This appendix is available online at https://github.com/alherm/TSO-DSO_coordination.

Appendix A
Modeling assumptions

We collect here all modeling assumptions made. We make no specific assumptions to the design of the local DSO markets that are employed. It is merely assumed that these markets are efficient and work to maximize social welfare. Therefore the model in (9) is a generic market model that we cast as a stochastic optimization problem that maximizes expected social welfare.

Renewable production is the only source of uncertainty. The production of each renewable energy source (RES) r is capped by an uncertain parameter $W^{\mathrm{RT}}_{r\omega}$ that is dependent on scenario ω (i.e., RES can be freely spilled as required). The DA market is deterministic, and the offer of each RES is assumed to be the expected value of its production. The price offer of each RES, i.e., π^{R}, is assumed to be zero in the DA stage.

Although stochastic market-clearing setups depend on the used scenarios and a thorough definition of who generates them is usually pertinent, we here consider them as an external parameter. A scenario generation method, which correlates geographically close renewable sources is used – more information can be found in section F of this appendix.

The RT market is assumed to be any market that changes the DA dispatch. This re-dispatch is assumed to incur an additional cost, due to premiums charged by the market participants. The premiums do not have to be symmetric, such that up- and down-regulation can have different costs.

We take the same view on network modeling as [9], that the meshed HV transmission network is adequately modeled by linear power flow approximations, while the radial LV distribution feeders are best represented by a convex relaxation of the AC power flow equations. Specifically, in this paper a second-order cone program (SOCP) will be used, as explained in more detail in section IV.

Ramping constraints, energy storage and other inter-temporal couplings are ignored. Also, binary variables such as the commitment status of conventional generators are ignored, such that both DA and RT market-clearing problems are convex.

In order to be able to calculate the RT re-dispatch cost, topology information of both TSO and DSO networks is necessary. Both TSO and DSOs may be unwilling to share data about their network topology. It is assumed that this information is available to the PCC optimizer, which is a reasonable assumptions as we are examining the best possible outcome. Decentralized optimization such as the proposals in [9] and [14] may in the future make it easier to coordinate in RT without sharing specific network-related proprietary information.

Appendix B
DSO Market Lower Level Problem

The DSO pre-qualification optimization problem has both constraints from the DA-market and the scenarios for the Real-time realization. Every DSO has its own separate problem such that Cost_e contains one value for every DSO e. The day ahead market is cleared for each distribution network separately, where the day ahead market has no nodal information. The real time realization is a stochastic SOCP problem.

$$\begin{aligned}\max_{\Xi^{\mathrm{E}}} \quad & \mathcal{SW}_e = \sum_{d\in D^{\mathrm{D}}_e} \pi^{\mathrm{DA}}_d \widetilde{p}^{\mathrm{DA}}_d - \sum_{g\in G^{\mathrm{D}}_e} \pi^{\mathrm{DA}}_g \widetilde{p}^{\mathrm{DA}}_g \\ & - VOLL^{\mathrm{DA}}_e s^{\mathrm{DA}}_e - \sum_{r\in R^{\mathrm{D}}_e} \pi^{R} w^{\mathrm{DA}}_r - \pi^{\mathrm{PCC,DA}}_e p^{\mathrm{PCC,DA}}_e \\ & - \sum_{\omega}\phi_\omega \Bigg[\sum_{g\in G^{\mathrm{D}}_e} \Big(\pi^{\mathrm{DA}}_g (p^{\mathrm{RT}}_{g\omega} - \widetilde{p}^{\mathrm{DA}}_g) + \pi^{\uparrow}_g p^{\uparrow}_{g\omega} \\ & + \pi^{\downarrow}_g p^{\downarrow}_{g\omega}\Big) + \sum_{d\in D^{\mathrm{D}}_e} \Big(\pi^{\mathrm{DA}}_d (\widetilde{p}^{\mathrm{DA}}_d - p^{\mathrm{RT}}_{d\omega}) \\ & + \pi^{\uparrow}_d p^{\uparrow}_{d\omega} + \pi^{\downarrow}_d p^{\downarrow}_{d\omega}\Big) + \sum_{n\in N^{\mathrm{D}}_e} VOLL^{\mathrm{RT}}_n s^{\mathrm{RT}}_{n\omega} \\ & + \pi^{\mathrm{PCC,DA}}_e (p^{\mathrm{PCC,RT}}_{e\omega} - p^{\mathrm{PCC,DA}}_e) \\ & + \pi^{\uparrow PCC}_e p^{\uparrow PCC}_{e\omega} + \pi^{\downarrow PCC}_e p^{\downarrow PCC}_{e\omega} \\ & + \sum_{r\in R^{\mathrm{D}}_e} \left(\pi^{R} (w^{\mathrm{RT}}_{r\omega} - w^{\mathrm{DA}}_r) + \pi^{\uparrow R} w^{\uparrow}_{r\omega} + \pi^{\downarrow R} w^{\downarrow}_{r\omega}\right) \Bigg]\end{aligned} \tag{9a}$$

subject to:

DA-level constraints:

$$\sum_{g\in G_e} \widetilde{p}^{\mathrm{DA}}_g - \sum_{d\in D^{\mathrm{D}}_e} \widetilde{p}^{\mathrm{DA}}_d + \sum_{r\in R^{\mathrm{D}}_e} w^{\mathrm{DA}}_r + s^{\mathrm{DA}}_e + p^{\mathrm{PCC,DA}}_e = 0, \quad : (\lambda^{\mathrm{DA}}_e) \tag{9b}$$

$$\underline{P}_g \le \widetilde{p}^{\mathrm{DA}}_g \le \overline{P}_g, \quad \forall g \in G^{\mathrm{D}}_e \quad : (\varsigma^{\mathrm{DA}-}_g, \varsigma^{\mathrm{DA}+}_g) \tag{9c}$$

$$\underline{P}_d \le \widetilde{p}^{\mathrm{DA}}_d \le \overline{P}_d, \quad \forall d \in D^{\mathrm{D}}_e \quad : (\varsigma^{\mathrm{DA}-}_d, \varsigma^{\mathrm{DA}+}_d) \tag{9d}$$

$$0 \le w^{\mathrm{DA}}_r \le W^{\mathrm{DA}}_r, \ \forall r \in R^{\mathrm{D}}_e \quad : (\iota^-_r, \iota^+_r) \tag{9e}$$

$$\underline{f}_e \le p^{\mathrm{PCC,DA}}_e \le \overline{f}_e, \quad : (\rho^{\mathrm{DA}-}_e, \rho^{\mathrm{DA}+}_e) \tag{9f}$$

$$0 \le s^{\mathrm{DA}}_e \le \sum_d p^{\mathrm{DA}}_d, \quad : (\Upsilon^{\mathrm{DA}-}_e, \Upsilon^{\mathrm{DA}+}_e) \tag{9g}$$

Real-time constraints:

$$p^{\mathrm{RT}}_{g\omega} = p^{\mathrm{DA}}_g + p^{\uparrow}_{g\omega} - p^{\downarrow}_{g\omega}, \quad \forall \omega, g \in G^{\mathrm{D}}_e, \quad : (\zeta^{p}_{g\omega}) \tag{9h}$$

$$p^{\mathrm{RT}}_{d\omega} = p^{\mathrm{DA}}_d - p^{\uparrow}_{d\omega} + p^{\downarrow}_{d\omega}, \quad \forall \omega, d \in D^{\mathrm{D}}_e, \quad : (\zeta^{p}_{d\omega}) \tag{9i}$$

$$w^{\mathrm{RT}}_{r\omega} = w^{\mathrm{DA}}_r + w^{\uparrow}_{r\omega} - w^{\downarrow}_{r\omega}, \ \forall \omega, r \in R^{\mathrm{D}}_e, : (\zeta^{p}_{r\omega}) \tag{9j}$$

$$\sum_{g\in G_n} p^{\mathrm{RT}}_{g\omega} - \sum_{d\in D_n} p^{\mathrm{RT}}_{d\omega} + \sum_{r\in R_n} w^{\mathrm{RT}}_{r\omega} + p^{\mathrm{PCC,RT}}_{e\omega}|_{n=n^{\mathrm{LV}}_e} + s^{\mathrm{RT}}_{n\omega} = \sum_{l\in n\rightarrow} p^{\mathrm{RT}}_{l\omega} - \sum_{l\in \rightarrow n} p^{\mathrm{RT}}_{l\omega}, \ \forall \omega, n \in N^{\mathrm{D}}_e \ : (\lambda^{\mathrm{p,RT}}_{n\omega}) \tag{9k}$$

$$p^{\mathrm{PCC,RT}}_{e\omega} = p^{\mathrm{PCC,DA}}_e + p^{\uparrow PCC}_{e\omega} - p^{\downarrow PCC}_{e\omega}, \ \forall \omega, \ : (\zeta^{\mathrm{PCC}}_{e\omega}) \tag{9l}$$

$$\sum_{g\in G_n} q^{\mathrm{RT}}_{g\omega} - \sum_{d\in D_n} q^{\mathrm{RT}}_{d\omega} + s^{\mathrm{q,RT}}_{n\omega} + q^{\mathrm{PCC,RT}}_{e\omega}|_{n=n^{\mathrm{LV}}_e} = \sum_{l\in n\rightarrow} q^{\mathrm{RT}}_{l\omega} - \sum_{l\in \rightarrow n} q^{\mathrm{RT}}_{l\omega}, \ \forall \omega, n \in N^{\mathrm{D}}_e \ : (\lambda^{\mathrm{q,RT}}_{n\omega}) \tag{9m}$$

$$p^{(RT)2}_{l\omega} + q^{(RT)2}_{l\omega} \le \varphi^{\mathrm{RT}}_{l\omega} v^{\mathrm{RT}}_{n\omega}, \quad \forall \omega, l \in L^{\mathrm{D}}_e \ : (\gamma_{l\omega}) \tag{9n}$$

$$p^{\mathrm{RT}}_{l\omega} + p^{\mathrm{RT}}_{l'\omega} = R_l \varphi^{\mathrm{RT}}_{l\omega}, \quad \forall \omega, l \in L^{\mathrm{D}}_e \ : (\mu^{p}_{l\omega}) \tag{9o}$$

$$q^{\mathrm{RT}}_{l\omega} + q^{\mathrm{RT}}_{l'\omega} = X_l \varphi^{\mathrm{RT}}_{l\omega}, \quad \forall \omega, l \in L^{\mathrm{D}}_e \ : (\mu^{q}_{l\omega}) \tag{9p}$$

$$p_{l\omega}^{(RT)2} + q_{l\omega}^{(RT)2} \le S_l, \quad \forall \omega, l \in L_e^{\mathrm{D}} \; : (\eta_{l\omega}) \tag{9q}$$

$$v_{m\omega}^{\mathrm{RT}} = v_{n\omega}^{\mathrm{RT}} - 2(R_l p_{l\omega}^{\mathrm{RT}} + X_l q_{l\omega}^{\mathrm{RT}}) + (R_l^2 + X_l^2)\varphi_{l\omega}^{\mathrm{RT}}, \quad \forall \omega, l \in L_e^{\mathrm{D}} \quad : (\beta_{l\omega}) \tag{9r}$$

