12.818: INTRODUCTION TO ATMOSPHERIC DATA AND LARGE-SCALE DYNAMICS

PROJECT EIGHT: ESTIMATION OF VERTICAL VELOCITY

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In this project, we will make estimates for the vertical velocity of air parcels in the atmosphere using two different methods, by inference from the vorticity equation and by the use of isentropic analysis.

1 Estimation of vertical velocity from the vorticity equation

As we saw in class, the vorticity equation

$$f\frac{\partial \omega}{\partial p} = \frac{\partial \zeta_g}{\partial t} + \boldsymbol{u}_g \cdot \nabla(\zeta_g + f) + g\frac{\partial}{\partial p}(\hat{\boldsymbol{z}} \cdot \nabla \times \boldsymbol{\tau}) \tag{1}$$

relates the change in vorticity to the difference in vertical velocity at the top and bottom of the column, and so if the vertical velocity at one end of the column is known, the vertical velocity at the other end can be inferred from the vorticity equation. Here, $\zeta_g + f$ is the absolute vorticity, f is the planetary vorticity, ζ_g is the geostrophic relative vorticity evaluated on an isobaric surface, ω is the vertical velocity in pressure coordinates, $\boldsymbol{\tau}$ is the surface stress vector, g is the acceleration due to gravity, and $\boldsymbol{u}_g$ is the geostrophic wind velocity field.

The $\frac{\partial \zeta_g}{\partial t}$ term is the rate of change of relative vorticity (figure 1), while $\boldsymbol{u}_g \cdot \nabla(\zeta_g + f)$ represents the horizontal advection of absolute vorticity (figure 2) and $g\frac{\partial}{\partial p}(\hat{\boldsymbol{z}} \cdot \nabla \times \boldsymbol{\tau})$ the destruction of vorticity by surface stress, which becomes significant only near the Earth's surface and so we won't consider it here. For this section, we will be looking at the Great Lakes region.

Discretizing the $\partial\omega/\partial p$ derivative in (1) using a first-order finite difference we get

$$f\frac{\omega(850\,\text{hPa}) - \omega(700\,\text{hPa})}{\Delta p} = \frac{\partial \zeta_g}{\partial t} + \boldsymbol{u}_g \cdot \nabla(\zeta_g + f) + \underbrace{g\frac{\partial}{\partial p}(\hat{\boldsymbol{z}} \cdot \nabla \times \boldsymbol{\tau})}_{=\,0\text{ near the Earth's surface}}$$

where $\Delta p = 850\,\text{hPa} - 700\,\text{hPa} = 150\,\text{hPa}$ and we are ignoring effects due to surface stresses and wind stress curl. Thus we can estimate the vertical velocity at 850 hPa from the vertical velocity at 700 hPa using

$$\omega(850\,\text{hPa}) = \omega(700\,\text{hPa}) + \frac{\Delta p}{f}\left[\frac{\partial \zeta_g}{\partial t} + \boldsymbol{u}_g \cdot \nabla(\zeta_g + f)\right]$$

