Growing Critical: Self-Organized Criticality in a Developing Neural System

Ali Abolhassanzadeh Mahani

Sharif University of Technology

An article by:
Felipe Yaroslav Kalle Kossio, Sven Goedeke, Benjamin van den Akker, Borja Ibarz, and Raoul-Martin Memmesheimer

- Intro to criticality

An article by:
Felipe Yaroslav Kalle Kossio, Sven Goedeke, Benjamin van den Akker, Borja Ibarz, and Raoul-Martin Memmesheimer

- Intro to criticality
- Genetic development of neural networks: active neurons shrink while inactive ones grow

A bit of history

The dynamics is an inhomogeneous Poisson point process (?)
Notation:

- $f_i(t)$: instantaneous firing rate of neuron i
- gA_{ij}: time-dependent connection strength
- g: proportionality constant
- A_{ij}: overlap areas of the neurons
- τ: the decay time constant due to leak currents
- $\hat{t}_j$: spike times of neuron j

The dynamics is an inhomogeneous Poisson point process (?)
Notation:

- $f_i(t)$: instantaneous firing rate of neuron i
- gA_{ij}: time-dependent connection strength
- g: proportionality constant
- A_{ij}: overlap areas of the neurons
- τ: the decay time constant due to leak currents
- $\hat{t}_j$: spike times of neuron j

The Stationary State Dynamics is a follows:

$$\tau \dot{f}_i(t) = f_0 - f_i(t) + \tau g \sum_j A_{ij}(t^-) \sum_{\hat{t}_j} \delta(t - \hat{t}_j) \qquad (1)$$

Notation:

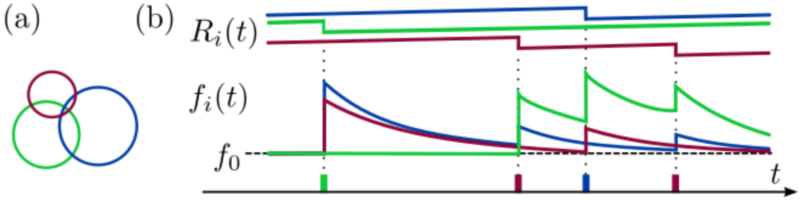
- $R_i(t)$: radius of disk representing a single neuron
- K: linear growth rate of neurons
- $\frac{K}{f_{sat}}$: neuron radii shrinkage at spike sending
- Growth takes much longer than decay of activity $\frac{1}{K} \gg \tau$,
- experiments $\Rightarrow f_{sat} \gg f_0$

Notation:

- $R_i(t)$: radius of disk representing a single neuron
- K: linear growth rate of neurons
- $\frac{K}{f_{sat}}$: neuron radii shrinkage at spike sending
- Growth takes much longer than decay of activity $\frac{1}{K} \gg \tau$,
- experiments $\Rightarrow f_{sat} \gg f_0$

The network growth dynamics is as follows:

$$\dot{R}_i(t) = K\left(1 - \frac{1}{f_{sat}} \sum_{\hat{t}_i} \delta(t - \hat{t}_i)\right) \tag{2}$$



Example of the network dynamics.

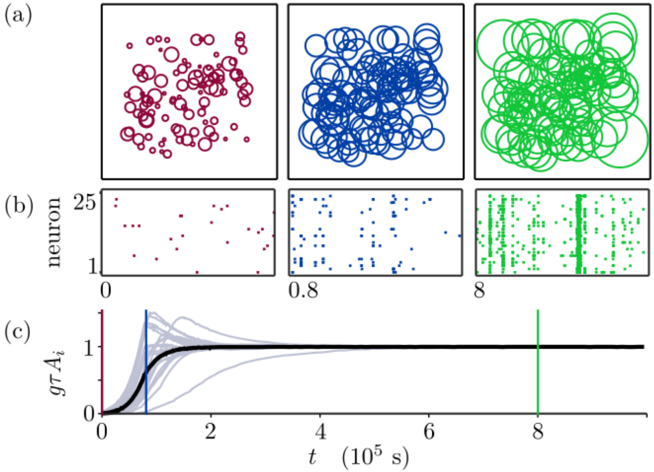
Averaged over the randomness of spike generation, each spike generates in total

$$\sigma = \tau g \sum_j \bar{A_{ij}} = 1 - \frac{f_0}{f_{sat}} \tag{3}$$

spikes.

Thus we have a age-dependent branching process with branching parameter σ. (Individuals –spikes– generate offspring at an age-dependent rate)

Ref: Crump-Mode-Jagers branching process


(a)
(b)
neuron
25
1
0
0.8
8
(c)
$g\tau A_i$
1
0
0
2
4
6
8
$t \quad (10^5$ s)

The avalanche sizes s follow the Borel distribution

$$P(s) = \frac{(s\sigma)^{s-1} e^{-s\sigma}}{s!} \tag{4}$$

where using the Stirling's approximation we get

$$P_{appr}(s) = \frac{1}{\sqrt{2\pi}\sigma} s^{-\frac{3}{2}} e^{-(\sigma - \ln\sigma - 1)s} \tag{5}$$

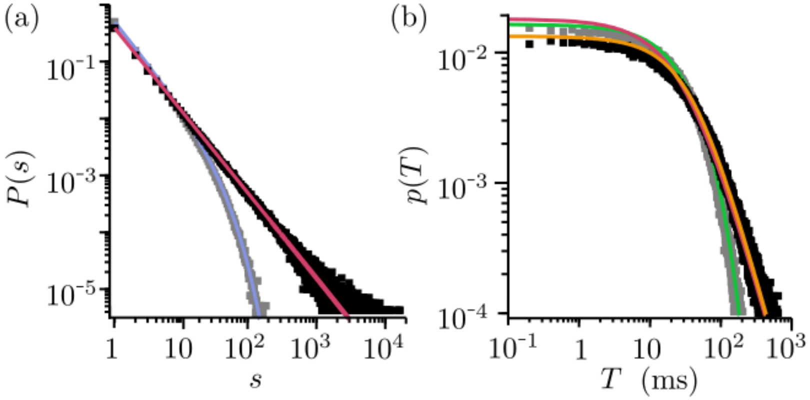
The avalanche sizes s follow the Borel distribution

$$P(s) = \frac{(s\sigma)^{s-1} e^{-s\sigma}}{s!} \tag{4}$$

where using the Stirling's approximation we get

$$P_{appr}(s) = \frac{1}{\sqrt{2\pi}\sigma} s^{-\frac{3}{2}} e^{-(\sigma - \ln\sigma - 1)s} \tag{5}$$

Notice the power-law tail with exponent $\frac{3}{2}$ of a critical branching process for $\sigma = 1$


(a)
$P(s)$
10^{-1}
10^{-3}
10^{-5}
1 10 10^2 10^3 10^4
s
(b)
$p(T)$
10^{-2}
10^{-3}
10^{-4}
10^{-1} 1 10 10^2 10^3
T (ms)

Thank you for your time :)