# INTRODUCTION TO FILTER CIRCUITS

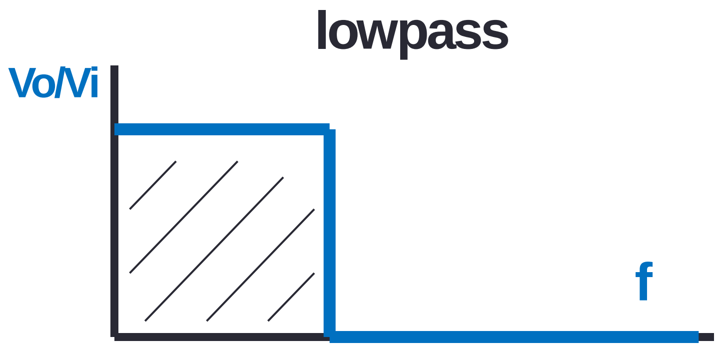
A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

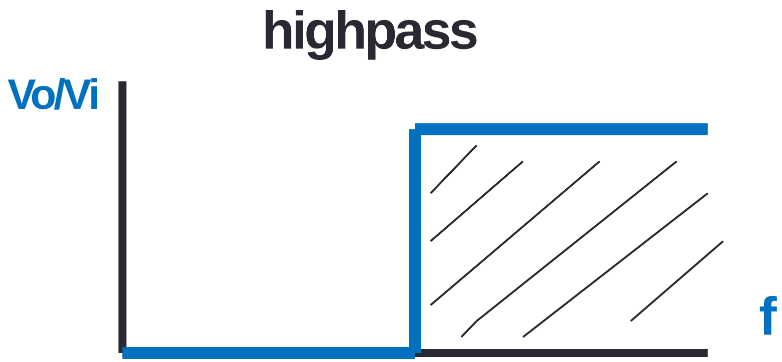
## Background:

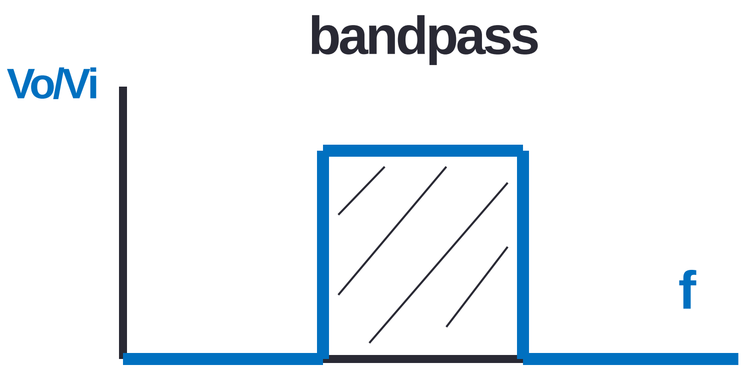
- Filters may be classified as either digital or analog.
- Digital filters are implemented using a digital computer or special purpose digital hardware.
- Analog filters may be classified as either passive or active and are usually implemented with R, L, and C components and operational amplifiers.
- An active filter is one that, along with R, L, and C components, also contains an energy source, such as that derived from an operational amplifier.
- A passive filter is one that contains only R, L, and C components.
- It is not necessary that all three be present. L is often omitted (on purpose) from passive filter design because of the size and cost of inductors – and they also carry along an R that must be included in the design.