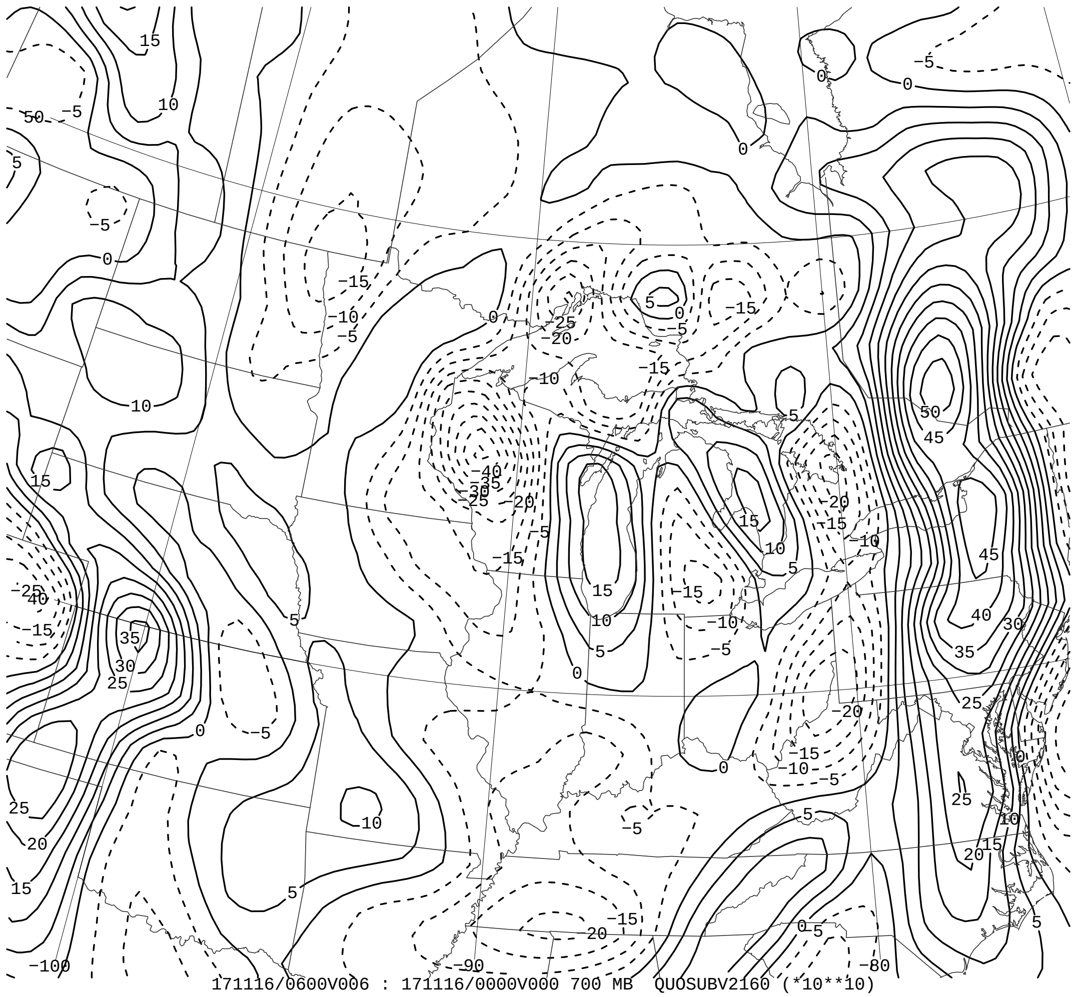

Figure 1: Contour plot of the rate of change of relative vorticity ($\times 10^{10}\,\text{s}^{-2}$) at 700 hPa over the Great Lakes region between November 16, 2017 0Z and 6Z.

