Lab 6 Report

The DFT and FFT

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Project 1: Sampling a Combination of Sinusoids

This project investigates the sampling of the following signal at multiple rates.

$$x(t) = \sin(200\pi t) + \sin(480\pi t) \tag{1}$$

MATLAB was used to determine whether aliasing occurs at the indicated sample rate. MATLAB was also used to calculate the minimum number of samples required to eliminate leakage (N_{min}). This data is shown below.

S	Aliasing?	N_{min}	$3N_{\text{min}}$	$3N_{\text{min}}+1$
1700Hz	no	$85^{\text{fig 1(a)}}$	$255^{\text{fig 1(b)}}$	$263^{\text{fig 1(c)}}$
360Hz	yes	$18^{\text{fig 2(a)}}$	$54^{\text{fig 2(b)}}$	$62^{\text{fig 2(c)}}$
120Hz	yes	$6^{\text{fig 3(a)}}$	$18^{\text{fig 3(b)}}$	$26^{\text{fig 3(c)}}$
50Hz	yes	$5^{\text{fig 4(a)}}$	$15^{\text{fig 4(b)}}$	$23^{\text{fig 4(c)}}$

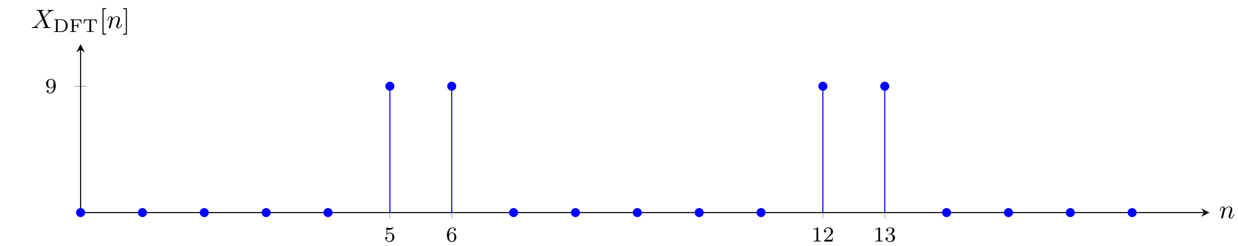
Numbers typeset in color indicate combinations of sampling rate and DFT size that will not result in identical reconstruction (i.e. $x(t) \neq x_{\text{r}}(t)$ due to aliasing if red or leakage if blue). Required plots are shown in figures 1, 2, 3 and 4. Questions (c), (f) and (g) from the lab hand-out are addressed in the figure captions.

(a) The 85-point DFT of $x(t)$ sampled at 1700Hz shows peaks at analog frequencies of ±100Hz and ±240Hz as expected. This is because the sampling frequency is high enough and the DFT window captures an integral number of periods of the underlying sinusoids.

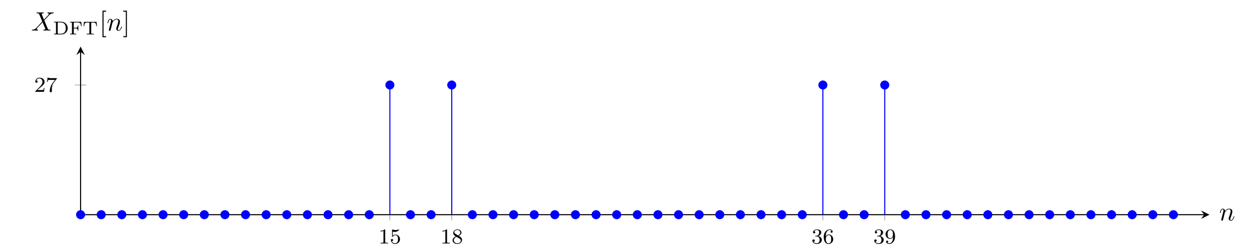
(b) The 255-point DFT of $x(t)$ sampled at 1700Hz. The commentary from Figure 1(a) (above) applies to this case as well.

(c) The 263-point DFT of $x(t)$ sampled at 1700Hz shows peaks at many analog frequencies near ±100Hz and ±240Hz. This is expected due to leakage.

