Lab 3 Report

Discrete Convolution

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Project 1

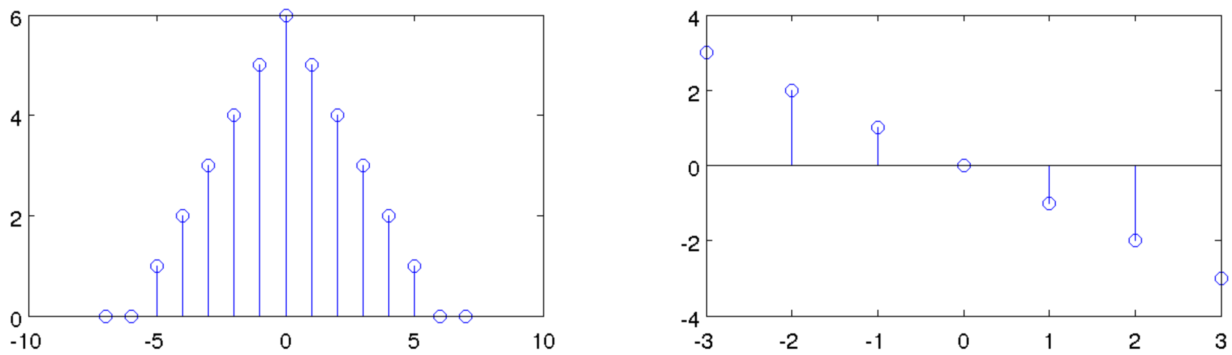


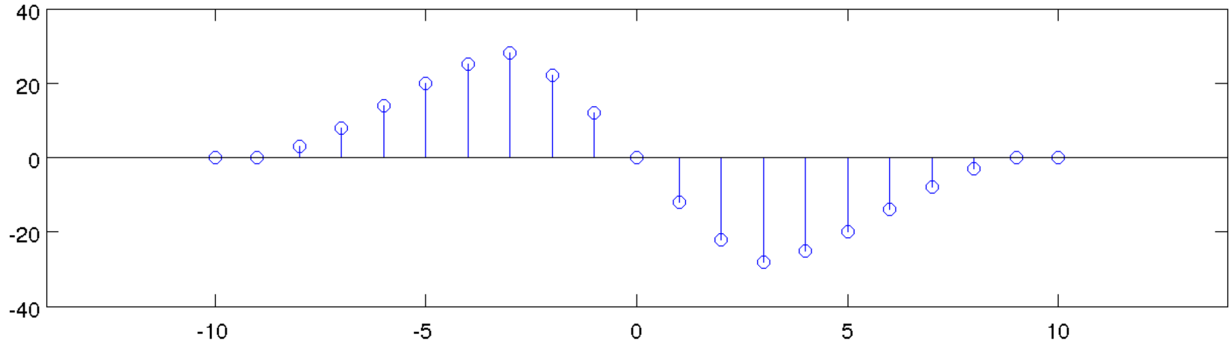
Figure 1: $x[n]$ (left) and $h[n]$ (right) used for convolution in Project 1.

The properties of convolution are verified using Figure 1 as follows:

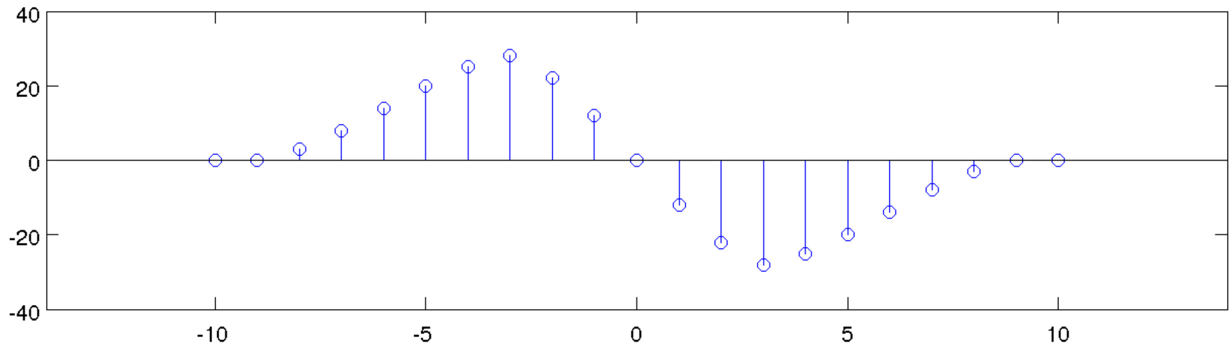
- The **order property is confirmed** because Figure 2(a) and Figure 2(b) are identical;
- The **length property is verified** by noting that Figure 2(a) and Figure 2(b) are equal in length;
- The **sum property is verified** using Listing 2 by noting that $\sum x[n] = 36$ and $\sum h[x] = 0$ and that the lengths of $y_1 \cdots y_4$ are in agreement with the sum property (i.e. all are zero except $\sum y_3 = 36^2 = 1296$).

