Table 1 Results for 30 runs of minimizing benchmark functions 1 to 7, Number of iterations = 1000, Number of dimensions = 30

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Function |  | GA | PSO | IPO |
| F1 | Average of best fitnesses | 468.94 |  |  |
| Median of best fitnesses | 235.03 |  |  |
| Average of mean fitnesses | 493.35 |  |  |
| Overall best fitness | 2.12 | 2.06×10-12 | 3.74×10-23 |
| F2 | Average of best fitnesses | 0.31 | 2.00 |  |
| Median of best fitnesses | 0.21 |  |  |
| Average of mean fitnesses | 0.32 | 2.00 |  |
| Overall best fitness | 0.01 | 7.25×10-8 | 4.62×10-20 |
| F3 | Average of best fitnesses | 1.50×104 | 348.77 |  |
| Median of best fitnesses | 1.44×104 | 146.74 |  |
| Average of mean fitnesses | 1.55×104 | 6289.89 |  |
| Overall best fitness | 0.74×104 | 38.00 | 3.57×10-5 |
| F4 | Average of best fitnesses | 63.33 | 2.73 |  |
| Median of best fitnesses | 64.70 | 2.56 |  |
| Average of mean fitnesses | 63.33 | 11.03 |  |
| Overall best fitness | 43.59 | 1.15 | 5.75×10-6 |
| F5 | Average of best fitnesses | 952.36 | 167.42 | 29.93 |
| Median of best fitnesses | 474.01 | 77.92 | 26.42 |
| Average of mean fitnesses | 1705.11 | 3365.86 | 30.05 |
| Overall best fitness | 72.86 | 14.93 | 23.87 |
| F6 | Average of best fitnesses | 5175.20 | 0 | 0 |
| Median of best fitnesses | 4774 | 0 | 0 |
| Average of mean fitnesses | 5175.22 | 6.43 | 0.81 |
| Overall best fitness | 2134 | 0 | 0 |
| F7 | Average of best fitnesses | 0.90 |  |  |
| Median of best fitnesses | 0.86 |  |  |
| Average of mean fitnesses | 4.84 | 0.62 | 0.50 |
| Overall best fitness | 0.41 | 9.12×10-3 | 4.71×10-3 |

Table 2 Results for 30 runs of minimizing benchmark functions 8 to 13, Number of iterations = 1000, Number of dimensions = 30

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Function |  | GA | PSO | IPO |
| F8 | Average of best fitnesses | -9406.14 | -9102.19 | -10403.51 |
| Median of best fitnesses | -9308.37 | -9056.66 | -10431.22 |
| Average of mean fitnesses | -9402.77 | -9102.18 | -10403.51 |
| Overall best fitness | -10979.96 | -10180.97 | -11496.56 |
| F9 | Average of best fitnesses | 10.42 | 44.45 | 3.59 |
| Median of best fitnesses | 8.96 | 42.78 | 3.12 |
| Average of mean fitnesses | 10.55 | 45.72 | 11.35 |
| Overall best fitness | 1.00 | 22.88 | 0.02 |
| F10 | Average of best fitnesses | 5.86 |  | 0.74 |
| Median of best fitnesses | 5.57 |  | 0.42 |
| Average of mean fitnesses | 5.89 |  | 0.74 |
| Overall best fitness | 2.03 | 2.74×10-7 | 8.70×10-8 |
| F11 | Average of best fitnesses | 63.85 |  |  |
| Median of best fitnesses | 67.24 |  |  |
| Average of mean fitnesses | 64.07 |  |  |
| Overall best fitness | 20.61 | 2.28×10-11 | 7.09×10-14 |
| F12 | Average of best fitnesses | 31.96 |  |  |
| Median of best fitnesses | 32.06 |  |  |
| Average of mean fitnesses | 60.69 |  |  |
| Overall best fitness | 8.77 | 4.59×10-14 | 5.45×10-15 |
| F13 | Average of best fitnesses | 8.31 |  |  |
| Median of best fitnesses | 4.59 |  |  |
| Average of mean fitnesses | 19.44 | 102.53 |  |
| Overall best fitness | 0.03 | 4.43×10-12 | 2.97×10-17 |

Table 3 Results for 30 runs of minimizing benchmark functions 14 to 23, Number of iterations = 1000

