Cairo University

Faculty of Engineering
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Communications and Computer Engineering

Credit Hours System

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PHYN211—Project 1

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To:

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Question 1:

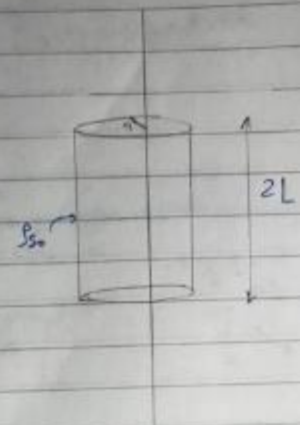
Hand Analysis:

We have driven analytical equations for the electric field and potential of this surface distributed charge cylinder using Gauss' Law.

Question: Given: $L = 5m$, $a = 0.5m$, $\rho_{s0} = 20\mu C/m^2$, $h = 15m$

By applying Gauss Law $\oint \underline{E} \cdot d\underline{S} = \frac{\Sigma Q_{en}}{\epsilon_0}$

$$E(2\pi\rho h) = \frac{\rho_{s0} \times 2\pi a h}{\epsilon_0} \quad \therefore \underline{E} = \frac{a\rho_{s0}\, a_\rho}{\epsilon_0 \rho}$$



$$V = -\int_{\infty}^{r} \underline{E} \cdot d\underline{L} = -\int_{\infty}^{r} \frac{a\rho_{s0}}{\epsilon_0 \rho} d\rho$$

$$= -\frac{a\rho_{s0}}{\epsilon_0} \int_{\infty}^{r} \frac{1}{\rho} d\rho = \frac{-\rho_{s0} a}{\epsilon_0} \ln(\rho)\Big|_{\infty}^{\rho}$$

as we've taken $\infty = 30L$ (150) in our project to get numerical approximation

$$V = -\frac{\rho_{s0} a}{\epsilon_0} \ln\left(\frac{\rho}{30L}\right)$$

$$= \frac{\rho_{s0} a}{\epsilon_0} \ln\left(\frac{30L}{\rho}\right) \quad \# = \frac{\rho_{s0} a}{\epsilon_0} \ln\left(\frac{150}{\rho}\right)\Big|_{L=5m} \quad \#$$

MATLAB Code:

```
% team 3
% problem 1
epsilon = 8.854e-12;
% defining our parameters
L = 5; % cylinder half-length
a = 0.5;% cylinder radius
Ps = 20e-6; % surface charge density
%h = 15; % point to measure at
%
%% 1)
syms z h
dE_symbolic = (a*Ps/(2*epsilon)) * ((h-z))/(a^2+(h-z)^2)^(3/2);
h = 15;
dE = matlabFunction(subs(dE_symbolic)); % subs(dE_symbolic) repaces all instances of h with 15 in dE_symbolic
N = [3 9 15 30 50 100]; % array of steps
Result = zeros;
% Numerical Integration using the trapezoidal rule
for i = 1:6
    step = (2*L)/N(i); % step by which n + 1 points are generated
    point_idx = 1;
    values = zeros;
    for j = -L : step : L
        values(point_idx) = dE(j);
        point_idx = point_idx + 1;
    end
    Result(i) = step * trapz(values);
end
%
disp(Result);

%% 2)
% syms h % discard h as 15 and make it symbolic again

% % for requirement b we should vary h from L to infinity(a reasonably large value)
% E = int(dE_symbolic, z, -L, L); % The integral is calculated wrt to z

% number_of_sample_points = 1000;
% H = linspace(6, 200, number_of_sample_points);
% Eplot = zeros;
% for i = 1:number_of_sample_points
%     h = H(i);
%     Eplot(i) = double(subs(E));
% end
% LineSpec = {'Color','red','LineWidth',2};
% %plot(H,Eplot,LineSpec{:});

%% 3)
```

```
%syms h; % return h as symbolic again
syms r;
E = piecewise(abs(r)<=a, 0, abs(r) > a, (Ps*a)/(epsilon*r) ); % E is a piecewise function

syms R; % restore r to be symbolic
% and initialize symbolic var R
V = int(E, r, R, 30*L); %3.a
%fplot(E,[0 150]); %3.b
%3.c
X = (-5:0.5:-0.5);
X = cat(2,X,0);
X = cat(2,X,0.5:0.5:5);
[x, y] = meshgrid(X);
u = x; v = y;
quiver(x, y, u, v);
%fplot(V); %3.d
```