Additionally, symmetry of the outputs is observed as shown in the below table.

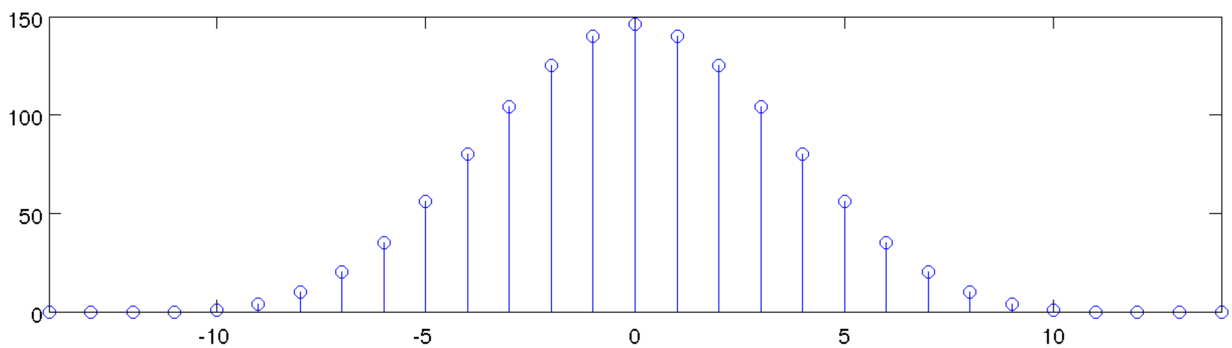
Input	Output
even $*$ odd	even i.e. y_1, y_2
odd $*$ odd	odd i.e. y_3
even $*$ even	even i.e. y_4



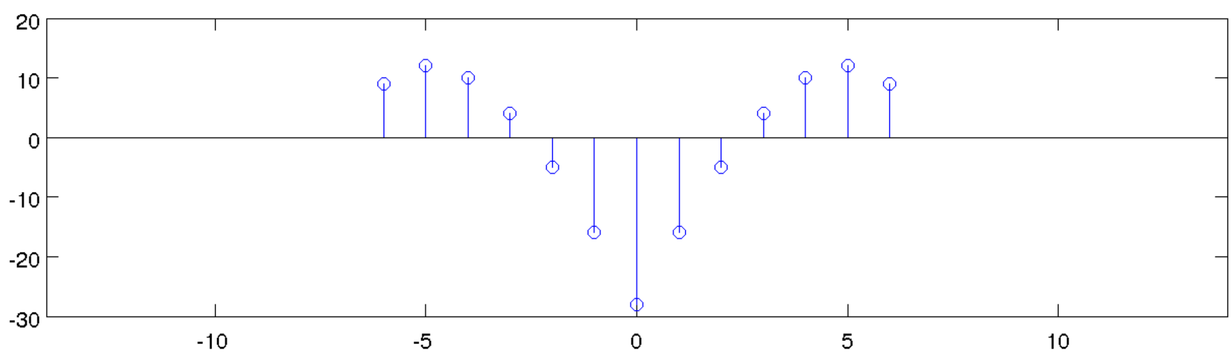
(a) The convolution result $y_1[n] = x[n] * h[n]$.



(b) The convolution result $y_2[n] = h[n] * x[n]$.



(c) The convolution result $y_3[n] = x[n] * x[n]$.



(d) The convolution result $y_4[n] = h[n] * h[n]$.

