Generating an Optimal Path for Variable Joint Stiffness in an Underactuated System

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I. Introduction

Underactuated tendon driven compliant grippers have a lot of advantages over conventional rigid body grippers. However, they lack direct control over joint angles which restricts them in generating any arbitrary desired trajectory in configuration space. The stiffness of joints plays a crucial role in how the grippers close around an object. Therefore, by actively varying the joint stiffness, a better control over the joint angles can be achieved.

Since realistic systems are highly non linear, an appropriate path for controlling stiffness needs to be developed for intermediate configurations to not only fulfill any desired objective but also comply with workspace restrictions. In this work, a framework is developed which searches through a space of control inputs corresponding to each configuration to return an optimal path according to an objective function. Generating optimal paths for variable joint stiffness in underactuated systems is the first step towards creating a control system for a variable joint stiffness gripper. Making this control system opens the possibility of adopting different types of grasps that extends the use for various different purposes.

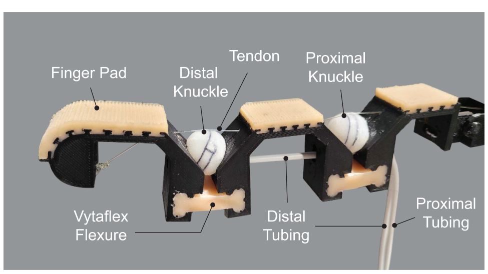

Fig. 1: Variable stiffness finger with knuckles that can be pressurized with water coming from the proximal and distal tubing. Fingerpads are on the finger to provide a stronger grip while grasping. Each finger contains two Vytaflex flexures.

II. Related Works

A paper by Troxler et al. is the initial inspiration for the project and will be the gripper analyzed in the project [1]. The gripper possess variable joint stiffness which allows it to achieve various different grasp primitives. The finger design is shown in Fig 1.

There have been some works regarding the development of control systems with variable stiffness actuators. An article by Zhakatayev et al. found the optimal control to reduce the time to preform a task [2]. Ji et al. improved the design by adjusting the trajectory planning [3]. The project also considered focusing in on increasing the weight the variable stiffness gripper can lift however, the speed and power are more vital to the project [4].

III. Problem Statement

The aim of the project is to move the system from a given starting configuration to any desired final configuration. The system of underactuated finger represented by three links connected with variable stiffness joints and actuated via a single tendon pulling on the links is shown in Fig. 2.

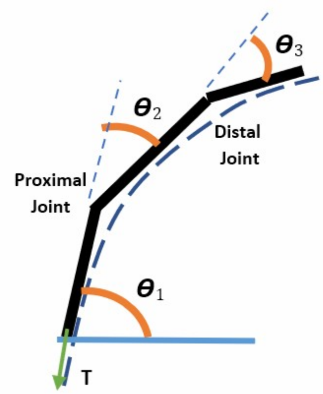

Fig. 2: 3-linked tendon driven finger with θ_i as joint angles and T as tension in the tendon

To achieve a desired configuration in equilibrium the proximal and distal joint stiffness and tension with which tendon is pulled is changed which act as control inputs to a controller. In this work, a framework is developed that generates the optimal path of these control inputs that changes the configuration according to a given objective.

IV. Methodology

In this work, a system of three links connected to each other and a base with soft flexures having some stiffness that act as revolute joints is considered as shown in Fig. 3. The stiffness of joints between links 1 and 2 (proximal) and 2 and 3 (distal) can be varied actively. Additionally, the system is tendon driven and there is no direct control over joint angles. For this tendon driven underactuated system, the stiffness of proximal and distal joints (K_2 and K_3) and tension in the tendon (T) are the control inputs for the system to change the configuration from any starting configuration to any desired configuration.

As these systems, can be highly non linear, the intermediate control points obtained from interpolating control inputs corresponding to desired configuration can lead to undesired intermediate configurations that maybe out of workspace or disregard any objective. Therefore, it is necessary to generate a path for control inputs that optimises a given objective and leads to meaningful and continuous intermediate configurations. To generate an optimal path, first the control inputs corresponding to each possible configuration is obtained, then virtual connections between mutually reachable configurations are made whose weights are dependent on the objective function and then a path is found that minimises the objective function.