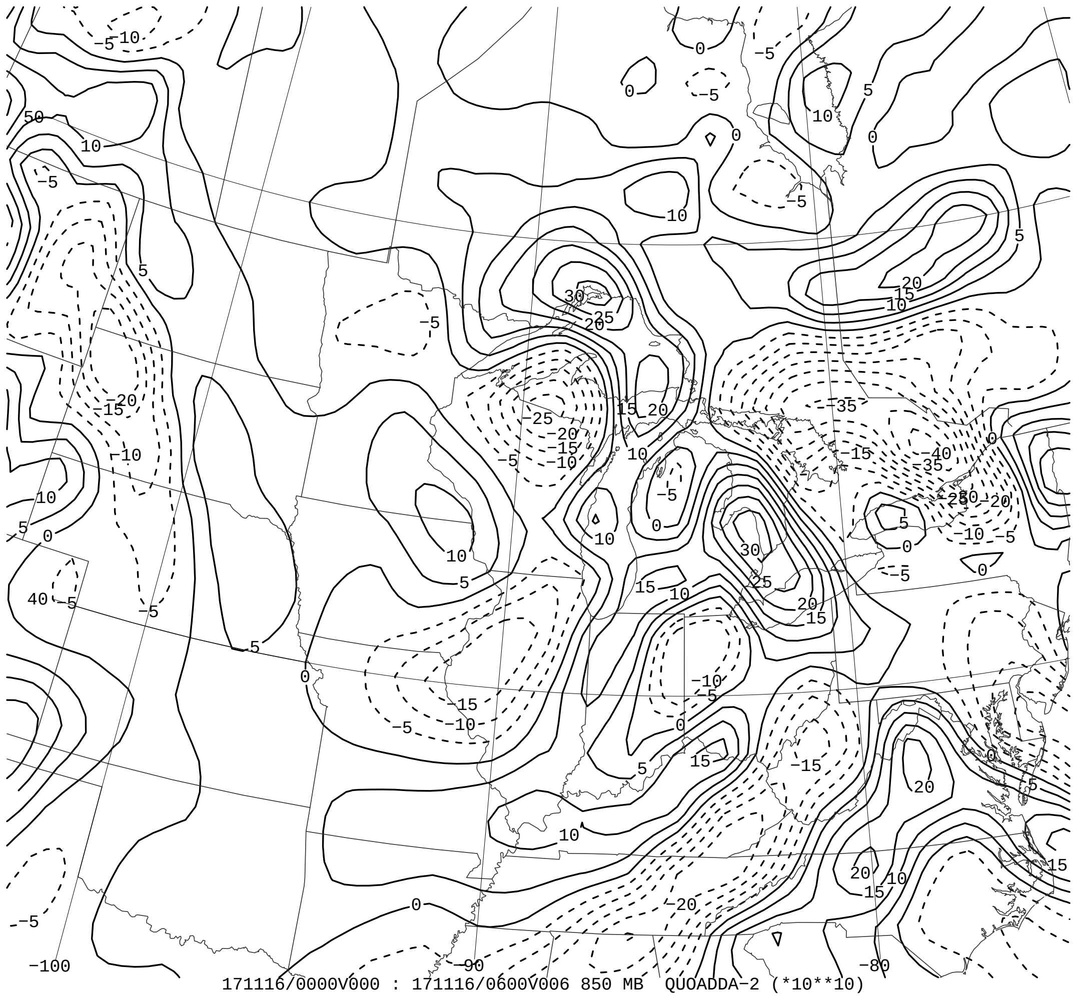

Figure 2: Contour plot of the Horizontal advection of absolute vorticity ($\times 10^{10}\,\mathrm{s}^{-2}$) at 850 hPa over the Great Lakes region between November 16, 2017 0Z and 6Z.

2 Estimation of vertical velocity using isentropic analysis

Another method of estimating vertical velocities is by inference from isentropic analysis. The premise of this method relies on the fact that away from clouds, potential temperature is approximately convserved for air parcels and so their trajectories lie on an isentropic surface, i.e. a surface of constant entropy. We can obtain an estimate of the vertical velocity from the change in pressure level of the surface along such a trajectory as $\omega = dp/dt$.

The reason we chose to look at the Great Lakes region in the previous section was due