1) **Perform numerical integration to find the electric field at a point on the z-axis outside the cylinder at (0,0,h). Use different steps ‘n’ for integration (i.e., n = 3, 9, 15, 30, 50, 100).**

MATLAB Code:

```
% team 3
% problem 1
epsilon = 8.854e-12;
% defining our parameters
L = 5; % cylinder half-length
a = 0.5;% cylinder radius
Ps = 20e-6; % surface charge density
%h = 15; % point to measure at
%
%% 1)
syms z h
dE_symbolic = (a*Ps/(2*epsilon)) * ((h-z))/(a^2+(h-z)^2)^(3/2);
h = 15;
dE = matlabFunction(subs(dE_symbolic)); % subs(dE_symbolic) repaces all instances of h with 15 in dE_symbolic
N = [3 9 15 30 50 100]; % array of steps
Result = zeros;
% Numerical Integration using the trapezoidal rule
```

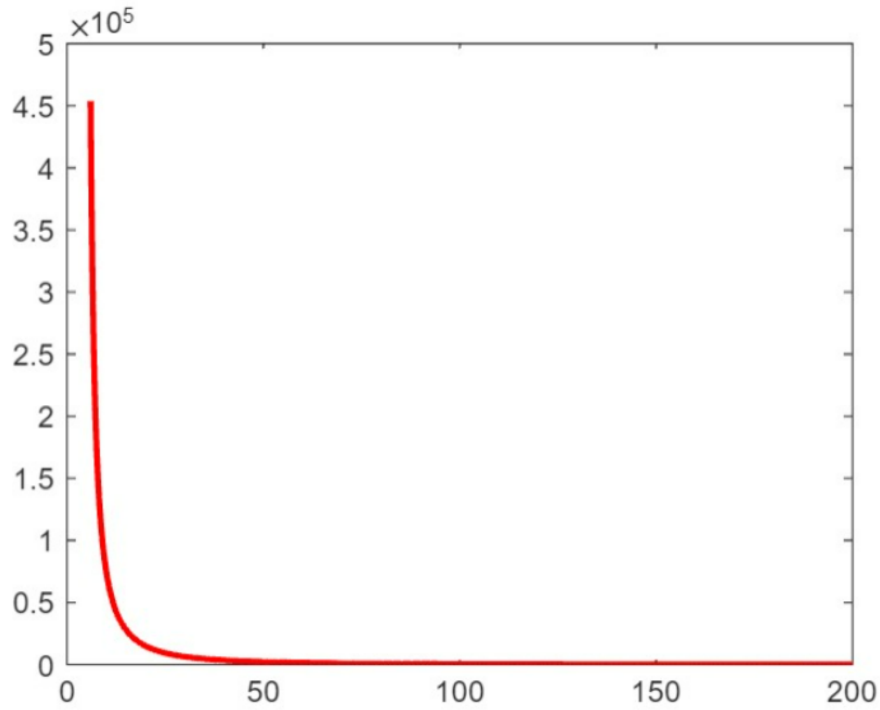
```
for i = 1:6
    step = (2*L)/N(i); % step by which n + 1 points are generated
    point_idx = 1;
    values = zeros;
    for j = -L : step : L
        values(point_idx) = dE(j);
        point_idx = point_idx + 1;
    end
    Result(i) = step * trapz(values);
end
%
disp(Result);
```

Results:

The results of the numerical integration as we increase the number of steps.

```
Q1
1.0e+04 *

 2.9061    2.8275    2.8210    2.8183    2.8177    2.8175
```

2) Plot the electric field magnitude vs. the z-axis where $|z| > L$.



3)

a) We have defined Electric Field as a piecewise function, so we have three regions of varying electric field.

→ from –∞ to -1/2, the electric field is given by this function:

```
V =

piecewise(R == 150, 0, R < -1/2, (967140655691703339764940 8*log(300))/8563063365494340625 - (9671406556917033397649408*log(-2*R))/8563063365494340625,
```

→ from -1/2 to 1/2, the electric field is given by this function:

