PSet6 Report

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1 Complex Networks

The Graphs and theirs degree and clustering distribution is shown in Fig1 for mean degree 0.8, Fig1 for mean degree 1.0 and Fig1 for mean degree 8.0.

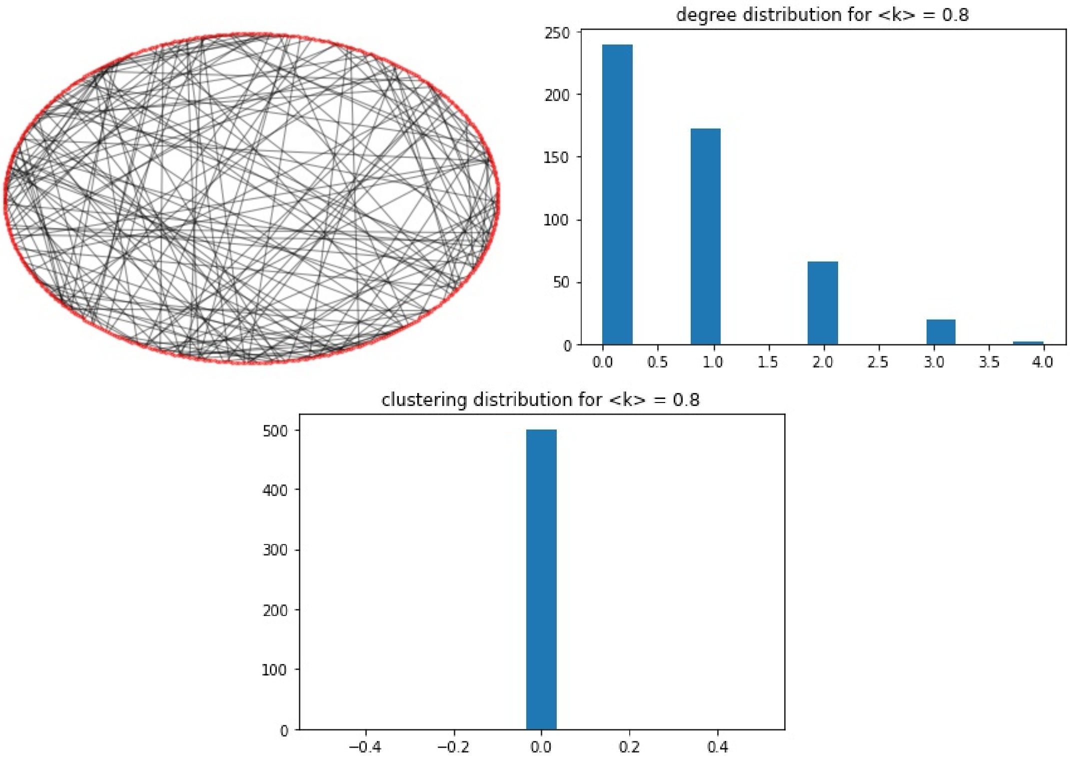

Figure 1: Illustrations for graph with mean degree of 0.8

2 Monte Carlo Integral(SS MC & IS MC)

I did the SS MC and IS MC as wanted. I defined their functions and used `time.time()` to find the runtime of the integration. The results are available in Table2
It's worth noting that the numeric value is roughly 0.8820814. And for IS MC, the result of the integral of $g(x)$ is:

$$\int_0^2 e^{-x} = 1 - e^{-2} \tag{1}$$

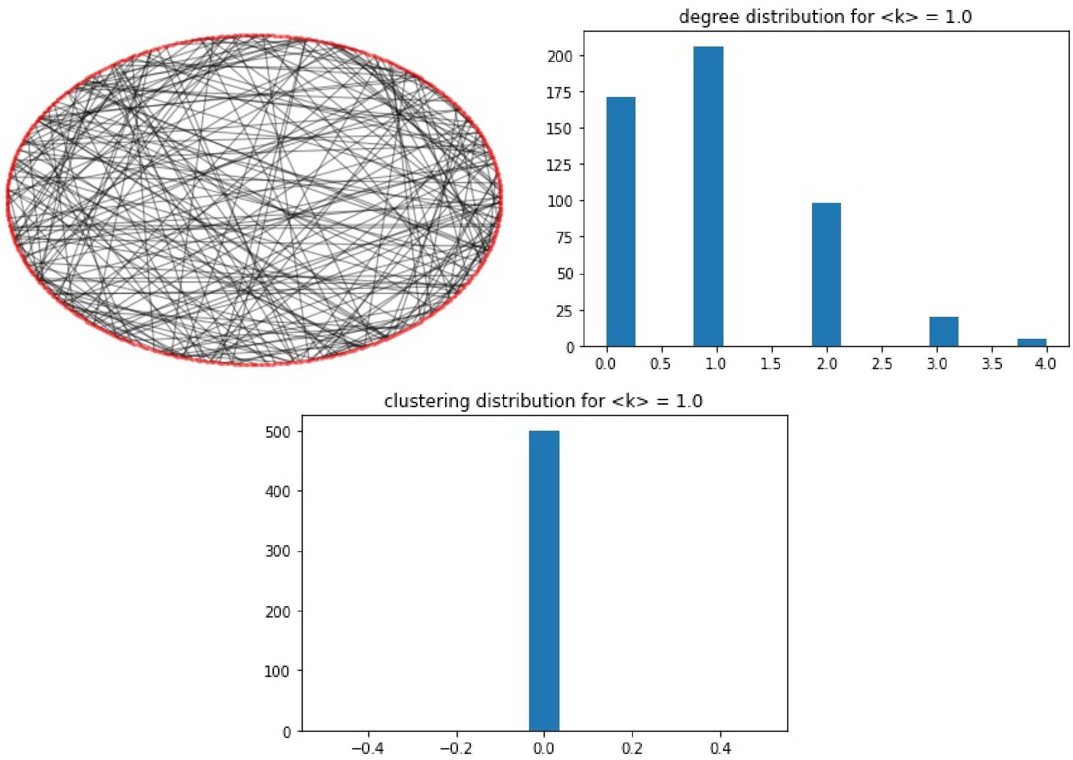

Figure 2: Illustrations for graph with mean degree of 1.0

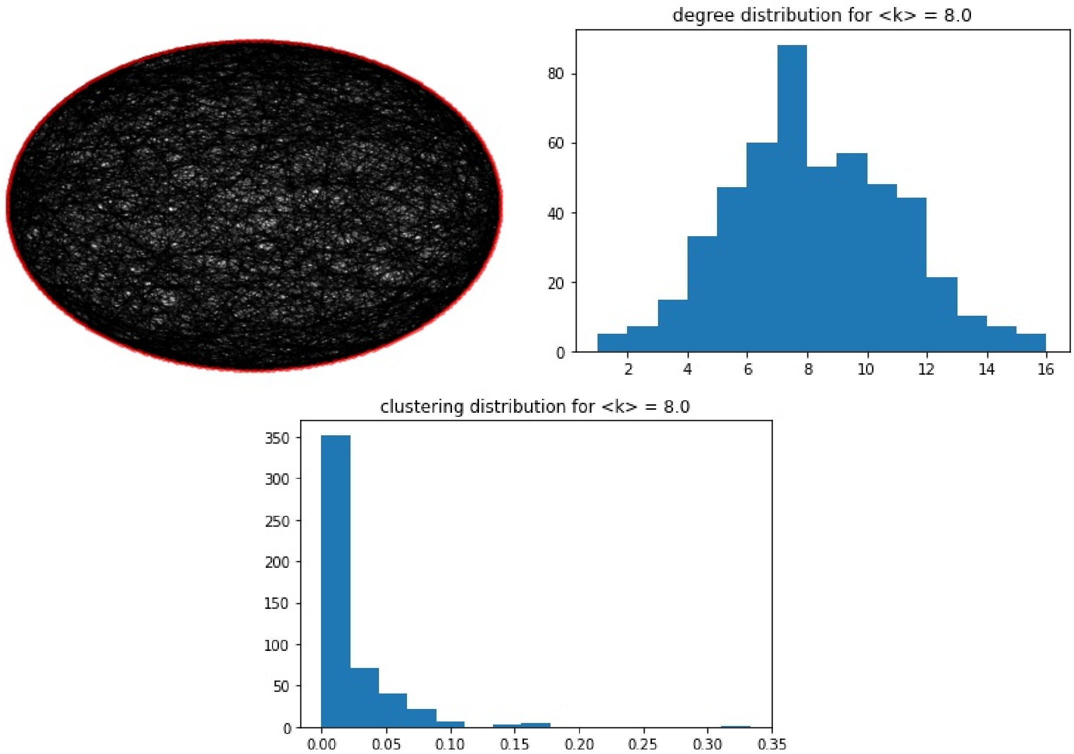

Figure 3: Illustrations for graph with mean degree of 8.0

3 The Sphere (Multi variable Integration SS MC)

Here the problem asks us to find the center of mass for the sphere with density $\rho(z) = \rho_0(1 - \frac{z}{4R})$ where R is the radius of the sphere.
This problem has a cylindrical symmetry. We take the sphere's bottom to lie on the origin and the

n	SS MC				IS MC			
n	I	$\Delta_{stat}I$	$\Delta_{real}I$	t	I	$\Delta_{stat}I$	$\Delta_{real}I$	t
1000	0.858522	0.546	0.023	0.02078	0.882866	0.0100	0.00078	0.01938
2000	0.861797	0.55	0.02	0.01930	0.871512	0.007	0.0106	0.03906
4000	0.884244	0.56	0.0022	0.05336	0.875264	0.005	0.007	0.08817
8000	0.886907	0.56	0.0048	0.07861	0.883804	0.003	0.002	0.11154
16000	0.883410	0.560	0.0013	0.14961	0.882986	0.002	0.0009	0.20379
32000	0.875776	0.557	0.0063	0.19460	0.881274	0.0017	0.0008	0.39223
64000	0.879419	0.558	0.0027	0.33243	0.881791	0.0012	0.0003	0.69962

Table 1: Data for numerical integration using SS MC and IS MC.

center of the sphere on $z = R$ on the z-axis. This way the solution is as follows:

$$\begin{aligned} MR_{CoM} = I &= \int_0^{2R} \rho(z) z \mathrm{d}v = \int_0^{2R} \rho(z) z \pi \left(R^2 - (R-z)^2\right) \mathrm{d}z \\ M &= \int_0^{2R} \mathrm{d}m = \int_0^{2R} \rho(z) \pi \left(R^2 - (R-z)^2\right) \mathrm{d}z \\ \Rightarrow R_{CoM} &= \frac{I}{M} \end{aligned} \tag{2}$$