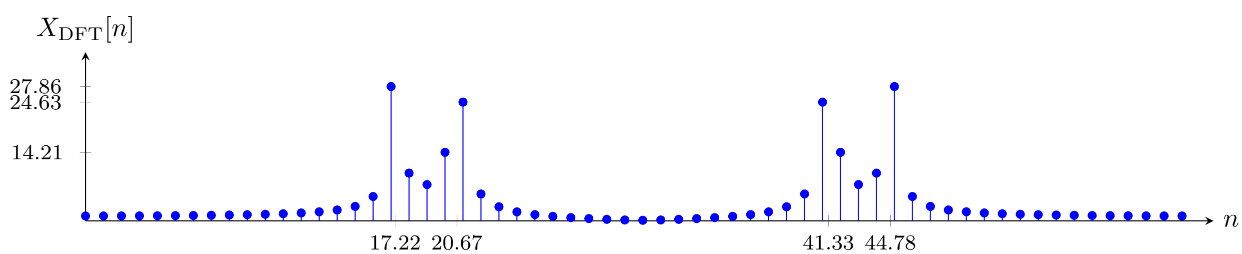
Figure 1: Sampling system (1) at 1700Hz.



(a) The 18-point DFT of $x(t)$ sampled at 360Hz shows peaks at $\pm$100Hz and $\mp$120Hz. These are as expected because the 240Hz frequency component in the original signal is aliased and phase-reversed (because $240\text{Hz} - 360\text{Hz} = -120\text{Hz}$).

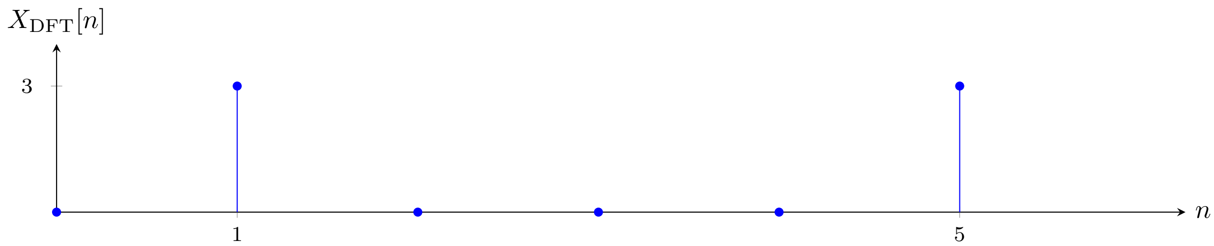


(b) The 54-point DFT of $x(t)$ sampled at 360Hz. The commentary from Figure 2(a) (above) applies to this case as well.

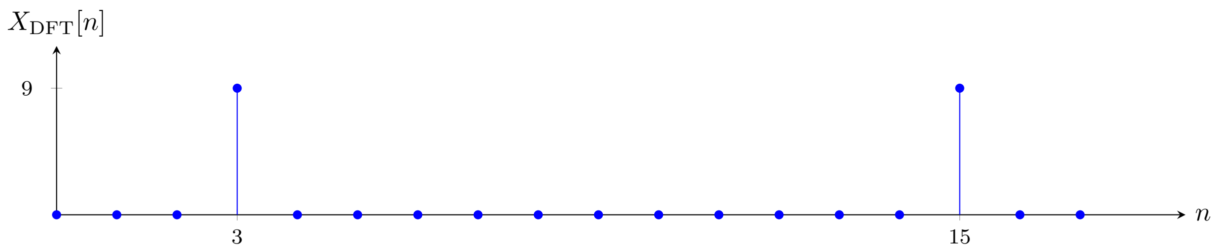


(c) The 62-point DFT of $x(t)$ sampled at 360Hz shows peaks at many analog frequencies near $\pm$100Hz and $\mp$120Hz. This is expected due to leakage.