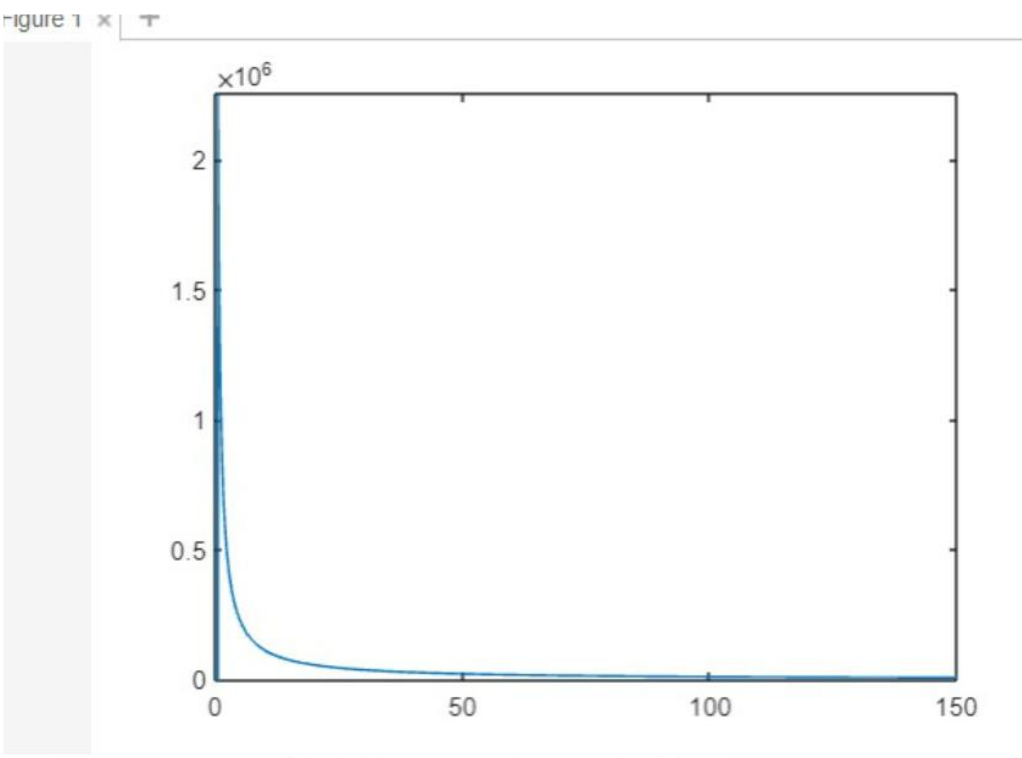
```
Interval([-1/2], [1/2]), (9671406556917033397649408*log(300))/8563063365494340625,
```

→ from ½ to ∞ the electric field is given by this function:

```
Interval([1/2], 150), (967140655691703339764940B*log(150))/8563063365494340625 - (967140655691703339764940B*log(R))/8563063365494340625)
```

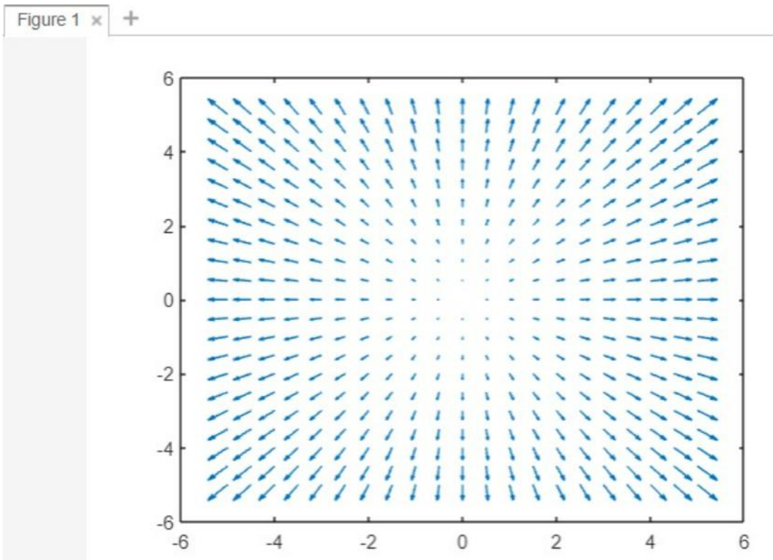
Note: We have substituted ∞ equals to 150 in this simulation, which is large enough to see approximate results without high MATLAB computational cost and time.

b) Plot the electric field magnitude vs. radius (inside and outside the cylinder).