Figure 2: The outputs from convolution in Project 1.

Project 2

The input and response ($x[n]$ and $y[n]$, respectively) are shown in Figure 3. **The period of both $x[n]$ and $y[n]$ is 10.** This makes sense because $h[n]$ is non-periodic and is an LTI system; this means a periodic input will result in an output that is periodic with the same frequency.

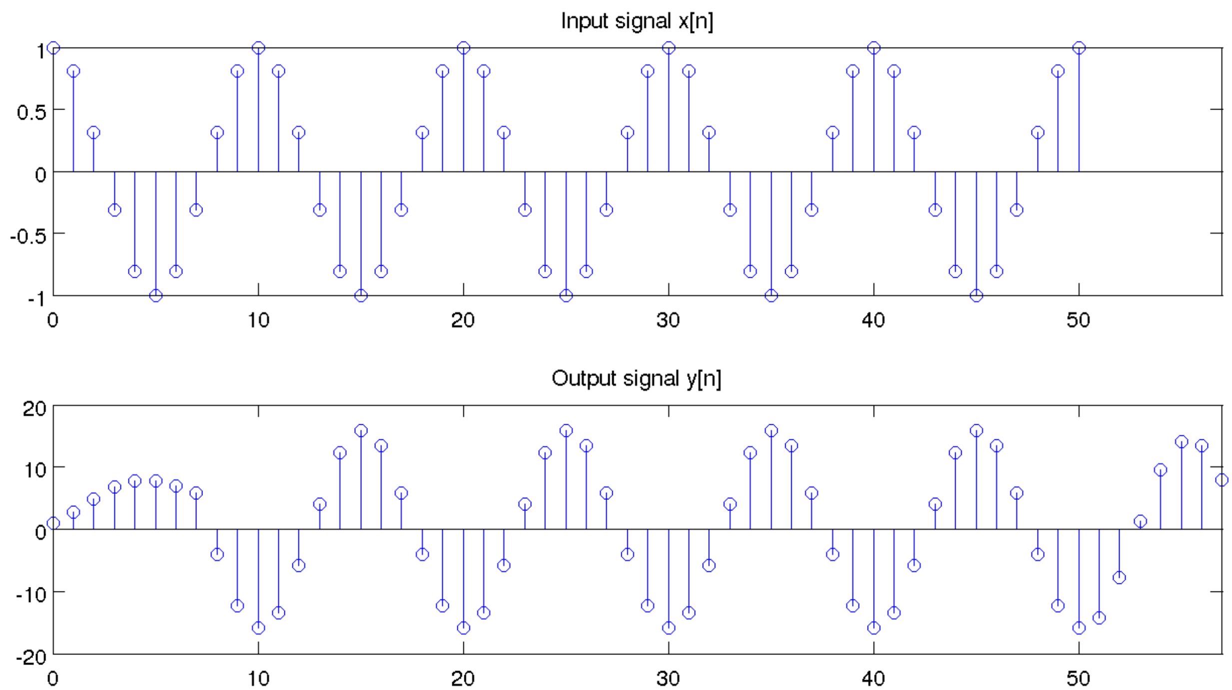
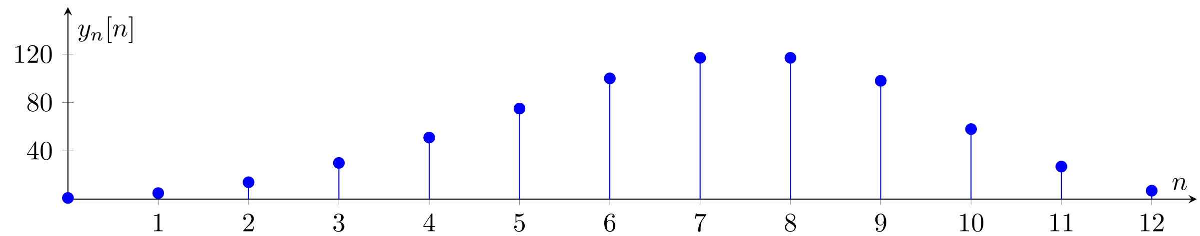


