	تمرین سری اول **درس یادگیری ماشین**	استاد درس: دکتر امیرخانی **مهلت تحویل: شنبه** ۱۸ اسفندماه

1. Complete the notebook which is attached.
2. Assume y is an exponentially distributed random variable with rate $\lambda > 0$, i.e., $f_y(y;\lambda) = \lambda e^{-\lambda y}$. Verify that the maximum-likelihood estimate of λ given N iid realizations is $\hat{\lambda} = \frac{N}{\sum_{i=1}^{N} y_i}$.

3. Let y be distributed according to the following distribution, and Assume we collect N iid observations $\{y_1, y_{2,} y_3, \dots, y_N\}$.

$$f_y(y;a) = a_y^{a-1} \quad , y \in (0,1) \ , a > 0$$

show that the Maximum-likelihood estimate of a is given by:

$$\hat{a}_{ML} = -\left(\frac{1}{N}\sum_{j=1}^{N} \ln(y_i)\right)^{-1}$$

4. Consider a collection of N independent Gaussian realizations $\{y_n\}$ with mean μ and unit variance. (i.e., $y_n \sim N(\mu, 1)$). The mean μ is unknown but arises from a Gaussian prior distribution μ ~ N (0, σ_μ^2) with known variance.
   (a) Determine the posterior distribution $f_{\mu|y_0,y_1,\dots,y_n}(\mu|y_0, y_1, \dots, y_n)$

   (b) Determine the MAP estimator for μ.

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5. Assume that δ is a known constant.
   Let $X_1, X_2, \dots, X_n$ be a random sample from a distribution with the probability density function:

$$f(x;\beta,\delta) = \beta\delta x^{\delta-1}e^{-\beta x^{\delta}}, x > 0, 0 \text{ otherwise.}$$

   a. Derive the maximum likelihood estimator (MLE) of β, denoted as $\hat{\beta}$.

6. Let $\tau > 0$, and let $X_1, X_2, \dots, X_n$ be a random sample drawn from a distribution with the probability density function:

$$f(x;\tau) = \tau e^{-\tau(x-3)}, x > 3, 0 \text{ otherwise}$$

   a. Derive the maximum likelihood estimator (MLE) of τ, denoted as $\hat{\tau}$.

   b. Given a sample of size $n = 4$ with observed values $X_1 = 3.15, X_2 = 3.65, X_3 = 3.10$, and $X_4 = 3.30$, compute the maximum likelihood estimate of τ.

❖ **Submission Guidelines:**
- Submit **only one** compressed file (**zip**, **rar**, etc.) containing:
  - A **PDF file** (typewritten is preferred; handwritten is acceptable).
  - **One notebook file** (**.ipynb**).
- **Do your homework independently.** Similarities with other students' work will not be tolerated.
- **Reminder:** Humans are always smarter than AI tools like ChatGPT. Use them wisely.

- **File Naming Convention:**
  - Compressed file: HWn_[studentNumber].zip
  - PDF file: HWn_[studentNumber].pdf
  - Notebook file: HWn_[studentNumber].ipynb
- **Important:**
  - Include your **name** and **student number** on the **first page** of your PDF file.

Good Luck