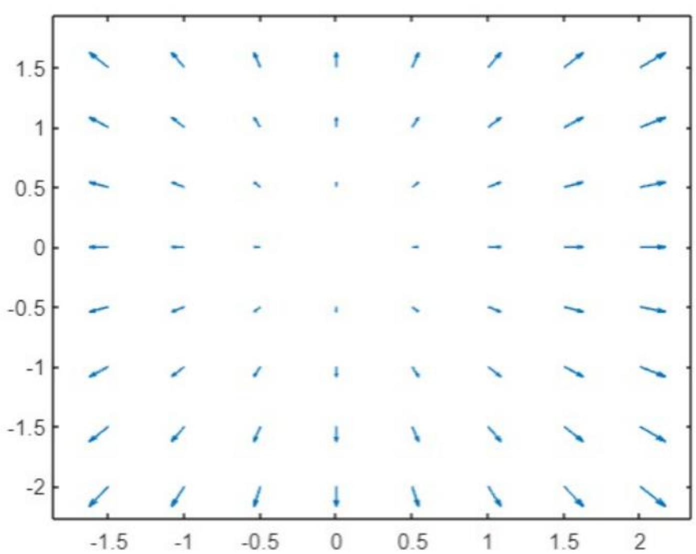
As expected, the electric fields decays as r grows very large.

c) Plot the electric field lines in xy-plane and show the electric field arrows.

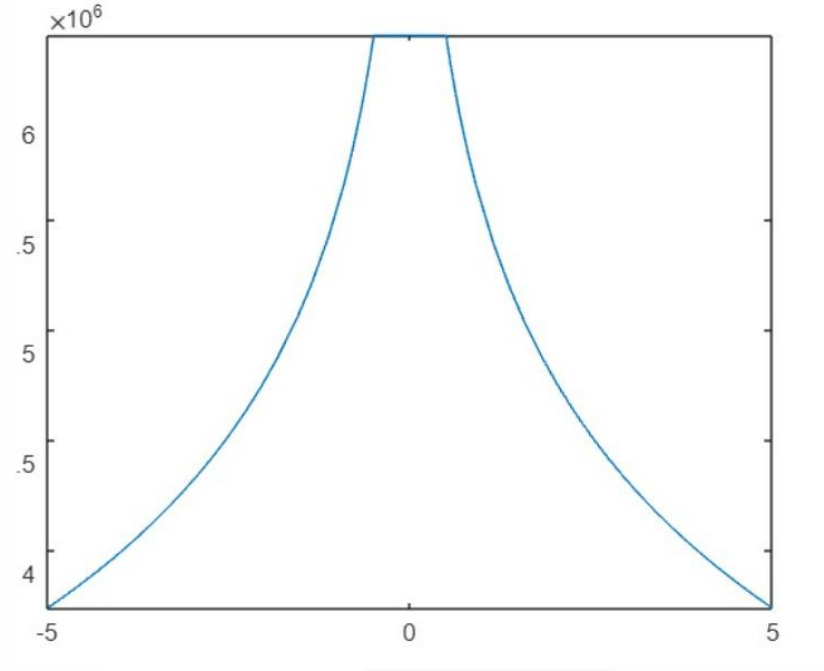


The electric field lines are only directed at r>=1/2, as the charge is distributed throughout the surface.

A zoomed-in image, where this can be seen more clearly:



d) Plot the electric potential in the xy plane.



The potential decays with infinity, so the figure has this shape.

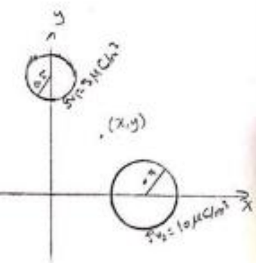
Question 2

Hand Analysis:

<u>Question2)</u>

<u>Given</u>: <u>sphere1</u>: $(0,5,0)$, $\rho_{v_1} = 5\mu\, C/m^3$, $r = 0.05$

<u>sphere2</u>: $(10,0,0)$, $\rho_{v_2} = 10\mu C/m^3$, $r = 0.07$



<u>Sol.</u>

<u>for sphere1</u>:

$$\oint \underline{E} \cdot d\underline{s} = \frac{\sum Q_{in}}{\epsilon_o}$$

R: Radius

$$E(4\pi R^2) = \frac{1}{\epsilon_o}\int \rho_{v_1}\, dV$$

$$D(4\pi R^2) = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_{v_1} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$D(4\pi R^2) = \rho_{v_1}(2\pi)(2)\frac{r^3}{3}$$

