Probability and Bayes' rule - Exercises

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1. *We extract a ball at random from an urn containing 2 red balls, 3 white balls and 2 black balls and deposit it into a second urn already containing 2 white balls and 3 black balls. What is the probability of extracting now a black ball from the second urn (after mixing)?*

2. *An urn contains 6 white balls labelled 1 to 6. We extract a ball and paint in black as many balls as the label of the extracted ball indicates. Then (after mixing) we extract a second ball. What is the probability that this second ball is white?*

3. *Two urns, U_1 and U_2, contain white (w) and red (r) balls in the following proportions:*

$$U_1 = (3w, 5r), \qquad U_2 = (2w, 1r).$$

We roll a die. If a 3 or a 6 turns up then a randomly selected ball from U_2 is transferred to U_1 and then (after mixing) we extract at random a ball from U_1. In the other cases a randomly selected ball from U_1 is transferred to U_2 and then (after mixing) we extract at random a ball from U_2. Compute the probabilities:

1. *That both balls are red.*
2. *That both balls are white.*

4. *30% of the people in a city are vaccinated against flu. The probability of catching flu is* 0.01 *for vaccinated individuals and* 0.1 *for non-vaccinated individuals.*

What is the probability that a patient with flu has been vaccinated?

What is the probability that a given individual who has not caught the flu has been vaccinated?

5. *A bag contains 8 red, 15 white, and 5 yellow balls. The experimenter extracts a ball at random and registers its color (does not tell). If the ball is red or yellow it is returned to the bag, adding 2 more balls of the same color; if the ball is white it is kept out of the bag. Then a second ball is extracted. Compute the probability:*

1. *That the second ball is red.*
2. *That the first ball was not red if we observe the second ball is red.*

6. *A box A contains* 9 *cards numbered* 1 *to* 9 *and another box B contains* 5 *cards numbered* 1 *to* 5*. A box is chosen at random and a card is extracted from it, also at random. If the card number is even, whithout returning it, a second card is extracted from the same box. If the card number is odd, a card is extracted from the other box.*

1. *What is the probability that both card numbers are even?*
2. *If both card numbers are even, what is the probability that they came from box A?*
3. *What is the probability that both card numbers are odd?*

7 (the Kahneman cab problem)**.** *A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. 85% of the cabs in the city are Green and 15% are Blue. A witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colours 80% of the time and failed 20% of the time.*

What is the probability that the cab involved in the accident was Blue rather than Green knowing that this witness identified it as Blue?

[from: Kahneman, Daniel, Thinking, fast and slow, *New York: Farrar, Straus and Giroux (2013)].*

8 (The Monty Hall problem)**.** *A TV game show. Choose one of three closed doors. One of them hides a hefty prize (a car). If you choose it you win the car. Behind each of the other two doors there is a goat. If you choose either of them you win nothing.*

Procedure: You choose a door. For the time being, it remains closed. The game show host (Monty Hall), who knows where the car is hidden, opens one of the unselected doors, showing a goat. Then you have the opportunity to maintain your initial choice or to switch to the remaining closed door.

Which is the best strategy? Stick to the first choice or change? Or, perhaps, it is indifferent? Once a door has been selected, for sure at least one of the other two hides a goat, hence opening one of them supplies no new information. Thus we can conclude that switching will not affect chances of winning. Or does it?

Compute the probability of winning the car, depending on the strategy (keep or switch), assuming:

- *The car is behind door A, written in red.*
- *Choices by M (Monty, the game show host) are as random as possible, e.g., if we select A he opens B or C with equal probability 0.5; if we select another door he has no choice.*
- *Our first selection is at random, the second one depends on the adopted strategy.*
