Available online at: `https://github.com/alherm/TSO-DSO_coordination`

This online companion includes four appendixes. Appendix A provides the mathematical formulation of the lower-level problem for DSO market, and then its corresponding Karush–Kuhn–Tucker (KKT) conditions are presented. Appendixes B and C provide the KKT conditions of the lower-level problems representing the day-ahead and real-time markets, respectively. Finally, Appendix D explains why the KKT conditions of the DSO market-clearing problem in Appendix A are redundant, proving this problem can be removed.

Appendix A. Lower-Level Problem Representing the DSO Market

Each distribution system operator (DSO) e clears a local market to pre-qualify the participation of flexible resources located in its operational domain in the wholesale day-ahead market. If necessary, the DSO imposes caps on the quantity bids of DSO-level flexible resources to the day-ahead market. The DSO market-clearing problem includes both day-ahead and real-time stages. Recall that, aligned with European market design, the network constraints in the day-ahead stage are not included. However, such constraints are enforced in real time using a conic relaxation of power flow equations in the distribution level. The resulting model is a stochastic second-order cone programming (SOCP) problem as below. Note that dual variables are provided alongside each constraint:

$$\max_{\Xi^{\mathrm{E}}} \quad \mathcal{SW}_e = \sum_{d\in D_e^{\mathrm{D}}} \pi_d^{\mathrm{DA}} \widetilde{p}_d^{\mathrm{DA}} - \sum_{g\in G_e^{\mathrm{D}}} \pi_g^{\mathrm{DA}} \widetilde{p}_g^{\mathrm{DA}} - VOLL\ s_e^{\mathrm{DA}} - \sum_{r\in R_e^{\mathrm{D}}} \pi^{\mathrm{R}} w_r^{\mathrm{DA}} - \pi_e^{\mathrm{IO,DA}} p_e^{\mathrm{IO,DA}}$$
$$- \sum_{\omega} \phi_\omega \Bigg[\sum_{g\in G_e^{\mathrm{D}}} \Big(\pi_g^{\mathrm{DA}} (p_{g\omega}^{\mathrm{RT}} - \widetilde{p}_g^{\mathrm{DA}}) + \pi_g^{\uparrow} p_{g\omega}^{\uparrow} + \pi_g^{\downarrow} p_{g\omega}^{\downarrow} \Big) + \sum_{d\in D_e^{\mathrm{D}}} \Big(\pi_d^{\mathrm{DA}} (\widetilde{p}_d^{\mathrm{DA}} - p_{d\omega}^{\mathrm{RT}})$$
$$+ \pi_d^{\uparrow} p_{d\omega}^{\uparrow} + \pi_d^{\downarrow} p_{d\omega}^{\downarrow} \Big) + \sum_{n\in N_e^{\mathrm{D}}} VOLL\ s_{n\omega}^{\mathrm{RT}} + \pi_e^{\mathrm{IO,DA}} (p_{e\omega}^{\mathrm{IO,RT}} - p_e^{\mathrm{IO,DA}})$$
$$+ \pi_e^{\uparrow\mathrm{IO}} p_{e\omega}^{\uparrow\mathrm{IO}} + \pi_e^{\downarrow\mathrm{IO}} p_{e\omega}^{\downarrow\mathrm{IO}} + \sum_{r\in R_e^{\mathrm{D}}} \Big(\pi^{\mathrm{R}} (w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}}) + \pi^{\uparrow\mathrm{R}} w_{r\omega}^{\uparrow} + \pi^{\downarrow\mathrm{R}} w_{r\omega}^{\downarrow} \Big) \Bigg] \tag{A.1a}$$

subject to:

Day-ahead constraints (deterministic and single-node):

$$\sum_{g\in G_e} \widetilde{p}_g^{\mathrm{DA}} - \sum_{d\in D_e^{\mathrm{D}}} \widetilde{p}_d^{\mathrm{DA}} + \sum_{r\in R_e^{\mathrm{D}}} w_r^{\mathrm{DA}} + s_e^{\mathrm{DA}} + p_e^{\mathrm{IO,DA}} = 0, \quad : (\lambda_e^{\mathrm{DA}}) \tag{A.1b}$$

$$\underline{P}_g \le \widetilde{p}_g^{\mathrm{DA}} \le \overline{P}_g, \quad \forall g \in G_e^{\mathrm{D}} \quad : (\varsigma_g^{\mathrm{DA}-}, \varsigma_g^{\mathrm{DA}+}) \tag{A.1c}$$

$$\underline{P}_d \le \widetilde{p}_d^{\mathrm{DA}} \le \overline{P}_d, \quad \forall d \in D_e^{\mathrm{D}} \quad : (\varsigma_d^{\mathrm{DA}-}, \varsigma_d^{\mathrm{DA}+}) \tag{A.1d}$$

$$0 \le w_r^{\mathrm{DA}} \le W_r^{\mathrm{DA}},\ \forall r \in R_e^{\mathrm{D}} \quad : (\iota_r^-, \iota_r^+) \tag{A.1e}$$

$$\underline{f}_e \le p_e^{\mathrm{IO,DA}} \le \overline{f}_e, \quad : (\rho_e^{\mathrm{DA}-}, \rho_e^{\mathrm{DA}+}) \tag{A.1f}$$

$$0 \le s_e^{\mathrm{DA}} \le \sum_d p_d^{\mathrm{DA}}, \quad : (\Upsilon_e^{\mathrm{DA}-}, \Upsilon_e^{\mathrm{DA}+}) \tag{A.1g}$$

Real-time constraints (stochastic and network-aware):

$$p_{g\omega}^{\mathrm{RT}} = p_g^{\mathrm{DA}} + p_{g\omega}^{\uparrow} - p_{g\omega}^{\downarrow}, \quad \forall \omega, g \in G_e^{\mathrm{D}}, \quad : (\zeta_{g\omega}^{\mathrm{p}}) \tag{A.1h}$$

$$p_{d\omega}^{\mathrm{RT}} = p_d^{\mathrm{DA}} - p_{d\omega}^{\uparrow} + p_{d\omega}^{\downarrow}, \quad \forall \omega, d \in D_e^{\mathrm{D}}, \quad : (\zeta_{d\omega}^{\mathrm{p}}) \tag{A.1i}$$

$$w_{r\omega}^{\mathrm{RT}} = w_r^{\mathrm{DA}} + w_{r\omega}^{\uparrow} - w_{r\omega}^{\downarrow}, \ \forall \omega, r \in R_e^{\mathrm{D}}, \quad : (\zeta_{r\omega}^{\mathrm{p}}) \tag{A.1j}$$

$$\sum_{g \in G_n} p_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} p_{d\omega}^{\mathrm{RT}} + \sum_{r \in R_n} w_{r\omega}^{\mathrm{RT}} + p_{e\omega}^{\mathrm{IO,RT}}|_{n=n_e^{\mathrm{LV}}} + s_{n\omega}^{\mathrm{RT}} = \sum_{l \in n \rightarrow} p_{l\omega}^{\mathrm{RT}} - \sum_{l \in \rightarrow n} p_{l\omega}^{\mathrm{RT}}, \ \forall \omega, n \in N_e^{\mathrm{D}} \ : (\lambda_{n\omega}^{\mathrm{p,RT}}) \tag{A.1k}$$

$$p_{e\omega}^{\mathrm{IO,RT}} = p_e^{\mathrm{IO,DA}} + p_{e\omega}^{\uparrow \mathrm{IO}} - p_{e\omega}^{\downarrow \mathrm{IO}}, \ \forall \omega, \ : (\zeta_{e\omega}^{\mathrm{IO}}) \tag{A.1l}$$

$$\sum_{g \in G_n} q_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} q_{d\omega}^{\mathrm{RT}} + s_{n\omega}^{\mathrm{q,RT}} + q_{e\omega}^{\mathrm{IO,RT}}|_{n=n_e^{\mathrm{LV}}} = \sum_{l \in n \rightarrow} q_{l\omega}^{\mathrm{RT}} - \sum_{l \in \rightarrow n} q_{l\omega}^{\mathrm{RT}}, \ \forall \omega, n \in N_e^{\mathrm{D}} \ : (\lambda_{n\omega}^{\mathrm{q,RT}}) \tag{A.1m}$$