<u>at $r<a$</u>: $D(4\pi r^2) = \rho_{v_1}(\frac{4\pi}{3})\, r^3 \big|_{R=r}$

$$E = \frac{\rho_{v_1} r}{3\epsilon_o}\, a_r$$

<u>at $r \geq a$</u>: $E = \frac{\rho_{v_1} a^3}{3\epsilon_o r^2}\, a_r$

$$E_1 = \begin{cases} \frac{\rho_{v_1} r_1}{3\epsilon_o}\, a_{r_1}, & r_1 < a \\ \frac{\rho_{v_1} a^3}{3\epsilon_o r_1^2}\, a_r, & r_1 \geq a \end{cases} \qquad \underline{E}_2 = \begin{cases} \frac{\rho_{v_2} r_2}{3\epsilon_o}\, a_{r_2}, & r_2 < a \\ \frac{\rho_{v_2} a^3}{3\epsilon_o r_2^2}\, a_{r_2}, & r \geq a \end{cases}$$

$$\sum \underline{E} = \underline{E}_1 + \underline{E}_2$$

$\therefore$ General Field Function is given by:

$$\underline{E} = \begin{cases} \frac{\rho_{v_1} + \rho_{v_2} b^3}{3\epsilon_o}\left(r_1 a_{r_1} + \frac{1}{r_2^2} a_{r_2}\right), & r < a \,\&\, r_2 \geq b \\ \frac{\rho_{v_1} a^3 + \rho_{v_2}}{3\epsilon_o}\left(\frac{1}{r_1^2} a_{r_1} + r_2 a_{r_2}\right), & r_1 \geq a \,\&\, r_2 < b \\ \frac{\rho_{v_1} a^3 + \rho_{v_2} b^3}{3\epsilon_o}\left(\frac{1}{r_1^2} a_{r_1} + \frac{1}{r_2^2} a_{r_2}\right), & r_1 \geq a \,\&\, r_2 \geq b \end{cases}$$

i) Find the point(s) where the electric field is maximum.

We’ve assumed that the maximum point of the electric field will be in the xy plane, as at certain regions, the components will not cancel each other out. The maximum points of the electric field will be confined in the rectangular region outside the area between the two spheres, as the components will add together, because the maximum circumference of both spheres lies in the xy plane so that’s a reasonable assumption.

MATLAB code:

```
% team 3
% problem 2
% problem parameters
a = 0.5;
b = 0.7;
epsilon = 8.854e-12;
pv1 = 5e-6;
pv2 = 10e-6;
%
% K1 and K2 are auxiliary constants for readability
K1 = pv1/(3*epsilon);
K2 = pv2/(3*epsilon);
%
E1_E2_dotproduct = @(x, y)((x - 10).* x + (y - 5) .* y); %this is the dot
product of E1 and E2 position vectors(r1, r2)
E1 = @(x, y)((K1 .(a.^3)).^2)./((x.^2+(y-5).^2).^2).(x.^2+(y-5).^2>a^2);
E2 = @(x, y)((K2 (b^3))^2)./(((x-10).^2+y.^2).^2).((x-10) .^2 + y.^2 >
(b^2));
E1_dash = @(x, y)(K1.*(a.^3)./((x.^2+(y-5).^2).^(3/2)));
E2_dash = @(x, y)(K2.*(b^3)./(((x-10).^2+y.^2).^(3/2)));
E12 = @(x,y)((2.E1_dash(x,y).*E2_dash(x, y) .
E1_E2_dotproduct(x,y)).(x.^2+(y-5).^2>(a^2)).((x-10).^2+y.^2>(b^2)));
E1_in = @(x,y)((K1).((x.^2+(y-5).^2)).(x.^2+(y-5).^2<=(a^2)));
E2_in = @(x,y)((K2).(((x-10).^2+y.^2)).((x-10).^2+y.^2<=(b^2)));
E1_in_dash = @(x,y)(2.*(E1_in(x,y).^2.^(a^2)).*sqrt(K1).*E2_dash(x,
y).*E1_E2_dotproduct(x,y));
E2_in_dash =
@(x,y)(2.*(E2_in(x,y).^2.^(a^2)).*sqrt(K2).*E1_dash(x,y).*E1_E2_dotproduct(
x,y));
```

```
E =
@(x,y)(sqrt(E1(x,y)+E2(x,y)+E12(x,y)+E1_in(x,y).^2+E2_in(x,y).^2+E1_in_dash
(x,y)+E2_in_dash(x,y)));
[X,Y] = meshgrid(9:0.0025:11,-1:0.0025:1);
E_values = E(X, Y);
[Ex, Ex_idx] = max(E_values); % we get the max maximum values of E in the x
direction and get their indices
[Exy, Exy_idx] = max(Ex); % for the maximum values in X direction, we also
get the one with the maximum value in y, and also its position
Exy_Max_pos = Ex_idx(:,Exy_idx);
X_Max_Idx = X(Exy_Max_pos, Exy_idx);
Y_Max_Idx = Y(Exy_Max_pos, Exy_idx);
fprintf("Electric field is maximum at X=%.4f, Y=%.4f, Z=%.4f\n", X_Max_Idx,
Y_Max_Idx, 0);
fprintf("Its value is = %.4f",Exy);
```

