Collaborative Filtering

Agenda

- **Collaborative Filtering (CF)**
  - Pure CF approaches
  - User-based nearest-neighbor
  - The Pearson Correlation similarity measure
  - Memory-based and model-based approaches
  - Item-based nearest-neighbor
  - The cosine similarity measure
  - Data sparsity problems
  - Recent methods (SVD, Association Rule Mining, Slope One, RF-Rec, …)
  - The Google News personalization engine
  - Discussion and summary
  - Literature

Collaborative Filtering (CF)

- **The most prominent approach to generate recommendations**
  - used by large, commercial e-commerce sites
  - well-understood, various algorithms and variations exist
  - applicable in many domains (book, movies, DVDs, ..)
- **Approach**
  - use the "wisdom of the crowd" to recommend items

- **Basic assumption and idea**
  - Users give ratings to catalog items (implicitly or explicitly)
  - Customers who had similar tastes in the past, will have similar tastes in the future

Pure CF Approaches

- **Input**
  - Only a matrix of given user–item ratings
- **Output types**
  - A (numerical) prediction indicating to what degree the current user will like or dislike a certain item
  - A top-N list of recommended items

User-based nearest-neighbor collaborative filtering (1)

- **The basic technique**
  - Given an "active user" (Alice) and an item i not yet seen by Alice
    - find a set of users (peers/nearest neighbors) who liked the same items as Alice in the past **and** who have rated item i
    - use, e.g. the average of their ratings to predict, if Alice will like item i
    - do this for all items Alice has not seen and recommend the best-rated
- **Basic assumption and idea**
  - If users had similar tastes in the past they will have similar tastes in the future
  - User preferences remain stable and consistent over time

User-based nearest-neighbor collaborative filtering (2)

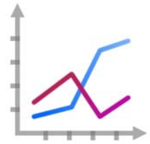
- **Example**
  - A database of ratings of the current user, Alice, and some other users is given:

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

  - Determine whether Alice will like or dislike *Item5*, which Alice has not yet rated or seen

User-based nearest-neighbor collaborative filtering (3)

- **Some first questions**
  - How do we measure similarity?
  - How many neighbors should we consider?
  - How do we generate a prediction from the neighbors' ratings?



	Item1	Item2	Item3	Item4	Item5
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User1	3	1	2	3	3
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User3	3	3	1	5	4
User4	1	5	5	2	1

Measuring user similarity (1)

- **A popular similarity measure in user-based CF: Pearson correlation**

  a, b : users

  $r_{a,p}$: rating of user a for item p

  P : set of items, rated both by a and b

  – Possible similarity values between -1 and 1

$$\boldsymbol{sim(a,b)} = \frac{\sum_{p \in P}(r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_{p \in P}(r_{a,p} - \bar{r}_a)^2}\sqrt{\sum_{p \in P}(r_{b,p} - \bar{r}_b)^2}}$$

Measuring user similarity (2)

- **A popular similarity measure in user-based CF: Pearson correlation**

  a, b : users

  $r_{a,p}$: rating of user a for item p

  P : set of items, rated both by a and b

  – Possible similarity values between -1 and 1

	Item1	Item2	Item3	Item4	Item5	
Alice	5	3	4	4	?	
User1	3	1	2	3	3	sim = 0,85
User2	4	3	4	3	5	sim = 0,00
User3	3	3	1	5	4	sim = 0,70
User4	1	5	5	2	1	sim = -0,79