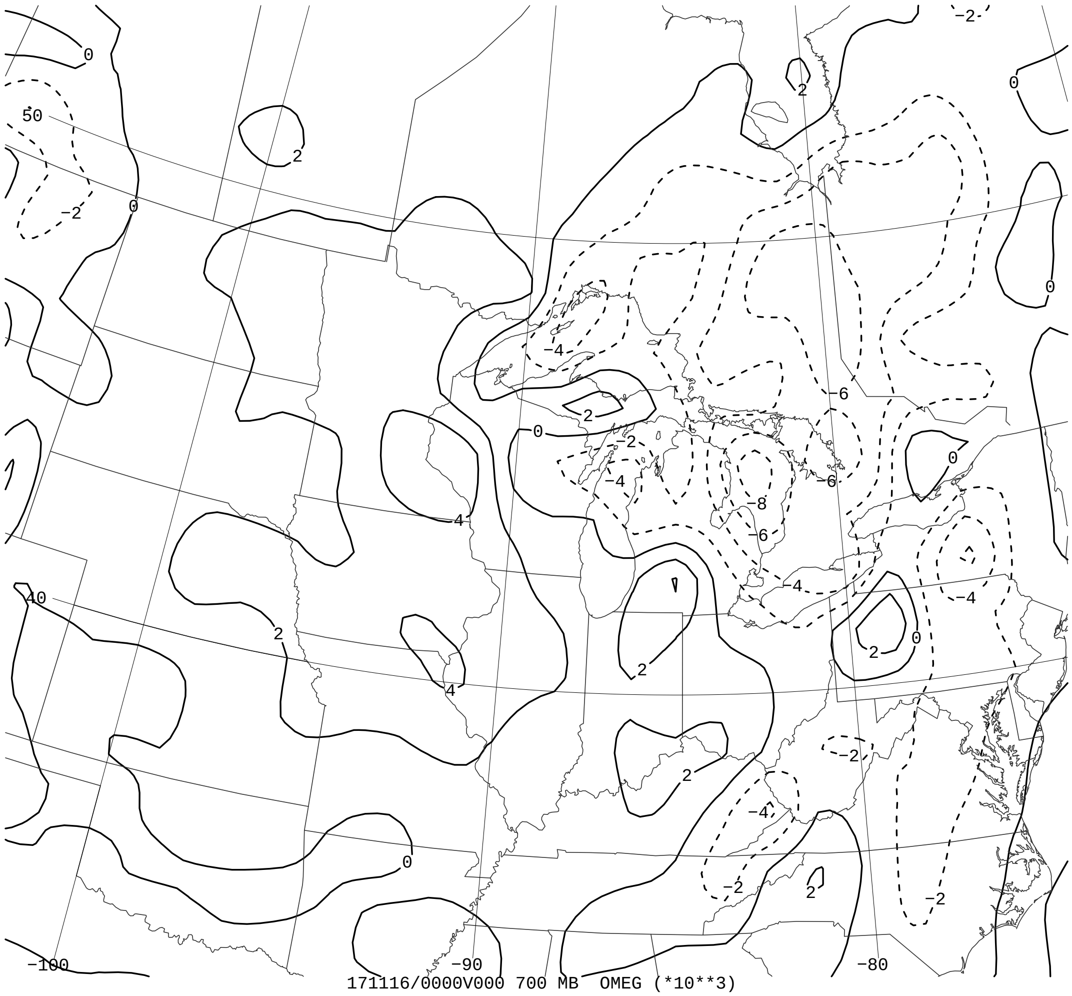

Figure 3: Contour plot of the vertical velocity in pressure coordinates ω ($\times 10^3$ Pa s^{-1}) at 700 hPa over the Great Lakes region between November 16, 2017 0Z and 6Z.

to the presence of a low-pressure system over the Great Lakes region on November 16, 2017 0Z (clearly seen in figures 8 and 9). The zonal cross-sections at 0Z and 6Z (figures 4 and 5) and the meridional cross-sections at 0Z and 6Z (figure 6 and 7) all suggest that the 300 K potential temperature surface would be a suitable one to track air parcel trajectories on as the pressure changes significantly on it, providing us with a clear signal for which we can estimate vertical velocities.