We use IS MC with the function $\rho(z)$ as our distribution $g(z)$. This way we have:

$$\begin{aligned} I &= C \times \int_0^{2R} f(z) g(z) \mathrm{d}z,\ C = \text{Const.}, \\ M &= C \times \int_0^{2R} h(z) g(z) \mathrm{d}z, \\ f(z) &= z\pi \left(R^2 - (R-z)^2\right), \\ h(z) &= \pi \left(R^2 - (R-z)^2\right) \\ \Rightarrow R_{CoM} &= \frac{\int_0^{2R} f(z) g(z) \mathrm{d}z}{\int_0^{2R} h(z) g(z) \mathrm{d}z} \end{aligned} \tag{3}$$

using the equations in 2, we find:

$$\begin{aligned} I &= \frac{14}{15} \pi \rho_0 R^4, \\ M &= \pi \rho_0 R^3 \\ \Rightarrow R_{CoM} &= \frac{14}{15} R \end{aligned} \tag{4}$$

I made the random generator by finding the inverse of $g(z)$, took $\rho_0 = 1$ and $R = 10$, this leads to the numerical answer $R_{num} = 9.99854$ and the analytical result of $R_{real} = 9.33333$ which raises the error to be about $\delta = \frac{R_{num} - R_{real}}{R_{real}} \simeq 0.71 = 7.1\%$

4 Metropolis Random Generation

I made the function to generate random numbers with a normal distribution using the Metropolis Algorithm.

Then, I fount the steps in which the acceptance rate is $a.r. = \{0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1\}$ Then I made a function to find the correlation length (gyro length). There I used the fact that this length is equal to the first j that gives an auto-correlation of less than e^{-1}. The result is in Table2.

a. r.	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
ξ	28	9	4	3	2	2	3	4	8

Table 2: The gyro length for each acceptance rate