$$\underline{V}_n^2 \le v_{n\omega}^{\mathrm{RT}} \le \overline{V}_n^2, \quad \forall \omega, n \in N_e^{\mathrm{D}} \quad : (\sigma_{n\omega}^-, \sigma_{n\omega}^+) \tag{9s}$$

$$0 \le w_{r\omega}^{\mathrm{RT}} \le W_{r\omega}^{\mathrm{RT}}, \quad \forall \omega, n \in N_e \quad : (\nu_{n\omega}^-, \nu_{n\omega}^+) \tag{9t}$$

$$\underline{P}_g \le p_{g\omega}^{\mathrm{RT}} \le \overline{P}_g, \quad \forall \omega, g \in G_e \quad : (\varsigma_{g\omega}^{\mathrm{RT}-}, \varsigma_{g\omega}^{\mathrm{RT}+}) \tag{9u}$$

$$\underline{P}_d \le p_{d\omega}^{\mathrm{RT}} \le \overline{P}_d, \quad \forall \omega, d \in D_e \quad : (\varsigma_{d\omega}^{\mathrm{RT}-}, \varsigma_{d\omega}^{\mathrm{RT}+}) \tag{9v}$$

$$\underline{Q}_g \le q_{g\omega}^{\mathrm{RT}} \le \overline{Q}_g, \quad \forall \omega, g \in G_e \quad : (\kappa_{g\omega}^{\mathrm{RT}-}, \kappa_{g\omega}^{\mathrm{RT}+}) \tag{9w}$$

$$\underline{Q}_d \le q_{d\omega}^{\mathrm{RT}} \le \overline{Q}_d, \quad \forall \omega, d \in D_e \quad : (\kappa_{d\omega}^{\mathrm{RT}-}, \kappa_{d\omega}^{\mathrm{RT}+}) \tag{9x}$$

$$\underline{f}_e \le p_{e\omega}^{\mathrm{PCC,RT}} \le \overline{f}_e, \quad \forall \omega \quad : (\rho_{e\omega}^{\mathrm{RT}-}, \rho_{e\omega}^{\mathrm{RT}+}) \tag{9y}$$

$$p_{g\omega}^{\uparrow} \ge 0, \; \forall \omega, g \;\; : (\epsilon_{g\omega}^{\mathrm{p}\uparrow}), \quad p_{g\omega}^{\downarrow} \ge 0, \; \forall \omega, g \;\; : (\epsilon_{g\omega}^{\mathrm{p}\downarrow}) \tag{9z}$$

$$p_{d\omega}^{\uparrow} \ge 0, \; \forall d, \omega \;\; : (\varepsilon_{d\omega}^{\mathrm{p}\uparrow}), \quad p_{d\omega}^{\downarrow} \ge 0, \; \forall d, \omega \;\; : (\varepsilon_{d\omega}^{\mathrm{p}\downarrow}) \tag{9aa}$$

$$p_{e\omega}^{\uparrow PCC} \ge 0, \; \forall \omega, \; : (\epsilon_{e\omega}^{\uparrow PCC}), \; p_{e\omega}^{\downarrow PCC} \ge 0, \; \forall \omega, \; : (\epsilon_{e\omega}^{\downarrow PCC}) \tag{9ab}$$

$$0 \le s_{n\omega}^{\mathrm{RT}} \le \sum_{d \in D_n} p_{d\omega}^{\mathrm{RT}}, \; \forall \omega, n \in N_e^{\mathrm{D}}, \; : (\Upsilon_{n\omega}^{\mathrm{RT}-}, \Upsilon_{n\omega}^{\mathrm{RT}+}) \tag{9ac}$$

$$w_{r\omega}^{\uparrow} \ge 0, \; \forall \omega, w \;\; : (\epsilon_{r\omega}^{\mathrm{p}\uparrow}), \quad w_{r\omega}^{\downarrow} \ge 0, \; \forall \omega, w \;\; : (\epsilon_{r\omega}^{\mathrm{p}\downarrow}) \tag{9ad}$$

Where $\Xi^E = \widetilde{p}_g^{\mathrm{DA}}, \widetilde{p}_d^{\mathrm{DA}}, p_{g\omega}^{\mathrm{RT}}, p_{g\omega}^{\uparrow}, p_{g\omega}^{\downarrow}, p_{d\omega}^{\mathrm{RT}}, p_{d\omega}^{\uparrow}, p_{d\omega}^{\downarrow}, q_{g\omega}^{\mathrm{RT}}, q_{d\omega}^{\mathrm{RT}}, s_{n\omega}^{\mathrm{RT}}, s_e^{\mathrm{DA}}, w_{n\omega}^{\mathrm{RT}}, p_{l\omega}^{\mathrm{RT}}, q_{l\omega}^{\mathrm{RT}}, \varphi_{l\omega}^{\mathrm{RT}}, v_{n\omega}^{\mathrm{RT}}, w_e^{\mathrm{DA}}, p_e^{\mathrm{PCC,DA}}, p_e^{\mathrm{PCC,RT}}, p_e^{\uparrow PCC}, p_e^{\downarrow PCC}, s_{n\omega}^{\mathrm{q,RT}}$ are the variables of the DSO-level combined day-ahead and real-time market clearing.

The Lagrangian of above problem is:

$$\mathcal{L}_e = \sum_{d \in D_e^{\mathrm{D}}} \pi_d^{\mathrm{DA}} \widetilde{p}_d^{\mathrm{DA}} - \sum_{g \in G_e^{\mathrm{D}}} \pi_g^{\mathrm{DA}} \widetilde{p}_g^{\mathrm{DA}}$$

$$- VOLL_e^{\mathrm{DA}} s_e^{\mathrm{DA}} - \sum_{r \in R_e^{\mathrm{D}}} \pi^R w_r^{\mathrm{DA}} - \pi_e^{\mathrm{PCC,DA}} p_e^{\mathrm{PCC,DA}}$$

$$- \sum_{\omega} \phi_\omega \Bigg[\sum_{g \in G_e^{\mathrm{D}}} \Big(\pi_g^{\mathrm{DA}} (p_{g\omega}^{\mathrm{RT}} - p_g^{\mathrm{DA}}) + \pi_g^{\uparrow} p_{g\omega}^{\uparrow} + \pi_g^{\downarrow} p_{g\omega}^{\downarrow} \Big)$$

$$+ \sum_{d \in D_e^{\mathrm{D}}} \Big(\pi_d^{\mathrm{DA}} (p_d^{\mathrm{DA}} - p_{d\omega}^{\mathrm{RT}}) + \pi_d^{\uparrow} p_{d\omega}^{\uparrow} + \pi_d^{\downarrow} p_{d\omega}^{\downarrow} \Big)$$

$$+ \sum_{n \in N_e^{\mathrm{D}}} VOLL_n^{\mathrm{RT}} s_{n\omega}^{\mathrm{RT}}$$

$$+ \sum_{r \in R_e^{\mathrm{D}}} \left(\pi^R (w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}}) + \pi^{\uparrow R} w_{r\omega}^{\uparrow} + \pi^{\downarrow R} w_{r\omega}^{\downarrow} \right)$$

$$+ \pi_e^{\mathrm{PCC,DA}} (p_{e\omega}^{\mathrm{PCC,RT}} - p_e^{\mathrm{PCC,DA}})$$

$$+ \pi_e^{\uparrow PCC} p_{e\omega}^{\uparrow PCC} + \pi_e^{\downarrow PCC} p_{e\omega}^{\downarrow PCC} \Bigg]$$

$$- \lambda_e^{\mathrm{DA}} \Bigg[\sum_{g \in G_e} \widetilde{p}_g^{\mathrm{DA}} - \sum_{d \in D_e^{\mathrm{D}}} \widetilde{p}_d^{\mathrm{DA}} + \sum_{r \in R_e^{\mathrm{D}}} w_r^{\mathrm{DA}}$$

$$+ s_e^{\mathrm{DA}} + p_e^{\mathrm{PCC,DA}} \Bigg]$$

$$+ \sum_{g \in G_e^{\mathrm{D}}} \left[\varsigma_g^{\mathrm{DA}-} \left(\underline{P}_g - \widetilde{p}_g^{\mathrm{DA}} \right) + \varsigma_g^{\mathrm{DA}+} \left(\widetilde{p}_g^{\mathrm{DA}} - \overline{P}_g \right) \right]$$

$$- \sum_{r \in R_e^{\mathrm{D}}} \left[\iota_r^- w_r^{\mathrm{DA}} - \iota_r^+ (w_r^{\mathrm{DA}} - W_r^{\mathrm{DA}}) \right]$$

$$+ \sum_{d \in D_e^{\mathrm{D}}} \left[\varsigma_d^{\mathrm{DA}-} \left(\underline{P}_d - \widetilde{p}_d^{\mathrm{DA}} \right) + \varsigma_d^{\mathrm{DA}+} \left(\widetilde{p}_d^{\mathrm{DA}} - \overline{P}_d \right) \right]$$

$$+ \rho_e^{\mathrm{DA}-} (\underline{f}_e - p_e^{\mathrm{PCC,DA}}) + \rho_e^{\mathrm{DA}+} (p_e^{\mathrm{PCC,DA}} - \overline{f}_e)$$

$$- \sum_{g \in G_e^{\mathrm{D}}} \zeta_{g\omega}^{\mathrm{p}} \left(p_{g\omega}^{\mathrm{RT}} - p_g^{\mathrm{DA}} - p_{g\omega}^{\uparrow} + p_{g\omega}^{\downarrow} \right)$$

$$- \sum_{d \in D_e^{\mathrm{D}}} \zeta_{d\omega}^{\mathrm{p}} \left(p_{d\omega}^{\mathrm{RT}} - p_d^{\mathrm{DA}} + p_{d\omega}^{\uparrow} - p_{d\omega}^{\downarrow} \right)$$

$$- \sum_{r \in R_e^{\mathrm{D}}} \zeta_{r\omega}^{p} \left(w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}} - w_{r\omega}^{\uparrow} + w_{r\omega}^{\downarrow} \right)$$

$$- \sum_{n \in N_e^{\mathrm{D}}, \omega} \lambda_{n\omega}^{\mathrm{p,RT}} \Big(\sum_{g \in G_n} p_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} p_{d\omega}^{\mathrm{RT}} + \sum_{r \in R_n} w_{r\omega}^{\mathrm{RT}}$$

$$+ s_{n\omega}^{\mathrm{RT}} + p_{e\omega}^{\mathrm{PCC,RT}}|_{n=n_e^{\mathrm{LV}}} - \sum_{l \in n \rightarrow} p_{l\omega}^{\mathrm{RT}} + \sum_{l \in \rightarrow n} p_{l\omega}^{\mathrm{RT}} \Big)$$

$$- \sum_{\omega} \zeta_{e\omega}^{\mathrm{PCC}} \left(p_{e\omega}^{\mathrm{PCC,RT}} - p_e^{\mathrm{PCC,DA}} - p_{e\omega}^{\uparrow PCC} + p_{e\omega}^{\downarrow PCC} \right)$$

$$- \sum_{n \in N_e^{\mathrm{D}}, \omega} \lambda_{n\omega}^{\mathrm{q,RT}} \Bigg(\sum_{g \in G_n} q_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} q_{d\omega}^{\mathrm{RT}} + s_{n\omega}^{\mathrm{q,RT}}$$

