Morris Lecar Equations to use for FORCE training

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The equations from the lecture series for the Morris Lecar model are as follows:

$$\begin{aligned}
C\frac{\mathrm{d}V_i}{\mathrm{d}t} &= I - g_L(V_i - E_L) = g_K n_i(V_i - E_K) - g_{Ca}m_\infty(V_i)(V_i - E_{Ca}) + I_{psp,i} \\
I_{psp,i} &= -\sum_{j=1}^{N} \bar{g}_{ij}s_j(t)(V_i - E_{ij}) \\
\frac{\mathrm{d}n_i}{\mathrm{d}t} &= \left[\frac{n_\infty(V_i) - n_i}{\tau_n(V_i)}\right] \\
m_\infty(V_i) &= \frac{1}{2}\left[1 + \tanh\left(\frac{V_i - V_1}{V_2}\right)\right] \\
n_\infty(V_i) &= \frac{1}{2}\left[1 + \tanh\left(\frac{V_i - V_3}{V_4}\right)\right] \\
\tau_n(V_i) &= \left[\phi\cosh\left(\frac{V_i - V_3}{2V_4}\right)\right]^{-1} \\
\frac{\mathrm{d}s_i}{\mathrm{d}t} &= a_r T(V_i)(1 - s_i) - a_d s_i \\
T(V_i) &= \frac{T_{max}}{1 + \exp\left(-\frac{V_i - V_T}{K_p}\right)}
\end{aligned} \tag{1}$$

where

$$E_{ij} = \{E_+, E_-\} \tag{2}$$

for E_+ for AMPA and E_- for GABA. and

$$E_{ij} = E_{kj}, \qquad \forall i, j, k \leq N \tag{3}$$

where the following **variables**

- V_i: The membrane potential of the neurons.
- n_i: recovery variable: probability that the K^+ channel is conducting.

and **parameters** in the **Hodgkin-Class I** with random coupling:

- $I = 100pA$: The applied current.

- $C = 20pF$: membrane capacitance.
- $g_L = 2, g_K = 8, g_{Ca} = 4nS$: leak, K, Ca conductances through the memeber ion channels.
- $E_L = -60, E_K = -84, E_{Ca} = 120mV$: equilibrium potential of relevant ion channels.
- $V_1 = -1.2, V_2 = 18, V_3 = 12, V_4 = 17.4mV$: tuning parameters for steady state and time constant.
- $\phi = 0.067Hz$: the reference frequency.
- $a_r = 1.1, a_d = 0.19ms^{-1}$: rise and decay inverse time constants for the post-synaptic weights.
- $V_T = 2, K_p = 5mV, T_{max} = 1$: Tuning parameters for the post-synaptic potential.
- $\bar{g}_{ij}$: weight matrix taken randomly from a normal distribution. Details to follow.

In vector form, the post-synaptic potential becomes

$$\vec{I}_{psp} = \left(\bar{\mathbf{g}} \ + \hat{\eta}.\hat{\phi}^{\intercal}\right) \otimes (\vec{V}.\mathbf{1}^{\intercal} - \hat{\mathbf{E}}).\vec{s}(t) \tag{4}$$