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Function |  | GA | PSO | IPO |
| F14 | Average of best fitnesses | 4.12 | 0.9980 | 0.9980 |
| Median of best fitnesses | 1.99 | 0.9980 | 0.9980 |
| Average of mean fitnesses | 4.12 | 0.9980 | 0.9980 |
| Overall best fitness | 0.9980 | 0.9980 | 0.9980 |
| F15 | Average of best fitnesses | 2.44×10-3 |  |  |
| Median of best fitnesses | 1.24×10-3 |  |  |
| Average of mean fitnesses | 20.79×10-3 | 19.71 |  |
| Overall best fitness | 6.47×10-4 | 3.07×10-4 | 3.07×10-4 |
| F16 | Average of best fitnesses | -1.0316 | -1.0316 | -1.0316 |
| Median of best fitnesses | -1.0316 | -1.0316 | -1.0316 |
| Average of mean fitnesses | -1.0316 | -0.9470 | -1.0316 |
| Overall best fitness | -1.0316 | -1.0316 | -1.0316 |
| F17 | Average of best fitnesses | 0.3979 | 0.3979 | 0.3979 |
| Median of best fitnesses | 0.3979 | 0.3979 | 0.3979 |
| Average of mean fitnesses | 0.3979 | 0.8000 | 0.3979 |
| Overall best fitness | 0.3979 | 0.3979 | 0.3979 |
| F18 | Average of best fitnesses | 3.90 | 3.00 | 3.00 |
| Median of best fitnesses | 3.00 | 3.00 | 3.00 |
| Average of mean fitnesses | 3.90 | 3.00 | 3.00 |
| Overall best fitness | 3.00 | 3.00 | 3.00 |
| F19 | Average of best fitnesses | -3.8628 | -3.8628 | -3.8628 |
| Median of best fitnesses | -3.8628 | -3.8628 | -3.8628 |
| Average of mean fitnesses | -3.8628 | -3.8628 | -3.8628 |
| Overall best fitness | -3.8628 | -3.8628 | -3.8628 |
| F20 | Average of best fitnesses | -3.2744 | -3.2683 | -3.2881 |
| Median of best fitnesses | -3.3220 | -3.3220 | -3.3220 |
| Average of mean fitnesses | -3.2744 | -3.2683 | -3.2881 |
| Overall best fitness | -3.3220 | -3.3220 | -3.3220 |
| F21 | Average of best fitnesses | -5.2121 | -6.0637 | -9.8164 |
| Median of best fitnesses | -5.0552 | -5.1008 | -10.1532 |
| Average of mean fitnesses | -5.2121 | -5.9120 | -9.8164 |
| Overall best fitness | -10.1532 | -10.1532 | -10.1532 |
| F22 | Average of best fitnesses | -6.8356 | -8.1109 | -10.0513 |
| Median of best fitnesses | -7.7659 | -10.4029 | -10.4029 |
| Average of mean fitnesses | -6.8356 | -7.9953 | -10.0513 |
| Overall best fitness | -10.4029 | -10.4029 | -10.4029 |
| F23 | Average of best fitnesses | -4.6898 | -9.7793 | -10.0254 |
| Median of best fitnesses | -2.8711 | -10.5364 | -10.5364 |
| Average of mean fitnesses | -4.6898 | -8.4290 | -10.0254 |
| Overall best fitness | -10.5364 | -10.5364 | -10.5364 |

Table 4 Average run time for 30 runs of functions 1 to 23, Number of iterations = 1000, Number of dimensions (for functions 1 to 13) = 30

|  |  |  |  |
| --- | --- | --- | --- |
| Function | GA | PSO | IPO |
| F1 | 13.38 | 0.65 | 7.01 |
| F2 | 13.29 | 0.64 | 8.92 |
| F3 | 25.36 | 1.06 | 8.97 |
| F4 | 12.65 | 0.64 | 8.69 |
| F5 | 13.36 | 0.66 | 8.80 |
| F6 | 12.75 | 0.66 | 6.47 |
| F7 | 17.73 | 1.14 | 9.19 |
| F8 | 12.90 | 0.80 | 8.85 |
| F9 | 12.96 | 0.76 | 8.84 |
| F10 | 13.43 | 0.76 | 8.09 |
| F11 | 17.92 | 0.97 | 6.66 |
| F12 | 16.96 | 1.81 | 9.42 |
| F13 | 16.51 | 1.75 | 8.80 |
| F14 | 54.96 | 1.92 | 3.12 |
| F15 | 5.85 | 0.23 | 1.56 |
| F16 | 5.63 | 0.20 | 1.33 |
| F17 | 5.57 | 0.19 | 1.14 |
| F18 | 5.75 | 0.18 | 1.33 |
| F19 | 18.79 | 0.48 | 1.78 |
| F20 | 19.08 | 0.50 | 1.38 |
| F21 | 15.43 | 0.41 | 1.55 |
| F22 | 19.60 | 0.50 | 1.77 |
| F23 | 25.44 | 0.63 | 1.84 |