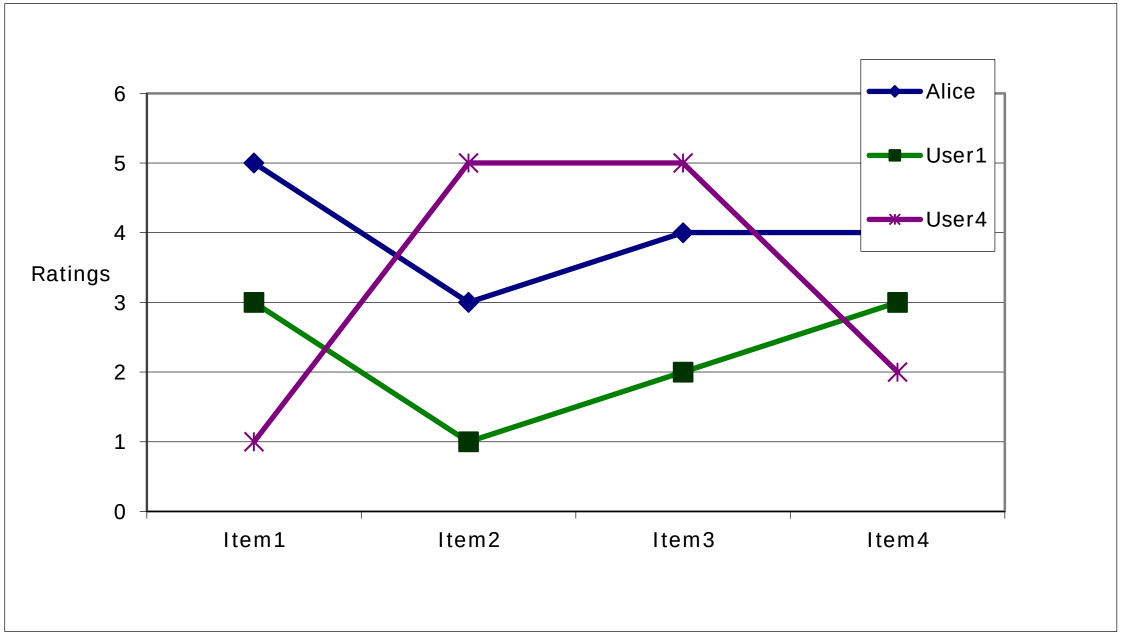
Z6

Folie 9

Z6 Punkt statt Komma als Trennzeichen.
Zeynep; 16.08.2011

Pearson correlation

- **Takes differences in rating behavior into account**



- **Works well in usual domains, compared with alternative measures**
  - such as cosine similarity

Making predictions

- A common prediction function:

$$\boldsymbol{pred(a,p) = \overline{r_a} + \frac{\sum_{b \in N} sim(a,b) * (r_{b,p} - \overline{r_b})}{\sum_{b \in N} sim(a,b)}}$$

- Calculate, whether the neighbors' ratings for the unseen item i are higher or lower than their average
- Combine the rating differences – use the similarity with a as a weight
- Add/subtract the neighbors' bias from the active user's average and use this as a prediction

Improving the metrics / prediction function

- **Not all neighbor ratings might be equally "valuable"**
  - Agreement on commonly liked items is not so informative as agreement on controversial items
  - **Possible solution**: Give more weight to items that have a higher variance
- **Value of number of co-rated items**
  - Use "significance weighting", by e.g., linearly reducing the weight when the number of co-rated items is low
- **Case amplification**
  - Intuition: Give more weight to "very similar" neighbors, i.e., where the similarity value is close to 1.
- **Neighborhood selection**
  - Use similarity threshold or fixed number of neighbors

Memory-based and model-based approaches

- **User-based CF is said to be "memory-based"**
  - the rating matrix is directly used to find neighbors / make predictions
  - does not scale for most real-world scenarios
  - large e-commerce sites have tens of millions of customers and millions of items
- **Model-based approaches**
  - based on an offline pre-processing or "model-learning" phase
  - at run-time, only the learned model is used to make predictions
  - models are updated / re-trained periodically
  - large variety of techniques used
  - model-building and updating can be computationally expensive
  - *item*-based CF is an example for model-based approaches

Item-based collaborative filtering

- **Basic idea:**
  - Use the similarity between items (and not users) to make predictions
- **Example:**
  - Look for items that are similar to Item5
  - Take Alice's ratings for these items to predict the rating for Item5

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

The cosine similarity measure

- **Produces better results in item-to-item filtering**
- **Ratings are seen as vector in n-dimensional space**
- **Similarity is calculated based on the angle between the vectors**

$$sim(\vec{a},\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}| * |\vec{b}|}$$

- **Adjusted cosine similarity**
  - take average user ratings into account, transform the original ratings
  - U: set of users who have rated both items a and b

$$sim(\vec{a},\vec{b}) = \frac{\sum_{u\in U}(r_{u,a}-\overline{r_u})(r_{u,b}-\overline{r_u})}{\sqrt{\sum_{u\in U}(r_{u,a}-\overline{r_u})^2}\sqrt{\sum_{u\in U}(r_{u,b}-\overline{r_u})^2}}$$

Making predictions

- A common prediction function:

$$\boldsymbol{pred(u,p)} = \frac{\sum_{\boldsymbol{i \in ratedItem(u)}} \boldsymbol{sim(i,p) * r_{u,i}}}{\sum_{\boldsymbol{i \in ratedItem(u)}} \boldsymbol{sim(i,p)}}$$

- Neighborhood size is typically also limited to a specific size
- Not all neighbors are taken into account for the prediction
- An analysis of the MovieLens dataset indicates that "in most real-world situations, a neighborhood of 20 to 50 neighbors seems reasonable" (Herlocker et al. 2002)