Results: the maximum electric field and the point.

```
>> q2
Electric field is maximum at X=10.5600, Y=-0.4200, Z=0.0000
Its value is = 263698.9948
```

This point is coherent with the fact that the maximum point of electric field would be right to the second sphere (of charge density 10 microC/m^3) as it has a higher charge density and larger volume.

ii) Find the point(s) where the electric field vanishes
Note: The point where we found to vanish is not exactly zero due to the approximations. But as r→∞, the electric field vanishes.

MATLAB code:

```
[maxE, maxIdx] =min(vecnorm(E_values));% dah el max; value beta3 el
electric field3 ely mafrod haytla3 hena bas ana msh 3aref akaren benhom
[Xmax,Ymax,Zmax] = ind2sub(size(X),maxIdx);
disp(Xmax);
disp(Zmax);
disp(maxE);
xm = X(Xmax, Ymax, Zmax);
ym = Y(Xmax, Ymax, Zmax);
zm = Z(Xmax, Ymax, Zmax);
disp("--------------------------------------------------------------------
---")
fprintf('The value of (x,y,z) is %d %d %d %d\n' ,xm,ym,zm);
```

Results:

```
-----------------------------------------------------------------------
fx The value of (x,y,z) at where the elctric field vanish is 9 -1 -10 >>
```

iii) Plot the Electric Field in the x-y plane.

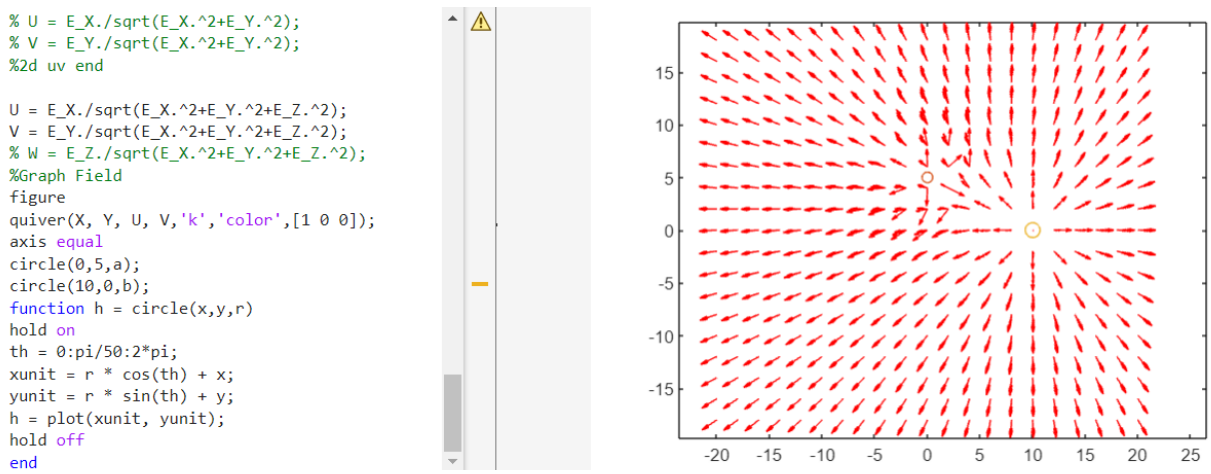
MATLAB code:

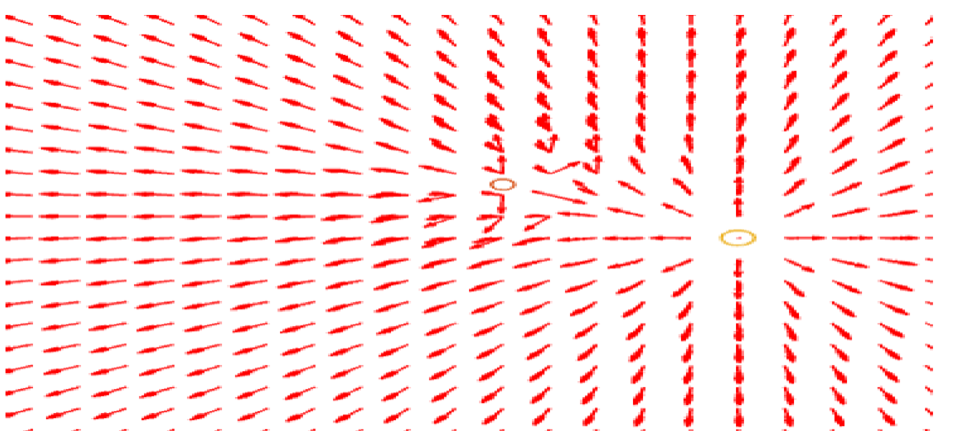
```
epsilon0=8.85*10^(-12);
pi=3.14159256;
k=1/(4*pi*epsilon0);
a=0.5;
b=0.7;
pv1=5*10^-6;
pv2=10*10^-6;
x = -20:2:20;
y = -20:2:20;
z=-20:5:20;
[X,Y,Z] = meshgrid(x,y,z);
E_X = zeros([21, 21]);
E_Y = zeros([21, 21]);
E_Z = zeros([21, 21]);
Theta_Variation = 0:pi/10:pi;
Phi_Variation= 0:pi/10:2*pi;
thetaCounter=0;
PhiCounter=0;
Q1=(a^3*pv1/(3*epsilon0))*10^-6;
Q2=(b^3*pv2/(3*epsilon0))*10^-6;

aCounter=0;
while aCounter<=a
while thetaCounter<=pi
while PhiCounter<=2*pi
q1point=[aCounter,thetaCounter,PhiCounter];
q1cart=[0,0,0];
q1cart(1)=q1point(1)*sin(q1point(2))*cos(q1point(3));
q1cart(2)=q1point(1)*sin(q1point(2))*sin(q1point(3));
q1cart(3)=q1point(1)*cos(q1point(2));
R1_x=(X-q1cart(1));
R1_y=(Y-q1cart(2)-5);
R1_z=(Z-q1cart(3));
R1= sqrt((R1_x).^2 + (R1_y).^2+(R1_z).^2);
%where do we multiply by the unit vectors
 E1_x = ((Q1/3465)./(R1.^3)).* R1_x;
 E1_y = ((Q1/3465)./(R1.^3)) .* R1_y;
 E1_z = ((Q1/3465)./(R1.^3)) .* R1_z;

    E_X = E_X + E1_x;
    E_Y = E_Y + E1_y;
```

```
    E_Z = E_Z + E1_z;

PhiCounter=PhiCounter+pi/20;
end
PhiCounter=0;
thetaCounter=thetaCounter+pi/20;
end
aCounter=aCounter+0.05;
end
bCounter=0;
thetaCounter=0;
PhiCounter=0;

while bCounter<=b
while thetaCounter<=pi
while PhiCounter<=2*pi
q2point=[bCounter,thetaCounter,PhiCounter];
q2cart=[0,0,0];
q1cart(1)=q2point(1)*sin(q2point(2))*cos(q2point(3));
q1cart(2)=q2point(1)*sin(q2point(2))*sin(q2point(3));
q1cart(3)=q2point(1)*cos(q2point(2));
R2_x=(X-q2cart(1)-10);
R2_y=(Y-q2cart(2));
R2_z=(Z-q2cart(3));
R2= sqrt((R2_x).^2 + (R2_y).^2+(R2_z).^2);
%where do we multiply by the unit vectors
 E2_x = ((Q2/3465)./(R2.^3)) .* R2_x;
 E2_y = ((Q2/3465)./(R2.^3)) .* R2_y;
 E2_z = ((Q2/3465)./(R2.^3)) .* R2_z;

    E_X = E_X + E2_x;
    E_Y = E_Y + E2_y;
    E_Z = E_Z + E2_z;

PhiCounter=PhiCounter+pi/20;
end
PhiCounter=0;
thetaCounter=thetaCounter+pi/20;
end
bCounter=bCounter+0.05;
end
U = E_X./sqrt(E_X.^2+E_Y.^2+E_Z.^2);
V = E_Y./sqrt(E_X.^2+E_Y.^2+E_Z.^2);
%Graph Field
figure
quiver(X, Y, U, V,'k','color',[1 0 0]);
axis equal
circle(0,5,a);
circle(10,0,b);
function h = circle(x,y,r)
hold on
th = 0:pi/50:2*pi;
```

```
xunit = r * cos(th) + x;
yunit = r * sin(th) + y;
h = plot(xunit, yunit);
hold off
end
```