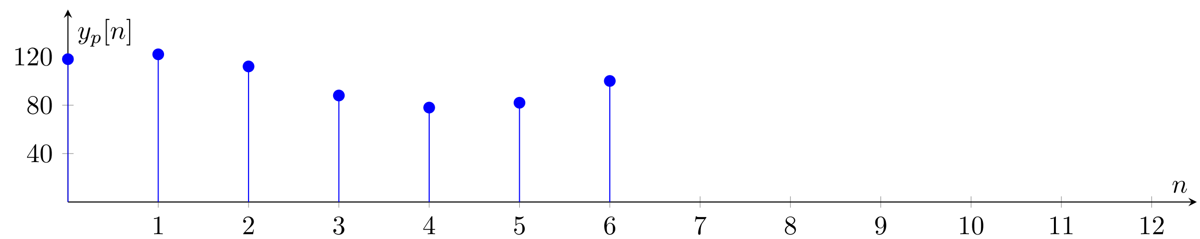
Figure 3: The input $x[n]$ and response $y[n]$ from Project 2.

Project 3

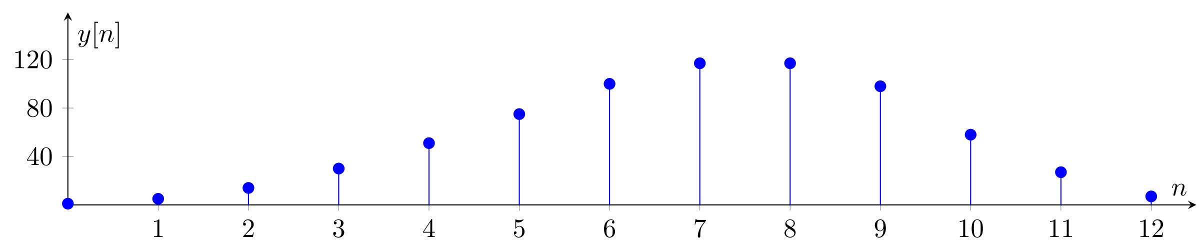
Shown in Figure 4 are the results of convolution as plotted from Listing 2. Note that it is expected for $y_n[n] = y[n]$, which is seen to be true.



(a) The convolution result $y_n[n] = x[n] * h[n]$.



(b) The convolution result $y_p[n] = x[n] \circledast h[n]$.



(c) The periodic convolution of the padded versions of $x[n]$ and $h[n]$ is identical to y_n.

Figure 4: The outputs from convolution in Project 3.

Project 4

(The hard copy from `chirpgui` is shown in Figure 7 in the appendices.)

- Figure 6(a) shows a bandpass filter with cutoff frequencies at approximately $F = 0.05$ and $F = 0.15$.
- Figure 6(b) shows a bandpass filter with an exponential drop-off (starting at approximately $F = 0.05$ and trailing off at $F = 0.4$.

The above filter types were rendered by looking to the power-spectral density (plotted in the bottom left of each quad) and comparing against known filter types (such as high-pass, low-pass, notch, etc.).