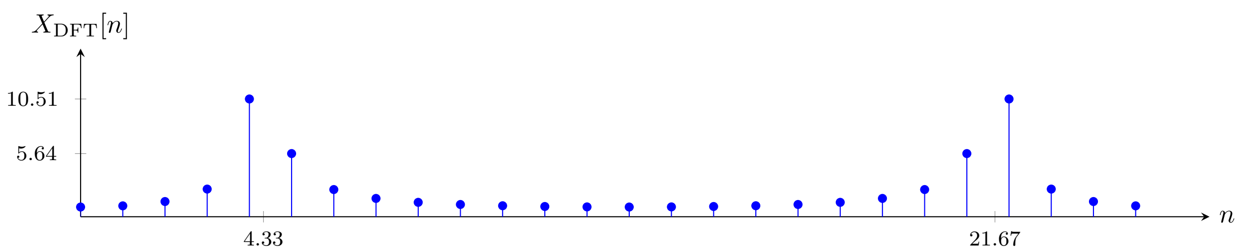
Figure 2: Sampling system (1) at 360Hz.



(a) The 6-point DFT of $x(t)$ sampled at 120Hz shows peaks at $\mp$20Hz. These are as expected because the 240Hz frequency component in the original signal is aliased to 0Hz (with the DC value cancling out due to the even number of samples) and the 100Hz component is aliased and phase-reversed to -20Hz (because 100Hz $-$ 120Hz $= -20$Hz).

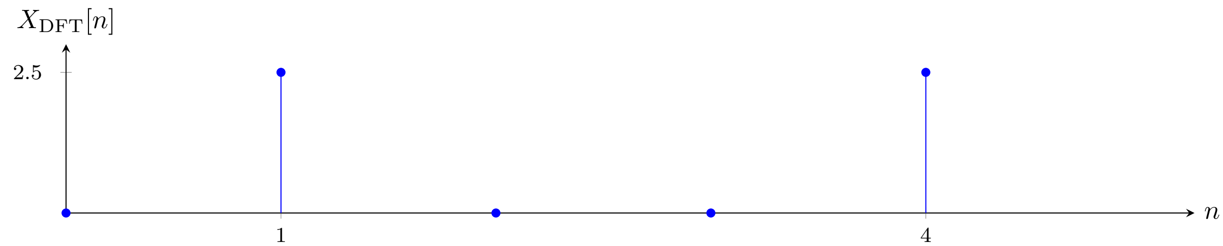


(b) The 18-point DFT of $x(t)$ sampled at 120Hz. The commentary from Figure 3(a) (above) applies to this case as well.

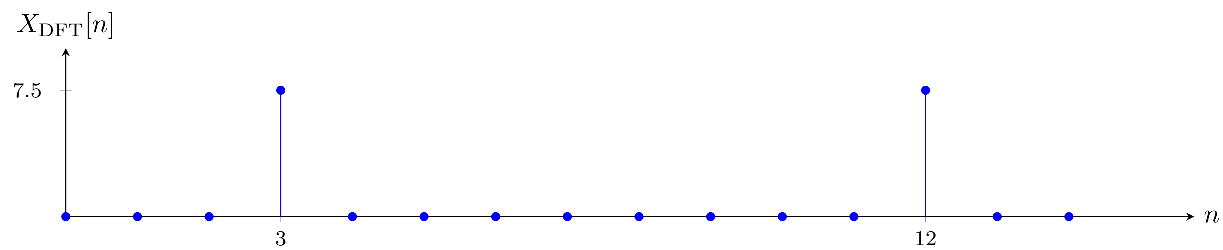


(c) The 26-point DFT of $x(t)$ sampled at 120Hz shows peaks at many analog frequencies near $\mp$20Hz. This is expected due to leakage.

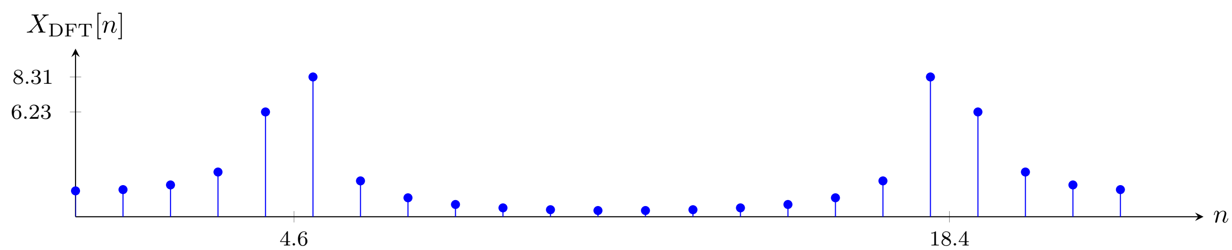
Figure 3: Sampling system (1) at 120Hz.



