathrm{T,DA}-} - \varsigma_{ge}^{\mathrm{T,DA}+} \right]_{g\in G_e^{\mathrm{D}}} \\
& - \left[\sigma_g^{\mathrm{T,DA}-} - \sigma_g^{\mathrm{T,DA}+} \right]_{g\in G^{\mathrm{T}}} \\
& - \lambda^{\mathrm{T,DA}} = 0, \quad \forall g \in G
\end{aligned} \tag{B.3a}
$$

$$
\begin{aligned}
(\widehat{p}_d^{\mathrm{DA}}): \quad & \pi_d^{\mathrm{DA}} - \left[\varsigma_{de}^{\mathrm{T,DA}-} - \varsigma_{de}^{\mathrm{T,DA}+} \right]_{d\in D_e^{\mathrm{D}}} \\
& - \left[\sigma_d^{\mathrm{T,DA}-} - \sigma_d^{\mathrm{T,DA}+} \right]_{d\in D^{\mathrm{T}}} - \rho^{\mathrm{T,DA}+} \\
& + \lambda^{\mathrm{T,DA}} = 0, \quad \forall d \in D
\end{aligned} \tag{B.3b}
$$

$$
\begin{aligned}
(s^{\mathrm{l,DA}}): \quad & -VOLL^{\mathrm{DA}} - \lambda^{\mathrm{T,DA}} - \rho^{\mathrm{T,DA}-} \\
& + \rho^{\mathrm{T,DA}+} = 0
\end{aligned} \tag{B.3c}
$$

$$
\begin{aligned}
(w_r^{\mathrm{DA}}): \quad & -\pi^R - \lambda^{\mathrm{T,DA}} \\
& - \left[\nu_r^{\mathrm{T,DA}-} - \nu_r^{\mathrm{T,DA}+} \right] = 0, \ \forall r \in R
\end{aligned} \tag{B.3d}
$$

The complimentary constraints are:

$$
0 \leq \varsigma_{ge}^{\mathrm{T,DA}-} \perp \widehat{p}_g^{\mathrm{DA}} - \underline{P}_g \geq 0 \quad \forall g, e \tag{B.4a}
$$

$$0 \leq \varsigma_{ge}^{\mathrm{T,DA+}} \perp \widehat{p}_g^{\mathrm{DA}} - \widehat{p}_g^{\mathrm{DA}} \geq 0 \quad \forall g \in G_e^{\mathrm{D}}, e \in E \tag{B.4b}$$

$$0 \leq \sigma_g^{\mathrm{T,DA+}} \perp \overline{P}_g - \widehat{p}_g^{\mathrm{DA}} \geq 0 \quad \forall g \in G^{\mathrm{T}} \tag{B.4c}$$

$$0 \leq \varsigma_{de}^{\mathrm{T,DA-}} \perp \widehat{p}_d^{\mathrm{DA}} - \underline{P}_d \geq 0 \quad \forall d, e \tag{B.4d}$$

$$0 \leq \varsigma_{de}^{\mathrm{T,DA+}} \perp \widehat{p}_d^{\mathrm{DA}} - \widehat{p}_d^{\mathrm{DA}} \geq 0 \quad \forall d \in D_e^{\mathrm{D}}, e \in E \tag{B.4e}$$

$$0 \leq \sigma_d^{\mathrm{T,DA+}} \perp \overline{P}_d - \widehat{p}_d^{\mathrm{DA}} \geq 0 \quad \forall d \in D^{\mathrm{T}} \tag{B.4f}$$

$$0 \leq \nu_r^{\mathrm{T,DA-}} \perp w_r^{\mathrm{DA}} \geq 0, \ \forall r \in R \tag{B.4g}$$

$$0 \leq \nu_r^{\mathrm{T,DA+}} \perp W_r^{\mathrm{DA}} - w_r^{\mathrm{DA}} \geq 0, \ \forall r \in R \tag{B.4h}$$

$$0 \leq \rho^{\mathrm{T,DA-}} \perp s^{\mathrm{l,DA}} \geq 0 \tag{B.4i}$$

$$0 \leq \rho^{\mathrm{T,DA+}} \perp \sum_d \widehat{p}_d^{\mathrm{DA}} - s^{\mathrm{l,DA}} \geq 0 \tag{B.4j}$$

Appendix C. KKT conditions of Real-Time Market

The KKT conditions of the SOCP problem for the real-time re-dispatch are not actually solved, because the Benders decomposition renders the scenarios solvable as single problems. However, they are used in a proof of equivalence between the DSO market and the global DA-RT combination.