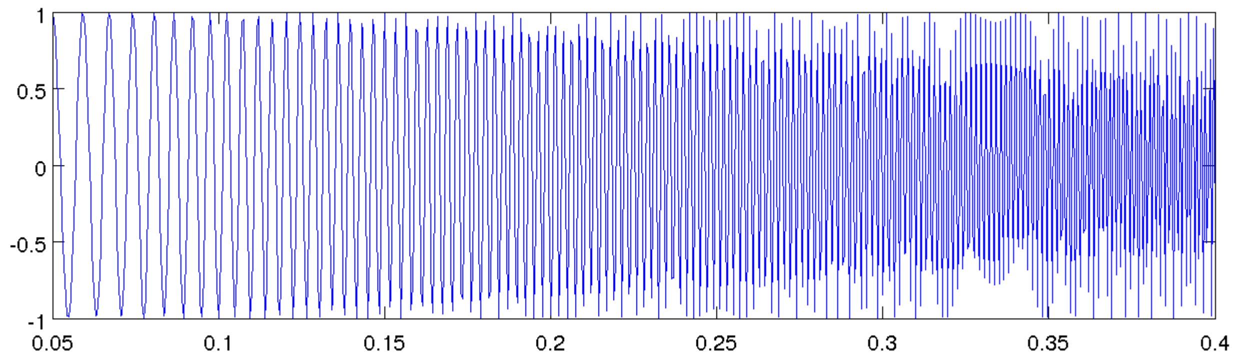
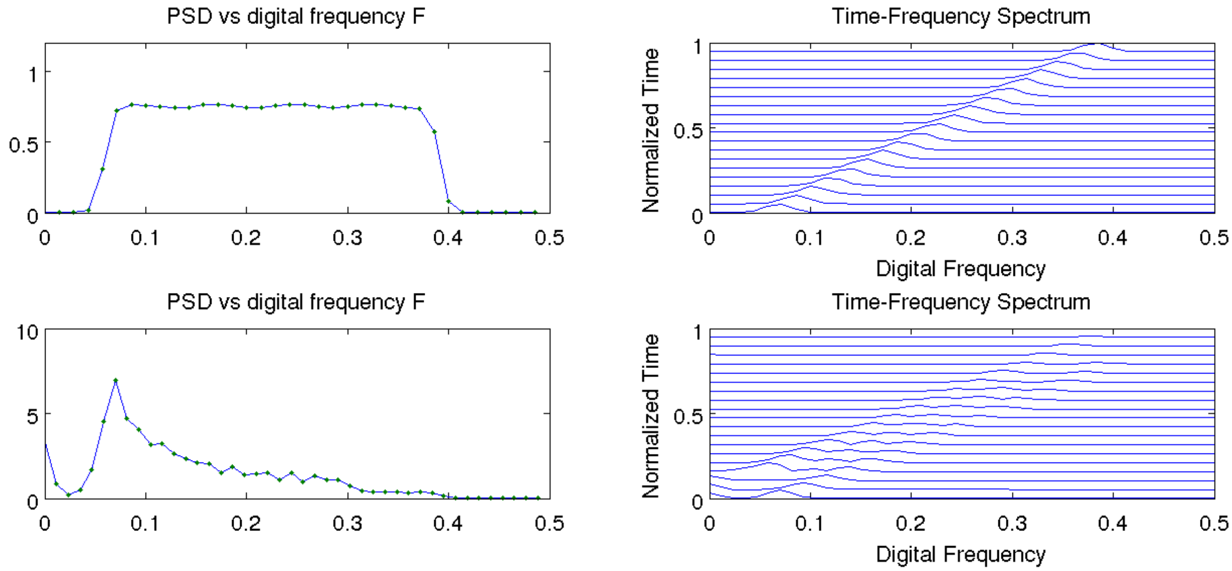
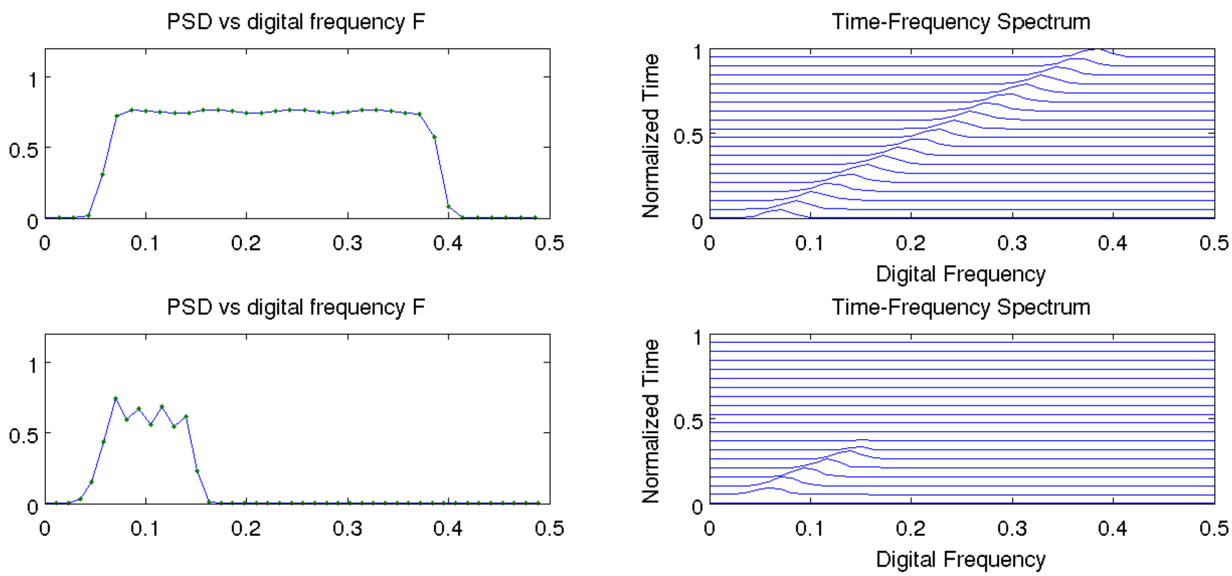