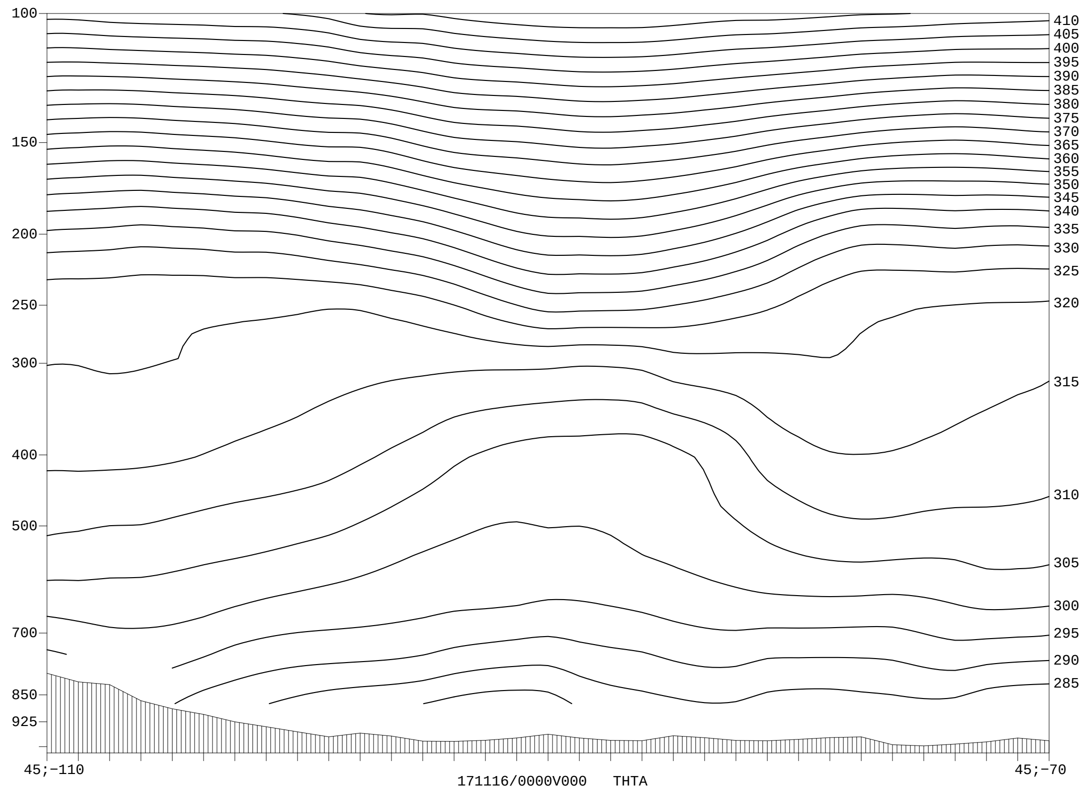

Figure 4: Zonal cross-section of the potential temperature at 45 °N and from 110 °W to 70 °W on November 16, 2017 0Z.

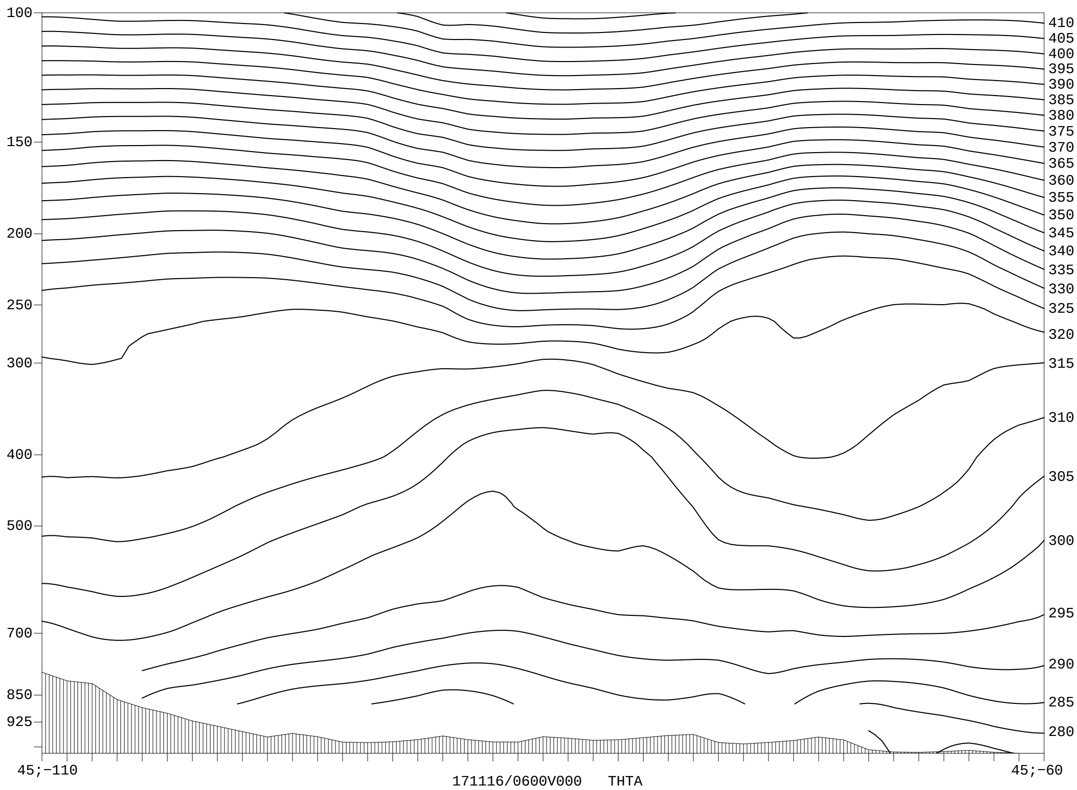

Figure 5: Zonal cross-section of the potential temperature at 45 °N and from 110 °W to 60 °W on November 16, 2017 6Z (6 hours later).