(a) The 5-point DFT of $x(t)$ sampled at 50Hz shows peaks at $\mp$10Hz. These are as expected because the 100Hz frequency component in the original signal is aliased to 0Hz (with the DC value canceling out due to the even number of samples) and the 240Hz component is aliased and phase-reversed to -10Hz (because $240\text{Hz} - 5 \cdot 50\text{Hz} = -10\text{Hz}$).



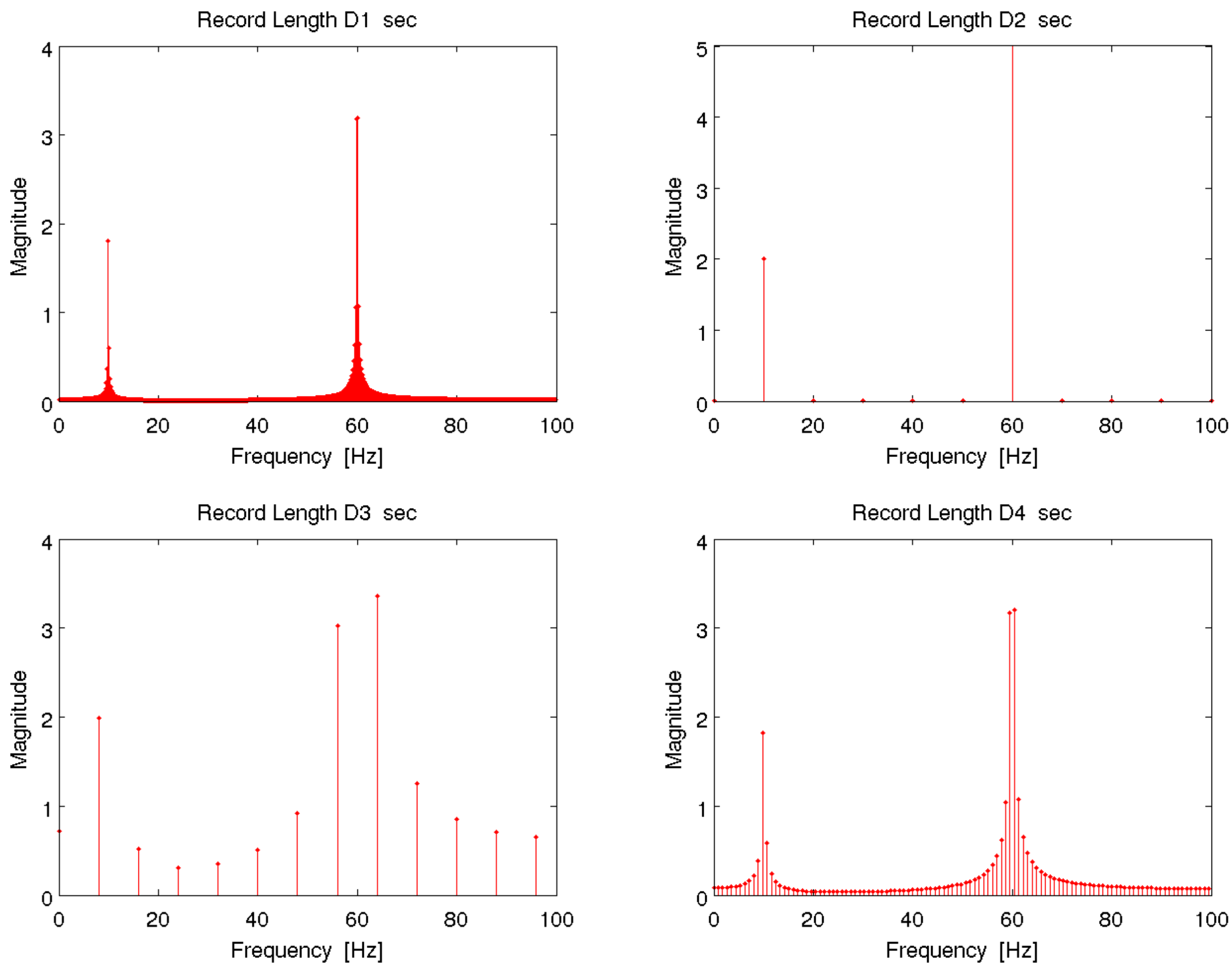
(b) The 5-point DFT of $x(t)$ sampled at 50Hz. The commentary from Figure 4(a) (above) applies to this case as well.