Table 5 Results for 5 runs of minimizing benchmark functions 1 to 13, Number of iterations = 1000 , Number of dimensions = 1000

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Function |  | GA | PSO | IPO |
| F1 | Average of best fitnesses | 1.91 × 106 | 5.90 × 105 | 2.04 × 104 |
| Median of best fitnesses | 1.90 × 106 | 5.86 × 105 | 1.99 × 104 |
| Average of mean fitnesses | 1.92 × 106 | 5.94 × 105 | 2.05 × 104 |
| Overall best fitness | 1.79 × 106 | 5.68 × 105 | 1.88 × 104 |
| F2 | Average of best fitnesses | 1165.69 | 923.17 | 438.70 |
| Median of best fitnesses | 1174.93 | 919.25 | 437.25 |
| Average of mean fitnesses | 1170.81 | 925.35 | 438.70 |
| Overall best fitness | 1128.65 | 883.80 | 435.73 |
| F3 | Average of best fitnesses | 3.21 × 107 | 8.43 × 106 | 1.79 × 105 |
| Median of best fitnesses | 3.19 × 107 | 8.63 × 106 | 1.82 × 105 |
| Average of mean fitnesses | 3.21 × 107 | 24.81 × 106 | 1.79 × 105 |
| Overall best fitness | 2.70 × 107 | 7.03 × 106 | 1.63 × 105 |
| F4 | Average of best fitnesses | 98.61 | 79.93 | 18.30 |
| Median of best fitnesses | 98.71 | 78.00 | 18.43 |
| Average of mean fitnesses | 98.61 | 90.99 | 18.30 |
| Overall best fitness | 98.16 | 76.93 | 17.88 |
| F5 | Average of best fitnesses | 5.86 × 109 | 1.31 × 109 | 5.67 × 106 |
| Median of best fitnesses | 5.77 × 109 | 1.26 × 109 | 5.56 × 106 |
| Average of mean fitnesses | 5.89 × 109 | 1.34 × 106 | 6.54 × 106 |
| Overall best fitness | 5.63 × 109 | 1.20 × 106 | 5.46 × 106 |
| F6 | Average of best fitnesses | 1.90 × 106 | 6.10 × 105 | 1.57 × 104 |
| Median of best fitnesses | 1.89 × 106 | 6.15 × 105 | 1.59 × 104 |
| Average of mean fitnesses | 1.90 × 106 | 6.13 × 105 | 1.61 × 104 |
| Overall best fitness | 1.83 × 106 | 5.91 × 105 | 1.50 × 104 |
| F7 | Average of best fitnesses | 8814.06 | 2.40 × 104 | 2892.85 |
| Median of best fitnesses | 8572.44 | 2.20 × 104 | 2786.45 |
| Average of mean fitnesses | 9923.61 | 2.42 × 104 | 2902.73 |
| Overall best fitness | 8097.48 | 1.99 × 104 | 2121.68 |
| F8 | Average of best fitnesses | -9.70 × 104 | -1.20 × 105 | -1.06 × 105 |
| Median of best fitnesses | -9.80 × 104 | -1.22 × 105 | -1.06 × 105 |
| Average of mean fitnesses | -9.69 × 104 | -1.18 × 105 | -1.04 × 105 |
| Overall best fitness | -10.22 × 104 | -1.24 × 105 | -1.11 × 105 |
| F9 | Average of best fitnesses | 7836.46 | 9098.96 | 6870.79 |
| Median of best fitnesses | 7792.10 | 9041.86 | 6846.23 |
| Average of mean fitnesses | 7946.43 | 9149.53 | 6995.40 |
| Overall best fitness | 7548.87 | 8814.01 | 6823.70 |
| F10 | Average of best fitnesses | 19.82 | 19.22 | 13.97 |
| Median of best fitnesses | 19.81 | 19.27 | 13.99 |
| Average of mean fitnesses | 19.85 | 19.24 | 13.97 |
| Overall best fitness | 19.67 | 19.05 | 13.75 |
| F11 | Average of best fitnesses | 1.96 × 104 | 5540.48 | 389.13 |
| Median of best fitnesses | 1.96 × 104 | 5622.17 | 394.21 |
| Average of mean fitnesses | 1.96 × 104 | 5573.36 | 389.13 |
| Overall best fitness | 1.92 × 104 | 5179.15 | 362.29 |
| F12 | Average of best fitnesses | 1.22 × 1010 | 2.06 × 109 | 6.99 |
| Median of best fitnesses | 1.23 × 1010 | 2.13 × 109 | 6.82 |
| Average of mean fitnesses | 1.23 × 1010 | 2.08 × 109 | 6.99 |
| Overall best fitness | 1.14 × 1010 | 1.36 × 109 | 6.64 |
| F13 | Average of best fitnesses | 2.77 × 1010 | 6.03 × 109 | 3049.78 |
| Median of best fitnesses | 2.78 × 1010 | 6.53 × 109 | 2693.64 |
| Average of mean fitnesses | 2.78 × 1010 | 6.10 × 109 | 3050.35 |
| Overall best fitness | 2.62 × 1010 | 4.58 × 109 | 2650.26 |