Pre-processing for item-based filtering

- **Item-based filtering does not solve the scalability problem itself**
- **Pre-processing approach by Amazon.com (in 2003)**
  - Calculate all pair-wise item similarities in advance
  - The neighborhood to be used at run-time is typically rather small, because only items are taken into account which the user has rated
  - Item similarities are supposed to be more stable than user similarities
- **Memory requirements**
  - Up to N^2 pair-wise similarities to be memorized (N = number of items) in theory
  - In practice, this is significantly lower (items with no co-ratings)
  - Further reductions possible
    - Minimum threshold for co-ratings
    - Limit the neighborhood size (might affect recommendation accuracy)

More on ratings – Explicit ratings

- Probably the most precise ratings
- Most commonly used (1 to 5, 1 to 7 Likert response scales)
- Research topics
  - Optimal granularity of scale; indication that 10-point scale is better accepted in movie dom.
  - An even more fine-grained scale was chosen in the joke recommender discussed by Goldberg et al. (2001), where a continuous scale (from −10 to +10) and a graphical input bar were used
    - No precision loss from the discretization
    - User preferences can be captured at a finer granularity
    - Users actually "like" the graphical interaction method
  - Multidimensional ratings (multiple ratings per movie such as ratings for actors and sound)
- Main problems
  - Users not always willing to rate many items
    - number of available ratings could be too small → sparse rating matrices → poor recommendation quality
  - How to stimulate users to rate more items?

More on ratings – Implicit ratings

- Typically collected by the web shop or application in which the recommender system is embedded
- When a customer buys an item, for instance, many recommender systems interpret this behavior as a positive rating
- Clicks, page views, time spent on some page, demo downloads …
- Implicit ratings can be collected constantly and do not require additional efforts from the side of the user
- Main problem
  - One cannot be sure whether the user behavior is correctly interpreted
  - For example, a user might not like all the books he or she has bought; the user also might have bought a book for someone else
- Implicit ratings can be used in addition to explicit ones; question of correctness of interpretation

Data sparsity problems

- **Cold start problem**
  - How to recommend new items? What to recommend to new users?
- **Straightforward approaches**
  - Ask/force users to rate a set of items
  - Use another method (e.g., content-based, demographic or simply non-personalized) in the initial phase
  - Default voting: assign default values to items that only one of the two users to be compared has rated (Breese et al. 1998)
- **Alternatives**
  - Use better algorithms (beyond nearest-neighbor approaches)
  - Example:
    - In nearest-neighbor approaches, the set of sufficiently similar neighbors might be too small to make good predictions
    - Assume "transitivity" of neighborhoods

Example algorithms for sparse datasets

- **Recursive CF** (Zhang and Pu 2007)
  - Assume there is a very close neighbor n of u who however has not rated the target item i yet.
  - Idea:
    - Apply CF-method recursively and predict a rating for item i for the neighbor
    - Use this predicted rating instead of the rating of a more distant direct neighbor

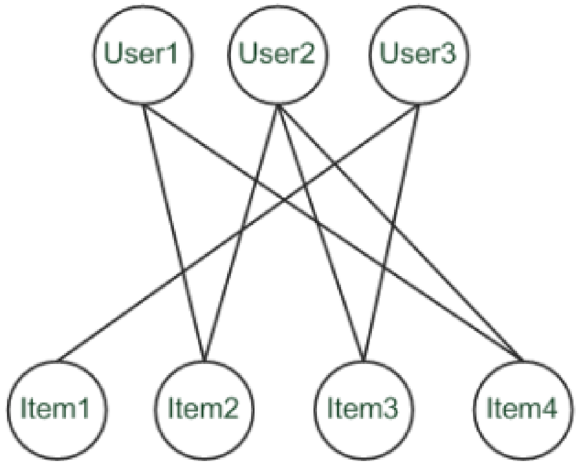
	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	?
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

sim = 0.85

Predict rating for User1

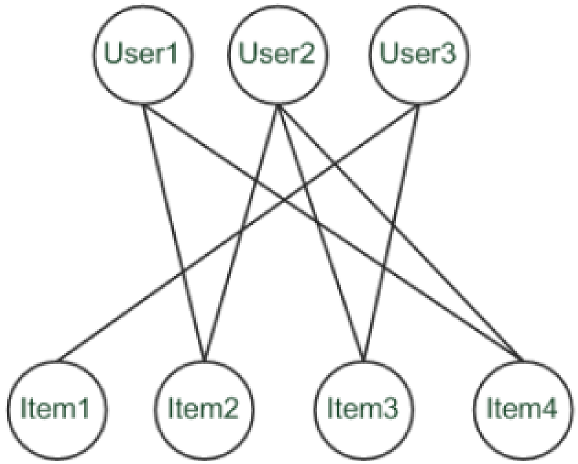
Graph-based methods (1)