(c) The 23-point DFT of $x(t)$ sampled at 50Hz shows peaks at many analog frequencies near $\mp$10Hz. This is expected due to leakage.

Figure 4: Sampling system (1) at 50Hz.

Project 2: Leakage (200Hz)



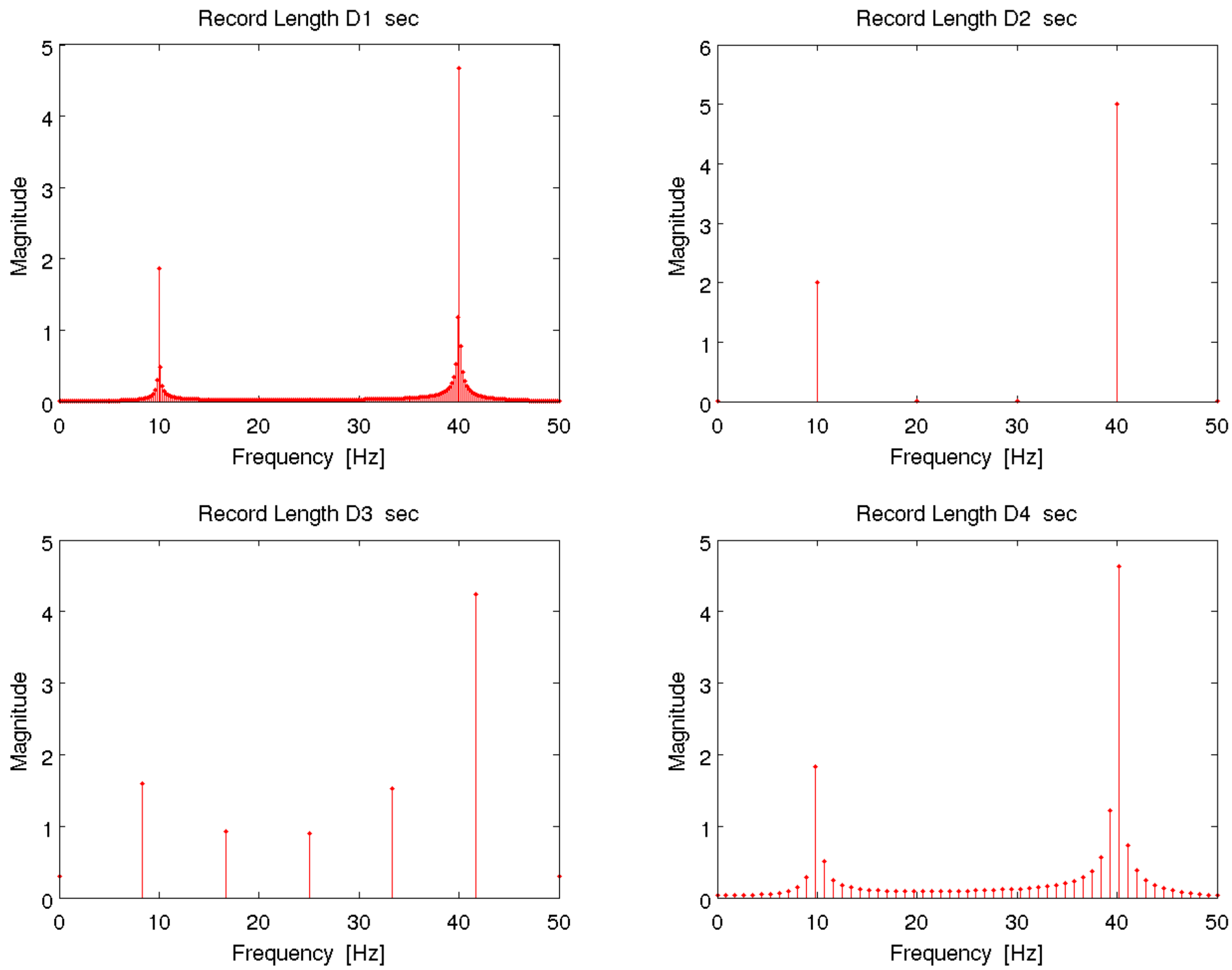
(a) Frequency components at 10Hz and 60Hz are expected. Aliasing does not occur in any plot because $f_s > 2f_{\max}$ (i.e. 200Hz $> 2 \cdot 60$Hz).

