## Random variables - Exercises

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## 1 Bivariate discrete distributions

**Exercise 1.** Consider the tables from last week's Psychophysiology example:

Table A

		$R_1$	$R_2$	R <sub>3</sub>	$R_4$
	$S_1$	3/4	1/4	0	0
	$S_2$	1/5	3/5	1/5	0
	$S_3$	0	0	4/5	1/5
	S <sub>4</sub>	0	0	1/6	5/6

Table B

	$R_1$	$R_2$	R <sub>3</sub>	R <sub>4</sub>	
$S_1$	3/16	1/16	0	0	1/4
$S_2$	1/20	3/20	1/20	0	1/4
S <sub>3</sub>	0	0	4/20	1/20	1/4
S <sub>4</sub>	0	0	1/24	5/24	1/4
	19/80	17/80	7/24	31/120	1

For the purpose of this exercise we will consider S and R as two r.v. with values on the set  $\{1,2,3,4\}$ . Table B contains the joint pmf of (S,R) and both marginal univariate pmf's, f=(1/4,1/4,1/4,1/4)', pmf of S and g=(19/80,17/80,7/120,31/120), pmf of R. Table A is the table of conditional pmf's of R with respect to S, i.e., each i-th row in Table B is the pmf of R conditioned to [S=i],  $(1 \le i \le 4)$ .

- 1. Compute the table of conditional pmf's of S with respect to R.
- 2. Compute the covariance and the correlation coefficient for (S, R).
- 3. Are S and R independent r.v.?

**Exercise 2.** Consider the following matrix (actually a sudoku):

$$h = \frac{1}{9 \cdot 9 \cdot 5} \begin{pmatrix} 7 & 6 & 8 & 4 & 5 & 9 & 2 & 1 & 3 \\ 9 & 5 & 4 & 1 & 3 & 2 & 8 & 6 & 7 \\ 2 & 3 & 1 & 6 & 7 & 8 & 4 & 5 & 9 \\ 6 & 8 & 7 & 9 & 4 & 5 & 3 & 2 & 1 \\ 5 & 4 & 9 & 2 & 1 & 3 & 7 & 8 & 6 \\ 3 & 1 & 2 & 8 & 6 & 7 & 9 & 4 & 5 \\ 1 & 2 & 3 & 7 & 8 & 6 & 5 & 9 & 4 \\ 8 & 7 & 6 & 5 & 9 & 4 & 1 & 3 & 2 \\ 4 & 9 & 5 & 3 & 2 & 1 & 6 & 7 & 8 \end{pmatrix}$$

1. Check that h can represent the bivariate pmf of a pair (X,Y) of discrete r.v. 's.



- 2. Compute both marginal pmf's, verifying that they are both discrete unifom (as it should be for a sudoku).
- 3. Compute the independence table  $h_0$  with the same marginals as h.
- 4. Compute both tables of conditional probabilities.
- 5. Assuming the vector of values of both X and Y is (1, 2, 3, 4, 5, 6, 7, 8, 9), compute the covariance and the correlation coefficient for (X, Y).

## 2 Bivariate continuous distributions

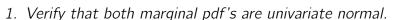
**Exercise 3.** The joint cdf of a pair (X,Y) of continuous r.v. is:

$$H(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)}, & \text{if } x \ge 0, y \ge 0, \\ 0, & \text{if } x < 0 \text{ or } y < 0. \end{cases}$$

- 1. Obtain both marginal cdf's, F(x) and G(y). Assess the possible independence of (X,Y).
- 2. Compute the joint pdf of (X,Y), h(x,y), and both marginal pdf's, f(x) and g(y).
- 3. Compute E(X), E(Y), var(X), var(Y), cov(X,Y), cor(X,Y).
- 4. Compute the pdf of Y, conditional to [X = 1].
- 5. Compute  $P([0 < X < 1] \cap [1 < Y < 2])$ .

**Exercise 4.** A pair (X,Y) of r.v. has a normal bivariate distribution with parameters  $(\mu_x,\mu_y,\sigma_x,\sigma_y,\rho)$ if their joint distribution is absolutely continuous, with joint pdf:

$$f_{(X,Y)}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)} \times \left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)\right]\right\}$$



- 2. Verify that the correlation coefficient of (X,Y) is equal to  $\rho$ .
- 3. Given  $x \in \mathbb{R}$ , compute the conditional pdf of (Y|X=x), verifying it is a univariate normal with parameters:

$$\mu_{y|x} = \mu_y + \frac{\rho \, \sigma_y}{\sigma_x} (x - \mu_x),$$

$$\sigma_{y|x}^2 = \sigma_y^2 (1 - \rho^2).$$

4. Similarly, given  $y \in \mathbb{R}$ , obtain the conditional pdf of (X|Y = y).