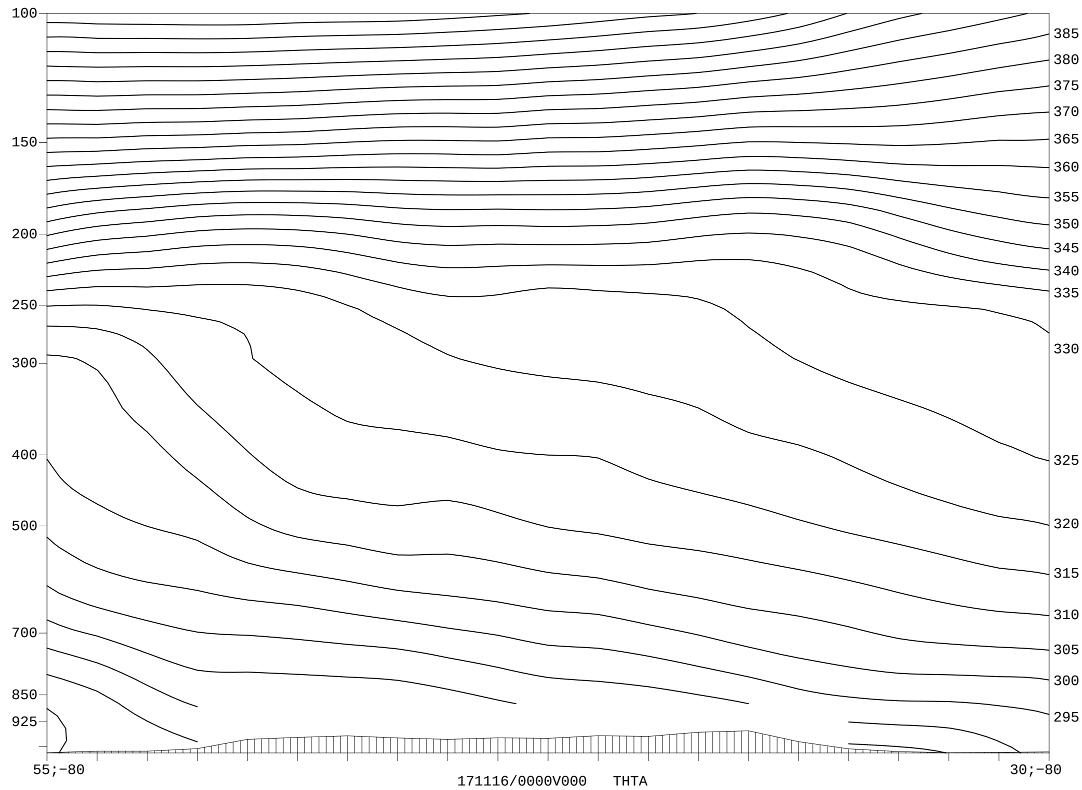

Figure 6: Meridional cross-section of the potential temperature at 80 °W and from 30 °N to 55 °N on November 16, 2017 0Z.

