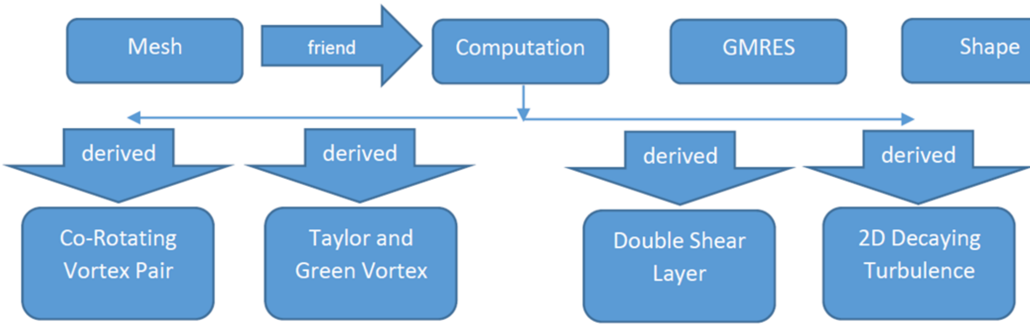
Object Oriented Programing:



Code description: This figure shows the map of the C++ code for the simulation of the 2D incompressible Navier-Stokes equations using incremental/non-incremental pressure correction schemes with an unstructured finite-element discretization (with Gmsh as a mesh generator) and a semi-implicit time integration method. This code can solve unsteady N.S. This code can solve unsteady N.S. equations in laminar regime for any 2D geometry (flow past a cylinder, flow over a backward facing step, Taylor-Green vortex in a box with a circular hole inside the domain, triangular cavity flow, and etc.). This code uses advanced data structure like compressed sparse row (CSR) techniques for data storing, matrix vector multiplication, and matrix assembling process. Also, it uses ILU(0) preconditioned GMRES(m) iterative solver. The code contains the coarse-grid-projection feature to such that it is able to solve the advection-diffusion and pressure Poisson equations in two different (unstructured) grid resolutions. Furthermore, the code is able to solve the energy equation as well as Lorentz force and electric potential Poisson equation leading to MHD flows.

The code has four main objects (class):

"**Mesh**" class: reading the mesh information from Gmsh and building the information of each element such as global and local coordinates, node numbering, connectivity, nodes on the boundary, and etc.

"**Shape**" class: containing Local shape functions (P1 and P2) and numerical integration for a desired operator such as Laplacin, divergence, gradient, and etc.

"**GMRES**" class: the iterative solver

"**Computation**" class: building global matrices, setting initial and boundary conditions, building the goal equations, time integration, and the rest of necessary calculations

For any specific problem, the user creates a derived class from the "Computation" class and imposes the desired boundary and initial conditions.

The following figures show the solution of the code for Taylor-green vortex problem, flow over a back-warding step, and flow past a cylinder.

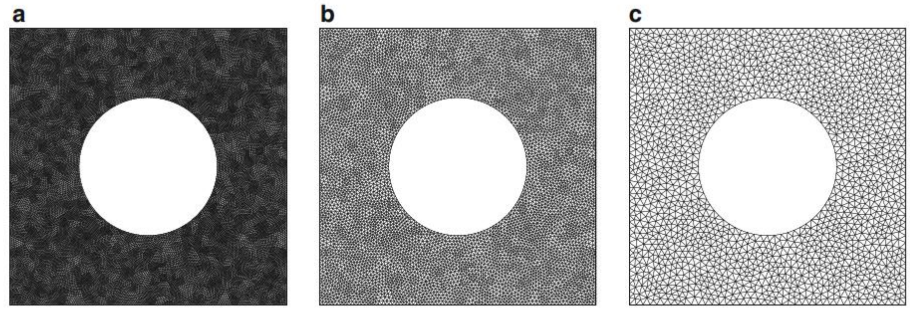


Fig. 2 Representation of the triangular finite element meshes used for solving Poisson's equation in the simulation of Taylor-Green vortex. **a** After one level coarsening ($l = 1$), 27520 nodes and 54144 elements; **b** After two levels coarsening ($l = 2$), 6992 nodes and 13536 elements; **c** After three levels coarsening ($l = 3$), 1804 nodes and 3384 elements