$$p_{l\omega}^{\mathrm{RT}^2} + q_{l\omega}^{\mathrm{RT}^2} \leq \varphi_{l\omega}^{\mathrm{RT}} v_{n\omega}^{\mathrm{RT}}, \quad \forall \omega, l \in L_e^{\mathrm{D}} \ : (\gamma_{l\omega}) \tag{A.1n}$$

$$p_{l\omega}^{\mathrm{RT}} + p_{l'\omega}^{\mathrm{RT}} = R_l \varphi_{l\omega}^{\mathrm{RT}}, \quad \forall \omega, l \in L_e^{\mathrm{D}} \ : (\mu_{l\omega}^{\mathrm{p}}) \tag{A.1o}$$

$$q_{l\omega}^{\mathrm{RT}} + q_{l'\omega}^{\mathrm{RT}} = X_l \varphi_{l\omega}^{\mathrm{RT}}, \quad \forall \omega, l \in L_e^{\mathrm{D}} \ : (\mu_{l\omega}^{\mathrm{q}}) \tag{A.1p}$$

$$p_{l\omega}^{\mathrm{RT}^2} + q_{l\omega}^{\mathrm{RT}^2} \leq S_l, \quad \forall \omega, l \in L_e^{\mathrm{D}} \ : (\eta_{l\omega}) \tag{A.1q}$$

$$v_{m\omega}^{\mathrm{RT}} = v_{n\omega}^{\mathrm{RT}} - 2(R_l p_{l\omega}^{\mathrm{RT}} + X_l q_{l\omega}^{\mathrm{RT}}) + (R_l^2 + X_l^2)\varphi_{l\omega}^{\mathrm{RT}}, \forall \omega, l \in L_e^{\mathrm{D}} \quad : (\beta_{l\omega}) \tag{A.1r}$$

$$\underline{V}_n^2 \leq v_{n\omega}^{\mathrm{RT}} \leq \overline{V}_n^2, \quad \forall \omega, n \in N_e^{\mathrm{D}} \quad : (\sigma_{n\omega}^{-}, \sigma_{n\omega}^{+}) \tag{A.1s}$$

$$0 \leq w_{r\omega}^{\mathrm{RT}} \leq W_{r\omega}^{\mathrm{RT}}, \quad \forall \omega, n \in N_e \quad : (\nu_{n\omega}^{-}, \nu_{n\omega}^{+}) \tag{A.1t}$$

$$\underline{P}_g \leq p_{g\omega}^{\mathrm{RT}} \leq \overline{P}_g, \quad \forall \omega, g \in G_e \quad : (\varsigma_{g\omega}^{\mathrm{RT}-}, \varsigma_{g\omega}^{\mathrm{RT}+}) \tag{A.1u}$$

$$\underline{P}_d \leq p_{d\omega}^{\mathrm{RT}} \leq \overline{P}_d, \quad \forall \omega, d \in D_e \quad : (\varsigma_{d\omega}^{\mathrm{RT}-}, \varsigma_{d\omega}^{\mathrm{RT}+}) \tag{A.1v}$$

$$\underline{Q}_g \leq q_{g\omega}^{\mathrm{RT}} \leq \overline{Q}_g, \quad \forall \omega, g \in G_e \quad : (\kappa_{g\omega}^{\mathrm{RT}-}, \kappa_{g\omega}^{\mathrm{RT}+}) \tag{A.1w}$$

$$\underline{Q}_d \leq q_{d\omega}^{\mathrm{RT}} \leq \overline{Q}_d, \quad \forall \omega, d \in D_e \quad : (\kappa_{d\omega}^{\mathrm{RT}-}, \kappa_{d\omega}^{\mathrm{RT}+}) \tag{A.1x}$$

$$\underline{f}_e \leq p_{e\omega}^{\mathrm{IO,RT}} \leq \overline{f}_e, \quad \forall \omega \quad : (\rho_{e\omega}^{\mathrm{RT}-}, \rho_{e\omega}^{\mathrm{RT}+}) \tag{A.1y}$$

$$p_{g\omega}^{\uparrow} \geq 0, \ \forall \omega, g \ : (\epsilon_{g\omega}^{\mathrm{p}\uparrow}), \quad p_{g\omega}^{\downarrow} \geq 0, \ \forall \omega, g \ : (\epsilon_{g\omega}^{\mathrm{p}\downarrow}) \tag{A.1z}$$

$$p_{d\omega}^{\uparrow} \geq 0, \ \forall d, \omega \ : (\varepsilon_{d\omega}^{\mathrm{p}\uparrow}), \quad p_{d\omega}^{\downarrow} \geq 0, \ \forall d, \omega \ : (\varepsilon_{d\omega}^{\mathrm{p}\downarrow}) \tag{A.1aa}$$

$$p_{e\omega}^{\uparrow \mathrm{IO}} \geq 0, \ \forall \omega, \ : (\epsilon_{e\omega}^{\uparrow \mathrm{IO}}), \ p_{e\omega}^{\downarrow \mathrm{IO}} \geq 0, \ \forall \omega, \ : (\epsilon_{e\omega}^{\downarrow \mathrm{IO}}) \tag{A.1ab}$$

$$0 \leq s_{n\omega}^{\mathrm{RT}} \leq \sum_{d \in D_n} p_{d\omega}^{\mathrm{RT}}, \ \forall \omega, n \in N_e^{\mathrm{D}}, \ : (\Upsilon_{n\omega}^{\mathrm{RT}-}, \Upsilon_{n\omega}^{\mathrm{RT}+}) \tag{A.1ac}$$

$$w_{r\omega}^{\uparrow} \geq 0, \ \forall \omega, w \ : (\epsilon_{r\omega}^{\mathrm{p}\uparrow}), \quad w_{r\omega}^{\downarrow} \geq 0, \ \forall \omega, w \ : (\epsilon_{r\omega}^{\mathrm{p}\downarrow}), \tag{A.1ad}$$

where the set of primal variables is $\Xi^{\mathrm{E}} = \{\widetilde{p}_g^{\mathrm{DA}}, \widetilde{p}_d^{\mathrm{DA}}, p_{g\omega}^{\mathrm{RT}}, p_{g\omega}^{\uparrow}, p_{g\omega}^{\downarrow}, p_{d\omega}^{\mathrm{RT}}, p_{d\omega}^{\uparrow}, p_{d\omega}^{\downarrow}, q_{g\omega}^{\mathrm{RT}}, q_{d\omega}^{\mathrm{RT}}, s_{n\omega}^{\mathrm{RT}}, s_e^{\mathrm{DA}}, w_{n\omega}^{\mathrm{RT}}, p_{l\omega}^{\mathrm{RT}}, q_{l\omega}^{\mathrm{RT}}, \varphi_{l\omega}^{\mathrm{RT}}, v_{n\omega}^{\mathrm{RT}}, w_e^{\mathrm{DA}}, p_e^{\mathrm{IO,DA}}, p_e^{\mathrm{IO,RT}}, p_e^{\uparrow \mathrm{IO}}, p_e^{\downarrow \mathrm{IO}}, s_{n\omega}^{\mathrm{q,RT}}\}$. The KKT conditions of lower-level problem (A.1) are

$$\text{(A.1b)}, \text{(A.1h)} - \text{(A.1m)}, \text{(A.1o)}, \text{(A.1p)}, \text{(A.1r)} \tag{A.2a}$$

$$(\widetilde{p}_g^{\mathrm{DA}}):\ \pi_g^{\mathrm{DA}} - \sum_{\omega} \phi_\omega \pi_g^{\mathrm{DA}} - \lambda_e^{\mathrm{DA}} - \varsigma_g^{\mathrm{DA}-} + \varsigma_g^{\mathrm{DA}+} + \sum_{\omega} \zeta_{g\omega}^{\mathrm{p}} = 0, \quad \forall g \in G_e^{\mathrm{D}} \tag{A.2b}$$