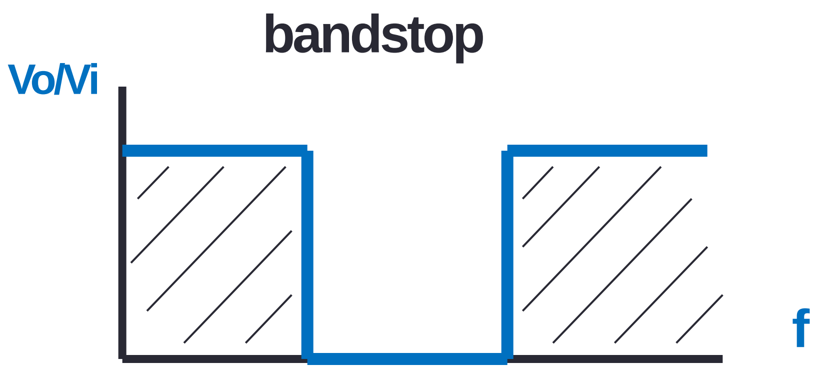
# Types Passive Analog Filters

## Four types of filters - “Ideal”



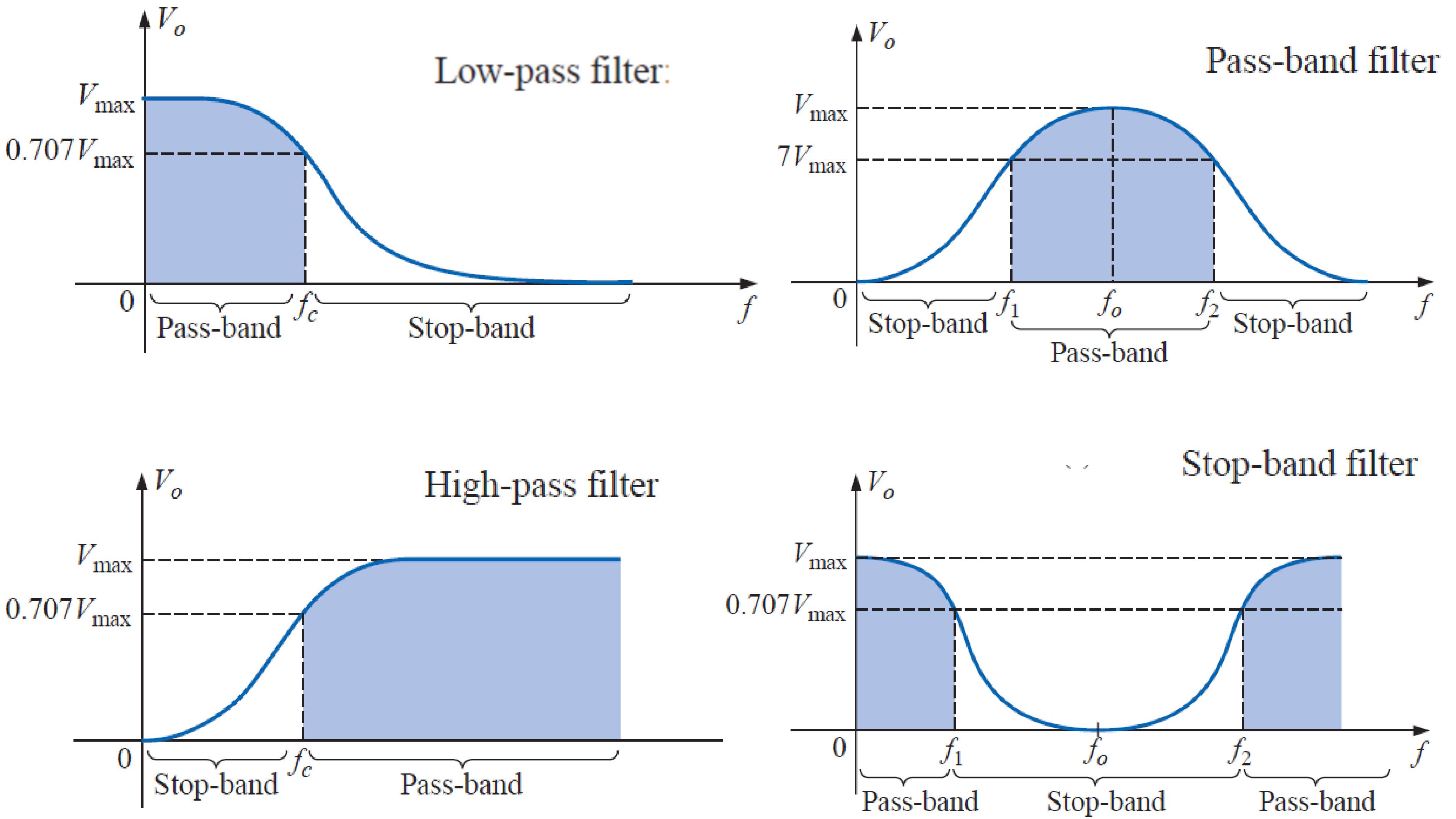



## Types of Passive Analog Filters

- A low pass filter passes low frequencies and stops high frequencies.
- A high pass filter passes high frequencies and rejects low frequencies.
- A band pass filter passes frequencies within a frequency band and blocks or attenuates frequencies outside the band.
- A band stop filter passes frequencies outside a frequency band and blocks or attenuates frequencies within the band.

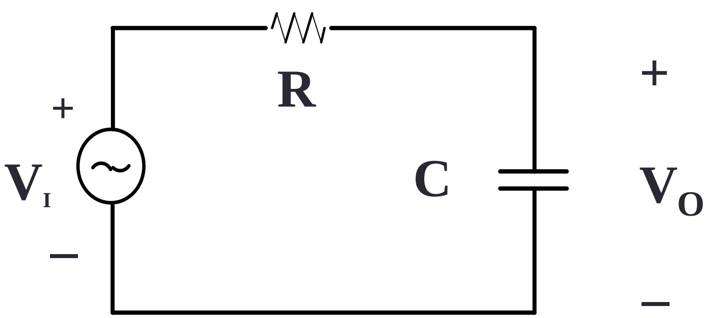
# Practical Filter Transfer Function vs. Frequency



# Analysis of Passive Analog Filters

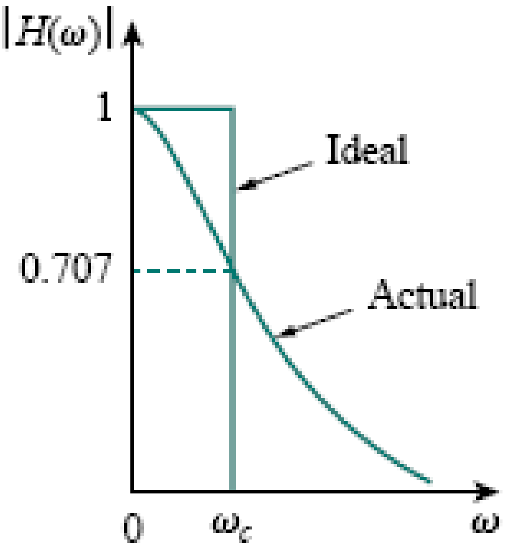
## 1.0 Low Pass Filter

Passes low frequencies
Attenuates high frequencies




$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$