The Real-Time problem from (4) is repeated here with dual variables added:

$$\begin{aligned}
\min_{\Xi^{\mathrm{RT}}} \ & \phi_\omega(\Delta \mathrm{Cost}_\omega^{\mathrm{RT}}) \qquad\qquad \text{(C.1a)} \\
& = \phi_\omega \Big[\sum_{g \in G} (\pi_g^{\mathrm{DA}}(p_{g\omega}^{\mathrm{RT}} - \widehat{p}_g^{\mathrm{DA}}) + \pi_g^{\uparrow} p_{g\omega}^{\uparrow} + \pi_g^{\downarrow} p_{g\omega}^{\downarrow}) \\
& + \sum_{d \in D} (\pi_d^{\mathrm{DA}}(\widehat{p}_d^{\mathrm{DA}} - p_{d\omega}^{\mathrm{RT}}) + \pi_d^{\uparrow} p_{d\omega}^{\uparrow} + \pi_d^{\downarrow} p_{d\omega}^{\downarrow}) \\
& + \sum_{n \in N} VOLL_n^{\mathrm{RT}} s_{n\omega}^{\mathrm{RT}} \\
& + \sum_r \Big(\pi^R (w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}}) + \pi^{\uparrow\mathrm{R}} w_{r\omega}^{\uparrow} + \pi^{\downarrow\mathrm{R}} w_{r\omega}^{\downarrow} \Big) \Big]
\end{aligned}$$

$$\text{s.t.} \quad p_{l\omega}^{\mathrm{RT}} = B_l(\theta_{n\omega} - \theta_{m\omega}), \quad \forall l \in L^{\mathrm{T}}, \ : (\gamma_{l\omega}^{\mathrm{T}}) \tag{C.1b}$$

$$p_{l\omega}^{\mathrm{RT}} \leq S_l, \ \forall l \in L^{\mathrm{T}}, \ : (\eta_{l\omega}^{\mathrm{T}}) \tag{C.1c}$$

$$p_{g\omega}^{\mathrm{RT}} = \widehat{p}_g^{\mathrm{DA}} + p_{g\omega}^{\uparrow} - p_{g\omega}^{\downarrow}, \quad \forall g \in G, \ : (\zeta_{g\omega}^{\mathrm{p,RT}}) \tag{C.1d}$$

$$p_{d\omega}^{\mathrm{RT}} = \widehat{p}_d^{\mathrm{DA}} - p_{d\omega}^{\uparrow} + p_{d\omega}^{\downarrow}, \quad \forall d \in D, \ : (\zeta_{d\omega}^{\mathrm{p,RT}}) \tag{C.1e}$$

$$w_{r\omega}^{\mathrm{RT}} = w_r^{\mathrm{DA}} + w_{r\omega}^{\uparrow} - w_{r\omega}^{\downarrow}, \ \forall r \in R, , \ : (\zeta_{r\omega}^{\mathrm{p,RT}}) \tag{C.1f}$$

$$\begin{aligned}
& \sum_{g \in G_n} p_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} p_{d\omega}^{\mathrm{RT}} + \sum_{r \in R_n} w_{r\omega}^{\mathrm{RT}} + s_{n\omega}^{\mathrm{RT}} \\
& \quad = \sum_{l \in n \rightarrow} p_{l\omega}^{\mathrm{RT}} - \sum_{l \in \rightarrow n} p_{l\omega}^{\mathrm{RT}}, \ \forall n \in N, \ : (\lambda_{n\omega}^{\mathrm{p,RT}}) \qquad \text{(C.1g)}
\end{aligned}$$

$$\sum_{g \in G_n} q_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} q_{d\omega}^{\mathrm{RT}} + s_{n\omega}^{\mathrm{q,RT}}$$

$$= \sum_{l\in n\rightarrow} q_{l\omega}^{\mathrm{RT}} - \sum_{l\in \rightarrow n} q_{l\omega}^{\mathrm{RT}},\ \forall n\in N_e^{\mathrm{D}},\ :(\lambda_{n\omega}^{\mathrm{q,RT}}) \tag{C.1h}$$

