Massive Scalar Fields

Thursday, January 2, 2025 10:10 PM

1. $G(x,y) \to$ Greens Function

Klein Gordan operator $(\partial_\mu \partial^\mu - m^2)\,\varphi(x)$

$$\to \quad \checkmark\varphi(x) = \int d^4y \; G(x,y)\, J(y) \quad \leftarrow \text{Given}$$

$$\underline{(\partial_\mu \partial^\mu - m^2)\,\varphi(x) = \boxed{J(x)}} \quad \leftarrow \text{To prove.}$$

L.H.S

$$\underbrace{(\partial_\mu \partial^\mu - m^2)}\int d^4y \; G(x,y)\, J(y)$$

$$\frac{d^2}{dx^2}\int dx\, x^2$$

DO

$$\int d^4y \cdot \underbrace{(\partial_\mu \partial^\mu - m^2)}\, G(x,y)\, J(y) \qquad \int dx \cdot \frac{d^2}{dx^2}\, x^2 .$$

$$\boxed{\int d^4y \; \delta^4(x-y)\, J(y)} \qquad x-y=0 \quad x=y.$$

$$LG(x,y) = \delta(x-y).$$

DO

$$\nabla^2 G(x,y) = \delta(x-y)$$

$$\boxed{\int dx \; \delta(x-3)\,(x^2+4)} \qquad x-3=0 \quad x=3$$

$$\to \quad x^2+4 \to (3)^2+4 = 13$$

$$\boxed{(\partial_t^2 - c^2\nabla^2)}\, G(x,x',t,t') = \delta(x-x')\,\delta(t-t').$$

$$\partial^\mu \to \underbrace{\partial^0, \partial^1, \partial^2, \partial^3}_{\partial^t,\ \partial^x,\ \partial^y,\ \partial^z}$$

$$\underbrace{\int dx\; \delta(x-y)\, f(x)}_{x-y=0,\ x=y} \Rightarrow f(y).$$

$$\int d^4y \; \delta^4(x-y)\, J(y) = J(x)$$

2.

$$L = \frac{1}{2}\overbrace{\partial_\mu\varphi_1 \partial^\mu\varphi_1}^{①} + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + \frac{1}{2}m^2\varphi_1^2 + \frac{1}{2}m^2\varphi_2^2 - (\varphi_1^2+\varphi_2^2)^2$$

(a) EL Equation

$$\partial_\mu\left[\frac{\partial L}{\partial(\partial_\mu\varphi)}\right] = \frac{\partial L}{\partial \varphi}$$

$$\partial_\mu\left[\frac{\partial L}{\partial(\partial_\nu \varphi)}\right] = \frac{\partial L}{\partial \varphi}$$

EL Equation for φ_1

$$\partial_\mu\left[\frac{\partial L}{\partial(\partial_\mu \varphi_1)}\right] = \frac{\partial L}{\partial \varphi_1}$$

$$\frac{\partial L}{\partial \varphi_1} = 0 + 0 + \frac{1}{\cancel{2}} m^2 \cdot \cancel{2}\varphi_1 - 2\varphi_1 + 0$$

$$\frac{\partial L}{\partial \varphi_1} = m^2\varphi_1 - 2\varphi_1$$

$$\partial_\mu\left[\frac{1}{2}\partial^\mu \varphi_1\right] = m^2\varphi_1 - 2\varphi_1$$

$$\frac{1}{2}\partial_\mu\partial^\mu\varphi_1 = m^2\varphi_1 - 2\varphi_1 \Rightarrow \partial_\mu\partial^\mu\varphi_1 = m^2\varphi_1 - 2\varphi_1$$

$$\partial_\mu\partial^\mu\varphi_2 = m^2\varphi_2 - 2\varphi_2.$$

EL-Equation for φ_2.