$$+ q_{e\omega}^{\mathrm{PCC,RT}}|_{n=n_e^{\mathrm{LV}}} - \sum_{l \in n \rightarrow} q_{l\omega}^{\mathrm{RT}} + \sum_{l \in \rightarrow n} q_{l\omega}^{\mathrm{RT}} \Bigg)$$

$$+ \sum_{l \in L_e^{\mathrm{D}}, \omega} \gamma_{l\omega} \left[p_{l\omega}^{(RT)2} + q_{l\omega}^{(RT)2} - \varphi_{l\omega}^{\mathrm{RT}} v_{n\omega}^{\mathrm{RT}} \right]$$

$$- \sum_{l \in L_e^{\mathrm{D}}, \omega} \Big[\mu_{l\omega}^{p} \left(p_{l\omega}^{\mathrm{RT}} + p_{l'\omega}^{\mathrm{RT}} - R_l \varphi_{l\omega}^{\mathrm{RT}} \right)$$

$$+ \mu_{l\omega}^{q} \left(q_{l\omega}^{\mathrm{RT}} + q_{l'\omega}^{\mathrm{RT}} - X_l \varphi_{l\omega}^{\mathrm{RT}} \right) \Big]$$

$$+ \sum_{l \in L_e^{\mathrm{D}}, \omega} \Big[\eta_{l\omega} \Big(p_{l\omega}^{(RT)2} + q_{l\omega}^{(RT)2} - S_l \Big) \Big]$$

$$- \sum_{l \in L_e^{\mathrm{D}}, \omega} \Big[\beta_{l\omega} \Big(v_{m\omega}^{\mathrm{RT}} - v_{n\omega}^{\mathrm{RT}} + 2(R_l p_{l\omega}^{\mathrm{RT}} + X_l q_{l\omega}^{\mathrm{RT}})$$

$$- (R_l^2 + X_l^2) \varphi_{l\omega}^{\mathrm{RT}} \Big) \Big]$$

$$+ \sum_{n \in N_e^{\mathrm{D}}, \omega} \Big[\sigma_{n\omega}^- \Big(\underline{V}_n^2 - v_{n\omega}^{\mathrm{RT}} \Big) + \sigma_{n\omega}^+ \Big(v_{n\omega}^{\mathrm{RT}} - \overline{V}_n^2 \Big) \Big]$$

$$- \sum_{r \in R_e^{\mathrm{D}}, \omega} \Big[\nu_{r\omega}^- w_{r\omega}^{\mathrm{RT}} - \nu_{r\omega}^+ \left(w_{r\omega}^{\mathrm{RT}} - W_{r\omega}^{\mathrm{RT}} \right) \Big]$$

$$+ \sum_{g \in G_e^{\mathrm{D}}, \omega} \left[\varsigma_{g\omega}^{\mathrm{RT}-} \left(\underline{P}_g - p_{g\omega}^{\mathrm{RT}} \right) + \varsigma_{g\omega}^{\mathrm{RT}+} \left(p_{g\omega}^{\mathrm{RT}} - \overline{P}_g \right) \right]$$

$$+ \sum_{d \in D_e^{\mathrm{D}}, \omega} \left[\varsigma_{d\omega}^{\mathrm{RT}-} \left(\underline{P}_d - p_{d\omega}^{\mathrm{RT}} \right) + \varsigma_{d\omega}^{\mathrm{RT}+} \left(p_{d\omega}^{\mathrm{RT}} - \overline{P}_d \right) \right]$$

$$+ \sum_{g \in G_e^{\mathrm{D}}, \omega} \Big[\kappa_{g\omega}^{\mathrm{RT}-} \Big(\underline{Q}_g - q_{g\omega}^{\mathrm{RT}} \Big) + \kappa_{g\omega}^{\mathrm{RT}+} \left(q_{g\omega}^{\mathrm{RT}} - \overline{Q}_g \right) \Big]$$

$$+ \sum_{d \in D_e^{\mathrm{D}}, \omega} \Big[\kappa_{d\omega}^{\mathrm{RT}-} \Big(\underline{Q}_d - q_{d\omega}^{\mathrm{RT}} \Big) + \kappa_{d\omega}^{\mathrm{RT}+} \left(q_{d\omega}^{\mathrm{RT}} - \overline{Q}_g \right) \Big]$$

$$+ \sum_{g \in G_e^{\mathrm{D}}, \omega} \left[-p_{g\omega}^{\uparrow} \epsilon_{g\omega}^{\mathrm{p}\uparrow} - p_{g\omega}^{\downarrow} \epsilon_{g\omega}^{\mathrm{p}\downarrow} \right]$$

$$+\sum_{r\in R_e^{\mathrm{D}},\omega}\left[-w_{r\omega}^{\uparrow}\epsilon_{r\omega}^{\mathrm{p}\uparrow}-w_{r\omega}^{\downarrow}\epsilon_{r\omega}^{\mathrm{p}\downarrow}\right]$$
$$+\sum_{d\in D_e^{\mathrm{D}},\omega}\left[-p_{d\omega}^{\uparrow}\varepsilon_{d\omega}^{\mathrm{p}\uparrow}-p_{d\omega}^{\downarrow}\varepsilon_{d\omega}^{\mathrm{p}\downarrow}\right]$$
$$+\sum_{\omega}\left[-p_{e\omega}^{\uparrow PCC}\epsilon_{e\omega}^{\uparrow PCC}-p_{e\omega}^{\downarrow PCC}\epsilon_{e\omega}^{\downarrow PCC}\right]$$
$$+\sum_{\omega}\left[\rho_{e\omega}^{\mathrm{RT}-}\left(\underline{f}_e-\sqrt{p_{e\omega}^{(PCC,RT)2}+q_{e\omega}^{(PCC,RT)2}}\right)\right.$$
$$\left.+\rho_{e\omega}^{\mathrm{RT}+}\left(\sqrt{p_{e\omega}^{(PCC,RT)2}+q_{e\omega}^{(PCC,RT)2}}-\overline{f}_e\right)\right]$$
$$+\sum_{n\in N_e^{\mathrm{D}},\omega}\left[-\Upsilon_{n\omega}^{\mathrm{RT}-}s_{n\omega}^{\mathrm{RT}}+\Upsilon_{n\omega}^{\mathrm{RT}+}\left(s_{n\omega}^{\mathrm{RT}}-\sum_{d\in D_n}p_{d\omega}^{\mathrm{RT}}\right)\right]$$
$$-\Upsilon_e^{\mathrm{DA}-}s_e^{\mathrm{DA}}+\Upsilon_e^{\mathrm{DA}+}\left(s_e^{\mathrm{DA}}-\sum_d p_d^{\mathrm{DA}}\right)\quad(10)$$

The KKT conditions of above problem are (excluding the primal constraints of 9):