$$p_{l\omega}^{(\mathrm{RT})2} + q_{l\omega}^{(\mathrm{RT})2} \le \varphi_{l\omega}^{\mathrm{RT}} v_{n\omega}^{\mathrm{RT}},\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\gamma_{l\omega}^{\mathrm{D,RT}}) \tag{C.1i}$$

$$p_{l\omega}^{\mathrm{RT}} + p_{l'\omega}^{\mathrm{RT}} = R_l\varphi_{l\omega}^{\mathrm{RT}},\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\mu_{l\omega}^{\mathrm{p,RT}}) \tag{C.1j}$$

$$q_{l\omega}^{\mathrm{RT}} + q_{l'\omega}^{\mathrm{RT}} = X_l\varphi_{l\omega}^{\mathrm{RT}},\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\mu_{l\omega}^{\mathrm{q,RT}}) \tag{C.1k}$$

$$p_{l\omega}^{(\mathrm{RT})2} + q_{l\omega}^{(\mathrm{RT})2} \le S_l^2,\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\eta_{l\omega}^{\mathrm{D}}) \tag{C.1l}$$

$$v_{m\omega}^{\mathrm{RT}} = v_{n\omega}^{\mathrm{RT}} - 2(R_l p_{l\omega}^{\mathrm{RT}} + X_l q_{l\omega}^{\mathrm{RT}}) + (R_l^2 + X_l^2)\varphi_{l\omega}^{\mathrm{RT}},\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\beta_{l\omega}^{\mathrm{RT}}) \tag{C.1m}$$

$$\underline{V}_n^2 \le v_{n\omega}^{\mathrm{RT}} \le \overline{V}_n^2,\quad \forall e, n\in N_e^{\mathrm{D}},\ :(\sigma_{n\omega}^{\mathrm{RT}-}, \sigma_{n\omega}^{\mathrm{RT}+}) \tag{C.1n}$$

$$0 \le w_{r\omega}^{\mathrm{RT}} \le W_{r\omega}^{\mathrm{RT}},\quad \forall r\in R,\ :(\nu_{r\omega}^{\mathrm{RT}-}, \nu_{r\omega}^{\mathrm{RT}+}) \tag{C.1o}$$

$$\underline{P}_g \le p_{g\omega}^{\mathrm{RT}} \le \overline{P}_g,\quad \forall g\in G,\ :(\varsigma_{g\omega}^{\mathrm{RT}-}, \varsigma_{g\omega}^{\mathrm{RT}+}) \tag{C.1p}$$

$$\underline{P}_d \le p_{d\omega}^{\mathrm{RT}} \le \overline{P}_d,\quad \forall d\in D,\ :(\varsigma_{d\omega}^{\mathrm{RT}-}, \varsigma_{d\omega}^{\mathrm{RT}+}) \tag{C.1q}$$

$$\underline{Q}_g \le q_{g\omega}^{\mathrm{RT}} \le \overline{Q}_g,\quad \forall g\in G_e^{\mathrm{D}},\ :(\kappa_{g\omega}^{\mathrm{RT}-}, \kappa_{g\omega}^{\mathrm{RT}+}) \tag{C.1r}$$

$$\underline{Q}_d \le q_{d\omega}^{\mathrm{RT}} \le \overline{Q}_d,\quad \forall d\in D_e^{\mathrm{D}},\ :(\kappa_{d\omega}^{\mathrm{RT}-}, \kappa_{d\omega}^{\mathrm{RT}+}) \tag{C.1s}$$

$$0 \le s_{n\omega}^{\mathrm{RT}} \le \sum_{d\in D_n} p_{d\omega}^{\mathrm{RT}},\ \forall n\in N,\ :(\Upsilon_{n\omega}^{\mathrm{RT}-}, \Upsilon_{n\omega}^{\mathrm{RT}+}) \tag{C.1t}$$

$$p_{g\omega}^{\uparrow} \ge 0,\ ,\quad p_{g\omega}^{\downarrow} \ge 0,\ \forall g,\ :(\epsilon_{g\omega}^{\uparrow,\mathrm{RT}}, \epsilon_{g\omega}^{\downarrow,\mathrm{RT}}) \tag{C.1u}$$