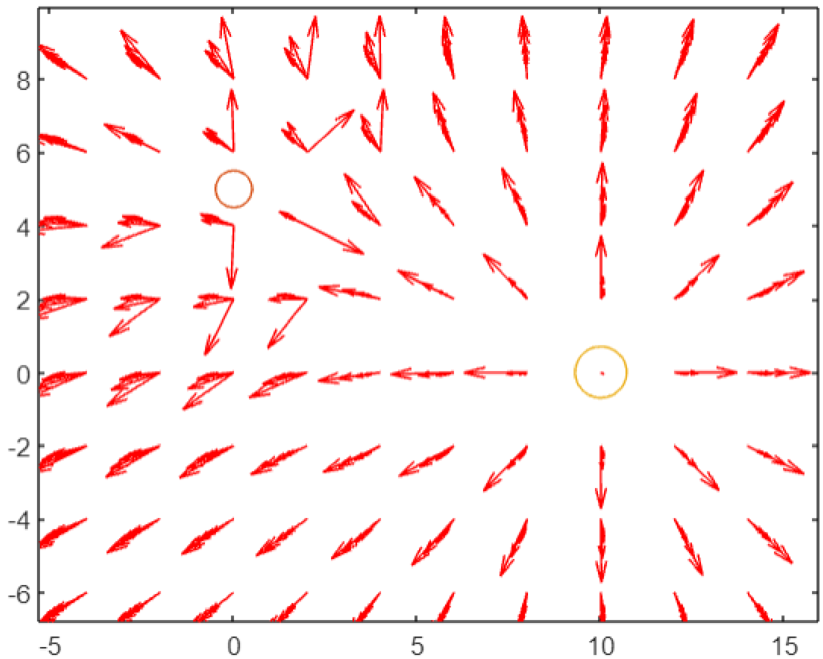
Results:





The electric Field lines are directed outward of the two circles in the radial direction—the vectors might be over-highlighted due to the large number of samples.

A zoomed-in image, where it is clear that the vectors directed from the two spheres superpose. As expected, the electric field lines would be more intense from the second sphere (drawn in yellow) as it has a larger radius and denser volumetric charge density.



iv) Plot the electric potential in the xy plane.

MATLAB code:

```
% Grid Size
x = linspace(-10, 20, 500);
y = linspace(-10, 10, 300);
```

```
z = linspace(-10, 10, 100);

% Meshgrid
[X,Y,Z] = meshgrid(x, y, z);
P = [X(:), Y(:), Z(:)];

% Constants
a = 0.5;
b = 0.7;
epsilon0 = 8.85e-12;
k = 1/(4*pi*epsilon0);
pvo1 = 5*10^(-6);
pvo2 = 10*10^(-6);

% We calculate the distance from each point in our grid to the two spheres
r1 = sqrt((P(:,1).^2) + (P(:,2)-5).^2 + (P(:,3).^2));
r2 = sqrt((P(:,1)-10).^2 + (P(:,2).^2) + (P(:,3).^2));

% Calculate the electric potential
V1 = k * (pvo1 .* a.^3) ./ r1;
V2 = k * (pvo2 .* b.^3) ./ r2;
% we add the electric potential
V = reshape(V1 + V2, size(X));

figure;
contourf(X(:,:,1), Y(:,:,1), V(:,:,50), 50, 'LineStyle', 'none');
colorbar;
xlabel('X');
ylabel('Y');
title('The electric potential of the two spheres in XY-Plane');
```

Results:

/MATLAB Drive/potential.m

```
% Grid Size
x = linspace(-10, 20, 500);
y = linspace(-10, 10, 300);
z = linspace(-10, 10, 100);

% Meshgrid
[X,Y,Z] = meshgrid(x, y, z);
P = [X(:), Y(:), Z(:)];

% Constants
a = 0.5;
b = 0.7;
epsilon0 = 8.85e-12;
k = 1/(4*pi*epsilon0);
pvo1 = 5*10^(-6);
pvo2 = 10*10^(-6);

% We calculate the distance from each point in our grid to th
r1 = sqrt((P(:,1).^2) + (P(:,2)-5).^2 + (P(:,3).^2));
r2 = sqrt((P(:,1)-10).^2 + (P(:,2).^2) + (P(:,3).^2));

```