Figure 5: Output signal versus time plot showing that frequency increases with time.



(a) $h_1[n] = \delta[n] - 0.4\cos(0.4n\pi)\,\mathrm{sinc}(0.2n)$.



(b) $h_2[n] = 0.3\,\mathrm{sinc}(0.3n)$.

Figure 6: Spectral information for the two systems under consideration in Project 4.

Appendix: Miscellaneous figures

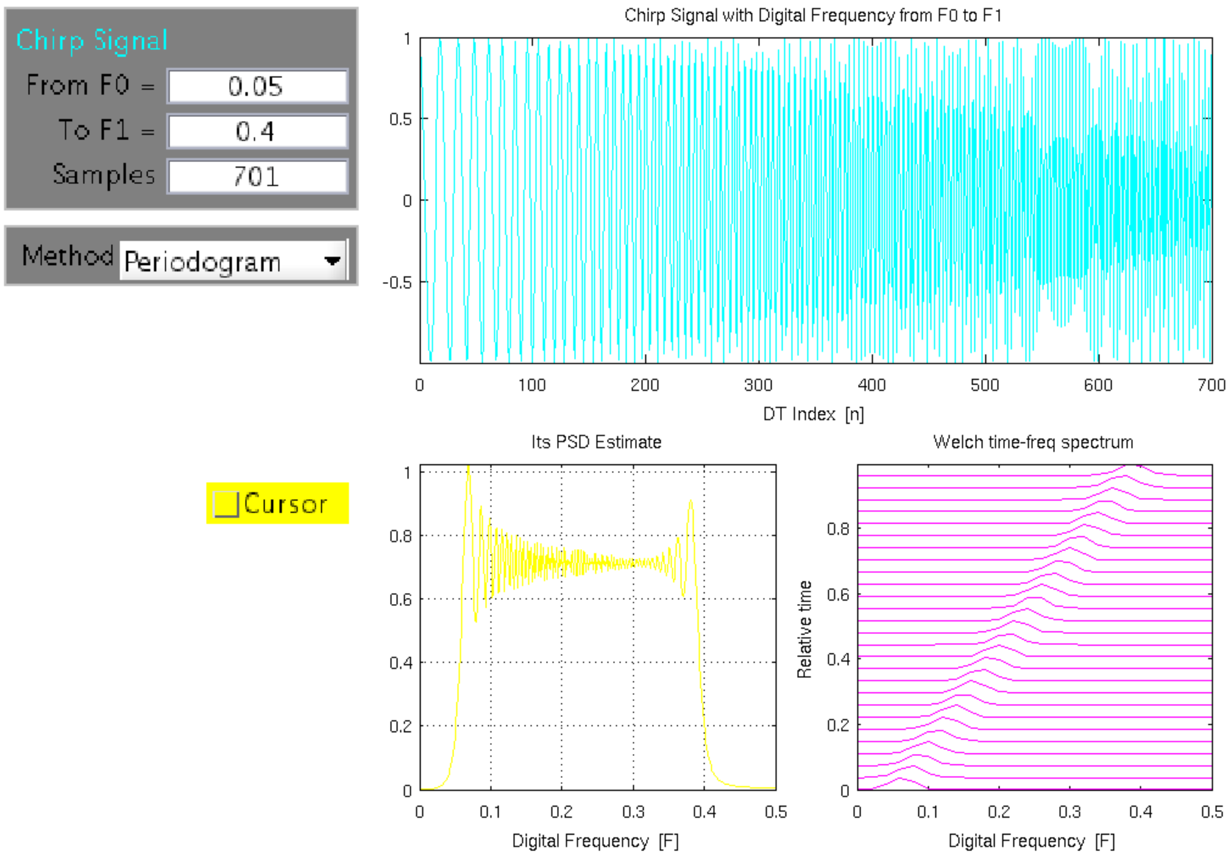


Figure 7: Hard copy of `chirpgui` for Project 4.