$$(\widetilde{p}_g^{\mathrm{DA}}):\ -\pi_g^{\mathrm{DA}}+\sum_\omega\phi_\omega\pi_g^{\mathrm{DA}}-\lambda_e^{\mathrm{DA}}-\varsigma_g^{\mathrm{DA}-}+\varsigma_g^{\mathrm{DA}+}+\sum_\omega\zeta_{g\omega}^{p}=0,\quad\forall g\in G_e^{\mathrm{D}}\quad(11a)$$
$$(\widetilde{p}_d^{\mathrm{DA}}):\sum_\omega\left(\zeta_{d\omega}^{p}-\phi_\omega\pi_d^{\mathrm{DA}}\right)+\pi_d^{\mathrm{DA}}+\lambda_e^{\mathrm{DA}}-\varsigma_d^{\mathrm{DA}-}+\varsigma_d^{\mathrm{DA}+}-\Upsilon_e^{\mathrm{DA}+}=0,\quad\forall d\in D_e^{\mathrm{D}}\quad(11b)$$
$$(p_{g\omega}^{\uparrow}):\ -\phi_\omega\pi_g^{\uparrow}+\zeta_{g\omega}^{p}-\epsilon_{g\omega}^{\mathrm{p}\uparrow}=0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(11c)$$
$$(p_{g\omega}^{\downarrow}):\ -\phi_\omega\pi_g^{\downarrow}-\zeta_{g\omega}^{p}-\epsilon_{g\omega}^{\mathrm{p}\downarrow}=0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(11d)$$
$$(p_{d\omega}^{\uparrow}):\ -\phi_\omega\pi_d^{\uparrow}-\zeta_{d\omega}^{p}-\varepsilon_{i\omega}^{\mathrm{p}\uparrow}=0,\quad\forall\omega,d\in D_e^{\mathrm{D}}\quad(11e)$$
$$(p_{d\omega}^{\downarrow}):\ -\phi_\omega\pi_d^{\downarrow}+\zeta_{d\omega}^{p}-\varepsilon_{d\omega}^{\mathrm{p}\downarrow}=0,\quad\forall\omega,d\in D_e^{\mathrm{D}}\quad(11f)$$
$$(w_{r\omega}^{\uparrow}):\ -\phi_\omega\pi^{\uparrow R}+\zeta_{r\omega}^{p}-\epsilon_{r\omega}^{\mathrm{p}\uparrow}=0,\quad\forall\omega,r\in R_e^{\mathrm{D}}\quad(11g)$$
$$(w_{r\omega}^{\downarrow}):\ -\phi_\omega\pi^{\downarrow R}-\zeta_{r\omega}^{p}-\epsilon_{r\omega}^{\mathrm{p}\downarrow}=0,\quad\forall\omega,r\in R_e^{\mathrm{D}}\quad(11h)$$
$$(s_e^{\mathrm{DA}}):-\mathrm{VOLL}_e-\lambda_e^{\mathrm{DA}}-\Upsilon_e^{\mathrm{DA}-}+\Upsilon_e^{\mathrm{DA}+}=0\quad(11i)$$
$$(s_{n\omega}^{\mathrm{RT}}):-\mathrm{VOLL}_n-\lambda_{n\omega}^{\mathrm{p,RT}}-\Upsilon_{n\omega}^{\mathrm{RT}-}+\Upsilon_{n\omega}^{\mathrm{RT}+}=0,\ \forall\omega,n\in N_e^{\mathrm{D}}\quad(11j)$$
$$(w_{r\omega}^{\mathrm{RT}}):-\phi_\omega\pi^{R}-\zeta_{r\omega}^{p}-\left[\lambda_{n\omega}^{\mathrm{p,RT}}\right]_{n_r}+\nu_{r\omega}^{+}-\nu_{r\omega}^{-}=0,\quad\forall\omega,r\in R_e^{\mathrm{D}}\quad(11k)$$
$$(w_r^{\mathrm{DA}}):-\pi^{R}+\sum_\omega\phi_\omega\pi^{R}-\lambda_e^{\mathrm{DA}}-\iota_r^{-}+\iota_r^{+}+\sum_\omega\zeta_{r\omega}^{p}=0,\forall r\in R_e^{\mathrm{D}}\quad(11l)$$
$$(p_{g\omega}^{\mathrm{RT}}):-\phi_\omega\pi_g^{\mathrm{DA}}-\zeta_{g\omega}^{p}-\varsigma_{g\omega}^{\mathrm{RT}-}+\varsigma_{g\omega}^{\mathrm{RT}+}-\left[\lambda_{n\omega}^{\mathrm{p,RT}}\right]_{n_g}=0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(11m)$$
$$(q_{g\omega}^{\mathrm{RT}}):-\kappa_{g\omega}^{\mathrm{RT}-}+\kappa_{g\omega}^{\mathrm{RT}+}-\left[\lambda_{n\omega}^{\mathrm{q,RT}}\right]_{n_g}=0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(11n)$$
$$(p_{d\omega}^{\mathrm{RT}}):\phi_\omega\pi_d^{\mathrm{DA}}-\zeta_{d\omega}^{p}-\varsigma_{d\omega}^{\mathrm{RT}-}+\varsigma_{d\omega}^{\mathrm{RT}+}+\left[\lambda_{n\omega}^{\mathrm{p,RT}}-\Upsilon_{n\omega}^{\mathrm{RT}+}\right]_{n_d}=0,\quad\forall\omega,d\in D_e^{\mathrm{D}}\quad(11o)$$
$$(q_{d\omega}^{\mathrm{RT}}):-\kappa_{d\omega}^{\mathrm{RT}-}+\kappa_{d\omega}^{\mathrm{RT}+}+\left[\lambda_{n\omega}^{\mathrm{q,RT}}\right]_{n_d}=0,\quad\forall\omega,d\in D_e^{\mathrm{D}}\quad(11p)$$
$$(p_{l\omega}^{\mathrm{RT}}):\lambda_{n\omega}^{\mathrm{p,RT}}-\lambda_{m\omega}^{\mathrm{p,RT}}+2\gamma_{l\omega}p_{l\omega}^{\mathrm{RT}}-\mu_{l\omega}^{p}-\mu_{l'\omega}^{p}+2\eta_{l\omega}p_{l\omega}^{\mathrm{RT}}-2\beta_{l\omega}R_l=0,\quad\forall\omega,l=(n,m)\in L_e^{\mathrm{D}}\quad(11q)$$
$$(q_{l\omega}^{\mathrm{RT}}):\lambda_{n\omega}^{\mathrm{q,RT}}-\lambda_{m\omega}^{\mathrm{q,RT}}+2\gamma_{l\omega}q_{l\omega}^{\mathrm{RT}}-\mu_{l\omega}^{q}-\mu_{l'\omega}^{q}+2\eta_{l\omega}q_{l\omega}^{\mathrm{RT}}-2\beta_{l\omega}X_l=0,\quad\forall\omega,l=(n,m)\in L_e^{\mathrm{D}}\quad(11r)$$
$$(\varphi_{l\omega}^{\mathrm{RT}}):-\gamma_{l\omega}v_{n\omega}^{\mathrm{RT}}+\mu_{l\omega}^{p}R_l+\mu_{l\omega}^{q}X_l+\beta_{l\omega}(R_l^2+X_l^2)=0,\quad\forall\omega,l=(n,m)\in L_e^{\mathrm{D}}\quad(11s)$$
$$(v_{n\omega}^{\mathrm{RT}}):-\gamma_{l\omega}\varphi_{l\omega}^{\mathrm{RT}}-\beta_{l'\omega}+\beta_{l\omega}-\sigma_{n\omega}^{-}+\sigma_{n\omega}^{+}=0,\quad\forall\omega,l=(n,m)\in L_e^{\mathrm{D}}\quad(11t)$$
$$(p_e^{\mathrm{PCC,DA}}):-\pi_e^{\mathrm{PCC,DA}}+\sum_\omega\left(\phi_\omega\pi_e^{\mathrm{PCC,DA}}+\zeta_{e\omega}^{\mathrm{PCC}}\right)-\lambda_e^{\mathrm{DA}}-\rho_e^{\mathrm{DA}-}+\rho_e^{\mathrm{DA}+}=0\quad(11u)$$
$$(p_{e\omega}^{\mathrm{PCC,RT}}):-\phi_\omega\pi_e^{\mathrm{PCC,DA}}-\left[\lambda_{n\omega}^{\mathrm{p,RT}}\right]_{n_e^{\mathrm{LV}}}-\zeta_{e\omega}^{\mathrm{PCC}}-\rho_{e\omega}^{\mathrm{RT}-}+\rho_{e\omega}^{\mathrm{RT}+}=0,\quad\forall\omega\quad(11v)$$
$$(q_{e\omega}^{\mathrm{PCC,RT}}):-\left[\lambda_{n\omega}^{\mathrm{q,RT}}\right]_{n_e^{\mathrm{LV}}}=0,\quad\forall\omega\quad(11w)$$
$$(p_{e\omega}^{\uparrow PCC}):-\phi_\omega\pi_e^{\uparrow PCC}+\zeta_{e\omega}^{\mathrm{PCC}}-\epsilon_{e\omega}^{\uparrow PCC}=0,\quad\forall\omega\quad(11x)$$
$$(p_{e\omega}^{\downarrow PCC}):-\phi_\omega\pi_e^{\downarrow PCC}-\zeta_{e\omega}^{\mathrm{PCC}}-\epsilon_{e\omega}^{\downarrow PCC}=0,\quad\forall\omega\quad(11y)$$

The complimentarity constraints are as follows:

$$0\le\varsigma_g^{\mathrm{DA}+}\perp\overline{P}_g-\widetilde{p}_g^{\mathrm{DA}}\ge0,\quad\forall g\in G_e^{\mathrm{D}}\quad(12a)$$
$$0\le\varsigma_g^{\mathrm{DA}-}\perp\widetilde{p}_g^{\mathrm{DA}}-\underline{P}_g\ge0,\quad\forall g\in G_e^{\mathrm{D}}\quad(12b)$$
$$0\le\varsigma_d^{\mathrm{DA}+}\perp\overline{P}_d-\widetilde{p}_d^{\mathrm{DA}}\ge0,\quad\forall d\in D_e^{\mathrm{D}}\quad(12c)$$
$$0\le\varsigma_d^{\mathrm{DA}-}\perp\widetilde{p}_d^{\mathrm{DA}}-\underline{P}_d\ge0,\quad\forall d\in D_e^{\mathrm{D}}\quad(12d)$$
$$0\le\iota_r^{-}\perp w_r^{\mathrm{DA}}\ge0,\quad\forall r\in R_e^{\mathrm{D}}\quad(12e)$$
$$0\le\iota_r^{+}\perp W_r^{\mathrm{DA}}-w_r^{\mathrm{DA}}\ge0,\quad\forall r\in R_e^{\mathrm{D}}\quad(12f)$$
$$0\le\rho_e^{\mathrm{DA}-}\perp p_e^{\mathrm{PCC,DA}}-\underline{f}_e\ge0\quad(12g)$$
$$0\le\rho_e^{\mathrm{DA}+}\perp\overline{f}_e-p_e^{\mathrm{PCC,DA}}\ge0\quad(12h)$$
$$0\le\gamma_{l\omega}\perp\varphi_{l\omega}^{\mathrm{RT}}v_{n\omega}^{\mathrm{RT}}-(p_{l\omega}^{(RT)2}+q_{l\omega}^{(RT)2})\ge0,\ \forall\omega,l\in L_e^{\mathrm{D}}\quad(12i)$$
$$0\le\eta_{l\omega}\perp S_l-p_{l\omega}^{(RT)2}-q_{l\omega}^{(RT)2}\ge0,\ \forall\omega,l\in L_e^{\mathrm{D}}\quad(12j)$$
$$0\le\sigma_{n\omega}^{-}\perp v_{n\omega}^{\mathrm{RT}}-\underline{V}_n^2\ge0,\quad\forall\omega,n\in N_e^{\mathrm{D}}\quad(12k)$$
$$0\le\sigma_{n\omega}^{+}\perp\overline{V}_n^2-v_{n\omega}^{\mathrm{RT}}\ge0,\quad\forall\omega,n\in N_e^{\mathrm{D}}\quad(12l)$$
$$0\le\nu_{r\omega}^{-}\perp w_{r\omega}^{\mathrm{RT}}\ge0,\quad\forall\omega,r\in R_e^{\mathrm{D}}\quad(12m)$$
$$0\le\nu_{r\omega}^{+}\perp W_{r\omega}^{\mathrm{RT}}-w_{r\omega}^{\mathrm{RT}}\ge0,\quad\forall\omega,r\in R_e^{\mathrm{D}}\quad(12n)$$
$$0\le\varsigma_{g\omega}^{\mathrm{RT}-}\perp p_{g\omega}^{\mathrm{RT}}-\underline{P}_g\ge0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(12o)$$
$$0\le\varsigma_{g\omega}^{\mathrm{RT}+}\perp\overline{P}_g-p_{g\omega}^{\mathrm{RT}}\ge0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(12p)$$
$$0\le\varsigma_{d\omega}^{\mathrm{RT}-}\perp p_{d\omega}^{\mathrm{RT}}-\underline{P}_d\ge0,\quad\forall\omega,d\in D_e^{\mathrm{D}}\quad(12q)$$
$$0\le\varsigma_{d\omega}^{\mathrm{RT}+}\perp\overline{P}_d-p_{d\omega}^{\mathrm{RT}}\ge0,\quad\forall\omega,d\in D_e^{\mathrm{D}}\quad(12r)$$
$$0\le\kappa_{g\omega}^{\mathrm{RT}+}\perp q_{g\omega}^{\mathrm{RT}}-\underline{Q}_g\ge0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(12s)$$
$$0\le\kappa_{g\omega}^{\mathrm{RT}-}\perp\overline{Q}_g-q_{g\omega}^{\mathrm{RT}}\ge0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(12t)$$
$$0\le\kappa_{d\omega}^{\mathrm{RT}+}\perp q_{d\omega}^{\mathrm{RT}}-\underline{Q}_d\ge0,\quad\forall\omega,d\in D_e^{\mathrm{D}}\quad(12u)$$
$$0\le\kappa_{d\omega}^{\mathrm{RT}-}\perp\overline{Q}_d-q_{d\omega}^{\mathrm{RT}}\ge0,\quad\forall\omega,d\in D_e^{\mathrm{D}}\quad(12v)$$
$$0\le\rho_{e\omega}^{\mathrm{RT}-}\perp p_{e\omega}^{\mathrm{PCC,RT}}-\underline{f}_e\ge0,\quad\forall\omega\quad(12w)$$
$$0\le\rho_{e\omega}^{\mathrm{RT}+}\perp\overline{f}_e-p_{e\omega}^{\mathrm{PCC,RT}}\ge0,\quad\forall\omega\quad(12x)$$
$$0\le\epsilon_{g\omega}^{\mathrm{p}\uparrow}\perp p_{g\omega}^{\uparrow}\ge0,\quad0\le\epsilon_{g\omega}^{\mathrm{p}\downarrow}\perp p_{g\omega}^{\downarrow}\ge0,\quad\forall\omega,g\in G_e^{\mathrm{D}}\quad(12y)$$

$$0 \le \epsilon^{\mathrm{p}\uparrow}_{r\omega} \perp w^{\uparrow}_{r\omega} \ge 0,\ 0 \le \epsilon^{\mathrm{p}\downarrow}_{r\omega} \perp w^{\downarrow}_{r\omega} \ge 0,\ \forall \omega, r \in R^{\mathrm{D}}_{e} \tag{12z}$$

$$0 \le \varepsilon^{\mathrm{p}\uparrow}_{d\omega} \perp p^{\uparrow}_{d\omega} \ge 0, \quad 0 \le \varepsilon^{\mathrm{p}\downarrow}_{d\omega} \perp p^{\downarrow}_{d\omega} \ge 0, \quad \forall \omega, d \in D^{\mathrm{D}}_{e} \tag{12aa}$$

$$0 \le \epsilon^{\uparrow PCC}_{e\omega} \perp p^{\uparrow PCC}_{e\omega} \ge 0, \quad 0 \le \epsilon^{\downarrow PCC}_{e\omega} \perp p^{\downarrow PCC}_{e\omega} \ge 0, \quad \forall \omega \tag{12ab}$$

$$0 \le \Upsilon^{\mathrm{RT}-}_{n\omega} \perp s^{\mathrm{RT}}_{n\omega} \ge 0, \forall n, \omega \tag{12ac}$$

$$0 \le \Upsilon^{\mathrm{RT}+}_{n\omega} \perp \sum_{d \in D_n} p^{\mathrm{RT}}_{d\omega} - s^{\mathrm{RT}}_{n\omega} \ge 0,\ \forall n, \omega \tag{12ad}$$

$$0 \le \Upsilon^{\mathrm{DA}-}_{e} \perp s^{\mathrm{DA}}_{e} \ge 0 \tag{12ae}$$

$$0 \le \Upsilon^{\mathrm{DA}+}_{e} \perp \sum_{d} p^{\mathrm{DA}}_{d} - s^{\mathrm{DA}}_{e} \ge 0 \tag{12af}$$