$$(\widetilde{p}_d^{\mathrm{DA}}):\ \sum_{\omega} \left(\zeta_{d\omega}^{\mathrm{p}} + \phi_\omega \pi_d^{\mathrm{DA}}\right) - \pi_d^{\mathrm{DA}} + \lambda_e^{\mathrm{DA}} - \varsigma_d^{\mathrm{DA}-} + \varsigma_d^{\mathrm{DA}+} - \Upsilon_e^{\mathrm{DA}+} = 0, \quad \forall d \in D_e^{\mathrm{D}} \tag{A.2c}$$

$$(p_{g\omega}^{\uparrow}):\ \phi_\omega \pi_g^{\uparrow} + \zeta_{g\omega}^{\mathrm{p}} - \epsilon_{g\omega}^{\mathrm{p}\uparrow} = 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2d}$$

$$(p_{g\omega}^{\downarrow}):\ \phi_\omega \pi_g^{\downarrow} - \zeta_{g\omega}^{\mathrm{p}} - \epsilon_{g\omega}^{\mathrm{p}\downarrow} = 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2e}$$

$$(p_{d\omega}^{\uparrow}):\ \phi_\omega \pi_d^{\uparrow} - \zeta_{d\omega}^{\mathrm{p}} - \varepsilon_{i\omega}^{\mathrm{p}\uparrow} = 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2f}$$

$$(p_{d\omega}^{\downarrow}):\ \phi_\omega \pi_d^{\downarrow} + \zeta_{d\omega}^{\mathrm{p}} - \varepsilon_{d\omega}^{\mathrm{p}\downarrow} = 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2g}$$

$$(w_{r\omega}^{\uparrow}):\ \phi_\omega \pi^{\uparrow \mathrm{R}} + \zeta_{r\omega}^{\mathrm{p}} - \epsilon_{r\omega}^{\mathrm{p}\uparrow} = 0, \quad \forall \omega, r \in R_e^{\mathrm{D}} \tag{A.2h}$$

$$(w_{r\omega}^{\downarrow}):\ \phi_\omega \pi^{\downarrow \mathrm{R}} - \zeta_{r\omega}^{\mathrm{p}} - \epsilon_{r\omega}^{\mathrm{p}\downarrow} = 0, \quad \forall \omega, r \in R_e^{\mathrm{D}} \tag{A.2i}$$

$$(s_e^{\mathrm{DA}}): VOLL - \lambda_e^{\mathrm{DA}} - \Upsilon_e^{\mathrm{DA}-} + \Upsilon_e^{\mathrm{DA}+} = 0 \tag{A.2j}$$

$$(s_{n\omega}^{\mathrm{RT}}): \phi_\omega VOLL - \lambda_{n\omega}^{\mathrm{p,RT}} - \Upsilon_{n\omega}^{\mathrm{RT}-} + \Upsilon_{n\omega}^{\mathrm{RT}+} = 0,\ \forall \omega, n \in N_e^{\mathrm{D}} \tag{A.2k}$$

$$(w_{r\omega}^{\mathrm{RT}}): \phi_\omega \pi^{\mathrm{R}} - \zeta_{r\omega}^{\mathrm{p}} - \left[\lambda_{n\omega}^{\mathrm{p,RT}}\right]_{n_r} + \nu_{r\omega}^{+} - \nu_{r\omega}^{-} = 0, \quad \forall \omega, r \in R_e^{\mathrm{D}} \tag{A.2l}$$

$$(w_r^{\mathrm{DA}}): \pi^{\mathrm{R}} - \sum_{\omega} \phi_\omega \pi^{\mathrm{R}} - \lambda_e^{\mathrm{DA}} - \iota_r^{-} + \iota_r^{+} + \sum_{\omega} \zeta_{r\omega}^{\mathrm{p}} = 0, \forall r \in R_e^{\mathrm{D}} \tag{A.2m}$$

$$(p_{g\omega}^{\mathrm{RT}}): \phi_\omega \pi_g^{\mathrm{DA}} - \zeta_{g\omega}^{\mathrm{p}} - \varsigma_{g\omega}^{\mathrm{RT}-} + \varsigma_{g\omega}^{\mathrm{RT}+} - \left[\lambda_{n\omega}^{\mathrm{p,RT}}\right]_{n_g} = 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2n}$$

$$(q_{g\omega}^{\mathrm{RT}}): -\kappa_{g\omega}^{\mathrm{RT}-} + \kappa_{g\omega}^{\mathrm{RT}+} - \left[\lambda_{n\omega}^{\mathrm{q,RT}}\right]_{n_g} = 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2o}$$

$$(p_{d\omega}^{\mathrm{RT}}): -\phi_\omega \pi_d^{\mathrm{DA}} - \zeta_{d\omega}^{\mathrm{p}} - \varsigma_{d\omega}^{\mathrm{RT}-} + \varsigma_{d\omega}^{\mathrm{RT}+} + \left[\lambda_{n\omega}^{\mathrm{p,RT}} - \Upsilon_{n\omega}^{\mathrm{RT}+}\right]_{n_d} = 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2p}$$

$$(q_{d\omega}^{\mathrm{RT}}): -\kappa_{d\omega}^{\mathrm{RT}-} + \kappa_{d\omega}^{\mathrm{RT}+} + \left[\lambda_{n\omega}^{\mathrm{q,RT}}\right]_{n_d} = 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2q}$$

$$\begin{aligned}(p_{l\omega}^{\mathrm{RT}}): &\ \lambda_{n\omega}^{\mathrm{p,RT}} - \lambda_{m\omega}^{\mathrm{p,RT}} + 2\gamma_{l\omega} p_{l\omega}^{\mathrm{RT}} - \mu_{l\omega}^{\mathrm{p}} - \mu_{l'\omega}^{\mathrm{p}} + 2\eta_{l\omega} p_{l\omega}^{\mathrm{RT}} \\ &- 2\beta_{l\omega} R_l = 0, \quad \forall \omega, l = (n,m) \in L_e^{\mathrm{D}}\end{aligned} \tag{A.2r}$$

$$\begin{aligned}(q_{l\omega}^{\mathrm{RT}}): &\ \lambda_{n\omega}^{\mathrm{q,RT}} - \lambda_{m\omega}^{\mathrm{q,RT}} + 2\gamma_{l\omega} q_{l\omega}^{\mathrm{RT}} - \mu_{l\omega}^{\mathrm{q}} - \mu_{l'\omega}^{\mathrm{q}} + 2\eta_{l\omega} q_{l\omega}^{\mathrm{RT}} \\ &- 2\beta_{l\omega} X_l = 0, \quad \forall \omega, l = (n,m) \in L_e^{\mathrm{D}}\end{aligned} \tag{A.2s}$$

$$\begin{aligned}(\varphi_{l\omega}^{\mathrm{RT}}): &-\gamma_{l\omega} v_{n\omega}^{\mathrm{RT}} + \mu_{l\omega}^{\mathrm{p}} R_l + \mu_{l\omega}^{\mathrm{q}} X_l + \beta_{l\omega}(R_l^2 + X_l^2) \\ &= 0, \quad \forall \omega, l = (n,m) \in L_e^{\mathrm{D}}\end{aligned} \tag{A.2t}$$

$$(v_{n\omega}^{\mathrm{RT}}): -\gamma_{l\omega} \varphi_{l\omega}^{\mathrm{RT}} - \beta_{l'\omega} + \beta_{l\omega} - \sigma_{n\omega}^{-} + \sigma_{n\omega}^{+} = 0, \quad \forall \omega, l = (n,m) \in L_e^{\mathrm{D}} \tag{A.2u}$$

$$(p_e^{\mathrm{IO,DA}}): \pi_e^{\mathrm{IO,DA}} + \sum_{\omega} \left(\phi_\omega \pi_e^{\mathrm{IO,DA}} + \zeta_{e\omega}^{\mathrm{IO}}\right) - \lambda_e^{\mathrm{DA}} - \rho_e^{\mathrm{DA}-} + \rho_e^{\mathrm{DA}+} = 0 \tag{A.2v}$$

