Lab 4 Report

The z-transform and its applications

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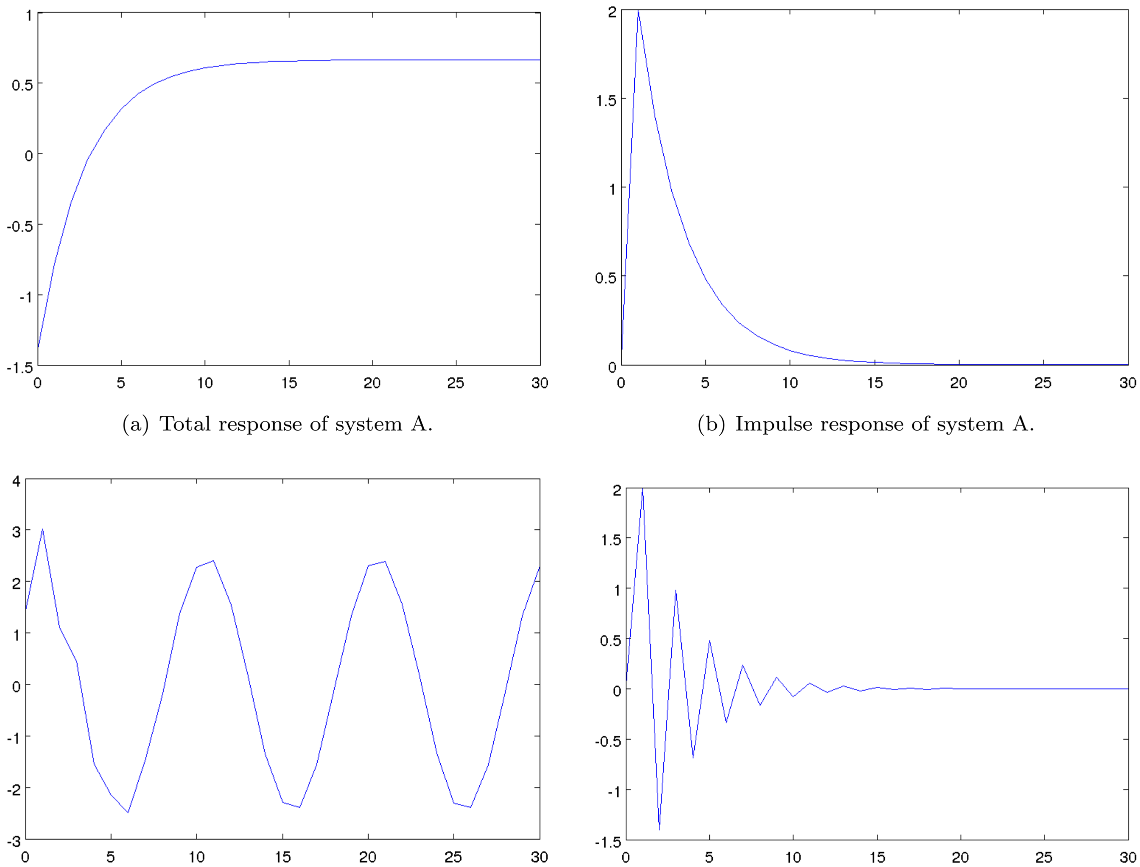
Project 1

The results of **ztr** and **izt** matched expectations; this can be confirmed by referring to **??** and noting the final symbolic representations are equivalent to the inputs. That is to say,

$$\texttt{(5*(2 .\^{}n)).*ustep(n)} \equiv 5(2)^n u(n)$$

and

$$\texttt{(5*(2 .\^{}n)-3*((-1).\^{}n)).*ustep(n)} \equiv 5(2)^n u(n) - 3(-1)^n u(n).$$



(a) Total response of system A.

(b) Impulse response of system A.

(c) Total response of system B.

(d) Impulse response of system B.

Figure 1: The outputs from convolution in Project 1.

The estimated steady-state value for system A is 0.6.