The half-power frequency,
is usually known as the cutoff frequency ωC ,
is obtained by setting the magnitude of H(ω)
equal to 1/√2, thus,

Note that |H(0)| = 1 , |H(∞)| = 0.

## Cut off frequency

The cutoff frequency is the frequency at which the transfer function H drops in magnitude to 70.71% of its maximum value. It is also regarded as the frequency at which the power dissipated in a circuit is half of its maximum value.

$|H_{max}| = 1$ at $\omega = 0$, at $w_c$ , $|H(\omega_c)| = 0.707\ H_{max}$

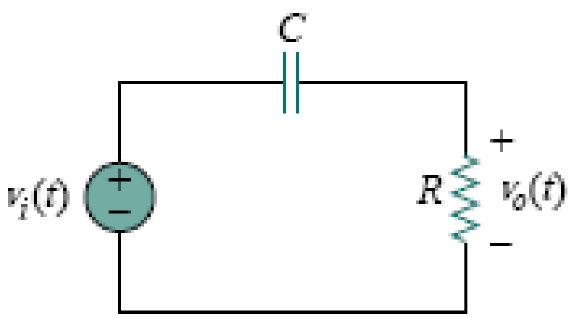
$$H(\omega_c) = \frac{1}{\sqrt{1+\omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}} \qquad \omega_c = \frac{1}{RC}$$

- A low_pass filter can also be formed when the output of an RL circuit is taken off the resistor.
- There are many other circuits for low_pass filters.