$$p_{d\omega}^{\uparrow} \ge 0,\ ,\quad p_{d\omega}^{\downarrow} \ge 0,\ \forall d,\ :(\epsilon_{d\omega}^{\uparrow,\mathrm{RT}}, \epsilon_{d\omega}^{\downarrow,\mathrm{RT}}) \tag{C.1v}$$

$$w_{r\omega}^{\uparrow} \ge 0,\quad w_{r\omega}^{\downarrow} \ge 0, \forall r,\ :(\varepsilon_{r\omega}^{\uparrow,\mathrm{RT}}, \varepsilon_{r\omega}^{\downarrow,\mathrm{RT}}) \tag{C.1w}$$

The Lagrangian of the real-time problem is as follows:

$$\begin{aligned}
\mathcal{L}^{\mathrm{RT}} = \sum_{\omega}\phi_\omega \Bigg[& \sum_{g\in G}\Big(\pi_g^{\mathrm{DA}}(p_{g\omega}^{\mathrm{RT}} - p_g^{\mathrm{DA}}) + \pi_g^{\uparrow}p_{g\omega}^{\uparrow} + \pi_g^{\downarrow}p_{g\omega}^{\downarrow}\Big) \\
& + \sum_{d\in D}\Big(\pi_d^{\mathrm{DA}}(p_d^{\mathrm{DA}} - p_{d\omega}^{\mathrm{RT}}) + \pi_d^{\uparrow}p_{d\omega}^{\uparrow} + \pi_d^{\downarrow}p_{d\omega}^{\downarrow}\Big) \\
& + \sum_{n\in N} VOLL_n^{\mathrm{RT}} s_{n\omega}^{\mathrm{RT}} \\
& + \sum_{r\in R}\Big(\pi^R(w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}}) + \pi^{\uparrow R}w_{r\omega}^{\uparrow} + \pi^{\downarrow R}w_{r\omega}^{\downarrow}\Big)\Bigg] \\
& + \sum_{l\in L^{\mathrm{T}}}\gamma_{l\omega}^{\mathrm{T}}\left(p_{l\omega}^{\mathrm{RT}} - B_l(\theta_{n\omega} - \theta_{m\omega})\right) \\
& + \sum_{l\in L^{\mathrm{T}}}\eta_{l\omega}^{\mathrm{T}}\left(p_{l\omega}^{\mathrm{RT}} - S_l\right) \\
& - \sum_{g\in G}\zeta_{g\omega}^{\mathrm{p}}\Big(p_{g\omega}^{\mathrm{RT}} - \widehat{p}_g^{\mathrm{DA}} - p_{g\omega}^{\uparrow} + p_{g\omega}^{\downarrow}\Big)
\end{aligned}$$