Appendix C
KKTs of DA market

For convenience problem (3) is repeated here, with dual varaibles for every constraint added.

$$\max_{\Xi^{\mathrm{DA}}} \mathcal{SW}^{\mathrm{DA}} = \sum_{d \in D} \pi^{\mathrm{DA}}_{d} \widehat{p}^{\mathrm{DA}}_{d} - \sum_{g \in G} \pi^{\mathrm{DA}}_{g} \widehat{p}^{\mathrm{DA}}_{g} - VOLL^{\mathrm{DA}} s^{\mathrm{DA}} - \pi^{R} \sum_{r} w^{\mathrm{DA}}_{r} \tag{13a}$$

subject to:

$$\sum_{g \in G} \widehat{p}^{\mathrm{DA}}_{g} - \sum_{d \in D} \widehat{p}^{\mathrm{DA}}_{d} + \sum_{r} w^{\mathrm{DA}}_{r} + s^{\mathrm{DA}} = 0,\ : (\lambda^{\mathrm{T,DA}}) \tag{13b}$$

$$\underline{P}_{g} \le \widehat{p}^{\mathrm{DA}}_{g} \le \widetilde{p}^{\mathrm{DA}}_{g},\ \forall g \in G^{\mathrm{D}}_{e},\ \forall e \in E\ : (\varsigma^{\mathrm{T,DA}-}_{ge}, \varsigma^{\mathrm{T,DA}+}_{ge}) \tag{13c}$$

$$\underline{P}_{g} \le \widehat{p}^{\mathrm{DA}}_{g} \le \overline{P}_{g},\ \forall g \in G^{\mathrm{T}}\ : (\sigma^{\mathrm{T,DA}-}_{g}, \sigma^{\mathrm{T,DA}+}_{g}) \tag{13d}$$

$$\underline{P}_{d} \le \widehat{p}^{\mathrm{DA}}_{d} \le \widetilde{p}^{\mathrm{DA}}_{d},\ \forall d \in D^{\mathrm{D}}_{e},\ \forall e \in E\ : (\varsigma^{\mathrm{T,DA}-}_{de}, \varsigma^{\mathrm{T,DA}+}_{de}) \tag{13e}$$

$$\underline{P}_{d} \le \widehat{p}^{\mathrm{DA}}_{d} \le \overline{P}_{d},\ \forall d \in D^{\mathrm{T}}\ : (\sigma^{\mathrm{T,DA}-}_{d}, \sigma^{\mathrm{T,DA}+}_{d}) \tag{13f}$$

$$0 \le w^{\mathrm{DA}}_{r} \le W^{\mathrm{DA}}_{r},\ \forall r \in R\ : (\nu^{\mathrm{T,DA}-}_{r}, \nu^{\mathrm{T,DA}+}_{r}) \tag{13g}$$

$$0 \le s^{\mathrm{l,DA}} \le \sum_{d} \widehat{p}^{\mathrm{DA}}_{d},\ : (\rho^{\mathrm{T,DA}-}, \rho^{\mathrm{T,DA}+}) \tag{13h}$$

$\widetilde{p}^{\mathrm{DA}}_{g}$ and $\widetilde{p}^{\mathrm{DA}}_{d}$ is the day ahead dispatch from problem 9.

The lagrangian of the TSO day-ahead market problem is as follows:

$$\begin{aligned}
\mathcal{L}^{\mathrm{DA}} &= \sum_{d \in D} \pi^{\mathrm{DA}}_{d} \widehat{p}^{\mathrm{DA}}_{d} - \sum_{g \in G} \pi^{\mathrm{DA}}_{g} \widehat{p}^{\mathrm{DA}}_{g} \\
&- VOLL^{\mathrm{DA}} s^{\mathrm{DA}} - \pi^{R} \sum_{r} w^{\mathrm{DA}}_{r} \\
&- \lambda^{\mathrm{T,DA}} \left[\sum_{g \in G} \widehat{p}^{\mathrm{DA}}_{g} - \sum_{d \in D} \widehat{p}^{\mathrm{DA}}_{d} + \sum_{r \in R} w^{\mathrm{DA}}_{r} + s^{\mathrm{DA}} \right] \\
&+ \sum_{g \in G^{\mathrm{D}}_{e}, e} \left[\varsigma^{\mathrm{T,DA}-}_{ge} \left(\underline{P}_{g} - \widehat{p}^{\mathrm{DA}}_{g} \right) + \varsigma^{\mathrm{T,DA}+}_{ge} \left(\widehat{p}^{\mathrm{DA}}_{g} - \widetilde{p}^{\mathrm{DA}}_{g} \right) \right] \\
&+ \sum_{g \in G^{\mathrm{T}}} \left[\sigma^{\mathrm{T,DA}-}_{g} \left(\underline{P}_{g} - \widehat{p}^{\mathrm{DA}}_{g} \right) + \sigma^{\mathrm{T,DA}+}_{g} \left(\widehat{p}^{\mathrm{DA}}_{g} - \overline{P}_{g} \right) \right] \\
&+ \sum_{d \in D^{\mathrm{D}}_{e}, e} \left[\varsigma^{\mathrm{T,DA}-}_{de} \left(\underline{P}_{d} - \widehat{p}^{\mathrm{DA}}_{d} \right) + \varsigma^{\mathrm{T,DA}+}_{de} \left(\widehat{p}^{\mathrm{DA}}_{d} - \widetilde{p}^{\mathrm{DA}}_{d} \right) \right] \\
&+ \sum_{d \in D^{\mathrm{T}}} \left[\sigma^{\mathrm{T,DA}-}_{d} \left(\underline{P}_{d} - \widehat{p}^{\mathrm{DA}}_{d} \right) + \sigma^{\mathrm{T,DA}+}_{d} \left(\widehat{p}^{\mathrm{DA}}_{d} - \overline{P}_{d} \right) \right] \\
&- \sum_{r \in R} \left[\nu^{\mathrm{T,DA}-}_{r} w^{\mathrm{DA}}_{r} - \nu^{\mathrm{T,DA}+}_{r} \left(w^{\mathrm{DA}}_{r} - W^{\mathrm{DA}}_{r} \right) \right] \\
&- \rho^{\mathrm{T,DA}-} s^{\mathrm{DA}} + \rho^{\mathrm{T,DA}+} (s^{\mathrm{DA}} - \sum_{d} \widehat{p}^{\mathrm{DA}}_{d})
\end{aligned} \tag{14}$$

The KKTs of the TSO day-ahead market (excluding primal constraints) are:

$$(\widehat{p}^{\mathrm{DA}}_{g}) : \quad -\pi^{\mathrm{DA}}_{g} - \left[\varsigma^{\mathrm{T,DA}-}_{ge} - \varsigma^{\mathrm{T,DA}+}_{ge} \right]_{g \in G^{\mathrm{D}}_{e}} - \left[\sigma^{\mathrm{T,DA}-}_{g} - \sigma^{\mathrm{T,DA}+}_{g} \right]_{g \in G^{\mathrm{T}}} - \lambda^{\mathrm{T,DA}} = 0, \quad \forall g \in G \tag{15a}$$

$$(\widehat{p}^{\mathrm{DA}}_{d}) : \quad \pi^{\mathrm{DA}}_{d} - \left[\varsigma^{\mathrm{T,DA}-}_{de} - \varsigma^{\mathrm{T,DA}+}_{de} \right]_{d \in D^{\mathrm{D}}_{e}} - \left[\sigma^{\mathrm{T,DA}-}_{d} - \sigma^{\mathrm{T,DA}+}_{d} \right]_{d \in D^{\mathrm{T}}} - \rho^{\mathrm{T,DA}+} + \lambda^{\mathrm{T,DA}} = 0, \quad \forall d \in D \tag{15b}$$

$$(s^{\mathrm{l,DA}}) : \quad -VOLL^{\mathrm{DA}} - \lambda^{\mathrm{T,DA}} - \rho^{\mathrm{T,DA}-} + \rho^{\mathrm{T,DA}+} = 0 \tag{15c}$$

$$(w^{\mathrm{DA}}_{r}) : \quad -\pi^{R} - \lambda^{\mathrm{T,DA}} - \left[\nu^{\mathrm{T,DA}-}_{r} - \nu^{\mathrm{T,DA}+}_{r} \right] = 0,\ \forall r \in R \tag{15d}$$

The complimentary constraints are:

$$0 \le \varsigma^{\mathrm{T,DA}-}_{ge} \perp \widehat{p}^{\mathrm{DA}}_{g} - \underline{P}_{g} \ge 0 \quad \forall g, e \tag{16a}$$

$$0 \le \varsigma^{\mathrm{T,DA}+}_{ge} \perp \widetilde{p}^{\mathrm{DA}}_{g} - \widehat{p}^{\mathrm{DA}}_{g} \ge 0 \quad \forall g \in G^{\mathrm{D}}_{e}, e \in E \tag{16b}$$

$$0 \le \sigma^{\mathrm{T,DA}+}_{g} \perp \overline{P}_{g} - \widehat{p}^{\mathrm{DA}}_{g} \ge 0 \quad \forall g \in G^{\mathrm{T}} \tag{16c}$$

$$0 \le \varsigma^{\mathrm{T,DA}-}_{de} \perp \widehat{p}^{\mathrm{DA}}_{d} - \underline{P}_{d} \ge 0 \quad \forall d, e \tag{16d}$$

$$0 \le \varsigma^{\mathrm{T,DA}+}_{de} \perp \widetilde{p}^{\mathrm{DA}}_{d} - \widehat{p}^{\mathrm{DA}}_{d} \ge 0 \quad \forall d \in D^{\mathrm{D}}_{e}, e \in E \tag{16e}$$

$$0 \le \sigma^{\mathrm{T,DA}+}_{d} \perp \overline{P}_{d} - \widehat{p}^{\mathrm{DA}}_{d} \ge 0 \quad \forall d \in D^{\mathrm{T}} \tag{16f}$$

$$0 \le \nu^{\mathrm{T,DA}-}_{r} \perp w^{\mathrm{DA}}_{r} \ge 0,\ \forall r \in R \tag{16g}$$

$$0 \le \nu^{\mathrm{T,DA}+}_{r} \perp W^{\mathrm{DA}}_{r} - w^{\mathrm{DA}}_{r} \ge 0,\ \forall r \in R \tag{16h}$$

$$0 \le \rho^{\mathrm{T,DA}-} \perp s^{\mathrm{l,DA}} \ge 0 \tag{16i}$$

$$0 \le \rho^{\mathrm{T,DA}+} \perp \sum_{d} \widehat{p}^{\mathrm{DA}}_{d} - s^{\mathrm{l,DA}} \ge 0 \tag{16j}$$

Appendix D
KKT conditions of Real-Time Market

The KKT conditions of the SOCP problem for the real-time re-dispatch are not actually solved, because the Benders decomposition renders the scenarios solvable as single problems. However, they are used in a proof of equivalence between the DSO market and the global DA-RT combination.