## 2.0 High Pass Filter

A highpass filter is designed to pass all frequencies above its cutoff frequency $\omega_c$.

- A high pass filter is formed when the output of an RC circuit is taken off the resistor as shown.




- The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

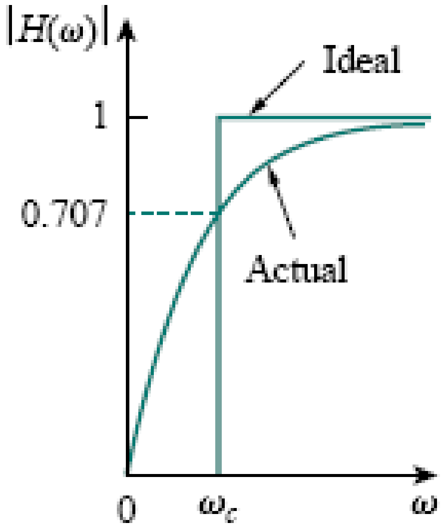
- Note that H(0) = 0, H(∞) = 1.

The corner or cutoff frequency is

$$\omega_c = \frac{1}{RC}$$

- A high pass filter can also be formed when the output of an RL circuit is taken off the inductor.

# High Pass Filters

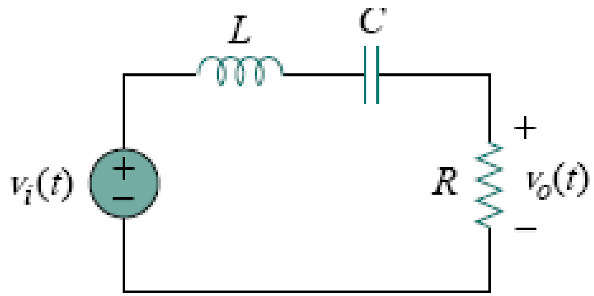



$$\boxed{\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}}$$

## Band Pass Filter

> A **bandpass filter** is designed to pass all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

The RLC series resonant circuit provides a band-pass filter when the output is taken off the resistor as shown.




The transfer function is

$$\boxed{\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}}$$

# Band Pass Filter

- We observe that |H(0)| = 0, |H(∞)| = 0. The bandpass filter passes
- a band of frequencies (ω1 < ω < ω2) centered on ω0,
- the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$w_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

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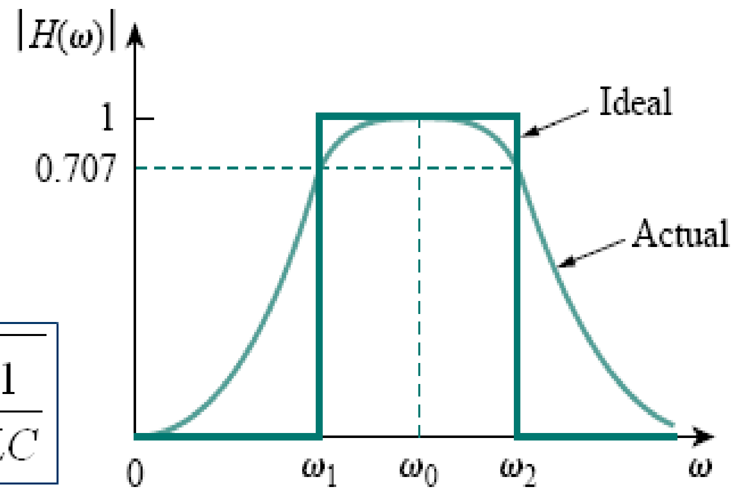
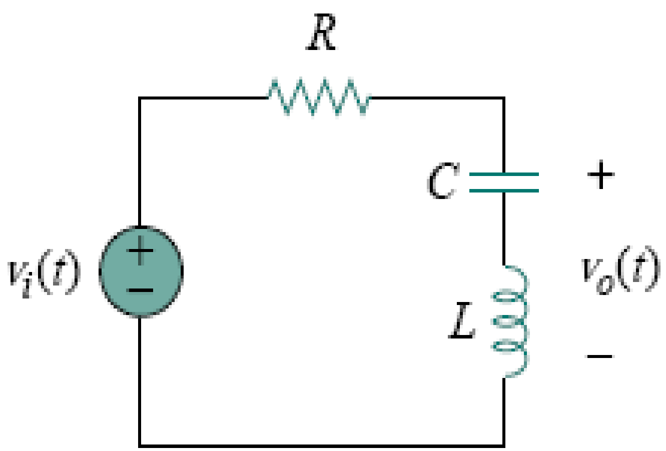