$$(p_{e\omega}^{\mathrm{IO,RT}}): \phi_\omega \pi_e^{\mathrm{IO,DA}} - \left[\lambda_{n\omega}^{\mathrm{p,RT}}\right]_{n_e^{\mathrm{LV}}} - \zeta_{e\omega}^{\mathrm{IO}} - \rho_{e\omega}^{\mathrm{RT}-} + \rho_{e\omega}^{\mathrm{RT}+} = 0, \quad \forall \omega \tag{A.2w}$$

$$(q_{e\omega}^{\mathrm{IO,RT}}): -\left[\lambda_{n\omega}^{\mathrm{q,RT}}\right]_{n_e^{\mathrm{LV}}} = 0, \quad \forall \omega \tag{A.2x}$$

$$(p_{e\omega}^{\uparrow \mathrm{IO}}): \phi_\omega \pi_e^{\uparrow \mathrm{IO}} + \zeta_{e\omega}^{\mathrm{IO}} - \epsilon_{e\omega}^{\uparrow \mathrm{IO}} = 0, \quad \forall \omega \tag{A.2y}$$

$$(p_{e\omega}^{\downarrow \mathrm{IO}}): \phi_\omega \pi_e^{\downarrow \mathrm{IO}} - \zeta_{e\omega}^{\mathrm{IO}} - \epsilon_{e\omega}^{\downarrow \mathrm{IO}} = 0, \quad \forall \omega \tag{A.2z}$$

$$0 \le \varsigma_g^{\mathrm{DA}+} \perp \overline{P}_g - \widetilde{p}_g^{\mathrm{DA}} \ge 0, \quad \forall g \in G_e^{\mathrm{D}} \tag{A.2aa}$$

$$0 \le \varsigma_g^{\mathrm{DA}-} \perp \widetilde{p}_g^{\mathrm{DA}} - \underline{P}_g \ge 0, \quad \forall g \in G_e^{\mathrm{D}} \tag{A.2ab}$$

$$0 \le \varsigma_d^{\mathrm{DA}+} \perp \overline{P}_d - \widetilde{p}_d^{\mathrm{DA}} \ge 0, \quad \forall d \in D_e^{\mathrm{D}} \tag{A.2ac}$$

$$0 \leq \varsigma_d^{\mathrm{DA}-} \perp \tilde{p}_d^{\mathrm{DA}} - \underline{P}_d \geq 0, \quad \forall d \in D_e^{\mathrm{D}} \tag{A.2ad}$$
$$0 \leq \iota_r^- \perp w_r^{\mathrm{DA}} \geq 0, \quad \forall r \in R_e^{\mathrm{D}} \tag{A.2ae}$$
$$0 \leq \iota_r^+ \perp W_r^{\mathrm{DA}} - w_r^{\mathrm{DA}} \geq 0, \quad \forall r \in R_e^{\mathrm{D}} \tag{A.2af}$$
$$0 \leq \rho_e^{\mathrm{DA}-} \perp p_e^{\mathrm{IO,DA}} - \underline{f}_e \geq 0 \tag{A.2ag}$$
$$0 \leq \rho_e^{\mathrm{DA}+} \perp \overline{f}_e - p_e^{\mathrm{IO,DA}} \geq 0 \tag{A.2ah}$$
$$0 \leq \gamma_{l\omega} \perp \varphi_{l\omega}^{\mathrm{RT}} v_{n\omega}^{\mathrm{RT}} - (p_{l\omega}^{\mathrm{RT}^2} + q_{l\omega}^{\mathrm{RT}^2}) \geq 0,\ \forall \omega, l \in L_e^{\mathrm{D}} \tag{A.2ai}$$
$$0 \leq \eta_{l\omega} \perp S_l - p_{l\omega}^{\mathrm{RT}^2} - q_{l\omega}^{\mathrm{RT}^2} \geq 0,\ \forall \omega, l \in L_e^{\mathrm{D}} \tag{A.2aj}$$
$$0 \leq \sigma_{n\omega}^- \perp v_{n\omega}^{\mathrm{RT}} - \underline{V}_n^2 \geq 0, \quad \forall \omega, n \in N_e^{\mathrm{D}} \tag{A.2ak}$$
$$0 \leq \sigma_{n\omega}^+ \perp \overline{V}_n^2 - v_{n\omega}^{\mathrm{RT}} \geq 0, \quad \forall \omega, n \in N_e^{\mathrm{D}} \tag{A.2al}$$
$$0 \leq \nu_{r\omega}^- \perp w_{r\omega}^{\mathrm{RT}} \geq 0, \quad \forall \omega, r \in R_e^{\mathrm{D}} \tag{A.2am}$$
$$0 \leq \nu_{r\omega}^+ \perp W_{r\omega}^{\mathrm{RT}} - w_{r\omega}^{\mathrm{RT}} \geq 0, \quad \forall \omega, r \in R_e^{\mathrm{D}} \tag{A.2an}$$
$$0 \leq \varsigma_{g\omega}^{\mathrm{RT}-} \perp p_{g\omega}^{\mathrm{RT}} - \underline{P}_g \geq 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2ao}$$
$$0 \leq \varsigma_{g\omega}^{\mathrm{RT}+} \perp \overline{P}_g - p_{g\omega}^{\mathrm{RT}} \geq 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2ap}$$
$$0 \leq \varsigma_{d\omega}^{\mathrm{RT}-} \perp p_{d\omega}^{\mathrm{RT}} - \underline{P}_d \geq 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2aq}$$
$$0 \leq \varsigma_{d\omega}^{\mathrm{RT}+} \perp \overline{P}_d - p_{d\omega}^{\mathrm{RT}} \geq 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2ar}$$
$$0 \leq \kappa_{g\omega}^{\mathrm{RT}+} \perp q_{g\omega}^{\mathrm{RT}} - \underline{Q}_g \geq 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2as}$$
$$0 \leq \kappa_{g\omega}^{\mathrm{RT}-} \perp \overline{Q}_g - q_{g\omega}^{\mathrm{RT}} \geq 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2at}$$
$$0 \leq \kappa_{d\omega}^{\mathrm{RT}+} \perp q_{d\omega}^{\mathrm{RT}} - \underline{Q}_d \geq 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2au}$$
$$0 \leq \kappa_{d\omega}^{\mathrm{RT}-} \perp \overline{Q}_d - q_{d\omega}^{\mathrm{RT}} \geq 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2av}$$
$$0 \leq \rho_{e\omega}^{\mathrm{RT}-} \perp p_{e\omega}^{\mathrm{IO,RT}} - \underline{f}_e \geq 0, \quad \forall \omega \tag{A.2aw}$$
$$0 \leq \rho_{e\omega}^{\mathrm{RT}+} \perp \overline{f}_e - p_{e\omega}^{\mathrm{IO,RT}} \geq 0, \quad \forall \omega \tag{A.2ax}$$
$$0 \leq \epsilon_{g\omega}^{\mathrm{p}\uparrow} \perp p_{g\omega}^{\uparrow} \geq 0, \quad 0 \leq \epsilon_{g\omega}^{\mathrm{p}\downarrow} \perp p_{g\omega}^{\downarrow} \geq 0, \quad \forall \omega, g \in G_e^{\mathrm{D}} \tag{A.2ay}$$
$$0 \leq \epsilon_{r\omega}^{\mathrm{p}\uparrow} \perp w_{r\omega}^{\uparrow} \geq 0,\ 0 \leq \epsilon_{r\omega}^{\mathrm{p}\downarrow} \perp w_{r\omega}^{\downarrow} \geq 0,\ \forall \omega, r \in R_e^{\mathrm{D}} \tag{A.2az}$$
$$0 \leq \varepsilon_{d\omega}^{\mathrm{p}\uparrow} \perp p_{d\omega}^{\uparrow} \geq 0, \quad 0 \leq \varepsilon_{d\omega}^{\mathrm{p}\downarrow} \perp p_{d\omega}^{\downarrow} \geq 0, \quad \forall \omega, d \in D_e^{\mathrm{D}} \tag{A.2ba}$$
$$0 \leq \epsilon_{e\omega}^{\uparrow\mathrm{IO}} \perp p_{e\omega}^{\uparrow\mathrm{IO}} \geq 0, \quad 0 \leq \epsilon_{e\omega}^{\downarrow\mathrm{IO}} \perp p_{e\omega}^{\downarrow\mathrm{IO}} \geq 0, \quad \forall \omega \tag{A.2bb}$$
$$0 \leq \Upsilon_{n\omega}^{\mathrm{RT}-} \perp s_{n\omega}^{\mathrm{RT}} \geq 0, \forall n, \omega \tag{A.2bc}$$
$$0 \leq \Upsilon_{n\omega}^{\mathrm{RT}+} \perp \sum_{d \in D_n} p_{d\omega}^{\mathrm{RT}} - s_{n\omega}^{\mathrm{RT}} \geq 0,\ \forall n, \omega \tag{A.2bd}$$
$$0 \leq \Upsilon_e^{\mathrm{DA}-} \perp s_e^{\mathrm{DA}} \geq 0 \tag{A.2be}$$
$$0 \leq \Upsilon_e^{\mathrm{DA}+} \perp \sum_d p_d^{\mathrm{DA}} - s_e^{\mathrm{DA}} \geq 0. \tag{A.2bf}$$