Project 2

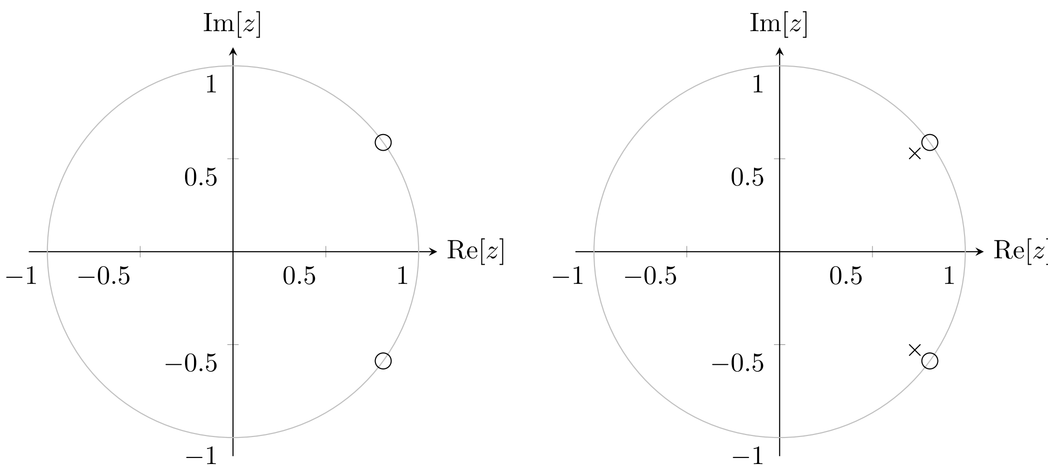
An FIR filter was created to attempt to remove the 60Hz noise from the ECG signal. Zeroes were placed at the locations in the z-plane corresponding to the 60Hz component when sampled at 300Hz. That is to say, conjugate zeroes were placed at $F = \pm 60/300 = 0.2$ on the unit circle, resulting in

$$H_{\text{FIR}}(z) = \frac{1}{(z - e^{j2\pi 60/300})(z - e^{-j2\pi 60/300})}. \tag{1}$$

The output of this filter is shown in **??**. A sister filter was designed by placing zeroes very near to these poles to reduce overshoot. This is an IIR filter because the representation of $h[n]$ (not shown) has only an infinite representation.

$$H_{\text{IIR}}(z) = \frac{(z - 0.99e^{j2\pi 60/300})(z - 0.99e^{-j2\pi 60/300})}{(z - e^{j2\pi 60/300})(z - e^{-j2\pi 60/300})}. \tag{2}$$

The output of this IIR filter is shown in **??**. Note that there is a *ringing* present at start-up which damps out over less than a quarter period. This ringing can be traded for overshoot by manipulating the radial distance of the poles[1]. Finally, an averaging filter was developed with a width of $360/60 = 5$ samples in hopes that it would eliminate the sinusoidal corruption. The output from this filter is shown in **??**.



(a) Pole-zero plot of **??**.

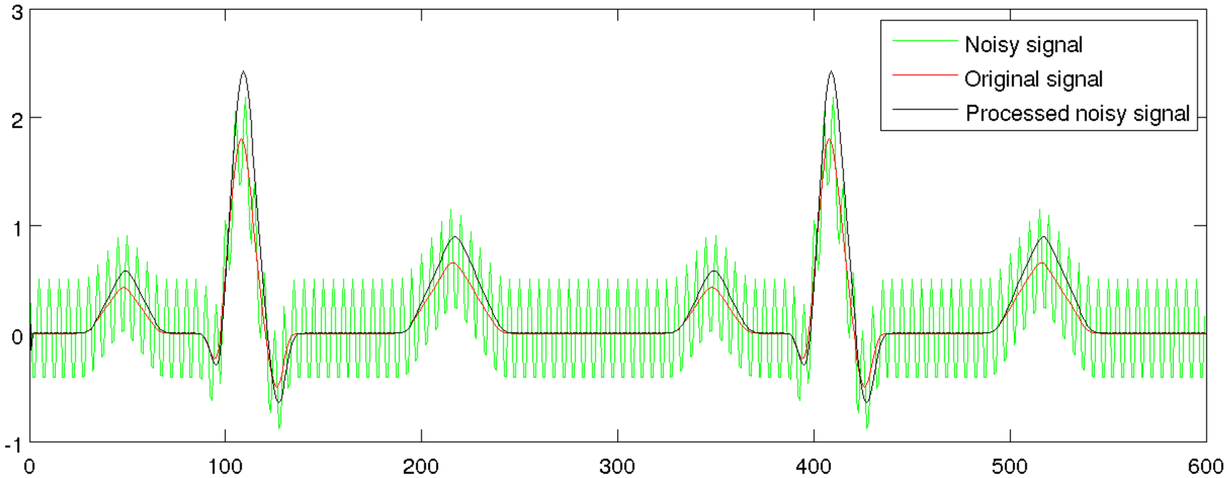
(b) Pole-zero plot of **??**.

Figure 2: Pole-zero plots of the FIR and IIR filters.

