Lab 5 Report

The DTFT

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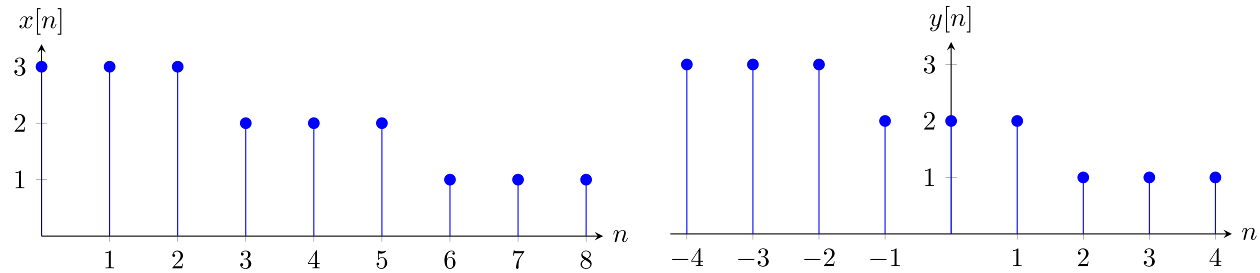
Project 1(a): The Shift Property



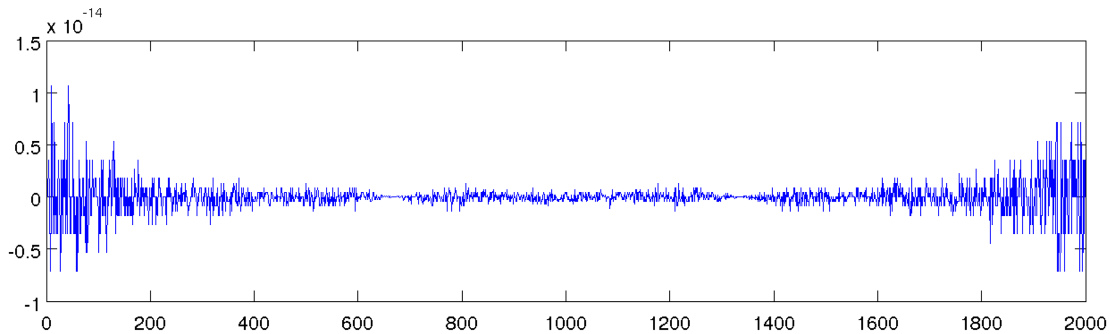
(a) The signal $x[n] = \{\overset{\Downarrow}{3}, 3, 3, 2, 2, 2, 1, 1, 1\}$.

(b) The signal $y[n] = \{3, 3, 3, 2, \overset{\Downarrow}{2}, 2, 1, 1, 1\}$.

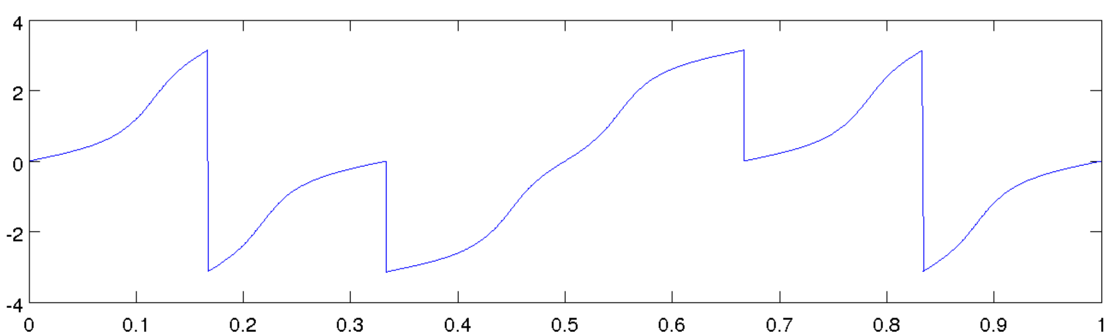
Figure 1: Input signals $x[n]$ and $y[n]$ for Project 1. They are shifted versions of one another related by $y[n] = x[n-4]$.

Comparing $x[n]$ and $y[n]$ from Figure 1,

(a) The magnitude spectra should be identical because the same frequencies are present in each signal; the only distinction between the signals in this respect is that each frequency component has peaks at different times; this amounts to a phase shift. The identicality of the magnitude spectra is shown by the below plot of $|X| - |Y|$, which has a 10^{-14} factor of scale on the y-axis.