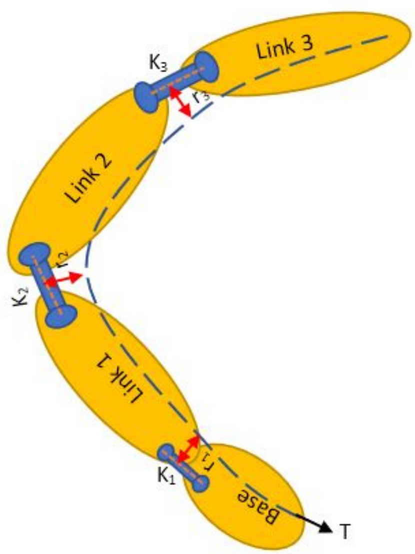

Fig. 3: Model of the system with three links, one base and blue soft flexures between each link. K_i is the stiffness and r_i is the distance of tendon from the center of the joint i. r_a is radius of actuator pulley and $\Delta\theta_a$ is the angular displacement of actuator pulley. $\Delta\theta_i$ is joint angular displacement of joint i from zero (for simplicity)

A. Dynamics

The system considered for system dynamics in this work is shown in Fig. 3. The equations [5] for kinematic constraints(1) and quasi static equilibrium constraints(2) provide four independent constraints.

$$r_a \Delta\theta_a = J_a \Delta\theta \tag{1}$$

$$K\Delta\theta + J_a^T f_a = 0 \tag{2}$$

where,

$$J_a = [r_1, r_2, r_3]$$
$$\Delta\theta = [\Delta\theta_1, \Delta\theta_2, \Delta\theta_3]^T$$
$$K = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix}$$

These constraints are used to calculate a set of K_2, K_3, $f_a(T)$ and $\Delta\theta_a$ for each possible joint configuration $(\Delta\theta_1, \Delta\theta_2, \Delta\theta_3)$. Other parameters are kept constant at K_1 = r_1 = r_2 = r_3 = r_a = 1.

Using the system dynamics, the stiffness of the proximal and distal joints and tendon tension is calculated to maintain the system at equilibrium in any particular configuration.

B. Graph Search

Each possible configuration along with the proximal and distal joint stiffness and tendon tension required to maintain that configuration at equilibrium is considered as a single node. Connections between nodes are made if either stiffness or tension value is same. This is done to connect mutually reachable configurations by changing any of control inputs discretely.

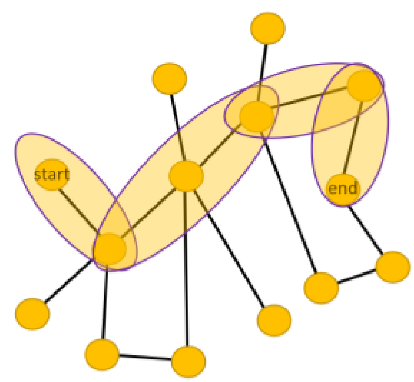

Fig. 4: Schematic representation of a graph along with an optimal path

$$\begin{aligned} X_j &= L_1 \cos(\theta_1)_j + L_2 \cos(\theta_1+\theta_2)_j + L_3 \cos(\theta_1+\theta_2+\theta_3)_j \\ Y_j &= L_1 \sin(\theta_1)_j + L_2 \sin(\theta_1+\theta_2)_j + L_3 \sin(\theta_1+\theta_2+\theta_3)_j \\ X_k &= L_1 \cos(\theta_1)_k + L_2 \cos(\theta_1+\theta_2)_k + L_3 \cos(\theta_1+\theta_2+\theta_3)_k \\ Y_k &= L_1 \sin(\theta_1)_k + L_2 \sin(\theta_1+\theta_2)_k + L_3 \sin(\theta_1+\theta_2+\theta_3)_k \\ D_{jk} &= \sqrt{(X_j - X_k)^2 + (Y_j - Y_k)^2} \end{aligned} \tag{3}$$

The edge length, or cost of the connection between nodes is set as per the objective. Here, the objective is to minimize the actual distance travelled by the end point. Therefore, the edge length (D_{jk}) was set as Cartesian distance of end point between two configurations as shown in Eqn. (3) where L_i are the link lengths and θ_i are the joint angles for any configuration j and k.

$A*$ graph search algorithm was then used to find an optimal path minimizing the distance travelled by end point. The heuristic used for the graph search was the Cartesian distance between the current intermediate configuration and the final desired configuration. This is the most optimistic heuristic that helps in the directional search of the space.

V. Experimentation and Results

For initial experiments, optimal paths were generated for achieving end configurations mimicking two commonly known grasp primitives i.e. wrap and pinch type grasps. The starting and end configurations for these grasp types given as input are shown in Table I.