- **"Spreading activation"** (Huang et al. 2004)
  - Exploit the supposed "transitivity" of customer tastes and thereby augment the matrix with additional information
  - Assume that we are looking for a recommendation for *User1*
  - When using a standard CF approach, *User2* will be considered a peer for *User1* because they both bought *Item2* and *Item4*
  - Thus *Item3* will be recommended to *User1* because the nearest neighbor, *User2*, also bought or liked it



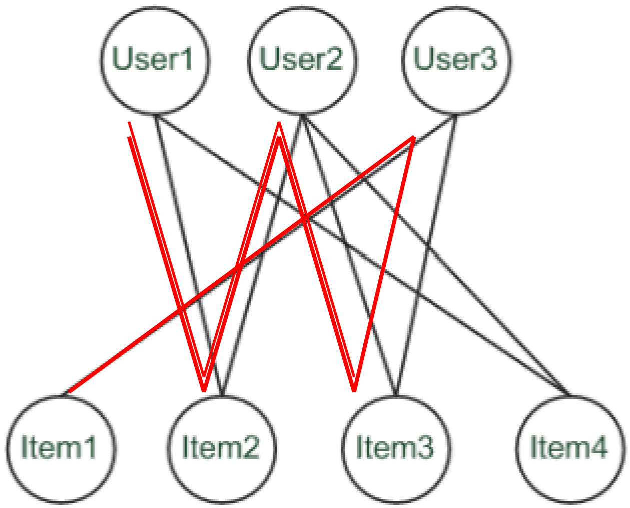
Graph-based methods (2)

- **"Spreading activation"** (Huang et al. 2004)
  - In a standard user-based or item-based CF approach, paths of length 3 will be considered – that is, *Item3* is relevant for *User1* because there exists a three-step path (*User1–Item2–User2–Item3*) between them
  - Because the number of such paths of length 3 is small in sparse rating databases, the idea is to also consider longer paths (indirect associations) to compute recommendations
  - Using path length 5, for instance



Graph-based methods (3)

- **"Spreading activation"** (Huang et al. 2004)
  - Idea: Use paths of lengths > 3
    to recommend items
  - Length 3: Recommend Item3 to User1
  - Length 5: Item1 also recommendable



More model-based approaches

- **Plethora of different techniques proposed in the last years, e.g.,**
  - Matrix factorization techniques, statistics
    - singular value decomposition, principal component analysis
  - Association rule mining
    - compare: shopping basket analysis
  - Probabilistic models
    - clustering models, Bayesian networks, probabilistic Latent Semantic Analysis
  - Various other machine learning approaches
- **Costs of pre-processing**
  - Usually not discussed
  - Incremental updates possible?

2000: *Application of Dimensionality Reduction in Recommender System*, B. Sarwar et al., WebKDD Workshop

- **Basic idea: Trade more complex offline model building for faster online prediction generation**
- **Singular Value Decomposition for dimensionality reduction of rating matrices**
  - Captures important factors/aspects and their weights in the data
  - factors can be genre, actors but also non-understandable ones
  - Assumption that k dimensions capture the signals and filter out noise (K = 20 to 100)
- **Constant time to make recommendations**
- **Approach also popular in IR (Latent Semantic Indexing), data compression,…**

Matrix factorization

- Informally, the SVD theorem (Golub and Kahan 1965) states that a given matrix M can be decomposed into a product of three matrices as follows

$$M = U \times \Sigma \times V^T$$

  - where U and V are called *left* and *right singular vectors* and the values of the diagonal of Σ are called the *singular values*
- We can approximate the full matrix by observing only the most important features – those with the largest singular values
- In the example, we calculate U, V, and Σ (with the help of some linear algebra software) but retain only the two most important features by taking only the first two columns of U and V^T

Example for SVD-based recommendation

- **SVD:** $$M_k = U_k \times \Sigma_k \times V_k^T$$

Z18

U_k	Dim1	Dim2
Alice	0.47	-0.30
Bob	-0.44	0.23
Mary	0.70	-0.06
Sue	0.31	0.93

V_k^T	Terminator	Die Hard	Twins	Eat Pray Love	Pretty Woman
Dim1	-0.44	-0.57	0.06	0.38	0.57
Dim2	0.58	-0.66	0.26	0.18	-0.36