The Real-Time problem from (4) is repeated here with dual variables added:

$$\min_{\Xi^{\mathrm{RT}}} \phi_{\omega} (\Delta \mathrm{Cost}^{\mathrm{RT}}_{\omega}) = \phi_{\omega} \Big[\sum_{g \in G} (\pi^{\mathrm{DA}}_{g} (p^{\mathrm{RT}}_{g\omega} - \widehat{p}^{\mathrm{DA}}_{g}) + \pi^{\uparrow}_{g} p^{\uparrow}_{g\omega} + \pi^{\downarrow}_{g} p^{\downarrow}_{g\omega}) \tag{17a}$$

$$+\sum_{d\in D}(\pi_d^{\mathrm{DA}}(\widehat{p}_d^{\mathrm{DA}}-p_{d\omega}^{\mathrm{RT}})+\pi_d^{\uparrow}p_{d\omega}^{\uparrow}+\pi_d^{\downarrow}p_{d\omega}^{\downarrow})$$
$$+\sum_{n\in N}VOLL_n^{\mathrm{RT}}s_{n\omega}^{\mathrm{RT}}$$
$$+\sum_{r}\left(\pi^R(w_{r\omega}^{\mathrm{RT}}-w_r^{\mathrm{DA}})+\pi^{\uparrow\mathrm{R}}w_{r\omega}^{\uparrow}+\pi^{\downarrow\mathrm{R}}w_{r\omega}^{\downarrow}\right)\Big]$$

$$\text{s.t.}\quad p_{l\omega}^{\mathrm{RT}}=B_l(\theta_{n\omega}-\theta_{m\omega}),\quad\forall l\in L^{\mathrm{T}},\ :(\gamma_{l\omega}^{\mathrm{T}}) \tag{17b}$$
$$p_{l\omega}^{\mathrm{RT}}\le S_l,\ \forall l\in L^{\mathrm{T}},\ :(\eta_{l\omega}^{\mathrm{T}}) \tag{17c}$$
$$p_{g\omega}^{\mathrm{RT}}=\widehat{p}_g^{\mathrm{DA}}+p_{g\omega}^{\uparrow}-p_{g\omega}^{\downarrow},\quad\forall g\in G,\ :(\zeta_{g\omega}^{\mathrm{p,RT}}) \tag{17d}$$
$$p_{d\omega}^{\mathrm{RT}}=\widehat{p}_d^{\mathrm{DA}}-p_{d\omega}^{\uparrow}+p_{d\omega}^{\downarrow},\quad\forall d\in D,\ :(\zeta_{d\omega}^{\mathrm{p,RT}}) \tag{17e}$$
$$w_{r\omega}^{\mathrm{RT}}=w_r^{\mathrm{DA}}+w_{r\omega}^{\uparrow}-w_{r\omega}^{\downarrow},\ \forall r\in R,,\ :(\zeta_{r\omega}^{\mathrm{p,RT}}) \tag{17f}$$
$$\sum_{g\in G_n}p_{g\omega}^{\mathrm{RT}}-\sum_{d\in D_n}p_{d\omega}^{\mathrm{RT}}+\sum_{r\in R_n}w_{r\omega}^{\mathrm{RT}}+s_{n\omega}^{\mathrm{RT}}=\sum_{l\in n\rightarrow}p_{l\omega}^{\mathrm{RT}}-\sum_{l\in\rightarrow n}p_{l\omega}^{\mathrm{RT}},\ \forall n\in N,\ :(\lambda_{n\omega}^{\mathrm{p,RT}}) \tag{17g}$$
$$\sum_{g\in G_n}q_{g\omega}^{\mathrm{RT}}-\sum_{d\in D_n}q_{d\omega}^{\mathrm{RT}}+s_{n\omega}^{\mathrm{q,RT}}=\sum_{l\in n\rightarrow}q_{l\omega}^{\mathrm{RT}}-\sum_{l\in\rightarrow n}q_{l\omega}^{\mathrm{RT}},\ \forall n\in N_e^{\mathrm{D}},\ :(\lambda_{n\omega}^{\mathrm{q,RT}}) \tag{17h}$$
$$p_{l\omega}^{(\mathrm{RT})2}+q_{l\omega}^{(\mathrm{RT})2}\le\varphi_{l\omega}^{\mathrm{RT}}v_{n\omega}^{\mathrm{RT}},\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\gamma_{l\omega}^{\mathrm{D,RT}}) \tag{17i}$$
$$p_{l\omega}^{\mathrm{RT}}+p_{l'\omega}^{\mathrm{RT}}=R_l\varphi_{l\omega}^{\mathrm{RT}},\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\mu_{l\omega}^{\mathrm{p,RT}}) \tag{17j}$$
$$q_{l\omega}^{\mathrm{RT}}+q_{l'\omega}^{\mathrm{RT}}=X_l\varphi_{l\omega}^{\mathrm{RT}},\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\mu_{l\omega}^{\mathrm{q,RT}}) \tag{17k}$$
$$p_{l\omega}^{(\mathrm{RT})2}+q_{l\omega}^{(\mathrm{RT})2}\le S_l^2,\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\eta_{l\omega}^{\mathrm{D}}) \tag{17l}$$
$$v_{m\omega}^{\mathrm{RT}}=v_{n\omega}^{\mathrm{RT}}-2(R_lp_{l\omega}^{\mathrm{RT}}+X_lq_{l\omega}^{\mathrm{RT}})+(R_l^2+X_l^2)\varphi_{l\omega}^{\mathrm{RT}},\ \forall l\in L_e^{\mathrm{D}}\cup l_e,\ :(\beta_{l\omega}^{\mathrm{RT}}) \tag{17m}$$
$$\underline{V}_n^2\le v_{n\omega}^{\mathrm{RT}}\le\overline{V}_n^2,\quad\forall e,n\in N_e^{\mathrm{D}},\ :(\sigma_{n\omega}^{\mathrm{RT}-},\sigma_{n\omega}^{\mathrm{RT}+}) \tag{17n}$$
$$0\le w_{r\omega}^{\mathrm{RT}}\le W_{r\omega}^{\mathrm{RT}},\quad\forall r\in R,\ :(\nu_{r\omega}^{\mathrm{RT}-},\nu_{r\omega}^{\mathrm{RT}+}) \tag{17o}$$
$$\underline{P}_g\le p_{g\omega}^{\mathrm{RT}}\le\overline{P}_g,\quad\forall g\in G,\ :(\varsigma_{g\omega}^{\mathrm{RT}-},\varsigma_{g\omega}^{\mathrm{RT}+}) \tag{17p}$$
$$\underline{P}_d\le p_{d\omega}^{\mathrm{RT}}\le\overline{P}_d,\quad\forall d\in D,\ :(\varsigma_{d\omega}^{\mathrm{RT}-},\varsigma_{d\omega}^{\mathrm{RT}+}) \tag{17q}$$
$$\underline{Q}_g\le q_{g\omega}^{\mathrm{RT}}\le\overline{Q}_g,\quad\forall g\in G_e^{\mathrm{D}},\ :(\kappa_{g\omega}^{\mathrm{RT}-},\kappa_{g\omega}^{\mathrm{RT}+}) \tag{17r}$$
$$\underline{Q}_d\le q_{d\omega}^{\mathrm{RT}}\le\overline{Q}_d,\quad\forall d\in D_e^{\mathrm{D}},\ :(\kappa_{d\omega}^{\mathrm{RT}-},\kappa_{d\omega}^{\mathrm{RT}+}) \tag{17s}$$
$$0\le s_{n\omega}^{\mathrm{RT}}\le\sum_{d\in D_n}p_{d\omega}^{\mathrm{RT}},\ \forall n\in N,\ :(\Upsilon_{n\omega}^{\mathrm{RT}-},\Upsilon_{n\omega}^{\mathrm{RT}+}) \tag{17t}$$
$$p_{g\omega}^{\uparrow}\ge 0,\ ,\quad p_{g\omega}^{\downarrow}\ge 0,\ \forall g,\ :(\epsilon_{g\omega}^{\uparrow,\mathrm{RT}},\epsilon_{g\omega}^{\downarrow,\mathrm{RT}}) \tag{17u}$$
$$p_{d\omega}^{\uparrow}\ge 0,\ ,\quad p_{d\omega}^{\downarrow}\ge 0,\ \forall d,\ :(\epsilon_{d\omega}^{\uparrow,\mathrm{RT}},\epsilon_{d\omega}^{\downarrow,\mathrm{RT}}) \tag{17v}$$
$$w_{r\omega}^{\uparrow}\ge 0,\quad w_{r\omega}^{\downarrow}\ge 0,\forall r,\ :(\varepsilon_{r\omega}^{\uparrow,\mathrm{RT}},\varepsilon_{r\omega}^{\downarrow,\mathrm{RT}}) \tag{17w}$$