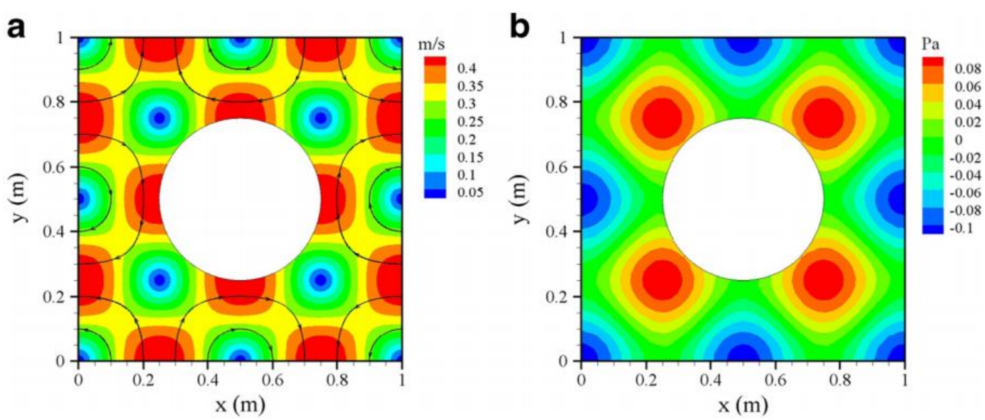


Fig. 3 The solution of Taylor-Green vortex in a non trivial geometry at $t = 1$ s obtained using the CGP ($l = 3$) method. The resolution of the nonlinear and linear equations is, respectively, 54144 and 3384 elements. **a** Velocity magnitude contour and streamlines; **b** Pressure field

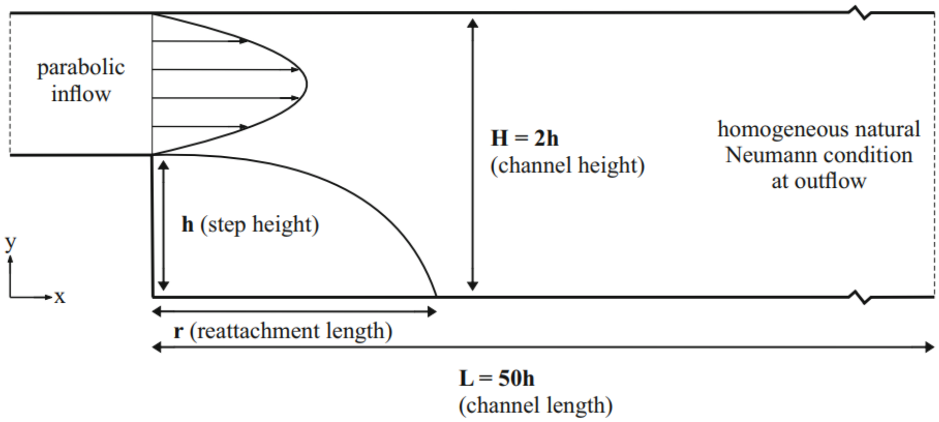


Fig. 4 Schematic view of flow past a backward-facing step

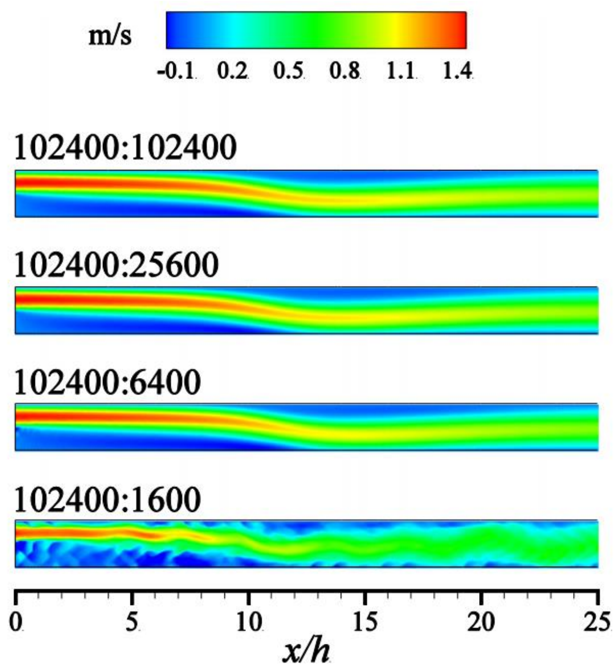


Fig. 6 Horizontal velocity component contour plot of flow over backward-facing step at $Re = 800$. Labels in the form of $M : N$ indicate the grid resolution of the advection-diffusion solver, M elements, and Poisson's equation, N elements