Appendix B. KKT Conditions of the Lower-Level Problem (3) Representing the Day-Ahead Market Clearing

For convenience, the lower-level problem (3) within the paper is repeated here by (B.1) below, while the dual variable for every constraint is defined:

$$\max_{\Xi^{\mathrm{DA}}} \mathcal{SW}^{\mathrm{DA}} = \sum_{d\in D} \pi_d^{\mathrm{DA}} \widehat{p}_d^{\mathrm{DA}} - \sum_{g\in G} \pi_g^{\mathrm{DA}} \widehat{p}_g^{\mathrm{DA}} - VOLL\ s^{\mathrm{DA}} - \pi^{\mathrm{R}} \sum_r w_r^{\mathrm{DA}} \tag{B.1a}$$

subject to:

$$\sum_{g\in G} \widehat{p}_g^{\mathrm{DA}} - \sum_{d\in D} \widehat{p}_d^{\mathrm{DA}} + \sum_r w_r^{\mathrm{DA}} + s^{\mathrm{DA}} = 0,\ : (\lambda^{\mathrm{T,DA}}) \tag{B.1b}$$

$$\underline{P}_g \le \widehat{p}_g^{\mathrm{DA}} \le \widetilde{p}_g^{\mathrm{DA}},\ \forall g \in G_e^{\mathrm{D}},\ \forall e \in E\ : (\varsigma_{ge}^{\mathrm{T,DA}-}, \varsigma_{ge}^{\mathrm{T,DA}+}) \tag{B.1c}$$

$$\underline{P}_g \le \widehat{p}_g^{\mathrm{DA}} \le \overline{P}_g,\ \forall g \in G^{\mathrm{T}}\ : (\sigma_g^{\mathrm{T,DA}-}, \sigma_g^{\mathrm{T,DA}+}) \tag{B.1d}$$

$$\underline{P}_d \le \widehat{p}_d^{\mathrm{DA}} \le \widetilde{p}_d^{\mathrm{DA}},\ \forall d \in D_e^{\mathrm{D}},\ \forall e \in E\ : (\varsigma_{de}^{\mathrm{T,DA}-}, \varsigma_{de}^{\mathrm{T,DA}+}) \tag{B.1e}$$

$$\underline{P}_d \le \widehat{p}_d^{\mathrm{DA}} \le \overline{P}_d,\ \forall d \in D^{\mathrm{T}}\ : (\sigma_d^{\mathrm{T,DA}-}, \sigma_d^{\mathrm{T,DA}+}) \tag{B.1f}$$

$$0 \le w_r^{\mathrm{DA}} \le W_r^{\mathrm{DA}},\ \forall r \in R\ : (\nu_r^{\mathrm{T,DA}-}, \nu_r^{\mathrm{T,DA}+}) \tag{B.1g}$$

$$0 \le s^{\mathrm{DA}} \le \sum_d \widehat{p}_d^{\mathrm{DA}},\ : (\rho^{\mathrm{T,DA}-}, \rho^{\mathrm{T,DA}+}). \tag{B.1h}$$

Recall that $\widetilde{p}_g^{\mathrm{DA}}$ and $\widetilde{p}_d^{\mathrm{DA}}$ are the outputs of problem (A.1), and treated as parameters within (B.1). Therefore, the KKT conditions associated with (B.1) are

$$\text{(B.1b)} \tag{B.2a}$$

$$\begin{aligned}(\widehat{p}_g^{\mathrm{DA}}): \quad & \pi_g^{\mathrm{DA}} - \left[\varsigma_{ge}^{\mathrm{T,DA}-} - \varsigma_{ge}^{\mathrm{T,DA}+}\right]_{g\in G_e^{\mathrm{D}}} - \left[\sigma_g^{\mathrm{T,DA}-} - \sigma_g^{\mathrm{T,DA}+}\right]_{g\in G^{\mathrm{T}}} \\ & - \lambda^{\mathrm{T,DA}} = 0, \quad \forall g \in G\end{aligned} \tag{B.2b}$$

$$\begin{aligned}(\widehat{p}_d^{\mathrm{DA}}): \quad & -\pi_d^{\mathrm{DA}} - \left[\varsigma_{de}^{\mathrm{T,DA}-} - \varsigma_{de}^{\mathrm{T,DA}+}\right]_{d\in D_e^{\mathrm{D}}} - \left[\sigma_d^{\mathrm{T,DA}-} - \sigma_d^{\mathrm{T,DA}+}\right]_{d\in D^{\mathrm{T}}} - \rho^{\mathrm{T,DA}+} \\ & + \lambda^{\mathrm{T,DA}} = 0, \quad \forall d \in D\end{aligned} \tag{B.2c}$$

$$(s^{\mathrm{DA}}): \quad VOLL - \lambda^{\mathrm{T,DA}} - \rho^{\mathrm{T,DA}-} + \rho^{\mathrm{T,DA}+} = 0 \tag{B.2d}$$

$$(w_r^{\mathrm{DA}}): \quad \pi^{\mathrm{R}} - \lambda^{\mathrm{T,DA}} - \left[\nu_r^{\mathrm{T,DA}-} - \nu_r^{\mathrm{T,DA}+}\right] = 0,\ \forall r \in R \tag{B.2e}$$

$$0 \le \varsigma_{ge}^{\mathrm{T,DA}-} \perp \widehat{p}_g^{\mathrm{DA}} - \underline{P}_g \ge 0 \quad \forall g, e \tag{B.2f}$$

$$0 \le \varsigma_{ge}^{\mathrm{T,DA}+} \perp \widetilde{p}_g^{\mathrm{DA}} - \widehat{p}_g^{\mathrm{DA}} \ge 0 \quad \forall g \in G_e^{\mathrm{D}}, e \in E \tag{B.2g}$$

$$0 \le \sigma_g^{\mathrm{T,DA}+} \perp \overline{P}_g - \widehat{p}_g^{\mathrm{DA}} \ge 0 \quad \forall g \in G^{\mathrm{T}} \tag{B.2h}$$

$$0 \le \varsigma_{de}^{\mathrm{T,DA}-} \perp \widehat{p}_d^{\mathrm{DA}} - \underline{P}_d \ge 0 \quad \forall d, e \tag{B.2i}$$

$$0 \le \varsigma_{de}^{\mathrm{T,DA}+} \perp \widetilde{p}_d^{\mathrm{DA}} - \widehat{p}_d^{\mathrm{DA}} \ge 0 \quad \forall d \in D_e^{\mathrm{D}}, e \in E \tag{B.2j}$$

$$0 \le \sigma_d^{\mathrm{T,DA}+} \perp \overline{P}_d - \widehat{p}_d^{\mathrm{DA}} \ge 0 \quad \forall d \in D^{\mathrm{T}} \tag{B.2k}$$

$$0 \le \nu_r^{\mathrm{T,DA}-} \perp w_r^{\mathrm{DA}} \ge 0,\ \forall r \in R \tag{B.2l}$$

$$0 \le \nu_r^{\mathrm{T,DA}+} \perp W_r^{\mathrm{DA}} - w_r^{\mathrm{DA}} \ge 0,\ \forall r \in R \tag{B.2m}$$

$$0 \le \rho^{\mathrm{T,DA}-} \perp s^{\mathrm{DA}} \ge 0 \tag{B.2n}$$

$$0 \leq \rho^{\mathrm{T,DA}+} \perp \sum_{d} \widehat{p}_d^{\mathrm{DA}} - s^{\mathrm{DA}} \geq 0. \tag{B.2o}$$

Appendix C. KKT Conditions of the Lower-Level Problem (4) Representing the Real-Time Market Clearing

Recall that the KKT conditions of the lower-level problem (4) for the real-time market clearing under each scenario ω are not used in the paper, because the proposed Benders' decomposition renders this lower-level problem to be solved as a single problem. However, the KKT conditions derived in this appendix for SOCP problem (4) are used for proving the redundancy of the DSO market-clearing problem (A.1).