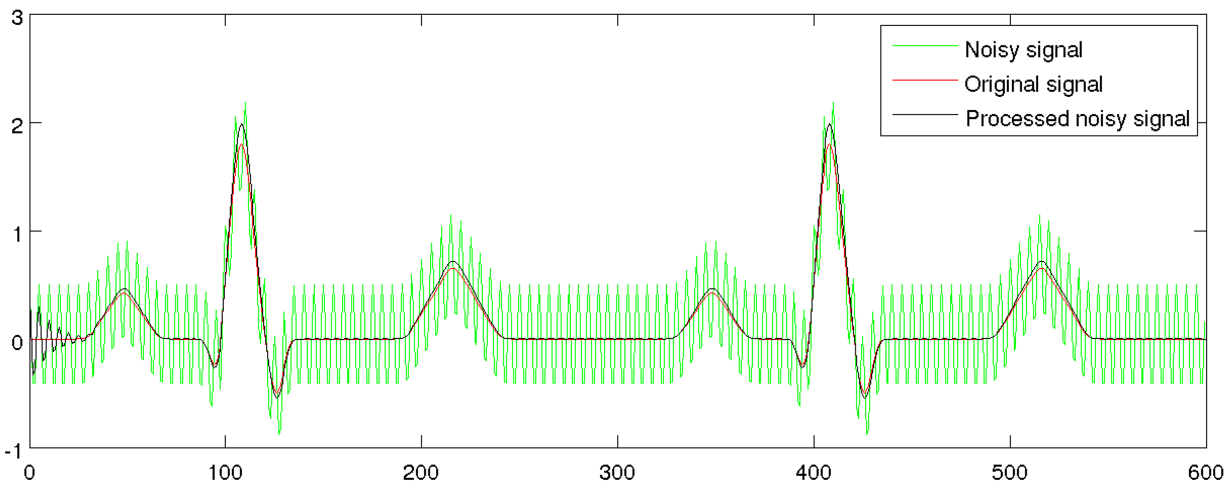
Conclusions

Of all approaches to filtering the 60Hz noise, it must be said that the moving average was all-round most effective except for the phase lag of 2.5 samples. If the end application of this filter cannot tolerate this distortion the next best choice is the IIR filter assuming the visible ringing on start-up can be ignored. (Refer to **??** on the left-hand side to view the ringing.)

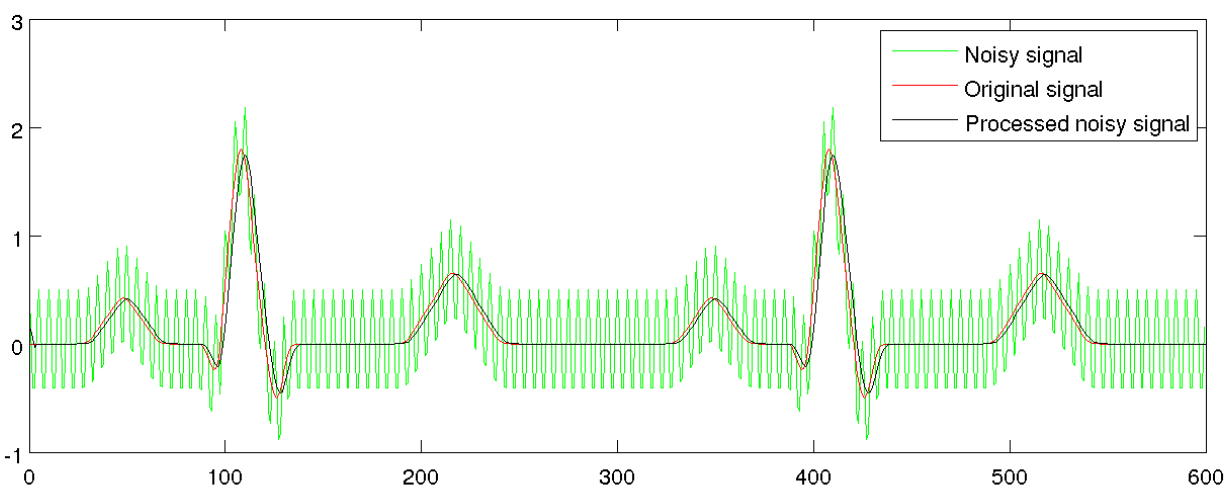
[1] that is to say, the closer the poles are to the unit circle, the less overshoot and more ringing is present; when the poles are moved further from the unit circle, ringing is reduced but overshoot is increased.



(a) Filtering the ECG signal with an FIR filter, **??**.



(b) Filtering the ECG signal with an IIR filter, **??**.