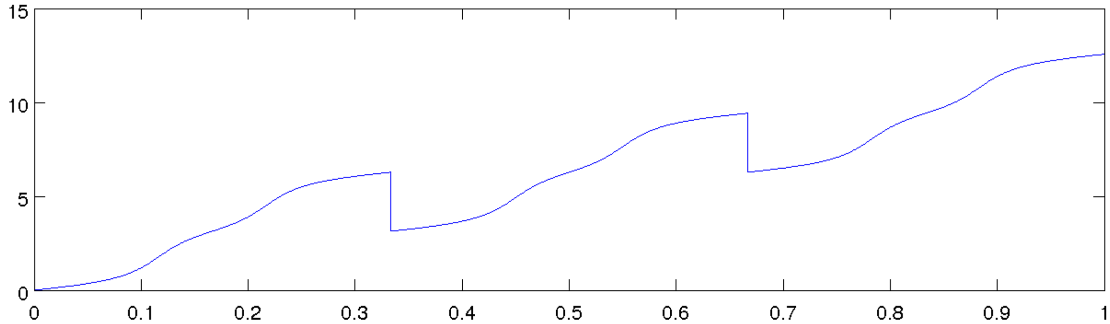


(b) The phase of the DFT of $x[n]$ will not be linear. This is due to the lack of symmetry[1] and is confirmed by the phase spectrum of $x[n]$ shown below.



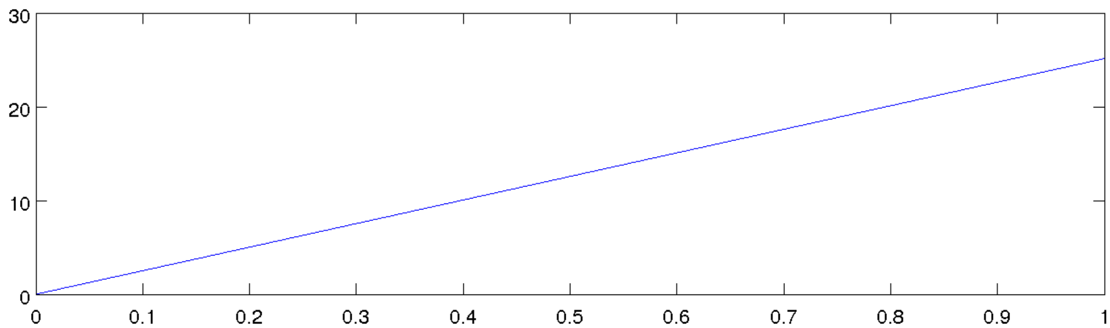
[1] i.e. $x[n]$ is not the shifted version of a signal that is even or odd.

(c) The phase spectrum of X does show jumps, but `unwrap` does not remove them all. Jumps remain which are less than 2π as shown below.



(d) The phase spectra for X and Y are not and should not be equal. They should be related by a linear difference due to the group delay but will not be equal.

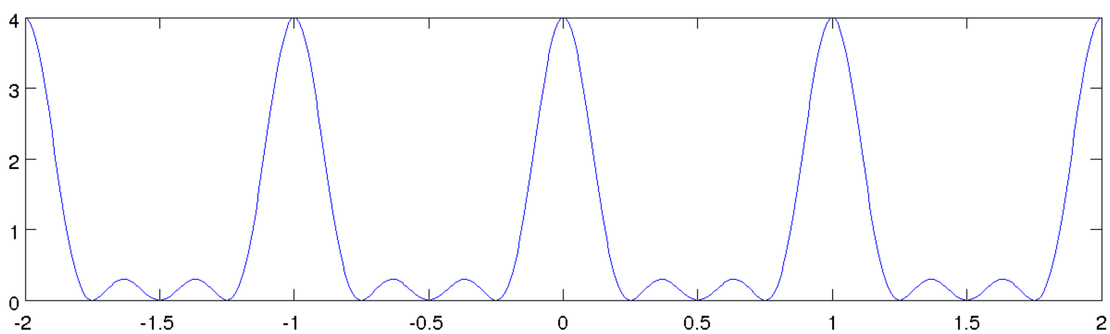
(e) Since the two signals are shifted version of one another in the discrete domain, they will have a constant group delay and so the below unwrapped plot of $\angle X - \angle Y$ is linear.