$$
\begin{aligned}
&- \sum_{d \in D} \zeta^{\mathrm{p}}_{d\omega} \left(p^{\mathrm{RT}}_{d\omega} - \widehat{p}^{\mathrm{DA}}_{d} + p^{\uparrow}_{d\omega} - p^{\downarrow}_{d\omega} \right) \\
&- \sum_{r \in R} \zeta^{p}_{r\omega} \left(w^{\mathrm{RT}}_{r\omega} - w^{\mathrm{DA}}_{r} - w^{\uparrow}_{r\omega} + w^{\downarrow}_{r\omega} \right) \\
&- \sum_{n \in N, \omega} \lambda^{\mathrm{p,RT}}_{n\omega} \Big(\sum_{g \in G_n} p^{\mathrm{RT}}_{g\omega} - \sum_{d \in D_n} p^{\mathrm{RT}}_{d\omega} + \sum_{r \in R_n} w^{\mathrm{RT}}_{r\omega} \\
&+ s^{\mathrm{RT}}_{n\omega} - \sum_{l \in n \rightarrow} p^{\mathrm{RT}}_{l\omega} + \sum_{l \in \rightarrow n} p^{\mathrm{RT}}_{l\omega} \Big) \\
&- \sum_{n \in N, \omega} \lambda^{\mathrm{q,RT}}_{n\omega} \Bigg(\sum_{g \in G_n} q^{\mathrm{RT}}_{g\omega} - \sum_{d \in D_n} q^{\mathrm{RT}}_{d\omega} + s^{\mathrm{q,RT}}_{n\omega} \\
&- \sum_{l \in n \rightarrow} q^{\mathrm{RT}}_{l\omega} + \sum_{l \in \rightarrow n} q^{\mathrm{RT}}_{l\omega} \Bigg) \\
&+ \sum_{l \in L^{\mathrm{D}}_e \cup l_e, \omega} \gamma_{l\omega} \left[p^{(RT)2}_{l\omega} + q^{(RT)2}_{l\omega} - \varphi^{\mathrm{RT}}_{l\omega} v^{\mathrm{RT}}_{n\omega} \right] \\
&- \sum_{l \in L^{\mathrm{D}}_e \cup l_e, \omega} \Big[\mu^{p}_{l\omega} \left(p^{\mathrm{RT}}_{l\omega} + p^{\mathrm{RT}}_{l'\omega} - R_l \varphi^{\mathrm{RT}}_{l\omega} \right) \\
&\quad + \mu^{q}_{l\omega} \left(q^{\mathrm{RT}}_{l\omega} + q^{\mathrm{RT}}_{l'\omega} - X_l \varphi^{\mathrm{RT}}_{l\omega} \right) \Big] \\
&+ \sum_{l \in L^{\mathrm{D}}_e \cup l_e, \omega} \left[\eta_{l\omega} \left(p^{(RT)2}_{l\omega} + q^{(RT)2}_{l\omega} - S_l \right) \right] \\
&- \sum_{l \in L^{\mathrm{D}}_e \cup l_e, \omega} \Big[\beta_{l\omega} \Big(v^{\mathrm{RT}}_{m\omega} - v^{\mathrm{RT}}_{n\omega} + 2 (R_l p^{\mathrm{RT}}_{l\omega} + X_l q^{\mathrm{RT}}_{l\omega}) \\
&\quad - (R_l^2 + X_l^2) \varphi^{\mathrm{RT}}_{l\omega} \Big) \Big] \\
&+ \sum_{n \in N^{\mathrm{D}}_e \cup n^{HV}_e, \omega} \left[\sigma^{-}_{n\omega} \Big(\underline{V}^2_n - v^{\mathrm{RT}}_{n\omega} \Big) + \sigma^{+}_{n\omega} \left(v^{\mathrm{RT}}_{n\omega} - \overline{V}^2_n \right) \right] \\
&- \sum_{r \in R, \omega} \left[\nu^{-}_{r\omega} w^{\mathrm{RT}}_{r\omega} - \nu^{+}_{r\omega} \left(w^{\mathrm{RT}}_{r\omega} - W^{\mathrm{RT}}_{r\omega} \right) \right] \\
&+ \sum_{g \in G, \omega} \left[\varsigma^{\mathrm{RT}-}_{g\omega} \left(\underline{P}_g - p^{\mathrm{RT}}_{g\omega} \right) + \varsigma^{\mathrm{RT}+}_{g\omega} \left(p^{\mathrm{RT}}_{g\omega} - \overline{P}_g \right) \right] \\
&+ \sum_{d \in D, \omega} \left[\varsigma^{\mathrm{RT}-}_{d\omega} \left(\underline{P}_d - p^{\mathrm{RT}}_{d\omega} \right) + \varsigma^{\mathrm{RT}+}_{d\omega} \left(p^{\mathrm{RT}}_{d\omega} - \overline{P}_d \right) \right] \\
&+ \sum_{g \in G, \omega} \left[\kappa^{\mathrm{RT}-}_{g\omega} \left(\underline{Q}_g - q^{\mathrm{RT}}_{g\omega} \right) + \kappa^{\mathrm{RT}+}_{g\omega} \left(q^{\mathrm{RT}}_{g\omega} - \overline{Q}_g \right) \right] \\
&+ \sum_{d \in D^{\mathrm{D}}_e, \omega} \left[\kappa^{\mathrm{RT}-}_{d\omega} \left(\underline{Q}_d - q^{\mathrm{RT}}_{d\omega} \right) + \kappa^{\mathrm{RT}+}_{d\omega} \left(q^{\mathrm{RT}}_{d\omega} - \overline{Q}_g \right) \right] \\
&+ \sum_{g \in G, \omega} \left[- p^{\uparrow}_{g\omega} \epsilon^{\mathrm{p}\uparrow}_{g\omega} - p^{\downarrow}_{g\omega} \epsilon^{\mathrm{p}\downarrow}_{g\omega} \right]
\end{aligned}
$$