(c) Filtering the ECG signal with a traditional moving average filter with 5 samples.

Figure 3: Various filtering techniques applied to an ECG signal contaminated with 60Hz noise.

Project 3

Note that since $S = 8192$Hz, τ is equivalent to 768 samples (and 2τ to 1536 samples), so in the z-domain,

$$c(t) = s(t) + \alpha_1 s(t-\tau) + \alpha_2 s(t-2\tau)$$
$$\therefore\ C(z) = S(z) + 0.9z^{-768}S(z) + 0.8z^{-1536}S(z)$$

and so the transfer function of the system can be found and inverted

$$H(z) = \frac{C(z)}{S(z)} = \frac{1}{1 + 0.9z^{-768} + 0.8z^{-1536}},$$
$$H_{\mathrm{I}}(z) = [H(z)]^{-1} = 1 + 0.9z^{-768} + 0.8z^{-1536}.$$

Applying this filter to the corrupted signal results in a computerized voice stating

$$\boxed{\textbf{"DSP is fun"},}$$

a statement with which this author generally agrees.

Appendix: MATLAB Source Code

What follows is a listing of the MATLAB source code (listing ??)—and the output of this code (listing ??)—used to generate the figures and other information presented in this report.

Listing 1: The MATLAB script used for this report, `Lab04_ahirzel.m`.

```
% EE4252 Lab 4
% Alex Hirzel <ahirzel@mtu.edu>
% 2012-10-18

addpath ../../ClassWorkspace;
delete 'generated/diary.txt'; diary 'generated/diary.txt'; diary on


% PROJECT 1
[n, d] = ztr([5, 2, 0, 0, 0]);                  disp(izt('tf', n, d))
[n, d] = ztr([5, 2, 0, 0, 0; -3, -1, 0, 0, 0]); disp(izt('tf', n, d))

n = 0:30;

[NX, DX]       = ztr([0.1 1 0 0 0 0]);
imp            = sysresp2('z', 2, [1, -0.7]);
[yt, yzs, yzi] = sysresp2('z', 2, [1, -0.7], NX, DX, -2);
plot(n, eval(imp)); mysaveas('p1_2a_ir',    6, 4);
plot(n, eval(yt));  mysaveas('p1_2a_total', 6, 4);

[NX, DX]       = ztr([2 1 0 0.2*pi 0 0]);
imp            = sysresp2('z', 2, [1, +0.7]);
[yt, yzs, yzi] = sysresp2('z', 2, [1, 0.7], NX, DX, -2);
plot(n, eval(imp)); mysaveas('p1_2b_ir',    6, 4);
plot(n, eval(yt));  mysaveas('p1_2b_total', 6, 4);

% PROJECT 2
load ecg; load ecgo

f = 60; fs = 300; F = f/fs; n = 0:599;

polepair = @(a, F) conv([1, -a*exp(2*pi*i*F)], [1, -a*exp(-2*pi*i*F)]);

% FIR filter
N = polepair(1, F);
D = [1];
plot(n, ecg, '-g', n, ecgo, '-r', n, filter(N, D, ecg), '-k');
legend('Noisy signal', 'Original signal', 'Processed noisy signal'); ylim([-1 3])
mysaveas('p2_fir', 10, 3.5);

% IIR filter
N = polepair(1, F);
D = polepair(0.9, F);
plot(n, ecg, '-g', n, ecgo, '-r', n, filter(N, D, ecg), '-k');
legend('Noisy signal', 'Original signal', 'Processed noisy signal'); ylim([-1 3])
mysaveas('p2_iir', 10, 3.5);

% MA filter
MA = @(N) ones(1, N)/N;
N = MA(300/60);
D = [1];
plot(n, ecg, '-g', n, ecgo, '-r', n, filter(N, D, ecg), '-k');
legend('Noisy signal', 'Original signal', 'Processed noisy signal'); ylim([-1 3])
mysaveas('p2_ma', 10, 3.5);


% PROJECT 3
S = 8192; alpha1 = 0.9; alpha2 = 0.8; tau = 0.09375;

load echosig;
H = [1 zeros(1,767) 0.9 zeros(1,767) 0.8];
sound(filter([1], H, echosig))

diary off
```

Listing 2: The output of listing ??, `diary.txt`.

```
S=1700   [  1/17  12/85 ]   Alias=0   N_min=85
S= 360   [  5/18   2/ 3 ]   Alias=1   N_min=18
S= 120   [  5/ 6   2/ 1 ]   Alias=1   N_min=6
S=  50   [  2/ 1  24/ 5 ]   Alias=1   N_min=5
```