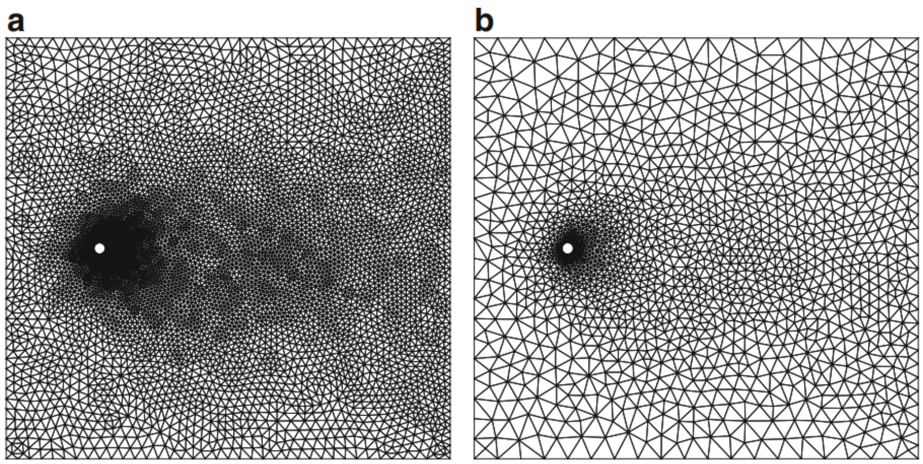


Fig. 8 Computational mesh for the Poisson equation solution of the flow past a circular cylinder. **a** After two levels coarsening ($l = 2$); **b** After three levels coarsening ($l = 3$). Details of the grids are reported

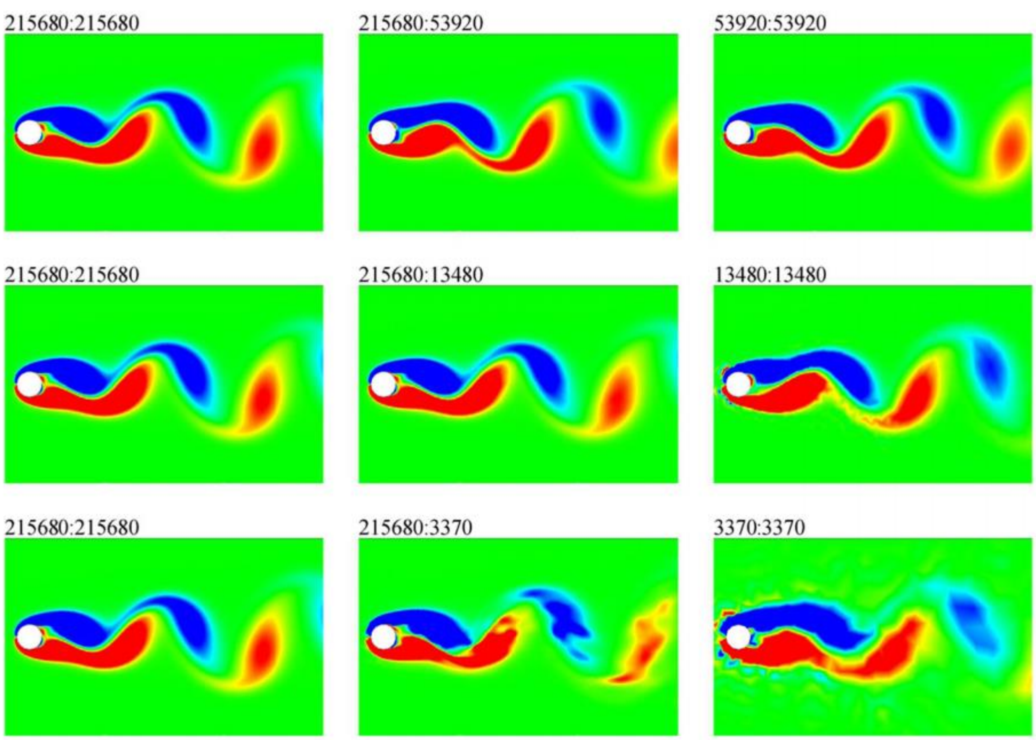


Fig. 9 Vorticity fields for the flow past a circular cylinder at $t = 150$ s. Labels in the form of $M : N$ illustrate the spatial resolution of the advection-diffusion grid, M elements, and the Poisson equation mesh, N elements