(b) No leakage occurs in plot D_2 because there is no "skirt" around frequency components in this plot.

(c) The remaining plots show leakage (i.e. D_1, D_3 and D_4). This is concluded because there are no sharp peaks at 10Hz or 60Hz in these plots.

(d) Plot D_1 has the longest duration of any of the plots mentioned in (c); this is because it is the most "sharp".

Project 2: Leakage (100Hz)



(a) Frequency components at 10Hz and 60Hz are expected. Aliasing is visible in all plots because the 60Hz frequency component is aliased to 40Hz and phase-reversed. (This is because 60Hz − 100Hz = −40Hz.)

(b) No leakage occurs in plot D_2 because there is no "skirt" around frequency components in this plot.

(c) The remaining plots show leakage (i.e. D_1, D_3 and D_4). This is concluded because there are no sharp peaks at 10Hz or 40Hz in these plots.

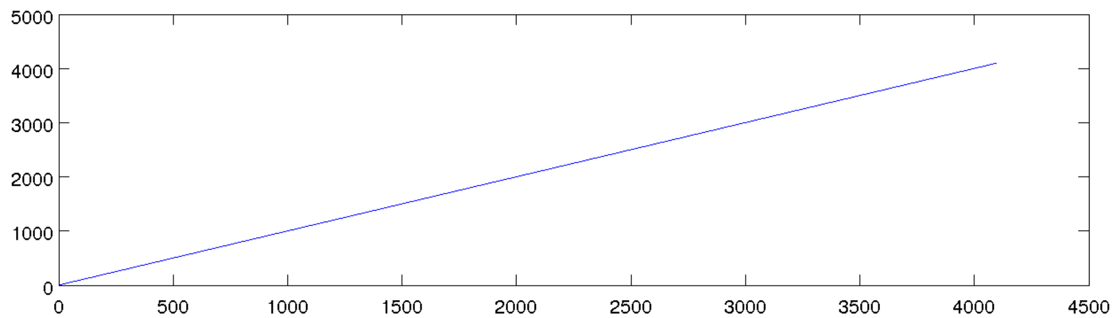
(d) Plot D_1 has the longest duration of any of the plots mentioned in (c); this is because it is the most "sharp".

Project 3: Sampling a Chirp Signal

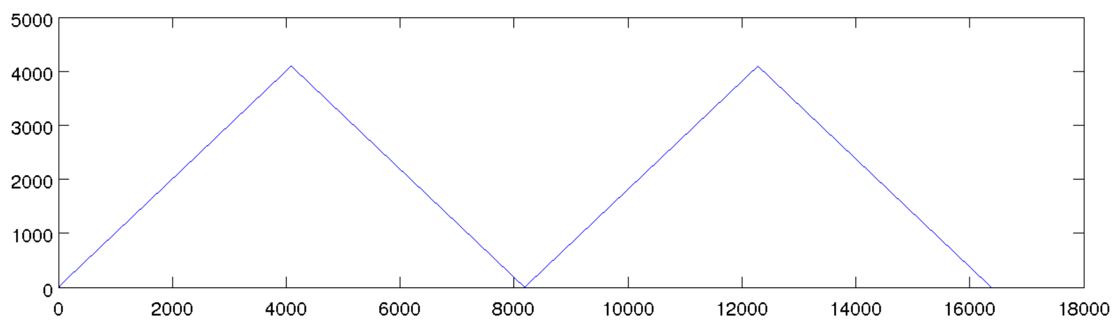
(a) The highest frequency present is $2048 \cdot 2 = 4096$Hz, which will not be aliased by sampling at 8192Hz. The frequency seems to increase with time in a smooth way, eventually fading due to the nonlinearity of my human hearing.