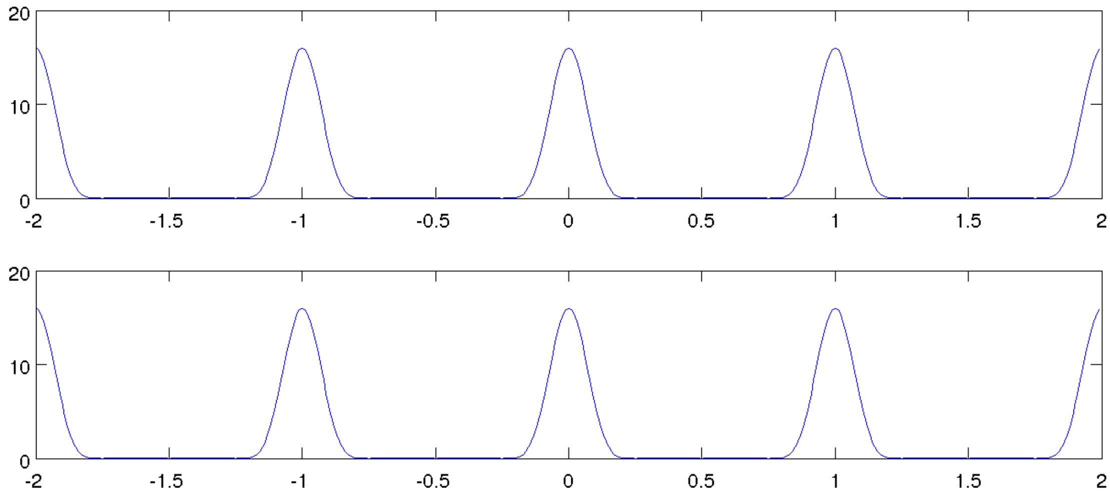
(f) From the plot above, delay can be found by calculating the slope. In this case, the slope is ≈ 25 and $25/(2\pi) \approx 3.979$. This matches the discrete-domain delay which we know to be 4 samples.

Project 1(b): Periodicity, Convolution and Multiplication

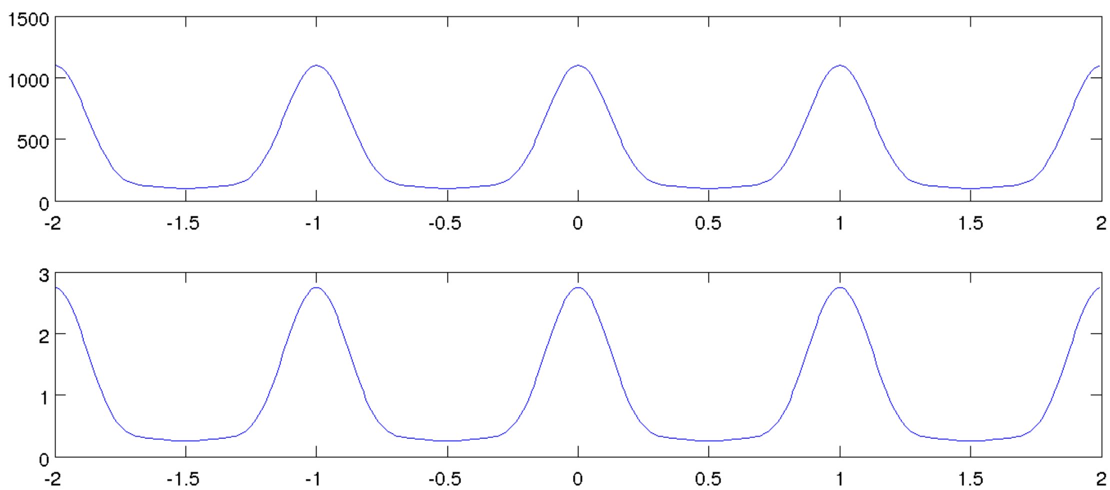
(a) $|X(F)|$ should be (and is) periodic with $N = 1$. This is because $X(F)$ is generated from a discrete signal $x[n]$.



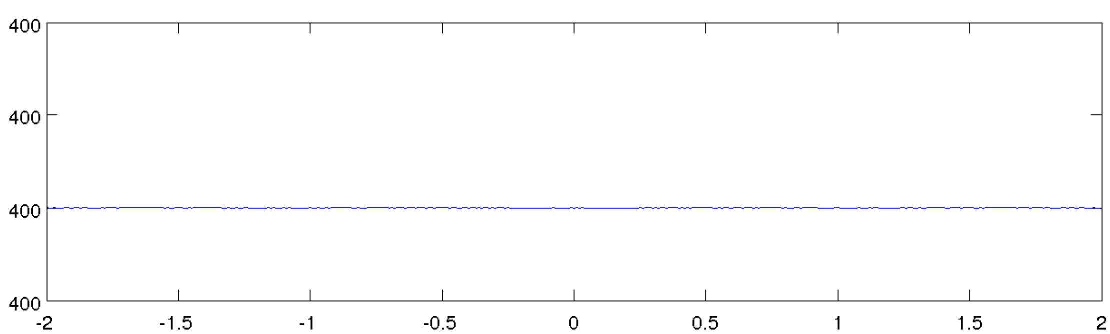
(b) $|X(F)|^2$ should be identical to $|G(F)|$ because $g[n] = x[n] * x[n]$ and because convolution in one domain translates to multiplication in another. The ratio of the below plots is 1.



