Probability and Bayes' rule - Exercises

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2018-09-19

1 (The Monty Hall problem)**.** *A TV game show. Choose one of three closed doors. One of them hides a hefty prize (a car). If you choose it you win the car. Behind each of the other two doors there is a goat. If you choose either of them you win nothing.*

Procedure: You choose a door. For the time being, it remains closed. The game show host (Monty Hall), who knows where the car is hidden, opens one of the unselected doors, showing a goat. Then you have the opportunity to maintain your initial choice or to switch to the remaining closed door.

Which is the best strategy? Stick to the first choice or change? Or, perhaps, it is indifferent? Once a door has been selected, for sure at least one of the other two hides a goat, hence opening one of them supplies no new information. Thus we can conclude that switching will not affect chances of winning. Or does it?

Compute the probability of winning the car, depending on the strategy (keep or switch), assuming:

- *The car is behind door A, written in red.*
- *Choices by M (Monty, the game show host) are as random as possible, e.g., if we select A he opens B or C with equal probability 0.5; if we select another door he has no choice.*
- *Our first selection is at random, the second one depends on the adopted strategy.*

2 (The double dice problem)**.** *Suppose I have a box that contains one each of 4-sided, 6-sided, 8-sided, and 12-sided dice. I choose a die at random, and roll it twice without letting you see the die or the outcome.*

1. What is the probability of obtaining the same outcome of both rolls?

2. Now, I report that I got the same outcome on both rolls. What is the posterior probability that I rolled each of the dice?

3. If I roll the same die again, what is the probability that I get the same outcome a third time?