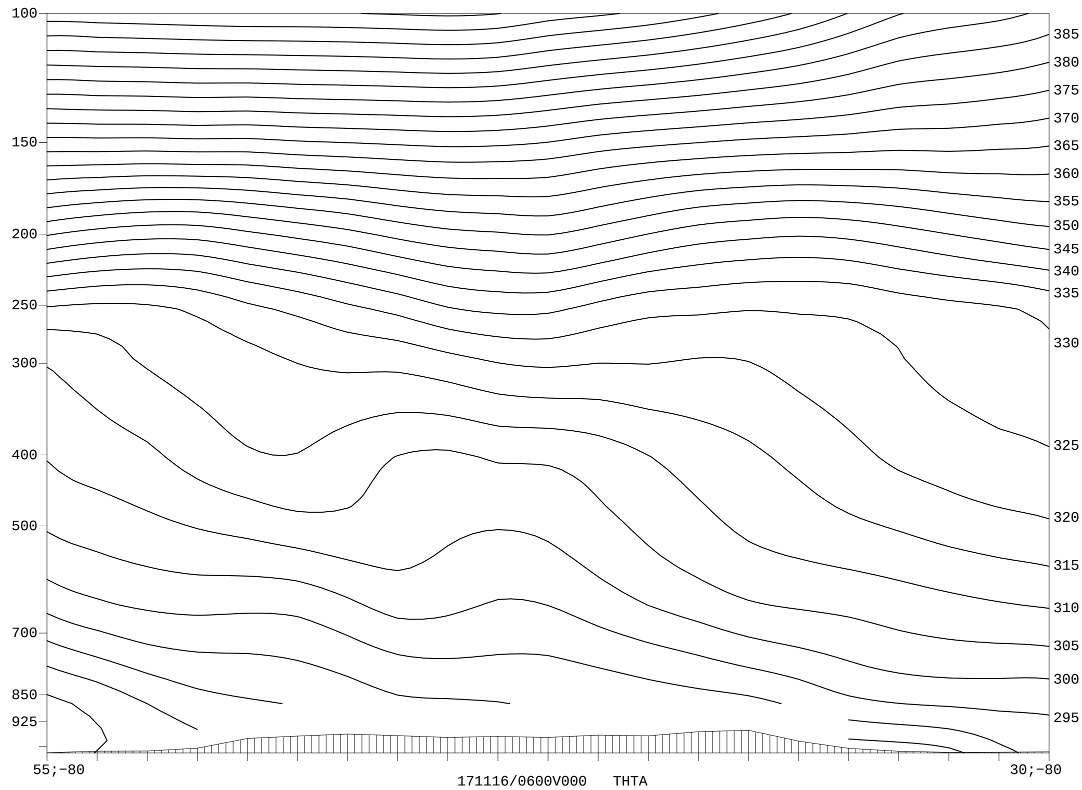

Figure 7: Meridional cross-section of the potential temperature at 80 °W and from 30 °N to 55 °N on November 16, 2017 6Z.

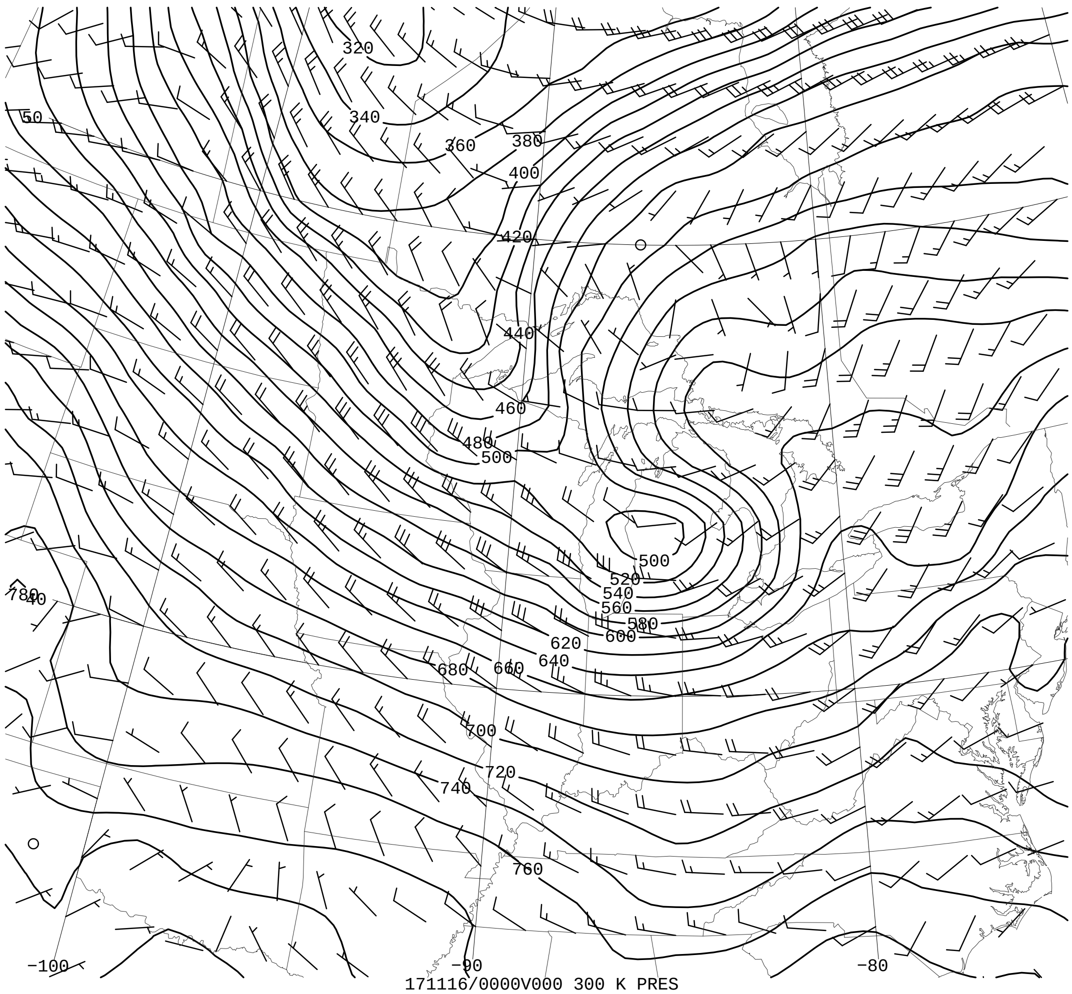

Figure 8: Contour plot of the pressure at the 300 K potential temperature isentropic surface with the observed wind velocity field superimposed, on November 16, 2017 0Z.