Σ_k	Dim1	Dim2
Dim1	5.63	0
Dim2	0	3.23

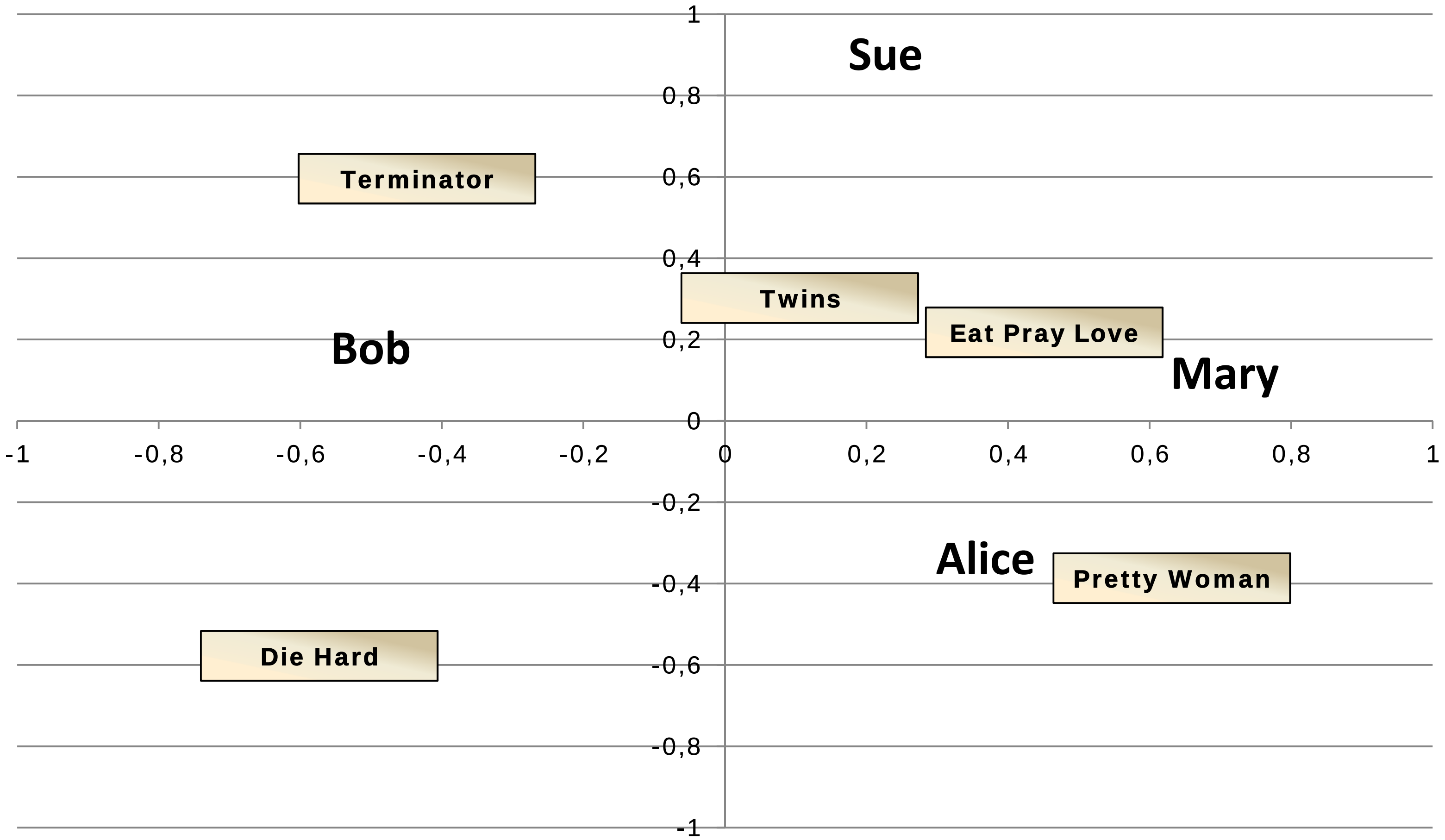
- **Prediction:** $$\hat{r}_{ui} = \bar{r}_u + U_k(Alice) \times \Sigma_k \times V_k^T(EPL)$$

= 3 + 0.84 = 3.84

Folie 28

Z18 Tabellenformatierung einheitlich.
Vielleicht auch kleinere Schriftgröße nehmen. Wirkt etwas voll und unübersichtlich.
Zeynep; 16.08.2011

The projection of U and V^T in the 2 dimensional space (U_2, V_2^T)



Discussion about dimensionality reduction (Sarwar et al. 2000a)