Appendix: MATLAB Source Code

What follows is a listing of the MATLAB source code (listing 1)—and the output of this code (listing 2)—used to generate the figures and other information presented in this report.

Listing 1: The MATLAB script used for this report, `Lab03_ahirzel.m`.

```
% EE4252 Lab 3
% Alex Hirzel <ahirzel@mtu.edu>
% 2012-10-04

addpath ../../ClassWorkspace;
delete 'generated/diary.txt'; diary 'generated/diary.txt'; diary on


% Project 1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x = 6*tri((-7:7)/6);
h = 4*tri(((-3:3) + 4)/4) - 4*tri(((-3:3) - 4)/4);

disp(['Sums: ' num2str(cellfun(@sum, {x h conv(x, h) conv(h, x) conv(x, x) conv(h, h)}))]);

subplot(1, 2, 1); stem(-7:7, x);
subplot(1, 2, 2); stem(-3:3, h);
mysaveas('p1_inputs', 10, 2.4);
```

```
stem(-10:10, conv(x, h), 'b'); xlim([-14 14]); mysaveas('p1_y1', 10, 2.4)
stem(-10:10, conv(h, x), 'b'); xlim([-14 14]); mysaveas('p1_y2', 10, 2.4)
stem(-14:14, conv(x, x), 'b'); xlim([-14 14]); mysaveas('p1_y3', 10, 2.4)
stem( -6:6,  conv(h, h), 'b'); xlim([-14 14]); mysaveas('p1_y4', 10, 2.4)


% Project 2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x = cos(0.2*pi*(0:50));
y = conv(x, 1:8);

subplot(2, 1, 1); stem(0:50, x, 'o'); xlim([0 57]); title('Input signal x[n]')
subplot(2, 1, 2); stem(0:57, y, 'o'); xlim([0 57]); title('Output signal y[n]')
mysaveas('p2', 10, 5);


% Project 3 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x = [1 3 5 7 5 3 1];
h = 1:7;

disp('Regular convolution:');           disp(conv(x, h))
disp('Periodic convolution:');          disp(convp(x, h))
disp('Regular convolution via convp:'); disp(convp([x zeros(1, 6)], [h zeros(1, 6)]))


% Project 4 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

load('project4.mat'); x = chirpsig;
m = 700;
dF = (0.4 - 0.05)/m;
F = 0.05 + (0:m)*dF;
plot(F, x)
mysaveas('p4_linear_freq', 10, 2.5)

h1 = @(n) 1 - 0.4*cos(0.4*n*pi)*sinc(0.2*n);
h2 = @(n) 0.3*sinc(0.3*n);

y1 = conv(x, arrayfun(h1, -80:80));
y2 = conv(x, arrayfun(h2, -80:80));

subplot(2,2,1); psdwelch(x);  axis([0 0.5 0 1.2]); subplot(2,2,2); timefreq(x);
subplot(2,2,3); psdwelch(y1); axis([0 0.5 0 10]);  subplot(2,2,4); timefreq(y1);
mysaveas('p4_y1', 10, 4)

subplot(2,2,1); psdwelch(x);  axis([0 0.5 0 1.2]); subplot(2,2,2); timefreq(x);
subplot(2,2,3); psdwelch(y2); axis([0 0.5 0 1.2]); subplot(2,2,4); timefreq(y2);
mysaveas('p4_y2', 10, 4)

diary off
```

Listing 2: The output of listing 1, `diary.txt`.

```
Sums: 36              0  8.8817842e-16  8.8817842e-16           1296              0
Regular convolution:
     1     5    14    30    51    75   100   117   117    98    58    27     7

Periodic convolution:
   118   122   112    88    78    82   100

Regular convolution via convp:
     1     5    14    30    51    75   100   117   117    98    58    27     7
```