Pursuing convenience, the lower-level problem (4) for each scenario ω is repeated here by (C.1) below, while the objective function is multiplied by the probability of the corresponding scenario, i.e., ϕ_ω, and the dual variable for every constraint is defined:

$$\min_{\Xi^{\mathrm{RT}}} \phi_\omega \ \Delta\mathrm{Cost}_\omega^{\mathrm{RT}} =$$

$$\phi_\omega \Bigg[\sum_{g \in G} \Big(\pi_g^{\mathrm{DA}} (p_{g\omega}^{\mathrm{RT}} - \widehat{p}_g^{\mathrm{DA}}) + \pi_g^{\uparrow} p_{g\omega}^{\uparrow} + \pi_g^{\downarrow} p_{g\omega}^{\downarrow} \Big) + \sum_{d \in D} \Big(\pi_d^{\mathrm{DA}} (\widehat{p}_d^{\mathrm{DA}} - p_{d\omega}^{\mathrm{RT}}) + \pi_d^{\uparrow} p_{d\omega}^{\uparrow} + \pi_d^{\downarrow} p_{d\omega}^{\downarrow} \Big)$$

$$+ \sum_{n \in N} VOLL \ s_{n\omega}^{\mathrm{RT}} + \sum_{r} \Big(\pi^{\mathrm{R}} (w_{r\omega}^{\mathrm{RT}} - w_r^{\mathrm{DA}}) + \pi^{\uparrow\mathrm{R}} w_{r\omega}^{\uparrow} + \pi^{\downarrow\mathrm{R}} w_{r\omega}^{\downarrow} \Big) \Bigg] \tag{C.1a}$$

subject to:

$$p_{l\omega}^{\mathrm{RT}} = B_l (\theta_{n\omega} - \theta_{m\omega}), \quad \forall l \in L^{\mathrm{T}}, \ : (\gamma_{l\omega}^{\mathrm{T}}) \tag{C.1b}$$

$$p_{l\omega}^{\mathrm{RT}} \leq S_l, \ \forall l \in L^{\mathrm{T}}, \ : (\eta_{l\omega}^{\mathrm{T}}) \tag{C.1c}$$

$$p_{g\omega}^{\mathrm{RT}} = \widehat{p}_g^{\mathrm{DA}} + p_{g\omega}^{\uparrow} - p_{g\omega}^{\downarrow}, \quad \forall g \in G, \ : (\zeta_{g\omega}^{\mathrm{p,RT}}) \tag{C.1d}$$

$$p_{d\omega}^{\mathrm{RT}} = \widehat{p}_d^{\mathrm{DA}} - p_{d\omega}^{\uparrow} + p_{d\omega}^{\downarrow}, \quad \forall d \in D, \ : (\zeta_{d\omega}^{\mathrm{p,RT}}) \tag{C.1e}$$

$$w_{r\omega}^{\mathrm{RT}} = w_r^{\mathrm{DA}} + w_{r\omega}^{\uparrow} - w_{r\omega}^{\downarrow}, \ \forall r \in R, \ : (\zeta_{r\omega}^{\mathrm{p,RT}}) \tag{C.1f}$$

$$\sum_{g \in G_n} p_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} p_{d\omega}^{\mathrm{RT}} + \sum_{r \in R_n} w_{r\omega}^{\mathrm{RT}} + s_{n\omega}^{\mathrm{RT}} = \sum_{l \in n\rightarrow} p_{l\omega}^{\mathrm{RT}} - \sum_{l \in \rightarrow n} p_{l\omega}^{\mathrm{RT}}, \ \forall n \in N, \ : (\lambda_{n\omega}^{\mathrm{p,RT}}) \tag{C.1g}$$

$$\sum_{g \in G_n} q_{g\omega}^{\mathrm{RT}} - \sum_{d \in D_n} q_{d\omega}^{\mathrm{RT}} + s_{n\omega}^{\mathrm{q,RT}} = \sum_{l \in n\rightarrow} q_{l\omega}^{\mathrm{RT}} - \sum_{l \in \rightarrow n} q_{l\omega}^{\mathrm{RT}}, \ \forall n \in N_e^{\mathrm{D}}, \ : (\lambda_{n\omega}^{\mathrm{q,RT}}) \tag{C.1h}$$

$$p_{l\omega}^{\mathrm{RT}^2} + q_{l\omega}^{\mathrm{RT}^2} \leq \varphi_{l\omega}^{\mathrm{RT}} v_{n\omega}^{\mathrm{RT}}, \ \forall l \in L_e^{\mathrm{D}} \cup l_e, \ : (\gamma_{l\omega}^{\mathrm{D,RT}}) \tag{C.1i}$$

$$p_{l\omega}^{\mathrm{RT}} + p_{l'\omega}^{\mathrm{RT}} = R_l \varphi_{l\omega}^{\mathrm{RT}}, \ \forall l \in L_e^{\mathrm{D}} \cup l_e, \ : (\mu_{l\omega}^{\mathrm{p,RT}}) \tag{C.1j}$$

$$q_{l\omega}^{\mathrm{RT}} + q_{l'\omega}^{\mathrm{RT}} = X_l \varphi_{l\omega}^{\mathrm{RT}}, \ \forall l \in L_e^{\mathrm{D}} \cup l_e, \ : (\mu_{l\omega}^{\mathrm{q,RT}}) \tag{C.1k}$$

$$p_{l\omega}^{\mathrm{RT}^2} + q_{l\omega}^{\mathrm{RT}^2} \leq S_l^2, \ \forall l \in L_e^{\mathrm{D}} \cup l_e, \ : (\eta_{l\omega}^{\mathrm{D}}) \tag{C.1l}$$

$$v_{m\omega}^{\mathrm{RT}} = v_{n\omega}^{\mathrm{RT}} - 2(R_l p_{l\omega}^{\mathrm{RT}} + X_l q_{l\omega}^{\mathrm{RT}}) + (R_l^2 + X_l^2) \varphi_{l\omega}^{\mathrm{RT}}, \ \forall l \in L_e^{\mathrm{D}} \cup l_e, \ : (\beta_{l\omega}^{\mathrm{RT}}) \tag{C.1m}$$

$$\underline{V}_n^2 \leq v_{n\omega}^{\mathrm{RT}} \leq \overline{V}_n^2, \quad \forall e, n \in N_e^{\mathrm{D}}, \ : (\sigma_{n\omega}^{\mathrm{RT}-}, \sigma_{n\omega}^{\mathrm{RT}+}) \tag{C.1n}$$

$$0 \leq w_{r\omega}^{\mathrm{RT}} \leq W_{r\omega}^{\mathrm{RT}}, \quad \forall r \in R, \ : (\nu_{r\omega}^{\mathrm{RT}-}, \nu_{r\omega}^{\mathrm{RT}+}) \tag{C.1o}$$

$$\underline{P}_g \le p^{\mathrm{RT}}_{g\omega} \le \overline{P}_g, \quad \forall g \in G, \; : (\varsigma^{\mathrm{RT}-}_{g\omega}, \varsigma^{\mathrm{RT}+}_{g\omega}) \tag{C.1p}$$

$$\underline{P}_d \le p^{\mathrm{RT}}_{d\omega} \le \overline{P}_d, \quad \forall d \in D, \; : (\varsigma^{\mathrm{RT}-}_{d\omega}, \varsigma^{\mathrm{RT}+}_{d\omega}) \tag{C.1q}$$