- **Matrix factorization**
  - Generate low-rank approximation of matrix
  - Detection of latent factors
  - Projecting items and users in the same n-dimensional space
- **Prediction quality can decrease because…**
  - the original ratings are not taken into account
- **Prediction quality can increase as a consequence of…**
  - filtering out some "noise" in the data and
  - detecting nontrivial correlations in the data
- **Depends on the right choice of the amount of data reduction**
  - number of singular values in the SVD approach
  - Parameters can be determined and fine-tuned only based on experiments in a certain domain
  - Koren et al. 2009 talk about 20 to 100 factors that are derived from the rating patterns

Association rule mining

- **Commonly used for shopping behavior analysis**
  - aims at detection of rules such as
    "If a customer purchases beer then he also buys diapers in 70% of the cases"
- **Association rule mining algorithms**
  - can detect rules of the form X → Y (e.g., beer → diapers) from a set of sales transactions $D = \{t_1, t_2, \dots t_n\}$
  - measure of quality: support, confidence
    - used e.g. as a threshold to cut off unimportant rules
  - let $\sigma(X) = \frac{|\{x | x \subseteq ti,\ ti \in D\}|}{|D|}$
  - support = $\frac{\sigma(X \cup Y)}{|D|}$, confidence = $\frac{\sigma(X \cup Y)}{\sigma(X)}$

Recommendation based on Association Rule Mining

	Item1	Item2	Item3	Item4	Item5
Alice	1	0	0	0	?
User1	1	0	1	0	1
User2	1	0	1	0	1
User3	0	0	0	1	1
User4	0	1	1	0	0

- **Simplest approach**
  - transform 5-point ratings into binary ratings (1 = above user average)
- **Mine rules such as**
  - Item1 → Item5
    - support (2/4), confidence (2/2) (without Alice)
- **Make recommendations for Alice (basic method)**
  - Determine "relevant" rules based on Alice's transactions (the above rule will be relevant as Alice bought Item1)
  - Determine items not already bought by Alice
  - Sort the items based on the rules' confidence values
- **Different variations possible**
  - dislike statements, user associations ..

Probabilistic methods

- **Basic idea (simplistic version for illustration):**
  - given the user/item rating matrix
  - determine the probability that user Alice will like an item i
  - base the recommendation on such these probabilities
- **Calculation of rating probabilities based on Bayes Theorem**
  - How probable is rating value "1" for Item5 given Alice's previous ratings?
  - Corresponds to conditional probability P(Item5=1 | X), where
    - X = Alice's previous ratings = (Item1 =1, Item2=3, Item3= …)
  - Can be estimated based on Bayes' Theorem

$$P(Y|X) = \frac{P(X|Y) \times P(Y)}{P(X)} \qquad P(Y|X) = \frac{\prod_{i=1}^{d} P(X_i|Y) \times P(Y)}{P(X)}$$

  - Assumption: Ratings are independent (?)

Calculation of probabilities in simplistic approach

	Item1	Item2	Item3	Item4	Item5
Alice	1	3	3	2	?
User1	2	4	2	2	4
User2	1	3	3	5	1
User3	4	5	2	3	3
User4	1	1	5	2	1

X = (Item1 =1, Item2=3, Item3= …)

$$\begin{aligned}
\boldsymbol{P(X|Item5 = 1)} \\
&= \boldsymbol{P(Item1 = 1|Item5 = 1) \times P(Item2 = 3|Item5 = 1)} \\
&\times \boldsymbol{P(Item3 = 3|Item5 = 1) \times P(Item4 = 2|Item5 = 1)} \\
&= \boldsymbol{\frac{2}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \approx 0.125}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{P(X|Item5 = 2)} \\
&= \boldsymbol{P(Item1 = 1|Item5 = 2) \times P(Item2 = 3|Item5 = 2)} \\
&\times \boldsymbol{P(Item3 = 3|Item5 = 2) \times P(Item4 = 2|Item5 = 2)} \\
&= \boldsymbol{\frac{0}{0} \times \cdots \times \cdots \times \cdots = 0}
\end{aligned}$$

- **More to consider**
  - Zeros (smoothing required)
  - like/dislike simplification possible

Practical probabilistic approaches

- **Use a cluster-based approach** (Breese et al. 1998)
  - assume users fall into a small number of subgroups (clusters)
  - Make predictions based on estimates
    - probability of Alice falling into cluster c
    - probability of Alice liking item i given a certain cluster and her previous ratings
    - $P(C = c, v_1, \ldots, v_n) = P(C = c) \prod_{i=1}^{n} P(v_i | C = c)$
  - Based on model-based clustering (mixture model)
    - Number of classes and model parameters have to be learned from data in advance (EM algorithm)
- **Others:**
  - Bayesian Networks, Probabilistic Latent Semantic Analysis, ….
- **Empirical analysis shows:**
  - Probabilistic methods lead to relatively good results (movie domain)
  - No consistent winner; small memory-footprint of network model