- Since the bandpass filter is a series resonant circuit, the halfpower
- frequencies, the bandwidth, and the quality factor are determined as the resonance chapter.
- A bandpass filter can also be formed by cascading the lowpass filter (where $\omega_2 = \omega_c$) with the highpass filter (where $\omega_1 = \omega_c$).

# Band Stop Filter

A bandstop filter is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

- A filter that prevents a band of frequencies between two designated values ($\omega_1$ and $\omega_2$) from passing is variably known as a bandstop, bandreject, or notch filter.
- A bandstop filter is formed when the output RLC series resonant circuit is taken off the LC series combination as shown.



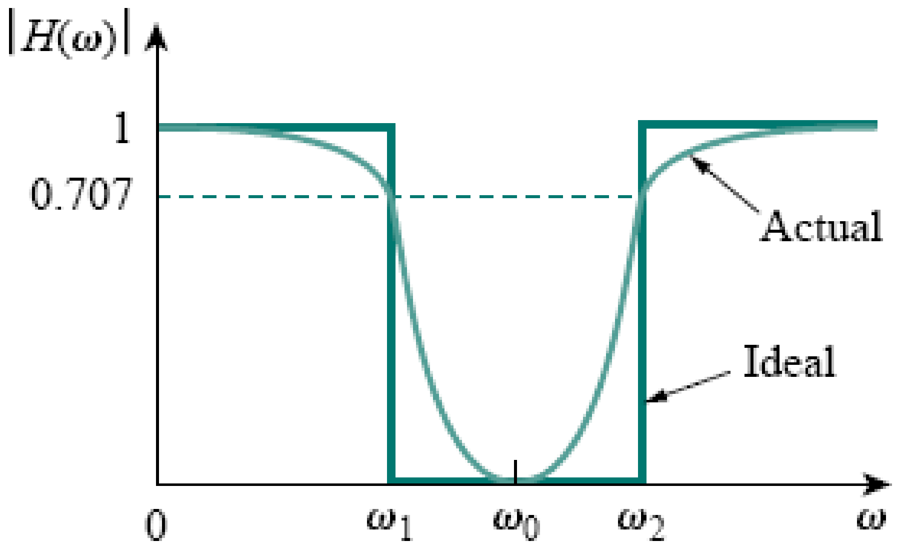

- The transfer function is:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$w_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$w_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$




- $\omega_0$ is called the frequency of rejection, while the corresponding
- bandwidth (BW = $\omega_2 - \omega_1$) is known as the bandwidth of rejection.
- The maximum gain of a passive filter is unity.
- To generate a gain greater than unity, one should use an active filter .

# Summary

Summary of the characteristics of filters.

| Type of Filter | $H(0)$ | $H(\infty)$ | $H(\omega_c)$ or $H(\omega_0)$ |
|---|---|---|---|
| Lowpass | 1 | 0 | $1/\sqrt{2}$ |
| Highpass | 0 | 1 | $1/\sqrt{2}$ |
| Bandpass | 0 | 0 | 1 |
| Bandstop | 1 | 1 | 0 |

# ACTIVE FILTERS

- There are three major limits to the passive filters considered in the previous section.
- First, they cannot generate gain greater than 1; passive elements cannot add energy to the network.
- Second, they may require bulky and expensive inductors.
- Third, they perform poorly at frequencies below the audio frequency range (300 Hz < f <3000 Hz).
- Nevertheless, passive filters are useful at high frequencies.