$$+\sum_{r\in R,\omega}\left[-w^{\uparrow}_{r\omega}\epsilon^{\mathrm{p}\uparrow}_{r\omega}-w^{\downarrow}_{r\omega}\epsilon^{\mathrm{p}\downarrow}_{r\omega}\right]$$
$$+\sum_{d\in D,\omega}\left[-p^{\uparrow}_{d\omega}\varepsilon^{\mathrm{p}\uparrow}_{d\omega}-p^{\downarrow}_{d\omega}\varepsilon^{\mathrm{p}\downarrow}_{d\omega}\right]$$
$$+\sum_{n\in N,\omega}\left[-\Upsilon^{\mathrm{RT}-}_{n\omega}s^{\mathrm{RT}}_{n\omega}+\Upsilon^{\mathrm{RT}+}_{n\omega}\left(s^{\mathrm{RT}}_{n\omega}-\sum_{d\in D_n}p^{\mathrm{RT}}_{d\omega}\right)\right] \tag{C.2}$$

The KKT conditions of above Lagrangian are (excluding the primal constraints of C.1):

$$(p^{\uparrow}_{g\omega}):\ \phi_\omega\pi^{\uparrow}_g+\zeta^p_{g\omega}-\epsilon^{\mathrm{p}\uparrow}_{g\omega}=0,\quad \forall g\in G^{\mathrm{D}}_e \tag{C.3a}$$
$$(p^{\downarrow}_{g\omega}):\ \phi_\omega\pi^{\downarrow}_g-\zeta^p_{g\omega}-\epsilon^{\mathrm{p}\downarrow}_{g\omega}=0,\quad \forall g\in G^{\mathrm{D}}_e \tag{C.3b}$$
$$(p^{\uparrow}_{d\omega}):\ \phi_\omega\pi^{\uparrow}_d-\zeta^p_{d\omega}-\varepsilon^{\mathrm{p}\uparrow}_{i\omega}=0,\quad \forall d\in D^{\mathrm{D}}_e \tag{C.3c}$$
$$(p^{\downarrow}_{d\omega}):\ \phi_\omega\pi^{\downarrow}_d+\zeta^p_{d\omega}-\varepsilon^{\mathrm{p}\downarrow}_{d\omega}=0,\quad \forall d\in D^{\mathrm{D}}_e \tag{C.3d}$$
$$(w^{\uparrow}_{r\omega}):\ \phi_\omega\pi^{\uparrow R}+\zeta^p_{r\omega}-\epsilon^{\mathrm{p}\uparrow}_{r\omega}=0,\quad \forall r\in R^{\mathrm{D}}_e \tag{C.3e}$$
$$(w^{\downarrow}_{r\omega}):\ \phi_\omega\pi^{\downarrow R}-\zeta^p_{r\omega}-\epsilon^{\mathrm{p}\downarrow}_{r\omega}=0,\quad \forall r\in R^{\mathrm{D}}_e \tag{C.3f}$$
$$\begin{aligned}(s^{\mathrm{RT}}_{n\omega}):\ &\phi_\omega\mathrm{VOLL}_n-\lambda^{\mathrm{p,RT}}_{n\omega}-\Upsilon^{\mathrm{RT}-}_{n\omega}\\&+\Upsilon^{\mathrm{RT}+}_{n\omega}=0,\ \forall n\in N\end{aligned} \tag{C.3g}$$
$$\begin{aligned}(w^{\mathrm{RT}}_{r\omega}):\ &\phi_\omega\pi^R-\zeta^p_{r\omega}-\left[\lambda^{\mathrm{p,RT}}_{n\omega}\right]_{n_r}+\nu^+_{r\omega}\\&-\nu^-_{r\omega}=0,\quad \forall r\in R^{\mathrm{D}}_e\end{aligned} \tag{C.3h}$$