Slope One predictors (Lemire and Maclachlan 2005)

- **Idea of Slope One predictors is simple and is based on a *popularity differential* between items for users**
- **Example:**

	Item1	**Item5**
Alice	2	**?**
User1	1	2

-

- **p(Alice, Item5) = 2 + (2 - 1) = 3**
- **Basic scheme: Take the average of these differences of the co-ratings to make the prediction**
- **In general: Find a function of the form f(x) = x + b**
  - **That is why the name is "Slope One"**

RF-Rec predictors (Gedikli et al. 2011)

- **Idea: Take rating frequencies into account for computing a prediction**
- **Basic scheme:** $\hat{r}_{u,i} = \arg\max_{v \in R} f_{user}(u, v) * f_{item}(i, v)$
  - R: Set of all rating values, e.g., $R = \{1,2,3,4,5\}$ on a 5-point rating scale
  - $f_{user}(u, v)$ and $f_{item}(i, v)$ basically describe *how often* a rating v was assigned by user u and to item i resp.
- **Example:**

	Item1	Item2	Item3	Item4	Item5
Alice	1	1	?	5	4
User1	2		5	5	5
User2			1	1	
User3		5	2		2
User4	3		1	1	
User5	1	2	2		4

- **p(Alice, Item3) = 1**

2008: *Factorization meets the neighborhood: a multifaceted collaborative filtering model*, Y. Koren, ACM SIGKDD

- **Stimulated by work on Netflix competition**
  - Prize of $1,000,000 for accuracy improvement of 10% RMSE compared to own Cinematch system
  - Very large dataset (~100M ratings, ~480K users , ~18K movies)
  - Last ratings/user withheld (set K)
- **Root mean squared error metric optimized to 0.8567**
- **Metrics measure error rate**
  - Mean Absolute Error (*MAE*) computes the deviation between predicted ratings and actual ratings

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|p_i - r_i|$$

  - Root Mean Square Error (*RMSE*) is similar to *MAE*, but places more emphasis on larger deviation

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(p_i - r_i)^2}$$

2008: ***Factorization meets the neighborhood: a multifaceted collaborative filtering model*, Y. Koren, ACM SIGKDD**

- **Merges neighborhood models with latent factor models**
- **Latent factor models**
  - **good to capture weak signals in the overall data**
- **Neighborhood models**
  - **good at detecting strong relationships between close items**
- **Combination in one prediction single function**
  - Local search method such as stochastic gradient descent to determine parameters
  - Add penalty for high values to avoid over-fitting

$$\hat{r}_{ui} = \mu + b_u + b_i + p_u^T q_i$$

$$\min_{p_*, q_*, b_*} \sum_{(u,i) \in K} (r_{ui} - \mu - b_u - b_i - p_u^T q_i)^2 + \lambda (\|p_u\|^2 + \|q_i\|^2 + b_u^2 + b_i^2)$$