## ACTIVE FILTERS

- Active filters consist of combinations of resistors, capacitors, and
- Opamps. They offer some advantages over passive RLC filters.
- **First**, they are often smaller and less expensive, because they do not require inductors. This makes feasible the integrated circuit realizations of filters.
- **Second,** they can provide amplifier gain in addition to providing the same frequency response as RLC filters.
- **Third,** active filters can be combined with buffer amplifiers (voltage followers) to isolate each stage of the filter from source and load impedance effects. This isolation allows designing the stages independently and then cascading them to realize the desired transfer function.
- Active filters are less reliable and less stable.  The practical limit of most active filters is about 100 kHz, most active filters operate well below that frequency.

# First-Order Low pass Filter

- A typical active low-pass filter is shown.
- For this filter, the transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$

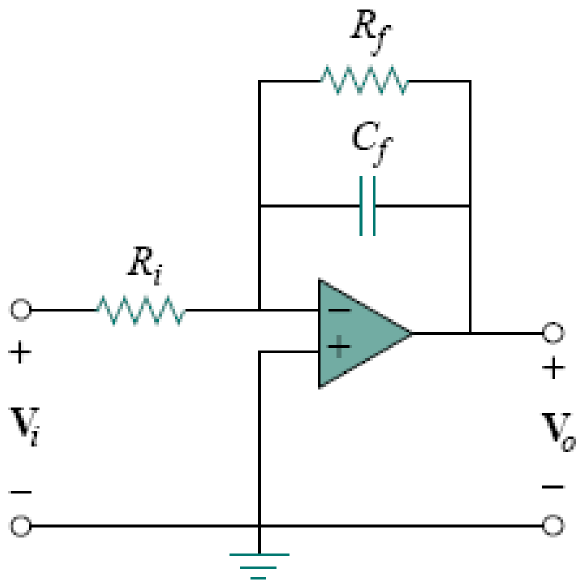
$$\mathbf{Z}_i = R_i$$

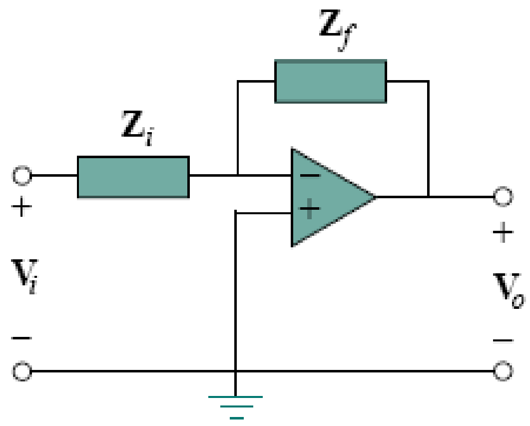
$$\mathbf{Z}_f = R_f \,\Big\|\, \frac{1}{j\omega C_f} = \frac{R_f/j\omega C_f}{R_f + 1/j\omega C_f} = \frac{R_f}{1 + j\omega C_f R_f}$$

Therefore,

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f}$$

- At low frequency (ω → 0) , the gain or dc gain = −Rf /Ri .
- The corner frequency is $\omega_c = \dfrac{1}{R_f C_f}$