$$\begin{aligned}(p^{\mathrm{RT}}_{g\omega}):\ &\phi_\omega\pi^{\mathrm{DA}}_g-\zeta^p_{g\omega}-\varsigma^{\mathrm{RT}-}_{g\omega}+\varsigma^{\mathrm{RT}+}_{g\omega}\\&-\left[\lambda^{\mathrm{p,RT}}_{n\omega}\right]_{n_g}=0,\quad \forall g\in G\end{aligned} \tag{C.3i}$$
$$(q^{\mathrm{RT}}_{g\omega}):-\kappa^{\mathrm{RT}-}_{g\omega}+\kappa^{\mathrm{RT}+}_{g\omega}-\left[\lambda^{\mathrm{q,RT}}_{n\omega}\right]_{n_g}=0,\quad \forall g\in G^{\mathrm{D}}_e \tag{C.3j}$$
$$\begin{aligned}(p^{\mathrm{RT}}_{d\omega}):\ &-\phi_\omega\pi^{\mathrm{DA}}_d-\zeta^p_{d\omega}-\varsigma^{\mathrm{RT}-}_{d\omega}+\varsigma^{\mathrm{RT}+}_{d\omega}\\&+\left[\lambda^{\mathrm{p,RT}}_{n\omega}-\Upsilon^{\mathrm{RT}+}_{n\omega}\right]_{n_d}=0,\quad \forall d\in D\end{aligned} \tag{C.3k}$$
$$(q^{\mathrm{RT}}_{d\omega}):-\kappa^{\mathrm{RT}-}_{d\omega}+\kappa^{\mathrm{RT}+}_{d\omega}+\left[\lambda^{\mathrm{q,RT}}_{n\omega}\right]_{n_d}=0,\quad \forall d\in D^{\mathrm{D}}_e \tag{C.3l}$$
$$\begin{aligned}(p^{\mathrm{RT}}_{l\omega}):\ &\lambda^{\mathrm{p,RT}}_{n\omega}-\lambda^{\mathrm{p,RT}}_{m\omega}+\Big[2\gamma_{l\omega}p^{\mathrm{RT}}_{l\omega}-\mu^{\mathrm{p}}_{l\omega}-\mu^{\mathrm{p}}_{l'\omega}+2\eta_{l\omega}p^{\mathrm{RT}}_{l\omega}\\&-2\beta_{l\omega}R_l\Big]_{l\in L^{\mathrm{D}}_e}+\Big[\gamma^{\mathrm{T}}_{l\omega}+\eta^{\mathrm{T}}_{l\omega}\Big]_{l\in L^{\mathrm{T}}}=0,\quad \forall l\in L\end{aligned} \tag{C.3m}$$
$$\begin{aligned}(q^{\mathrm{RT}}_{l\omega}):\ &\lambda^{\mathrm{q,RT}}_{n\omega}-\lambda^{\mathrm{q,RT}}_{m\omega}+\Big[2\gamma_{l\omega}q^{\mathrm{RT}}_{l\omega}-\mu^{q}_{l\omega}-\mu^{q}_{l'\omega}+2\eta_{l\omega}q^{\mathrm{RT}}_{l\omega}\\&-2\beta_{l\omega}X_l\Big]_{l\in L^{\mathrm{D}}_e}=0,\quad \forall l\in L\end{aligned} \tag{C.3n}$$
$$\begin{aligned}(\varphi^{\mathrm{RT}}_{l\omega}):\ &-\gamma_{l\omega}v^{\mathrm{RT}}_{n\omega}+\mu^p_{l\omega}R_l+\mu^q_{l\omega}X_l+\beta_{l\omega}(R^2_l+X^2_l)\\&=0,\quad \forall\omega,l=(n,m)\in L^{\mathrm{D}}_e\end{aligned} \tag{C.3o}$$
$$(v^{\mathrm{RT}}_{n\omega}):-\gamma_{l\omega}\varphi^{\mathrm{RT}}_{l\omega}-\beta_{l'\omega}+\beta_{l\omega}-\sigma^-_{n\omega}+\sigma^+_{n\omega}$$