$$\underline{Q}_g \le q^{\mathrm{RT}}_{g\omega} \le \overline{Q}_g, \quad \forall g \in G^{\mathrm{D}}_e, \; : (\kappa^{\mathrm{RT}-}_{g\omega}, \kappa^{\mathrm{RT}+}_{g\omega}) \tag{C.1r}$$

$$\underline{Q}_d \le q^{\mathrm{RT}}_{d\omega} \le \overline{Q}_d, \quad \forall d \in D^{\mathrm{D}}_e, \; : (\kappa^{\mathrm{RT}-}_{d\omega}, \kappa^{\mathrm{RT}+}_{d\omega}) \tag{C.1s}$$

$$0 \le s^{\mathrm{RT}}_{n\omega} \le \sum_{d \in D_n} p^{\mathrm{RT}}_{d\omega}, \; \forall n \in N, \; : (\Upsilon^{\mathrm{RT}-}_{n\omega}, \Upsilon^{\mathrm{RT}+}_{n\omega}) \tag{C.1t}$$

$$p^{\uparrow}_{g\omega} \ge 0, \quad p^{\downarrow}_{g\omega} \ge 0, \; \forall g, \; : (\epsilon^{\uparrow,\mathrm{RT}}_{g\omega}, \epsilon^{\downarrow,\mathrm{RT}}_{g\omega}) \tag{C.1u}$$

$$p^{\uparrow}_{d\omega} \ge 0, \quad p^{\downarrow}_{d\omega} \ge 0, \; \forall d, \; : (\epsilon^{\uparrow,\mathrm{RT}}_{d\omega}, \epsilon^{\downarrow,\mathrm{RT}}_{d\omega}) \tag{C.1v}$$

$$w^{\uparrow}_{r\omega} \ge 0, \quad w^{\downarrow}_{r\omega} \ge 0, \forall r, \; : (\varepsilon^{\uparrow,\mathrm{RT}}_{r\omega}, \varepsilon^{\downarrow,\mathrm{RT}}_{r\omega}). \tag{C.1w}$$

The KKT conditions associated with (C.1) are

$$\text{(C.1b)}, \text{(C.1d)} - \text{(C.1h)}, \text{(C.1j)}, \text{(C.1k)}, \text{(C.1m)} \tag{C.2a}$$

$$(p^{\uparrow}_{g\omega}): \; \phi_\omega \pi^{\uparrow}_g + \zeta^{\mathrm{p}}_{g\omega} - \epsilon^{\mathrm{p}\uparrow}_{g\omega} = 0, \quad \forall g \in G^{\mathrm{D}}_e \tag{C.2b}$$

$$(p^{\downarrow}_{g\omega}): \; \phi_\omega \pi^{\downarrow}_g - \zeta^{\mathrm{p}}_{g\omega} - \epsilon^{\mathrm{p}\downarrow}_{g\omega} = 0, \quad \forall g \in G^{\mathrm{D}}_e \tag{C.2c}$$

$$(p^{\uparrow}_{d\omega}): \; \phi_\omega \pi^{\uparrow}_d - \zeta^{\mathrm{p}}_{d\omega} - \varepsilon^{\mathrm{p}\uparrow}_{i\omega} = 0, \quad \forall d \in D^{\mathrm{D}}_e \tag{C.2d}$$

$$(p^{\downarrow}_{d\omega}): \; \phi_\omega \pi^{\downarrow}_d + \zeta^{\mathrm{p}}_{d\omega} - \varepsilon^{\mathrm{p}\downarrow}_{d\omega} = 0, \quad \forall d \in D^{\mathrm{D}}_e \tag{C.2e}$$

$$(w^{\uparrow}_{r\omega}): \; \phi_\omega \pi^{\uparrow\mathrm{R}} + \zeta^{\mathrm{p}}_{r\omega} - \epsilon^{\mathrm{p}\uparrow}_{r\omega} = 0, \quad \forall r \in R^{\mathrm{D}}_e \tag{C.2f}$$

$$(w^{\downarrow}_{r\omega}): \; \phi_\omega \pi^{\downarrow\mathrm{R}} - \zeta^{\mathrm{p}}_{r\omega} - \epsilon^{\mathrm{p}\downarrow}_{r\omega} = 0, \quad \forall r \in R^{\mathrm{D}}_e \tag{C.2g}$$

$$(s^{\mathrm{RT}}_{n\omega}): \phi_\omega VOLL - \lambda^{\mathrm{p,RT}}_{n\omega} - \Upsilon^{\mathrm{RT}-}_{n\omega} + \Upsilon^{\mathrm{RT}+}_{n\omega} = 0, \; \forall n \in N \tag{C.2h}$$

$$(w^{\mathrm{RT}}_{r\omega}): \phi_\omega \pi^{\mathrm{R}} - \zeta^{\mathrm{p}}_{r\omega} - \left[\lambda^{\mathrm{p,RT}}_{n\omega}\right]_{n_r} + \nu^{+}_{r\omega} - \nu^{-}_{r\omega} = 0, \quad \forall r \in R^{\mathrm{D}}_e \tag{C.2i}$$

$$(p^{\mathrm{RT}}_{g\omega}): \phi_\omega \pi^{\mathrm{DA}}_g - \zeta^{\mathrm{p}}_{g\omega} - \varsigma^{\mathrm{RT}-}_{g\omega} + \varsigma^{\mathrm{RT}+}_{g\omega} - \left[\lambda^{\mathrm{p,RT}}_{n\omega}\right]_{n_g} = 0, \quad \forall g \in G \tag{C.2j}$$

$$(q^{\mathrm{RT}}_{g\omega}): -\kappa^{\mathrm{RT}-}_{g\omega} + \kappa^{\mathrm{RT}+}_{g\omega} - \left[\lambda^{\mathrm{q,RT}}_{n\omega}\right]_{n_g} = 0, \quad \forall g \in G^{\mathrm{D}}_e \tag{C.2k}$$

$$(p^{\mathrm{RT}}_{d\omega}): -\phi_\omega \pi^{\mathrm{DA}}_d - \zeta^{\mathrm{p}}_{d\omega} - \varsigma^{\mathrm{RT}-}_{d\omega} + \varsigma^{\mathrm{RT}+}_{d\omega} + \left[\lambda^{\mathrm{p,RT}}_{n\omega} - \Upsilon^{\mathrm{RT}+}_{n\omega}\right]_{n_d} = 0, \quad \forall d \in D \tag{C.2l}$$

$$(q^{\mathrm{RT}}_{d\omega}): -\kappa^{\mathrm{RT}-}_{d\omega} + \kappa^{\mathrm{RT}+}_{d\omega} + \left[\lambda^{\mathrm{q,RT}}_{n\omega}\right]_{n_d} = 0, \quad \forall d \in D^{\mathrm{D}}_e \tag{C.2m}$$

$$\begin{aligned}(p^{\mathrm{RT}}_{l\omega}): \lambda^{\mathrm{p,RT}}_{n\omega} - \lambda^{\mathrm{p,RT}}_{m\omega} &+ \left[2\gamma_{l\omega} p^{\mathrm{RT}}_{l\omega} - \mu^{\mathrm{p}}_{l\omega} - \mu^{\mathrm{p}}_{l'\omega} + 2\eta_{l\omega} p^{\mathrm{RT}}_{l\omega} - 2\beta_{l\omega} R_l\right]_{l \in L^{\mathrm{D}}_e} \\ &+ \left[\gamma^{\mathrm{T}}_{l\omega} + \eta^{\mathrm{T}}_{l\omega}\right]_{l \in L^{\mathrm{T}}} = 0, \quad \forall l \in L\end{aligned} \tag{C.2n}$$

$$(q^{\mathrm{RT}}_{l\omega}): \lambda^{\mathrm{q,RT}}_{n\omega} - \lambda^{\mathrm{q,RT}}_{m\omega} + \left[2\gamma_{l\omega} q^{\mathrm{RT}}_{l\omega} - \mu^{\mathrm{q}}_{l\omega} - \mu^{\mathrm{q}}_{l'\omega} + 2\eta_{l\omega} q^{\mathrm{RT}}_{l\omega} - 2\beta_{l\omega} X_l\right]_{l \in L^{\mathrm{D}}_e} = 0, \quad \forall l \in L \tag{C.2o}$$