Summarizing recent methods

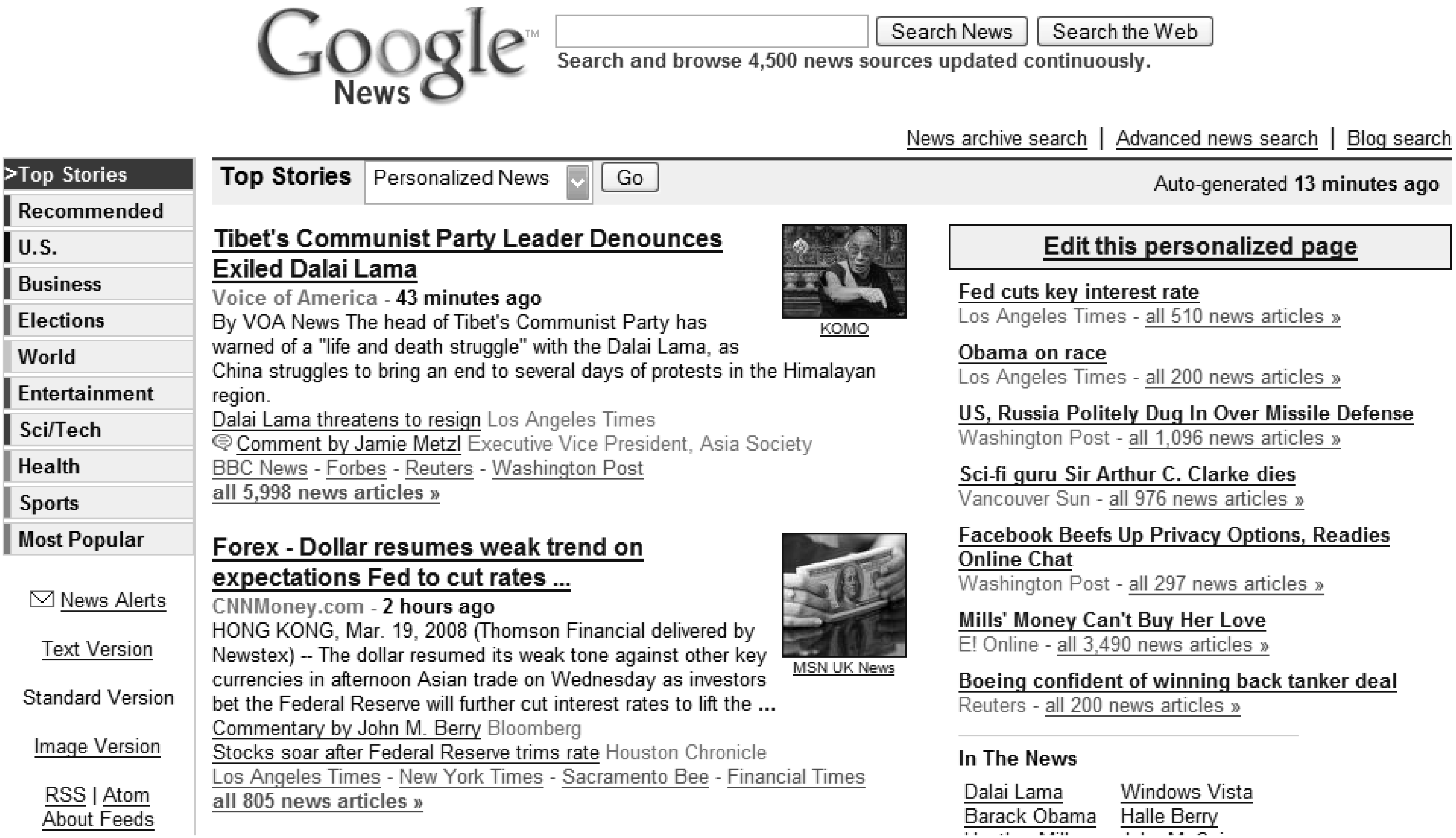
- **Recommendation is concerned with learning from noisy observations (x,y), where** $f(x) = \hat{y}$ **has to be determined such that** $\sum_{\hat{y}} (\hat{y} - y)^2$ **is minimal.**

- **A huge variety of different learning strategies have been applied trying to estimate f(x)**
  - Non parametric neighborhood models
  - MF models, SVMs, Neural Networks, Bayesian Networks,…

Collaborative Filtering Issues

- **Pros:**
  - well-understood, works well in some domains, no knowledge engineering required
- **Cons:**
  - requires user community, sparsity problems, no integration of other knowledge sources, no explanation of results
- **What is the best CF method?**
  - In which situation and which domain? Inconsistent findings; always the same domains and data sets; differences between methods are often very small (1/100)
- **How to evaluate the prediction quality?**
  - MAE / RMSE: What does an MAE of 0.7 actually mean?
  - Serendipity (novelty and surprising effect of recommendations)
    - Not yet fully understood
- **What about multi-dimensional ratings?**

The Google News personalization engine



Google News portal (1)

- **Aggregates news articles from several thousand sources**
- **Displays them to signed-in users in a personalized way**
- **Collaborative recommendation approach based on**
  - the click history of the active user and
  - the history of the larger community
- **Main challenges**
  - Vast number of articles and users
  - Generate recommendation list in real time (at most one second)
  - Constant stream of new items
  - Immediately react to user interaction
- **Significant efforts with respect to algorithms, engineering, and parallelization are required**

Google News portal (2)

- **Pure memory-based approaches are not directly applicable and for model-based approaches, the problem of continuous model updates must be solved**
- **A combination of model- and memory-based techniques is used**
- **Model-based part: Two clustering techniques are used**
  - Probabilistic Latent Semantic Indexing (PLSI) as proposed by (Hofmann 2004)
  - MinHash as a hashing method
- **Memory-based part: Analyze story *co-visits* for dealing with new users**
- **Google's MapReduce technique is used for parallelization in order to make computation scalable**

Literature (1)

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