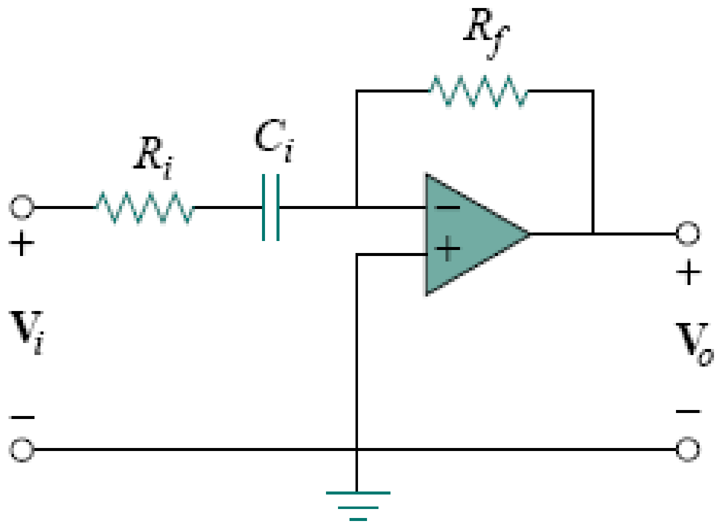



# First-Order High-Pass Filter

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$

where $\mathbf{Z}_i = R_i + 1/j\omega C_i$ and $\mathbf{Z}_f = R_f$ so that

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i + 1/j\omega C_i} = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i}$$




- This is similar to the active low pass filter, except that at very high frequencies ($\omega \rightarrow \infty$), the gain tends to $-R_f / R_i$ . The corner frequency is

$$\omega_c = \frac{1}{R_i C_i}$$

## Example

- Design a low-pass active filter with a dc gain of 4 and a corner frequency of 500 Hz.

**Solution:**

$$\omega_c = 2\pi f_c = 2\pi(500) = \frac{1}{R_f C_f}$$

The dc gain is

$$H(0) = -\frac{R_f}{R_i} = -4$$

- We have two equations and three unknowns. If we select $C_f = 0.2$ μF, then

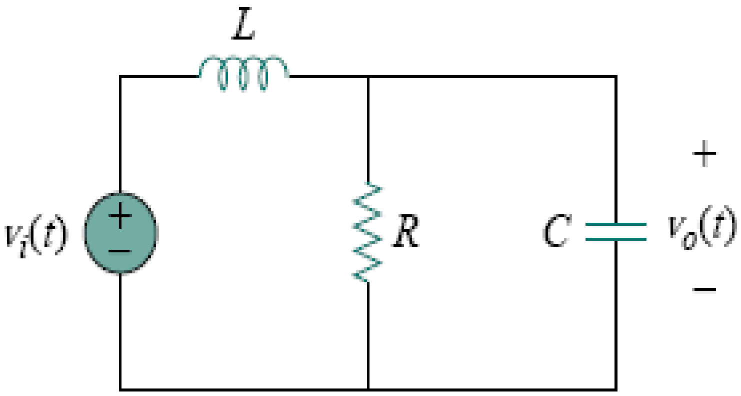
$$R_f = \frac{1}{2\pi(500)0.2 \times 10^{-6}} = 1.59\ \text{k}\Omega$$

$$R_i = \frac{R_f}{4} = 397.5\ \Omega$$

- We use a 1.6 kΩ resistor for $R_f$ and a 400Ω resistor for $R_i$ .

# Example

- Determine what type of filter in the circuit shown. Calculate the corner
- or cutoff frequency.
- Take R = 2 kW, L = 2 H, and C = 2 μF.




## Solution:

$$\boxed{\mathbf{H}(\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R}}$$

- The magnitude of H is

$$\boxed{H = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}}}$$

- Since H(0) = 1 and H(∞) = 0, we conclude that the circuit in is a second-order low pass filter.

- The corner frequency is the same as the half-power frequency, where
- H is reduced by a factor of 1√2. Since the dc value of H(ω) is 1, at the corner frequency,

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2} \longrightarrow 2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of $R$, $L$, and $C$, we obtain

$$2 = \left(1 - \omega_c^2\, 4 \times 10^{-6}\right)^2 + (\omega_c\, 10^{-3})^2$$

Assuming that $\omega_c$ is in krad/s,

$$2 = (1 - 4\omega_c)^2 + \omega_c^2 \qquad \text{or} \qquad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in $\omega_c^2$, we get $\omega_c^2 = 0.5509$, or

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$