Table 6 Average run time for 5 runs of functions 1 to 23, Number of iterations = 1000, Number of dimensions = 1000

|  |  |  |  |
| --- | --- | --- | --- |
| Function | GA | PSO | IPO |
| F1 | 1339.88 | 16.05 | 223.89 |
| F2 | 1441.35 | 15.89 | 175.86 |
| F3 | 2860.44 | 177.61 | 392.71 |
| F4 | 1396.29 | 15.64 | 219.24 |
| F5 | 1290.12 | 17.40 | 216.84 |
| F6 | 1272.29 | 17.22 | 225.00 |
| F7 | 1607.52 | 23.94 | 235.38 |
| F8 | 1301.90 | 18.94 | 229.27 |
| F9 | 1361.99 | 18.27 | 226.51 |
| F10 | 1347.18 | 18.55 | 169.19 |
| F11 | 1446.16 | 20.59 | 223.77 |
| F12 | 1373.86 | 39.58 | 251.87 |
| F13 | 1363.06 | 38.44 | 250.27 |

Table 7 Results for 30 runs of minimizing benchmark functions 1 to 7 using local version of algorithms, Number of iterations = 1000, Number of dimensions = 30

|  |  |  |  |
| --- | --- | --- | --- |
| Function |  | PSO-lbest | IPO-local |
| F1 | Average of best fitnesses | 2335.79 | 7.43×10-12 |
| Median of best fitnesses | 2326.96 | 6.52×10-12 |
| Average of mean fitnesses | 7030.86 | 2.17×10-10 |
| Overall best fitness | 1957.65 | 2.09×10-12 |
| F2 | Average of best fitnesses | 10.34 | 9.91×10-13 |
| Median of best fitnesses | 10.32 | 1.74×10-20 |
| Average of mean fitnesses | 18.50 | 9.91×10-13 |
| Overall best fitness | 9.01 | 1.08×10-20 |
| F3 | Average of best fitnesses | 5602.37 | 1.68 |
| Median of best fitnesses | 5488.77 | 1.36 |
| Average of mean fitnesses | 93206.24 | 699.54 |
| Overall best fitness | 4404.18 | 0.34 |
| F4 | Average of best fitnesses | 20.05 | 0.45 |
| Median of best fitnesses | 20.21 | 2.16×10-6 |
| Average of mean fitnesses | 36.70 | 0.45 |
| Overall best fitness | 17.93 | 1.29×10-6 |
| F5 | Average of best fitnesses | 3.97×105 | 29.68 |
| Median of best fitnesses | 4.04×105 | 25.32 |
| Average of mean fitnesses | 3.92×106 | 30.08 |
| Overall best fitness | 1.88×105 | 23.61 |
| F6 | Average of best fitnesses | 2344.87 | 2.17 |
| Median of best fitnesses | 2344.50 | 2 |
| Average of mean fitnesses | 7112.70 | 1003.76 |
| Overall best fitness | 1912 | 0 |
| F7 | Average of best fitnesses | 0.37 | 0.02 |
| Median of best fitnesses | 0.37 | 0.02 |
| Average of mean fitnesses | 2.78 | 0.50 |
| Overall best fitness | 0.21 | 0.01 |