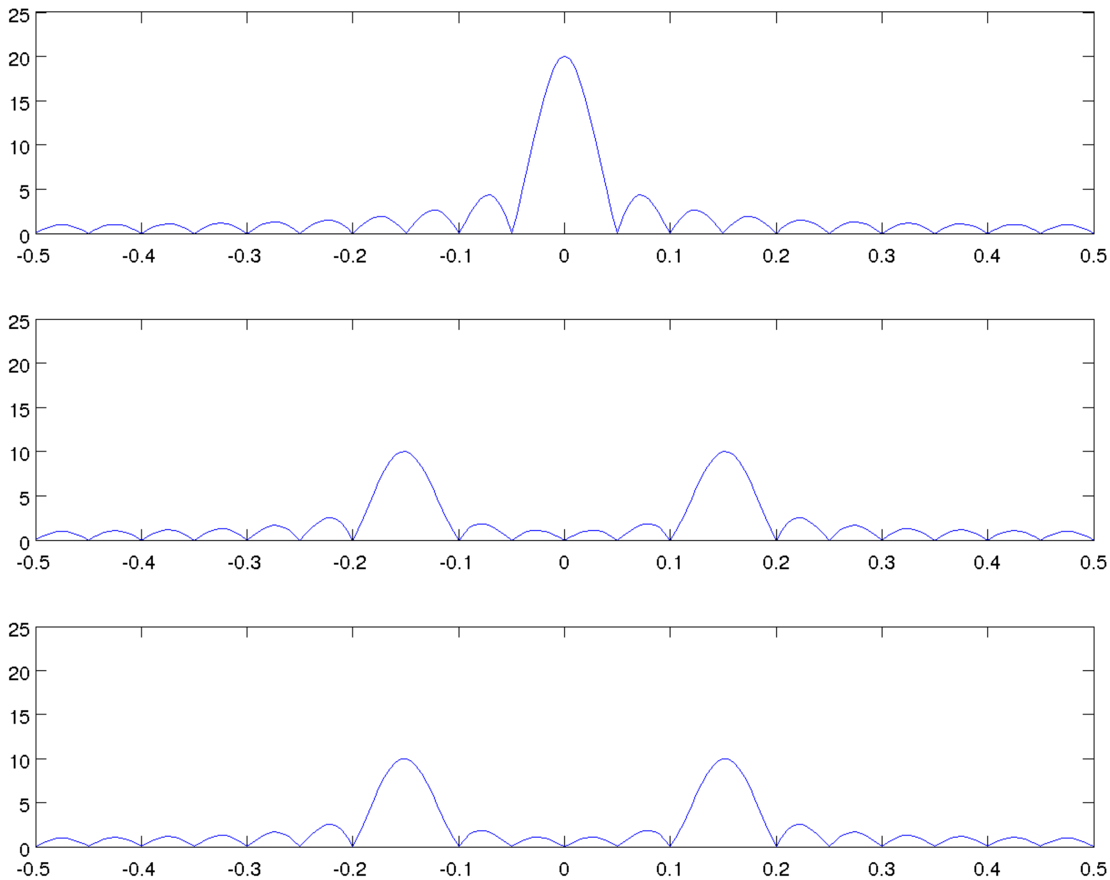
(c) $|Y_p(F)|$ and $|H(F)|$ are not identical (as shown below).



(d) $|Y_p(F)|$ and $|H(F)|$ have a ratio of $\boxed{400}$.



Project 1(c): Modulation



(a) $|Y_1(F)|$ should (and does) consist of two half-height, shifted versions of $X(F)$ (because the time-domain multiplication of a rect $x[t]$ and a $\cos(\cdots)$ results in the convolution of a $\mathrm{sinc}(\cdots)$ and a pair of impulses in the frequency domain).

(b) $|Y_1(F)|$ and $|Y_2(F)|$ should be (and are) identical because $\cos(2.3n\pi)$ aliases to $\cos(0.3n\pi)$ when n is an integer.