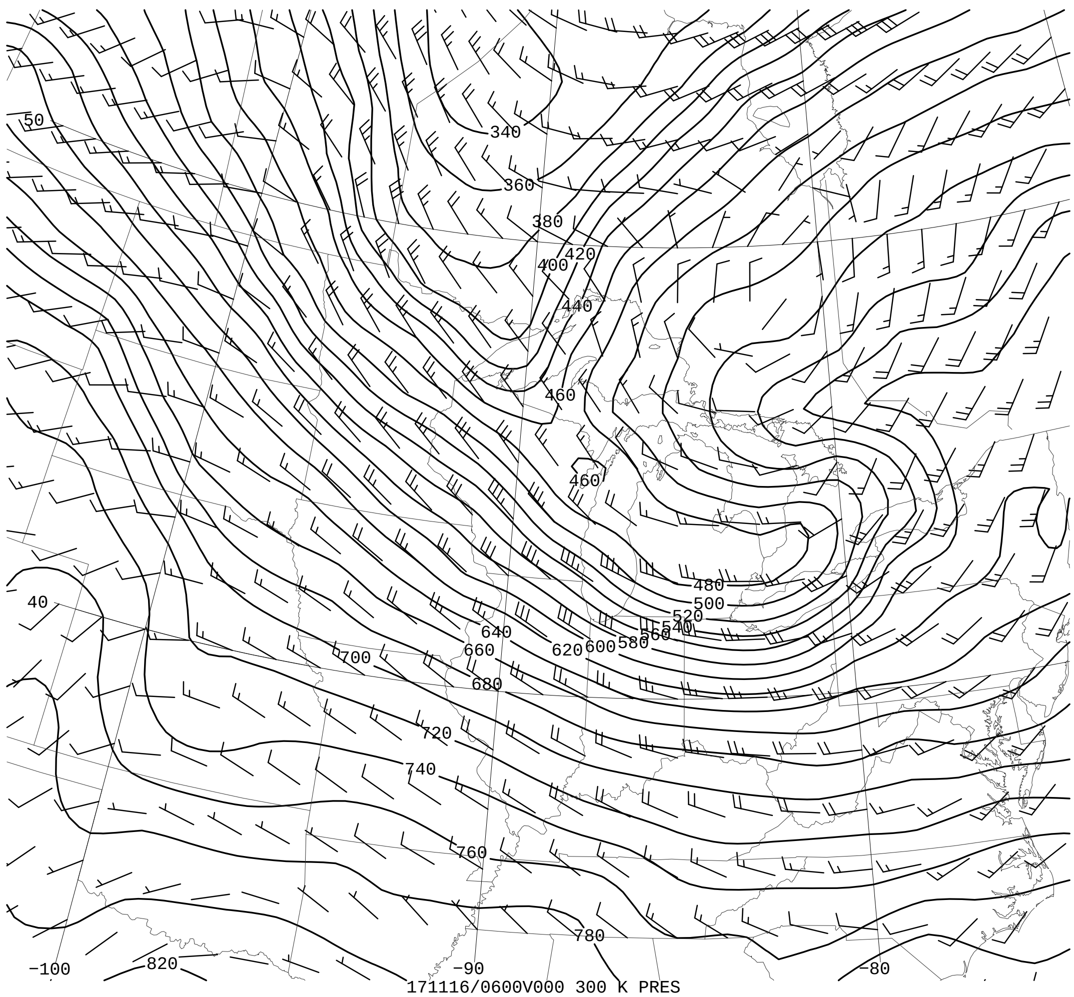

Figure 9: Contour plot of the pressure at the 300 K potential temperature isentropic surface with the observed wind velocity field superimposed, on November 16, 2017 6Z (6 hours later).