$$= 0, \quad \forall \omega, l = (n, m) \in L_e^{\mathrm{D}} \tag{C.3p}$$

/

Appendix D. Congestion Level

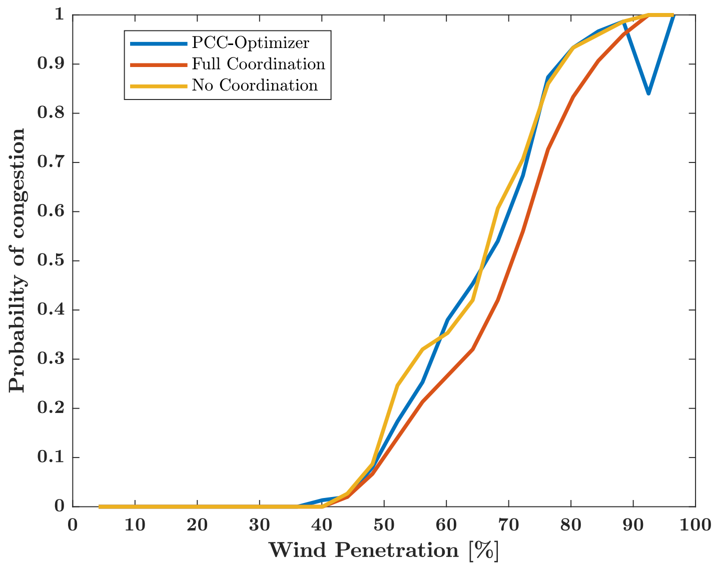

Figure D.9: Probability of at least two transmission lines being congested in RT.

The Congestion level of the colored dots in Fig. 4 is here plotted as a line plot in Fig. D.9. The data for the two plots is the same.

Appendix E. Computational Performance

Here we show some results pertaining to the Benders decomposition approach that was presented in section 4. In Fig. E.10 we present the convergence of the suggested multicut benders decomposition for a sample point of wind-penetration.

The upper bound of the benders decomposed problems in iteration (i) is found as:

$$UB^{(\mathrm{i})} = \mathcal{SW}^{\mathrm{DA}(i)} - \sum_{\omega} \phi_\omega \Delta \mathrm{Cost}_\omega^{\mathrm{RT(i)}} \tag{E.1}$$

The lower bound in iteration (i) is found via:

$$LB^{(\mathrm{i})} = \mathcal{SW}^{\mathrm{DA}(i)} - \sum_{\omega} \phi_\omega \psi_\omega^{(\mathrm{i})} \tag{E.2}$$

The computational burden of the decomposed problem is analyzed by logging the time it takes Mosek 8.0 to solve every master-problem and sub-problem for every scenario. The implementation we use in this paper relies on the CVX plugin for Matlab, which yields large overhead due to the time it takes to initialize every master-problem and sub-problem. Therefore the results in table 1 only give the time that the solver actually spent, while the full time including the overhead for the initialization is about two to four times this number. In the future we wish to use an implementation that does not rely on CVX which can help solving larger case studies.

The number of binary variables in the master-problem depend on the number of complimentarity constraints in (B.4). Because we choose to solve the complimentarity constraints with the Big-M approach every one of these constraints uses one binary. The number of complimentarity constraints in turn depend mainly on the number of generators, number of elastic demands and number of RES sources. In the case study for this work the master problem therefore contains 196 binary variables. As a result of the benders decomposition the conic constraints have all been moved to the subproblem, and therefore the master problem is MILP, while the subproblems are continuous SOCP.

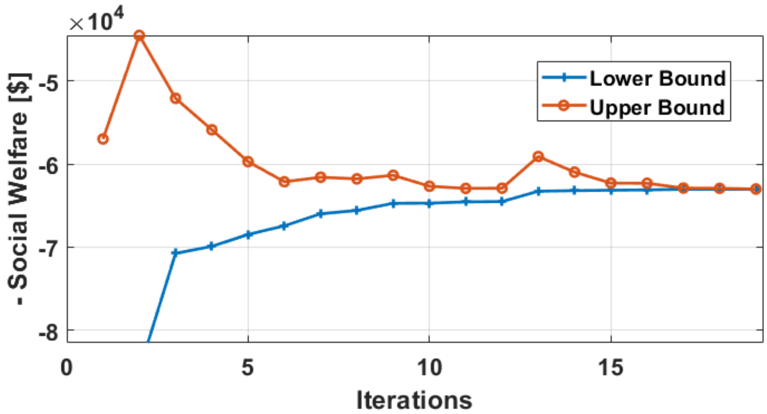

Figure E.10: Convergence of the Benders decomposition upper and lower bound over the iterations. Note, we minimize $-\mathcal{SW}$, which is equivalent to maximizing social welfare.