(b) The highest frequency present is $8192 \cdot 2 = 16384$Hz, which will most certainly be aliased by sampling at 8192Hz. The frequency increases and decreases twice in 2s, which agrees with the above plot.

(c) The below plot of `fa1` shows the frequency of the sound increases with time. My ears agree.



(d) The below plot of `fa2` shows the frequency of the sound oscillate twice. My ears agree.



Appendix: MATLAB Source Code

What follows is a listing of the MATLAB source code (Listing 1 and Listing 2)—and the output of this code (Listing 3)—used to generate the figures and other information presented in this report.

Listing 1: The MATLAB script used for this report, `Lab06_ahirzel.m`.

```
addpath ../../ClassWorkspace;


% Project 1: Sampling a Combination of Sinusoids %%%%%%%%%%%%%%%%%%%%%%%%%%

f1 = 100; f2 = 240;
x = @(t) sin(2*pi*f1*t) + sin(2*pi*f2*t);

delete 'generated/diary.txt'; diary 'generated/diary.txt'; diary on
for S = [1700 360 120 50]

    % Reduce fractions
    [n1, d1] = rat(f1/S);
    [n2, d2] = rat(f2/S);

    % Minimum number of samples
    N_min = lcm(d1, d2);

    fprintf('S=%4d   [ %2d/%2d  %2d/%2d ]   Alias=%1d   N_min=%3d, %3d, %3d\n', ...
        S, n1, d1, n2, d2, (2*n1 >= d1) || (2*n2 >= d2), N_min, 3*N_min, 3*N_min + 8);

    mydftplot(x, N_min,     S, 'min');
    mydftplot(x, 3*N_min,   S, '3min');
    mydftplot(x, 3*N_min+8, S, '3min8');
end
diary off


% Project 2: Leakage %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x = '2*sin(20*pi*t) + 5*sin(120*pi*t)';
leakage(x, 1/200); mysaveas('p2-200hz', 10, 7);
leakage(x, 1/100); mysaveas('p2-100hz', 10, 7);


% Project 3: Sampling a Chirp Signal %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x = @(t, alpha) cos(pi*alpha*t.^2);
w = @(t, alpha) alpha*t; % Hz

t = linspace(0, 2, 2*8192);

xn1 = x(t, 2048);  fi1 = w(t, 2048);  fa1 = abs(alias(fi1, 8192));
xn2 = x(t, 8192);  fi2 = w(t, 8192);  fa2 = abs(alias(fi2, 8192));

plot(fi1, fa1); mysaveas('p3-1', 10, 2.5);
plot(fi2, fa2); mysaveas('p3-2', 10, 2.5);
```

Listing 2: The MATLAB script used to plot the DFTs shown in this report, `mydftplot.m`.

```
function mydftplot(x, N, S, id)
    n = 0:(N - 1);
    X_DFT = fft(x(n/S), N);
    fn = ['generated/dft' num2str(S) '-' id '.txt'];
    csvwrite(fn, abs(X_DFT)');   % transposed so each sample is one row
    %plot(n/S, ifft(X_DFT, N)); plot(n/S, x(n/S));
```

Listing 3: The output of listing 1, `diary.txt`.

```
S=1700   [  1/17  12/85 ]   Alias=0   N_min= 85, 255, 263
S= 360   [  5/18   2/ 3 ]   Alias=1   N_min= 18,  54,  62
S= 120   [  5/ 6   2/ 1 ]   Alias=1   N_min=  6,  18,  26
S=  50   [  2/ 1  24/ 5 ]   Alias=1   N_min=  5,  15,  23
```