$$\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu \varphi_2)}\right) = \frac{\partial L}{\partial \varphi_2}$$

$$\partial_\mu\left[\frac{\cancel{1}}{\cancel{2}}\partial^\mu \varphi_2\right] = m^2\varphi_2 - 2\varphi_2$$

$$\frac{1}{2}\partial_\mu\partial^\mu\varphi_2 - m^2\varphi_2 + 2\varphi_2 = 0$$

(b) symmetry. small parameter $\cancel{\omega^2}$

$$\varphi_1 \rightarrow \varphi_1 - \omega\varphi_2 \quad , \quad \varphi_2 \rightarrow \varphi_2 + \omega\varphi_1$$

$$L = \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + \frac{1}{2}m^2\varphi_1^2 + \frac{1}{2}m^2\varphi_2^2 - (\varphi_1^2 + \varphi_2^2).$$

$$L' = \underbrace{\frac{1}{2}\partial_\mu[\varphi_1 - \omega\varphi_2]\,\partial^\mu[\varphi_1 - \omega\varphi_2]}_{} + \overbrace{\frac{1}{2}\partial_\mu[\varphi_2 + \omega\varphi_1]\,\partial^\mu[\varphi_2 + \omega\varphi_1]}^{\text{A}} + \frac{1}{2}m^2[\varphi_1 - \omega\varphi_2]^2$$

$$+ \overbrace{\frac{1}{2}m^2[\varphi_2 + \omega\varphi_1]^2}^{\text{C}} - \overbrace{\left[(\varphi_1 - \omega\varphi_2)^2 + (\varphi_2 + \omega\varphi_1)^2\right]}^{\text{B}}.$$

(A)

$$\ldots \Big] + \frac{1}{2}\Big[(\partial_\mu\varphi_2 + \omega\,\partial_\mu\varphi_1)(\partial^\mu\varphi_2 + \omega\partial^\mu\varphi_1)\Big].$$

(A)

$$\frac{1}{2}\left[(\partial_\mu\varphi_1-\omega\partial_\mu\varphi_2)(\partial^\mu\varphi_1-\omega\partial^\mu\varphi_2)\right]+\frac{1}{2}\left[(\partial_\mu\varphi_2+\omega\,\partial_\mu\varphi_1)(\partial^\mu\varphi_2+\omega\partial^\mu\varphi_1)\right].$$

$$\frac{1}{2}\left[\partial_\mu\varphi_1\partial^\mu\varphi_1-\underbrace{\omega\partial_\mu\varphi_1\,\partial^\mu\varphi_2-\omega\partial_\mu\varphi_2\partial^\mu\varphi_1}+\omega^2(\partial_\mu\varphi_2)^2\right]$$

$$+\frac{1}{2}\left[\partial_\mu\varphi_2\partial^\mu\varphi_2+\omega\partial_\mu\varphi_1\partial^\mu\varphi_2+\omega\,\partial_\mu\varphi_2\partial^\mu\varphi_1+\omega^2(\partial_\mu\varphi_1)^2\right]$$

$$\frac{1}{2}\left[\partial_\mu\varphi_1\,\partial^\mu\varphi_1-2\omega\,\partial_\mu\varphi_1\partial^\mu\varphi_2\right]+\frac{1}{2}\left[\partial_\mu\varphi_2\partial^\mu\varphi_2+2\omega\,\partial_\mu\varphi_1\partial^\mu\varphi_2\right].$$

$$\frac{1}{2}\left[\partial_\mu\varphi_1\,\partial^\mu\varphi_1\right]+\frac{1}{2}\left[\partial_\mu\varphi_2\,\partial^\mu\varphi_2\right] \quad \text{(A)}$$

(B)

$$\frac{1}{2}m^2(\varphi_1-\omega\varphi_2)^2+\frac{1}{2}m^2(\varphi_2+\omega\varphi_1)^2$$

$$\frac{1}{2}m^2\left[\varphi_1^2+\cancel{(\omega\varphi_2)^2}-2(\varphi_1)(\omega\varphi_2)\right]+\frac{1}{2}m^2\left[\varphi_2^2+\cancel{(\omega\varphi_1)^2}+2(\omega\varphi_1)(\varphi_2)\right].$$

$$\frac{1}{2}m^2\left[\varphi_1^2-2\omega\,\varphi_1\varphi_2\right]+\frac{1}{2}m^2\left[\varphi_2^2+2\omega\varphi_1\varphi_2\right].$$

$$\frac{1}{2}m^2\varphi_1^2-\cancel{m^2\omega\varphi_1\varphi_2}+\frac{1}{2}m^2\varphi_2^2+\cancel{m^2\omega\varphi_1\varphi_2}$$

$$\frac{1}{2}m^2\varphi_1^2+\frac{1}{2}m^2\varphi_2^2 \quad \text{(B)}$$

(C)