$$(\varphi^{\mathrm{RT}}_{l\omega}): -\gamma_{l\omega} v^{\mathrm{RT}}_{n\omega} + \mu^{\mathrm{p}}_{l\omega} R_l + \mu^{\mathrm{q}}_{l\omega} X_l + \beta_{l\omega}(R^2_l + X^2_l) = 0, \quad \forall \omega, l = (n,m) \in L^{\mathrm{D}}_e \tag{C.2p}$$

$$(v^{\mathrm{RT}}_{n\omega}): -\gamma_{l\omega} \varphi^{\mathrm{RT}}_{l\omega} - \beta_{l'\omega} + \beta_{l\omega} - \sigma^{-}_{n\omega} + \sigma^{+}_{n\omega} = 0, \quad \forall \omega, l = (n,m) \in L^{\mathrm{D}}_e \tag{C.2q}$$

$$0 \le \gamma_{l\omega} \perp \varphi^{\mathrm{RT}}_{l\omega} v^{\mathrm{RT}}_{n\omega} - (p^{\mathrm{RT}^2}_{l\omega} + q^{\mathrm{RT}^2}_{l\omega}) \ge 0, \; \forall \omega, l \in L^{\mathrm{D}}_e \tag{C.2r}$$

$$0 \le \eta_{l\omega} \perp S_l - p^{\mathrm{RT}^2}_{l\omega} - q^{\mathrm{RT}^2}_{l\omega} \ge 0, \; \forall \omega, l \in L^{\mathrm{D}}_e \tag{C.2s}$$

$$0 \le \sigma^{-}_{n\omega} \perp v^{\mathrm{RT}}_{n\omega} - \underline{V}^2_n \ge 0, \quad \forall \omega, n \in N^{\mathrm{D}}_e \tag{C.2t}$$

$$0 \leq \sigma^{+}_{n\omega} \perp \overline{V}^{2}_{n} - v^{\mathrm{RT}}_{n\omega} \geq 0, \quad \forall \omega, n \in N^{\mathrm{D}}_{e} \tag{C.2u}$$

$$0 \leq \nu^{-}_{r\omega} \perp w^{\mathrm{RT}}_{r\omega} \geq 0, \quad \forall \omega, r \in R^{\mathrm{D}}_{e} \tag{C.2v}$$

$$0 \leq \nu^{+}_{r\omega} \perp W^{\mathrm{RT}}_{r\omega} - w^{\mathrm{RT}}_{r\omega} \geq 0, \quad \forall \omega, r \in R^{\mathrm{D}}_{e} \tag{C.2w}$$

$$0 \leq \varsigma^{\mathrm{RT}-}_{g\omega} \perp p^{\mathrm{RT}}_{g\omega} - \underline{P}_{g} \geq 0, \quad \forall \omega, g \in G^{\mathrm{D}}_{e} \tag{C.2x}$$

$$0 \leq \varsigma^{\mathrm{RT}+}_{g\omega} \perp \overline{P}_{g} - p^{\mathrm{RT}}_{g\omega} \geq 0, \quad \forall \omega, g \in G^{\mathrm{D}}_{e} \tag{C.2y}$$

$$0 \leq \varsigma^{\mathrm{RT}-}_{d\omega} \perp p^{\mathrm{RT}}_{d\omega} - \underline{P}_{d} \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_{e} \tag{C.2z}$$

$$0 \leq \varsigma^{\mathrm{RT}+}_{d\omega} \perp \overline{P}_{d} - p^{\mathrm{RT}}_{d\omega} \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_{e} \tag{C.2aa}$$

$$0 \leq \kappa^{\mathrm{RT}+}_{g\omega} \perp q^{\mathrm{RT}}_{g\omega} - \underline{Q}_{g} \geq 0, \quad \forall \omega, g \in G^{\mathrm{D}}_{e} \tag{C.2ab}$$

$$0 \leq \kappa^{\mathrm{RT}-}_{g\omega} \perp \overline{Q}_{g} - q^{\mathrm{RT}}_{g\omega} \geq 0, \quad \forall \omega, g \in G^{\mathrm{D}}_{e} \tag{C.2ac}$$

$$0 \leq \kappa^{\mathrm{RT}+}_{d\omega} \perp q^{\mathrm{RT}}_{d\omega} - \underline{Q}_{d} \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_{e} \tag{C.2ad}$$

$$0 \leq \kappa^{\mathrm{RT}-}_{d\omega} \perp \overline{Q}_{d} - q^{\mathrm{RT}}_{d\omega} \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_{e} \tag{C.2ae}$$

$$0 \leq \rho^{\mathrm{RT}-}_{e\omega} \perp p^{\mathrm{IO,RT}}_{e\omega} - \underline{f}_{e} \geq 0, \quad \forall \omega \tag{C.2af}$$

$$0 \leq \rho^{\mathrm{RT}+}_{e\omega} \perp \overline{f}_{e} - p^{\mathrm{IO,RT}}_{e\omega} \geq 0, \quad \forall \omega \tag{C.2ag}$$

$$0 \leq \epsilon^{\mathrm{p}\uparrow}_{g\omega} \perp p^{\uparrow}_{g\omega} \geq 0, \quad 0 \leq \epsilon^{\mathrm{p}\downarrow}_{g\omega} \perp p^{\downarrow}_{g\omega} \geq 0, \quad \forall \omega, g \in G^{\mathrm{D}}_{e} \tag{C.2ah}$$

$$0 \leq \epsilon^{\mathrm{p}\uparrow}_{r\omega} \perp w^{\uparrow}_{r\omega} \geq 0,\ 0 \leq \epsilon^{\mathrm{p}\downarrow}_{r\omega} \perp w^{\downarrow}_{r\omega} \geq 0,\ \forall \omega, r \in R^{\mathrm{D}}_{e} \tag{C.2ai}$$

$$0 \leq \varepsilon^{\mathrm{p}\uparrow}_{d\omega} \perp p^{\uparrow}_{d\omega} \geq 0, \quad 0 \leq \varepsilon^{\mathrm{p}\downarrow}_{d\omega} \perp p^{\downarrow}_{d\omega} \geq 0, \quad \forall \omega, d \in D^{\mathrm{D}}_{e} \tag{C.2aj}$$

$$0 \leq \epsilon^{\uparrow\mathrm{IO}}_{e\omega} \perp p^{\uparrow\mathrm{IO}}_{e\omega} \geq 0, \quad 0 \leq \epsilon^{\downarrow\mathrm{IO}}_{e\omega} \perp p^{\downarrow\mathrm{IO}}_{e\omega} \geq 0, \quad \forall \omega \tag{C.2ak}$$

$$0 \leq \Upsilon^{\mathrm{RT}-}_{n\omega} \perp s^{\mathrm{RT}}_{n\omega} \geq 0, \forall n, \omega \tag{C.2al}$$

$$0 \leq \Upsilon^{\mathrm{RT}+}_{n\omega} \perp \sum_{d \in D_n} p^{\mathrm{RT}}_{d\omega} - s^{\mathrm{RT}}_{n\omega} \geq 0,\ \forall n, \omega \tag{C.2am}$$

$$0 \leq \Upsilon^{\mathrm{DA}-}_{e} \perp s^{\mathrm{DA}}_{e} \geq 0 \tag{C.2an}$$

$$0 \leq \Upsilon^{\mathrm{DA}+}_{e} \perp \sum_{d} p^{\mathrm{DA}}_{d} - s^{\mathrm{DA}}_{e} \geq 0. \tag{C.2ao}$$

Appendix D. Proof

Every condition within (A.2) related to the DSO market is included within either (B.2) associated with the day-ahead market clearing or (C.2) corresponding to the real-time market clearing under each scenario ω, or can be built by a combination of conditions within (B.2) and (C.2). This concludes that the DSO market-clearing problem is redundant, and therefore, it can be removed.