The Lagrangian of the real-time problem is as follows:

$$\mathcal{L}^{\mathrm{RT}}=\sum_{\omega}\phi_\omega\Bigg[\sum_{g\in G}\left(\pi_g^{\mathrm{DA}}(p_{g\omega}^{\mathrm{RT}}-p_g^{\mathrm{DA}})+\pi_g^{\uparrow}p_{g\omega}^{\uparrow}+\pi_g^{\downarrow}p_{g\omega}^{\downarrow}\right)$$
$$+\sum_{d\in D}\left(\pi_d^{\mathrm{DA}}(p_d^{\mathrm{DA}}-p_{d\omega}^{\mathrm{RT}})+\pi_d^{\uparrow}p_{d\omega}^{\uparrow}+\pi_d^{\downarrow}p_{d\omega}^{\downarrow}\right)$$
$$+\sum_{n\in N}VOLL_n^{\mathrm{RT}}s_{n\omega}^{\mathrm{RT}}$$
$$+\sum_{r\in R}\left(\pi^R(w_{r\omega}^{\mathrm{RT}}-w_r^{\mathrm{DA}})+\pi^{\uparrow R}w_{r\omega}^{\uparrow}+\pi^{\downarrow R}w_{r\omega}^{\downarrow}\right)\Bigg]$$
$$+\sum_{l\in L^{\mathrm{T}}}\gamma_{l\omega}^{\mathrm{T}}\left(p_{l\omega}^{\mathrm{RT}}-B_l(\theta_{n\omega}-\theta_{m\omega})\right)$$
$$+\sum_{l\in L^{\mathrm{T}}}\eta_{l\omega}^{\mathrm{T}}\left(p_{l\omega}^{\mathrm{RT}}-S_l\right)$$
$$-\sum_{g\in G}\zeta_{g\omega}^{\mathrm{p}}\left(p_{g\omega}^{\mathrm{RT}}-\widehat{p}_g^{\mathrm{DA}}-p_{g\omega}^{\uparrow}+p_{g\omega}^{\downarrow}\right)$$
$$-\sum_{d\in D}\zeta_{d\omega}^{\mathrm{p}}\left(p_{d\omega}^{\mathrm{RT}}-\widehat{p}_d^{\mathrm{DA}}+p_{d\omega}^{\uparrow}-p_{d\omega}^{\downarrow}\right)$$
$$-\sum_{r\in R}\zeta_{r\omega}^{p}\left(w_{r\omega}^{\mathrm{RT}}-w_r^{\mathrm{DA}}-w_{r\omega}^{\uparrow}+w_{r\omega}^{\downarrow}\right)$$
$$-\sum_{n\in N,\omega}\lambda_{n\omega}^{\mathrm{p,RT}}\Big(\sum_{g\in G_n}p_{g\omega}^{\mathrm{RT}}-\sum_{d\in D_n}p_{d\omega}^{\mathrm{RT}}+\sum_{r\in R_n}w_{r\omega}^{\mathrm{RT}}+s_{n\omega}^{\mathrm{RT}}-\sum_{l\in n\rightarrow}p_{l\omega}^{\mathrm{RT}}+\sum_{l\in\rightarrow n}p_{l\omega}^{\mathrm{RT}}\Big)$$
$$-\sum_{n\in N,\omega}\lambda_{n\omega}^{\mathrm{q,RT}}\left(\sum_{g\in G_n}q_{g\omega}^{\mathrm{RT}}-\sum_{d\in D_n}q_{d\omega}^{\mathrm{RT}}+s_{n\omega}^{\mathrm{q,RT}}-\sum_{l\in n\rightarrow}q_{l\omega}^{\mathrm{RT}}+\sum_{l\in\rightarrow n}q_{l\omega}^{\mathrm{RT}}\right)$$
$$+\sum_{l\in L_e^{\mathrm{D}}\cup l_e,\omega}\gamma_{l\omega}\left[p_{l\omega}^{(RT)2}+q_{l\omega}^{(RT)2}-\varphi_{l\omega}^{\mathrm{RT}}v_{n\omega}^{\mathrm{RT}}\right]$$
$$-\sum_{l\in L_e^{\mathrm{D}}\cup l_e,\omega}\Big[\mu_{l\omega}^{p}\left(p_{l\omega}^{\mathrm{RT}}+p_{l'\omega}^{\mathrm{RT}}-R_l\varphi_{l\omega}^{\mathrm{RT}}\right)+\mu_{l\omega}^{q}\left(q_{l\omega}^{\mathrm{RT}}+q_{l'\omega}^{\mathrm{RT}}-X_l\varphi_{l\omega}^{\mathrm{RT}}\right)\Big]$$
$$+\sum_{l\in L_e^{\mathrm{D}}\cup l_e,\omega}\left[\eta_{l\omega}\left(p_{l\omega}^{(RT)2}+q_{l\omega}^{(RT)2}-S_l\right)\right]$$
$$-\sum_{l\in L_e^{\mathrm{D}}\cup l_e,\omega}\Big[\beta_{l\omega}\Big(v_{m\omega}^{\mathrm{RT}}-v_{n\omega}^{\mathrm{RT}}+2(R_lp_{l\omega}^{\mathrm{RT}}+X_lq_{l\omega}^{\mathrm{RT}})-(R_l^2+X_l^2)\varphi_{l\omega}^{\mathrm{RT}}\Big)\Big]$$
$$+\sum_{n\in N_e^{\mathrm{D}}\cup n_e^{HV},\omega}\left[\sigma_{n\omega}^{-}\left(\underline{V}_n^2-v_{n\omega}^{\mathrm{RT}}\right)+\sigma_{n\omega}^{+}\left(v_{n\omega}^{\mathrm{RT}}-\overline{V}_n^2\right)\right]$$
$$-\sum_{r\in R,\omega}\left[\nu_{r\omega}^{-}w_{r\omega}^{\mathrm{RT}}-\nu_{r\omega}^{+}\left(w_{r\omega}^{\mathrm{RT}}-W_{r\omega}^{\mathrm{RT}}\right)\right]$$
$$+\sum_{g\in G,\omega}\left[\varsigma_{g\omega}^{\mathrm{RT}-}\left(\underline{P}_g-p_{g\omega}^{\mathrm{RT}}\right)+\varsigma_{g\omega}^{\mathrm{RT}+}\left(p_{g\omega}^{\mathrm{RT}}-\overline{P}_g\right)\right]$$
$$+\sum_{d\in D,\omega}\left[\varsigma_{d\omega}^{\mathrm{RT}-}\left(\underline{P}_d-p_{d\omega}^{\mathrm{RT}}\right)+\varsigma_{d\omega}^{\mathrm{RT}+}\left(p_{d\omega}^{\mathrm{RT}}-\overline{P}_d\right)\right]$$
$$+\sum_{g\in G,\omega}\left[\kappa_{g\omega}^{\mathrm{RT}-}\left(\underline{Q}_g-q_{g\omega}^{\mathrm{RT}}\right)+\kappa_{g\omega}^{\mathrm{RT}+}\left(q_{g\omega}^{\mathrm{RT}}-\overline{Q}_g\right)\right]$$
$$+\sum_{d\in D_e^{\mathrm{D}},\omega}\left[\kappa_{d\omega}^{\mathrm{RT}-}\left(\underline{Q}_d-q_{d\omega}^{\mathrm{RT}}\right)+\kappa_{d\omega}^{\mathrm{RT}+}\left(q_{d\omega}^{\mathrm{RT}}-\overline{Q}_g\right)\right]$$
$$+\sum_{g\in G,\omega}\left[-p_{g\omega}^{\uparrow}\epsilon_{g\omega}^{\mathrm{p}\uparrow}-p_{g\omega}^{\downarrow}\epsilon_{g\omega}^{\mathrm{p}\downarrow}\right]$$

$$+\sum_{r\in R,\omega}\left[-w^{\uparrow}_{r\omega}\epsilon^{\mathrm{p}\uparrow}_{r\omega}-w^{\downarrow}_{r\omega}\epsilon^{\mathrm{p}\downarrow}_{r\omega}\right]$$
$$+\sum_{d\in D,\omega}\left[-p^{\uparrow}_{d\omega}\varepsilon^{\mathrm{p}\uparrow}_{d\omega}-p^{\downarrow}_{d\omega}\varepsilon^{\mathrm{p}\downarrow}_{d\omega}\right]$$
$$+\sum_{n\in N,\omega}\left[-\Upsilon^{\mathrm{RT}-}_{n\omega}s^{\mathrm{RT}}_{n\omega}+\Upsilon^{\mathrm{RT}+}_{n\omega}\left(s^{\mathrm{RT}}_{n\omega}-\sum_{d\in D_n}p^{\mathrm{RT}}_{d\omega}\right)\right] \tag{18}$$

The KKT conditions of above Lagrangian are (excluding the primal constraints of 17):

$$(p^{\uparrow}_{g\omega}):\ \phi_\omega\pi^{\uparrow}_g+\zeta^p_{g\omega}-\epsilon^{\mathrm{p}\uparrow}_{g\omega}=0,\quad \forall g\in G^{\mathrm{D}}_e \tag{19a}$$
$$(p^{\downarrow}_{g\omega}):\ \phi_\omega\pi^{\downarrow}_g-\zeta^p_{g\omega}-\epsilon^{\mathrm{p}\downarrow}_{g\omega}=0,\quad \forall g\in G^{\mathrm{D}}_e \tag{19b}$$
$$(p^{\uparrow}_{d\omega}):\ \phi_\omega\pi^{\uparrow}_d-\zeta^p_{d\omega}-\varepsilon^{\mathrm{p}\uparrow}_{i\omega}=0,\quad \forall d\in D^{\mathrm{D}}_e \tag{19c}$$
$$(p^{\downarrow}_{d\omega}):\ \phi_\omega\pi^{\downarrow}_d+\zeta^p_{d\omega}-\varepsilon^{\mathrm{p}\downarrow}_{d\omega}=0,\quad \forall d\in D^{\mathrm{D}}_e \tag{19d}$$
$$(w^{\uparrow}_{r\omega}):\ \phi_\omega\pi^{\uparrow R}+\zeta^p_{r\omega}-\epsilon^{\mathrm{p}\uparrow}_{r\omega}=0,\quad \forall r\in R^{\mathrm{D}}_e \tag{19e}$$
$$(w^{\downarrow}_{r\omega}):\ \phi_\omega\pi^{\downarrow R}-\zeta^p_{r\omega}-\epsilon^{\mathrm{p}\downarrow}_{r\omega}=0,\quad \forall r\in R^{\mathrm{D}}_e \tag{19f}$$
$$(s^{\mathrm{RT}}_{n\omega}):\phi_\omega\mathrm{VOLL}_n-\lambda^{\mathrm{p,RT}}_{n\omega}-\Upsilon^{\mathrm{RT}-}_{n\omega}+\Upsilon^{\mathrm{RT}+}_{n\omega}=0,\ \forall n\in N \tag{19g}$$
$$(w^{\mathrm{RT}}_{r\omega}):\phi_\omega\pi^{R}-\zeta^p_{r\omega}-\left[\lambda^{\mathrm{p,RT}}_{n\omega}\right]_{n_r}+\nu^{+}_{r\omega}-\nu^{-}_{r\omega}=0,\quad \forall r\in R^{\mathrm{D}}_e \tag{19h}$$
$$(p^{\mathrm{RT}}_{g\omega}):\phi_\omega\pi^{\mathrm{DA}}_g-\zeta^p_{g\omega}-\varsigma^{\mathrm{RT}-}_{g\omega}+\varsigma^{\mathrm{RT}+}_{g\omega}-\left[\lambda^{\mathrm{p,RT}}_{n\omega}\right]_{n_g}=0,\quad \forall g\in G \tag{19i}$$
$$(q^{\mathrm{RT}}_{g\omega}):-\kappa^{\mathrm{RT}-}_{g\omega}+\kappa^{\mathrm{RT}+}_{g\omega}-\left[\lambda^{\mathrm{q,RT}}_{n\omega}\right]_{n_g}=0,\quad \forall g\in G^{\mathrm{D}}_e \tag{19j}$$
$$(p^{\mathrm{RT}}_{d\omega}):-\phi_\omega\pi^{\mathrm{DA}}_d-\zeta^p_{d\omega}-\varsigma^{\mathrm{RT}-}_{d\omega}+\varsigma^{\mathrm{RT}+}_{d\omega}+\left[\lambda^{\mathrm{p,RT}}_{n\omega}-\Upsilon^{\mathrm{RT}+}_{n\omega}\right]_{n_d}=0,\quad \forall d\in D \tag{19k}$$
$$(q^{\mathrm{RT}}_{d\omega}):-\kappa^{\mathrm{RT}-}_{d\omega}+\kappa^{\mathrm{RT}+}_{d\omega}+\left[\lambda^{\mathrm{q,RT}}_{n\omega}\right]_{n_d}=0,\quad \forall d\in D^{\mathrm{D}}_e \tag{19l}$$
$$(p^{\mathrm{RT}}_{l\omega}):\lambda^{\mathrm{p,RT}}_{n\omega}-\lambda^{\mathrm{p,RT}}_{m\omega}+\left[2\gamma_{l\omega}p^{\mathrm{RT}}_{l\omega}-\mu^{\mathrm{p}}_{l\omega}-\mu^{\mathrm{p}}_{l'\omega}+2\eta_{l\omega}p^{\mathrm{RT}}_{l\omega}-2\beta_{l\omega}R_l\right]_{l\in L^{\mathrm{D}}_e}+\left[\gamma^{\mathrm{T}}_{l\omega}+\eta^{\mathrm{T}}_{l\omega}\right]_{l\in L^{\mathrm{T}}}=0,\quad \forall l\in L \tag{19m}$$
$$(q^{\mathrm{RT}}_{l\omega}):\lambda^{\mathrm{q,RT}}_{n\omega}-\lambda^{\mathrm{q,RT}}_{m\omega}+\left[2\gamma_{l\omega}q^{\mathrm{RT}}_{l\omega}-\mu^{q}_{l\omega}-\mu^{q}_{l'\omega}+2\eta_{l\omega}q^{\mathrm{RT}}_{l\omega}-2\beta_{l\omega}X_l\right]_{l\in L^{\mathrm{D}}_e}=0,\quad \forall l\in L \tag{19n}$$
$$(\varphi^{\mathrm{RT}}_{l\omega}):-\gamma_{l\omega}v^{\mathrm{RT}}_{n\omega}+\mu^{p}_{l\omega}R_l+\mu^{q}_{l\omega}X_l+\beta_{l\omega}(R^2_l+X^2_l)=0,\quad \forall\omega,l=(n,m)\in L^{\mathrm{D}}_e \tag{19o}$$
$$(v^{\mathrm{RT}}_{n\omega}):-\gamma_{l\omega}\varphi^{\mathrm{RT}}_{l\omega}-\beta_{l'\omega}+\beta_{l\omega}-\sigma^{-}_{n\omega}+\sigma^{+}_{n\omega}=0,\quad \forall\omega,l=(n,m)\in L^{\mathrm{D}}_e \tag{19p}$$