TABLE I: Desired Configurations

Grasp Type	Start Configuration	End Configuration
Pinch	(1°,1°,1°)	(90°,10°,5°)
Wrap	(1°,1°,1°)	(90°,15°,30°)

The generated optimal paths i.e. stiffness (K_2 and K_3) and tension (T) values for starting, intermediate and end configurations for pinch and wrap grasps are shown in Tables II and III respectively.

TABLE II: Optimal Path Pinch

K2	K3	T	Intermediate Configuration
1.0	1.0	-0.01745329	(1°,1°,1°)
1.0	1.5	-0.78539816	(45°,45°,30°)
2.4	1.5	-1.04719755	(60°,25°,40°)
1.33	6.0	-1.04719755	(60°,45°,10°)
18.0	6.0	-1.57079633	(90°,5°,15°)
9.0	18.0	-1.57079633	(90°,10°,5°)

TABLE III: Optimal Path Wrap

K2	K3	T	Intermediate Configuration
1.0	1.0	-0.01745329	(1°,1°,1°)
1.0	1.5	-0.78539816	(45°,45°,30°)
2.0	1.5	-1.04719755	(60°,30°,40°)
1.33	3.0	-1.04719755	(60°,45°,20°)
6.0	3.0	-1.57079633	(90°,15°,30°)

The generated control inputs for proximal and distal joint stiffness and tendon tension values for pinch and wrap type grasps are shown in Fig.s 5,6 and 7.

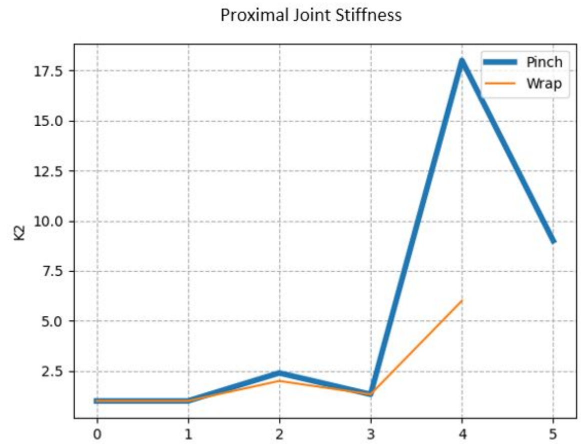

Fig. 5: Generated path for proximal joint stiffness

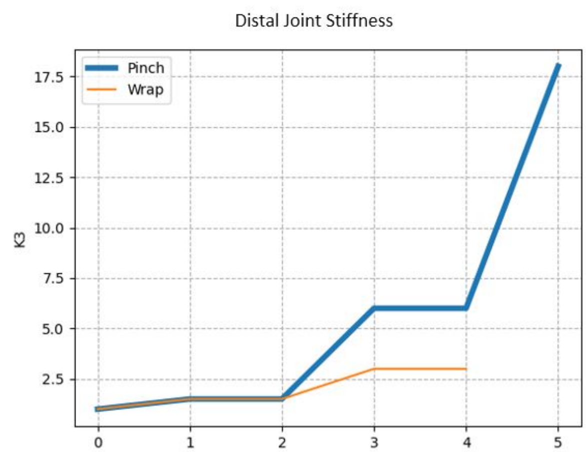

Fig. 6: Generated path for distal joint stiffness

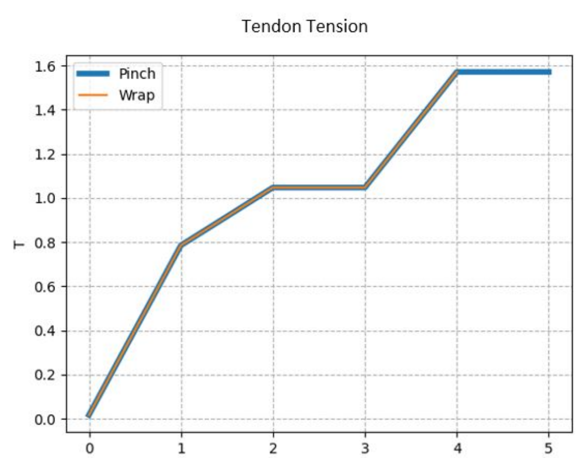

Fig. 7: Generated path for tendon tension

The stiffness plots are in agreement with the general idea that for a pinch grasp distal joints needs to be stiffer than a wrap grasp. The tension plots is similar for both grasp primitives which indicates that additional information of angular displacement of actuator pulling the tendon can be used as a control input.