Project 2: The DTFT of Digital Filters

1. (a) lowpass
   (b) IIR
   (c) nonlinear-phase
2. (a) bandstop
   (b) IIR
   (c) nonlinear-phase
3. (a) highpass
   (b) FIR
   (c) linear-phase
4. (a) bandpass
   (b) IIR
   (c) nonlinear-phase
5. (a) lowpass
   (b) IIR
   (c) linear-phase
6. (a) lowpass
   (b) IIR
   (c) linear-phase
7. (a) allpass
   (b) IIR
   (c) nonlinear-phase
8. (a) comb
   (b) FIR
   (c) linear-phase

Appendix: MATLAB Source Code

What follows is a listing of the MATLAB source code (listing 1) used to generate the figures and other information presented in this report.

Listing 1: The MATLAB script used for this report, `Lab05_ahirzel.m`.

```
% EE4252 Lab 5
% Alex Hirzel <ahirzel@mtu.edu>
% 2012-10-18

addpath ../../ClassWorkspace;


% Project 1(a): The Shift Property %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x = [3 3 3 2 2 2 1 1 1];
F = (0:1999)/2000; z = exp(1j*2*pi.*F);

X = polyval(x, z) ./ polyval([zeros(1, 4) 1 zeros(1, 4)], z);
Y = polyval(x, z) ./ polyval([1 zeros(1, 8)], z);
XM = abs(X); YM = abs(Y); XA = angle(X); YA = angle(Y); XU = unwrap(XA); YU = unwrap(YA);

plot(XM - YM);          mysaveas('p1a_magnitude', 10, 2.5)
plot(F, XA);            mysaveas('p1a_phase', 10, 2.5)
plot(F, XU);            mysaveas('p1a_phase_unwrapped', 10, 2.5)
plot(F, unwrap(XU-YU)); mysaveas('p1a_phase_diff', 10, 2.5)


% Project 1(b): Periodicity, Convolution and Multiplication %%%%%%%%%%%%%%%

n = 0:8; x = tri((n - 4)/4);
F = -2:0.01:1.99; W = 2*pi*F; z = exp(1j*W);
X = polyval(x, z) ./ polyval ([1 zeros(1, 8)], z);
G = polyval(conv(x, x), z) ./ polyval([1 zeros(1, 16)], z);
H = polyval(x .* x, z) ./ polyval([1 zeros(1, 8)], z);
Yp = convp(X, X);

XM = abs(X); XXM = XM .* XM; GM = abs(G); YpM = abs(Yp); HM = abs(H);

plot(F, XM);                    mysaveas('p1b_x', 10, 2.5);

subplot(2, 1, 1); plot(F, XXM);
subplot(2, 1, 2); plot(F, GM);  mysaveas('p1b_xx_and_g', 10, 4);

subplot(2, 1, 1); plot(F, YpM);
subplot(2, 1, 2); plot(F, HM);  mysaveas('p1b_ypm_and_hm', 10, 4);

plot(F, YpM ./ HM);             mysaveas('p1b_ypm_over_hm', 10, 2.5);


% Project 1(c): Modulation %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

n = 0:19;
x = ones(1, 20);
y1 = cos(0.3*n*pi);
y2 = cos(2.3*n*pi);
F = -0.5:0.005:0.5;
z = exp(1j*2*pi*F);

X  = polyval(x, z)  ./ polyval([1 zeros(1, 19)], z);
Y1 = polyval(y1, z) ./ polyval([1 zeros(1, 19)], z);
Y2 = polyval(y2, z) ./ polyval([1 zeros(1, 19)], z);
XM = abs(X); Y1M = abs(Y1); Y2M = abs(Y2);

subplot(3, 1, 1); plot(F, XM);  ylim([0 25]);
subplot(3, 1, 2); plot(F, Y1M); ylim([0 25]);
subplot(3, 1, 3); plot(F, Y2M); ylim([0 25]); mysaveas('p1c', 10, 7.5);

%%
dtf_analyze([1 0],             [1 (-0.6)]);  pause;
dtf_analyze([1 0.1 1],         [1 0.1 0.8]); pause;
dtf_analyze([1 (-2) 2 (-1)],   [1]);         pause;
dtf_analyze([1],               [1 0.4 0.8]); pause;
dtf_analyze([0.7 0],           [1 (-1)]);    pause;
dtf_analyze([(-0.7) 0],        [1 (-1)]);    pause;
dtf_analyze([2 3 1],           [1 3 2]);     pause;
dtf_analyze([1 0 0 0 0 0 0 1], [1]);         pause; close(gcf);
```