Appendix E
Proof of proposition 1

Proposition 1: The first order necessary conditions (KKT conditions) of the day-ahead market and the real-time market combined contain all the KKT conditions of the DSO market in (9) and solving (2) is equivalent to solving (1).

Proof of propositon 1: Problem (3) and (4) are explicitly convex. Therefore their KKT conditions are also optimality conditions. This is the first part of the proof. The KKT conditions of the DSO market in (9) are given in (11) and (12). The DSO market is also explicitly convex and the KKT conditions define optimality. The KKT conditions of the day-ahead market are presented in (15) and (16). All equations of (11) are contained in either (15) or (19) with exception of the variables related to the PCC injections in feeder e. The dual constraints with regards to those variables are (11u) through (11y). The PCC prices $\pi^{\mathrm{PCC,DA}}_e$, $\pi^{\uparrow\mathrm{PCC}}_e$, $\pi^{\downarrow\mathrm{PCC}}_e$ and the PCC flow limits $\overline{f}_e, \underline{f}_e$ are variables in the upper level problem (1) and therefore constitute slack variables that will not influence the optimality of (9). Therefore removing the DSO market lower level problem and solving (2) is equivalent to solving (1). This ends the proof. □

Appendix F
Scenario Generation

A simple scenario generation method is used for the uncertainties of the wind production. The distance of the wind farms are

$$D_{rw}=\left\|\begin{matrix}x_r-x_w\\ y_r-y_w\end{matrix}\right\|,\quad \forall r\in R,w\in R,r\neq w \tag{20}$$

The co-variance matrix is now given in (21).

$$\Sigma_{rw}=\frac{\sigma^2_r+\sigma^2_w}{2}e^{-D_{rw}},\forall r\in R,w\in R \tag{21}$$

The distributions of the wind generators are thus:

$$W\sim\mathcal{N}_r(\mu_r,\Sigma_{rw}) \tag{22}$$

Now the scenarios can be drawn by random sampling i.e.

$$W^{\mathrm{RT}}_{r\omega}\sim\sim\mathcal{N}_r(\mu_r,\Sigma_{rw}) \tag{23}$$

Appendix G
Modified 24 bus test netowk

The 24-bus power system – Single area RTS-96 is used here in a modified form. The mean and variance of the wind power plants in the network are given in table II. The locations of each wind farm is given in table III, which is used to claculate the distance between them as in equation (20).

TABLE II
THE MEAN OF THE FORECAST OF THE INSTALLED RES AND VARIANCE OF THE FORECAST.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7
Variance σ^2	750	740	760	300	300	300	200
Mean μ	200	200	200	40	40	40	10

Appendix H
Congestion Level

The Congestion level of the colored dots in Fig. 4 is here plotted as a line plot in Fig. 8. The data for the two plots is the same.

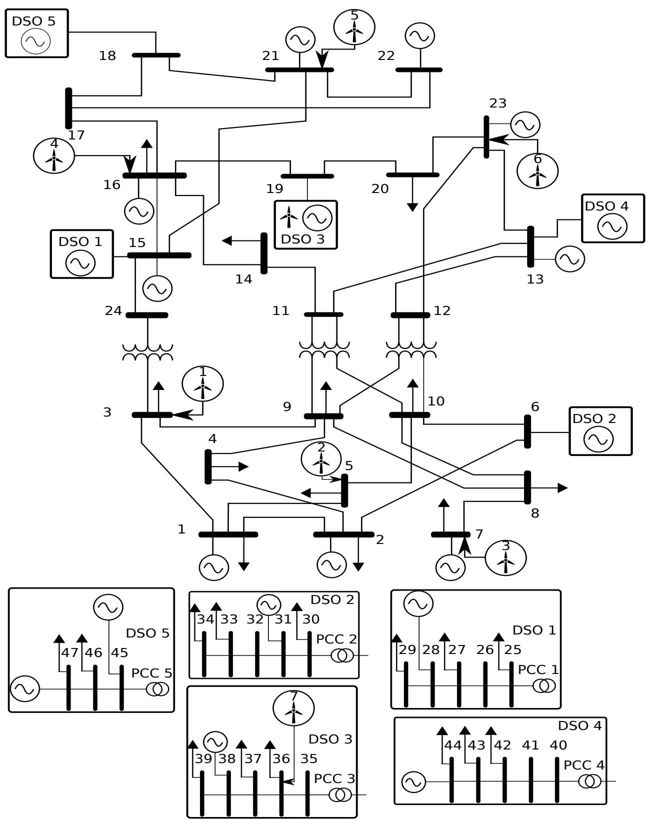

Fig. 7. Diagram of the 24-bus power system – Single area RTS-96. 5 DSO feeders have been added, as well as 7 Wind power plants. The loads from the buses where the DSO feeders are connected have been moved to the DSO feeders.

TABLE III
GEORGRAPHICAL LOCATION OF EACH WIND FARM.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7
X-coordinate	0	0.25	0.5	6	6.25	6.5	6.75
Y-Coordinate	0	0	0	5	5	5	5.2

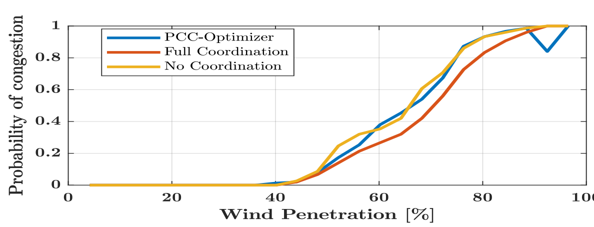

Fig. 8. Probability of at least two transmission lines being congested in RT.

Appendix I
Computational Performance

Here we show some results pertaining to the Benders decomposition approach that was presented in section IV. In Fig. 9 we present the convergence of the suggested multicut benders decomposition for a sample point of wind-penetration.

The upper bound of the benders decomposed problems in iteration (i) is found as:

$$UB^{(\mathrm{i})} = \mathcal{SW}^{\mathrm{DA}(i)} - \sum_{\omega} \phi_\omega \Delta \mathrm{Cost}_\omega^{\mathrm{RT(i)}} \tag{24}$$

The lower bound in iteration (i) is found via:

$$LB^{(\mathrm{i})} = \mathcal{SW}^{\mathrm{DA}(i)} - \sum_{\omega} \phi_\omega \psi_\omega^{(\mathrm{i})} \tag{25}$$

The computational burden of the decomposed problem is analyzed by logging the time it takes Mosek 8.0 to solve every master-problem and sub-problem for every scenario. The implementation we use in this paper relies on the CVX plugin for Matlab, which yields large overhead due to the time it takes to initialize every master-problem and sub-problem. Therefore the results in table IV only give the time that the solver actually spent, while the full time including the overhead for the initialization is about two to four times this number. In the future we wish to use an implementation that does not rely on CVX which can help solving larger case studies. The data provided in table IV is the average over all the different wind-penetration settings of the RES (i.e. it is the average of 24 different wind penetration settings). Because the sub-problems are independent, and can be solved in parallel, the number of scenarios do not affect the computational time as long as there are enough CPU-cores to solve them in parallel.

The number of binary variables in the master-problem depend on the number of complimentarity constraints in (16). Because we choose to solve the complimentarity constraints with the Big-M approach every one of these constraints uses one binary. The number of complimentarity constraints in turn depend mainly on the number of generators, number of elastic demands and number of RES sources. In the case study for this work the master problem therefore contains 196 binary variables. As a result of the benders decomposition the conic constraints have all been moved to the subproblem, and therefore the master problem is MILP, while the subproblems are continuous SOCP.

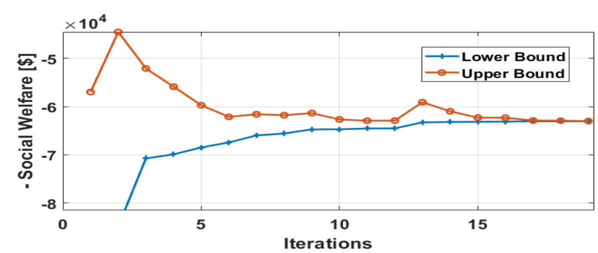

Fig. 9. Convergence of the Benders decomposition upper and lower bound over the iterations. Note, we minimize $-\mathcal{SW}$, which is equivalent to maximizing social welfare.

TABLE IV
COMPUTATIONAL BURDEN OF THE BENDERS MULTI-CUT SOLUTION STRATEGY. NOTE: WE AVERAGE FOR ALL SOLVED INSTANCES OF INCREASING WIND PENETRATION.

CPU times [s]				
Average subproblem	Average master	Initial master problem	Master in last iteration	Average #Iterations
0.31	0.54	0.67	0.91	29.5