Fig.s 8 and 9 show the plot of the start, intermediate and end configurations for the generated optimal paths in the cartesian space for wrap and pinch grasps respectively. The plots show that the end point moved almost linearly in the cartesian task space which shows that generated path is indeed optimal according to the specified objective.

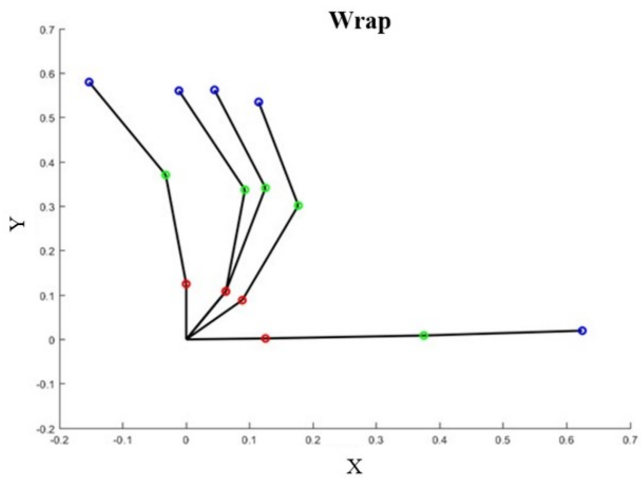

Fig. 8: Generated optimal path in Cartesian space for wrap grasp

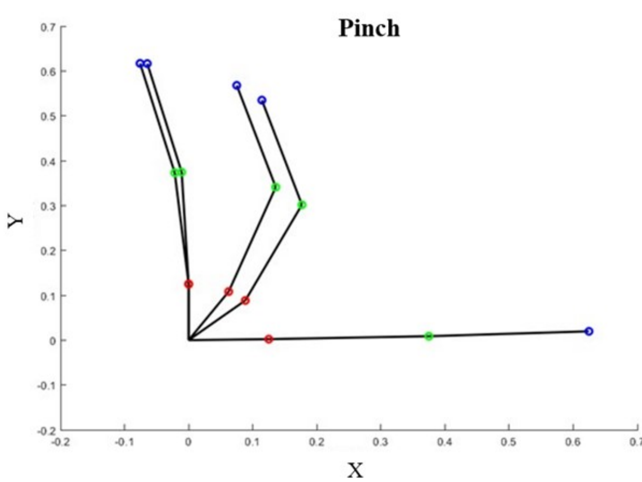

Fig. 9: Generated optimal path in Cartesian space for pinch grasp

VI. Conclusion/Discussion

The framework for generating optimal path of control inputs for an underactuated system where joint stiffness can be actively changed is developed. The possible configuration space was derived by varying state variables from θ_1: 1-90 degrees, θ_2 and θ_3: 1-45 degrees at interval of 5 degrees. The generated paths of joint stiffness were not smooth as the control inputs were calculated at these states, however if the granularity of the discrete configuration space is increased i.e. the interval is reduced the paths will be smoother. Additionally, the actual control inputs for actuators controlling stiffness can be made smooth by using an appropriate time scaling.

The tension with which the actuator pulls the tendon was seen to be similar for two grasp primitives, this shows that the information of tension alone will not suffice as control input. Additional information about the angular displacement of the actuator might be necessary which was also calculated using the dynamics in IV-A as $\Delta\theta_a$.

In this work, the objective was to minimize the distance travelled by end point. Another possible objective could be to optimise energy, for that knowledge about actuator efforts (actuator that changes the stiffness actively and actuator that pulls the tendon) to change the configuration from one to another are needed.

VII. Future Work

Currently, only the optimal path for control inputs were generated for limited state variables. A non linear expansion of discrete state variables can be done so that corresponding control inputs for possible configurations can be generated which do not have high first order derivatives between different configurations. Additionally, time scaling can be done to obtain smoother control trajectories.

Another future work is development of a control dynamics model that takes in the generated control inputs and using a dynamics model changes the configuration of the system. This can be used to measure the efficiency of the generated optimal path and finding a suitable time scaling.

Using knowledge of actuator efforts an energy optimal path can be generated as discussed earlier.

Currently, a known simplified static system was used to find control inputs corresponding to all possible configurations. However, in reality not all systems can be represented with dynamic equations. Therefore, a universal function approximator can be trained to get control inputs corresponding to configurations using experimental data values.

Contributions

Satyam Bhawsinghka

Developed the methodology and wrote the graph search algorithm.

Karina Puente

Wrote the dynamics equation solver code to generate control input for corresponding configurations.

Lissette Wilhelm

Plotted the generated optimal path in cartesian space.

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