Table 8 Results for 30 runs of minimizing benchmark functions 8 to 13 using local version of algorithms, Number of iterations = 1000, Number of dimensions = 30

|  |  |  |  |
| --- | --- | --- | --- |
| Function |  | PSO-lbest | IPO-local |
| F8 | Average of best fitnesses | -4891.67 | -10336.43 |
| Median of best fitnesses | -4877.71 | -10254.26 |
| Average of mean fitnesses | -1148.82 | -10312.39 |
| Overall best fitness | -5812.85 | -11498.59 |
| F9 | Average of best fitnesses | 183.28 | 9.57 |
| Median of best fitnesses | 184.52 | 9.09 |
| Average of mean fitnesses | 339.34 | 21.66 |
| Overall best fitness | 158.20 | 6.11 |
| F10 | Average of best fitnesses | 10.54 | 0.31 |
| Median of best fitnesses | 10.59 | 1.72×10-4 |
| Average of mean fitnesses | 14.18 | 0.68 |
| Overall best fitness | 9.78 | 1.42×10-4 |
| F11 | Average of best fitnesses | 22.12 | 0.01 |
| Median of best fitnesses | 22.45 | 0.01 |
| Average of mean fitnesses | 63.74 | 0.01 |
| Overall best fitness | 13.55 | 2.06×10-13 |
| F12 | Average of best fitnesses | 33.44 | 0.01 |
| Median of best fitnesses | 32.88 | 5.55×10-11 |
| Average of mean fitnesses | 1.07×106 | 2.19 |
| Overall best fitness | 15.07 | 1.01×10-11 |
| F13 | Average of best fitnesses | 9.02×104 | 1.07×10-3 |
| Median of best fitnesses | 8.44×104 | 6.92×10-10 |
| Average of mean fitnesses | 7.14×106 | 1.07×10-3 |
| Overall best fitness | 1.20×104 | 1.55×10-12 |

Table 9 Results for 30 runs of minimizing benchmark functions 14 to 23 using local version of algorithms, Number of iterations = 1000, Number of dimensions = 30

|  |  |  |  |
| --- | --- | --- | --- |
| Function |  | PSO-lbest | IPO-local |
| F14 | Average of best fitnesses | 1.0314 | 1.3618 |
| Median of best fitnesses | 0.9980 | 0.9980 |
| Average of mean fitnesses | 17.8787 | 1.3621 |
| Overall best fitness | 0.9980 | 0.9980 |
| F15 | Average of best fitnesses | 9.48×10-4 | 8.05×10-4 |
| Median of best fitnesses | 7.82×10-4 | 7.29×10-4 |
| Average of mean fitnesses | 0.23 | 12.10×10-4 |
| Overall best fitness | 6.48×10-4 | 4.07×10-4 |
| F16 | Average of best fitnesses | -1.0316 | -1.0316 |
| Median of best fitnesses | -1.0316 | -1.0316 |
| Average of mean fitnesses | -1.0241 | -1.0316 |
| Overall best fitness | -1.0316 | -1.0316 |
| F17 | Average of best fitnesses | 0.3979 | 0.3979 |
| Median of best fitnesses | 0.3979 | 0.3979 |
| Average of mean fitnesses | 0.4453 | 0.3979 |
| Overall best fitness | 0.3979 | 0.3979 |
| F18 | Average of best fitnesses | 3.00 | 3.00 |
| Median of best fitnesses | 3.00 | 3.00 |
| Average of mean fitnesses | 15.13 | 3.00 |
| Overall best fitness | 3.00 | 3.00 |
| F19 | Average of best fitnesses | -3.8628 | -3.8628 |
| Median of best fitnesses | -3.8628 | -3.8628 |
| Average of mean fitnesses | -3.3966 | -3.8628 |
| Overall best fitness | -3.8628 | -3.8628 |
| F20 | Average of best fitnesses | -3.3220 | -3.3220 |
| Median of best fitnesses | -3.3220 | -3.3220 |
| Average of mean fitnesses | -3.2689 | -3.3220 |
| Overall best fitness | -3.3220 | -3.3220 |
| F21 | Average of best fitnesses | -10.0132 | -10.1532 |
| Median of best fitnesses | -10.1532 | -10.1532 |
| Average of mean fitnesses | -6.4280 | -10.1532 |
| Overall best fitness | -10.1532 | -10.1532 |
| F22 | Average of best fitnesses | -10.4029 | -10.4029 |
| Median of best fitnesses | -10.4029 | -10.4029 |
| Average of mean fitnesses | -9.2092 | -10.4029 |
| Overall best fitness | -10.4029 | -10.4029 |
| F23 | Average of best fitnesses | -10.5364 | -9.8121 |
| Median of best fitnesses | -10.5364 | -9.8230 |
| Average of mean fitnesses | -9.8470 | -8.5833 |
| Overall best fitness | -10.5364 | -10.4093 |