$$(\varphi_1-\omega\varphi_2)^2+(\varphi_2+\omega\varphi_1)^2$$

$$\left[\varphi_1^2-(\omega\varphi_2)^2-2(\varphi_1)(\omega\varphi_2)\right]+\left[\varphi_2^2+(\omega\varphi_1)^2+2(\varphi_2)(\omega\varphi_1)\right].$$

$$\varphi_1^2+\varphi_2^2 \quad \text{(C)}$$

$$\varphi_1^2 - 2\omega\cancel{\varphi_1\varphi_2} + \varphi_2^2 + 2\omega\cancel{\varphi_1\varphi_2} = \varphi_1^2 + \varphi_2^2$$

$$\mathcal{L}' = \frac{1}{2}\partial^\mu\varphi_1\partial_\mu\varphi_1 + \frac{1}{2}\partial^\mu\varphi_2\partial_\mu\varphi_2 + \frac{1}{2}m^2\varphi_1^2 + \frac{1}{2}m^2\varphi_2^2 - (\varphi_1^2 + \varphi_2^2)$$

$$\mathcal{L}' = \mathcal{L} \qquad \left.\begin{aligned} \varphi_1 &\to \varphi_1 - \omega\varphi_2 \\ \varphi_2 &\to \varphi_2 + \omega\varphi_1 \end{aligned}\right\} \text{Symetries.}$$

(c)

Noether's Current.

$$J^\mu = \sum_{n=1}^{2} \frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi_n)}\delta\varphi_n$$

$\delta\varphi_1 = ?$

$\varphi_1 \to \varphi_1 - \cancel{\omega\varphi_2}$

$\varphi_2 \to \varphi_2 + \cancel{\omega\varphi_1}$

$\delta\varphi_2 = \omega\varphi_1$

$\delta\varphi_1 = -\omega\varphi_2$

$$J^\mu = \boxed{\frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi_1)}}\delta\varphi_1 + \boxed{\frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi_2)}}\delta\varphi_2.$$

$\partial^\mu\varphi_1 \qquad \partial^\mu\varphi_2$

$$J^\mu = \partial^\mu\varphi_1 \cdot (-\omega\varphi_2) + \partial^\mu\varphi_2 \cdot (\omega\varphi_1)$$

$$J^\mu = -\omega\varphi_2\partial^\mu\varphi_1 + \omega\varphi_1\partial^\mu\varphi_2$$

$$J^\mu = \omega\left[\varphi_1\partial^\mu\varphi_2 - \varphi_2\partial^\mu\varphi_1\right] \to \text{constant}$$

$$\frac{d}{dx}(2) = 0$$

(d) $\partial_\mu J^\mu = 0$

$$\partial_\mu J^\mu = \omega\left[\partial_\mu\left(\underset{①}{\varphi_1}\,\underset{②}{\partial^\mu\varphi_2}\right) - \partial_\mu\left(\underset{①}{\varphi_2}\,\underset{②}{\partial^\mu\varphi_1}\right)\right].$$

$$\partial_\mu J^\mu = \omega\left[\varphi_1 \partial_\mu \partial^\mu \varphi_2 + \cancel{\partial^\mu \varphi_2 \partial_\mu \varphi_1} - \varphi_2 \partial_\mu \partial^\mu \varphi_1 - \cancel{\partial^\mu \varphi_1 \partial_\mu \varphi_2}\right].$$

$$\partial_\mu J^\mu = \omega\left[\varphi_1 \underbrace{\partial_\mu \partial^\mu \varphi_2}_{m^2\varphi_2 - 2\varphi_2} - \varphi_2 \underbrace{\partial_\mu \partial^\mu \varphi_1}_{m^2\varphi_1 - 2\varphi_1}\right].$$

$$\partial_\mu J^\mu = \omega\left[\varphi_1 (m^2\varphi_2 - 2\varphi_2) - \varphi_2 (m^2\varphi_1 - 2\varphi_1)\right].$$

$$= \omega\left[\cancel{m^2\varphi_1\varphi_2} - \cancel{2\varphi_1\varphi_2} - \cancel{m^2\varphi_1\varphi_2} + \cancel{2\varphi_1\varphi_2}\right]$$

$\partial_\mu J^\mu = 0 \Rightarrow J^\mu$ is constant/conserved.

(e) Conserved Charge.

$$\boxed{Q = \int dx^3 J^0(x,t)}$$

$$J^\mu = \omega\left[\varphi_1 \partial_\mu \varphi_2 - \varphi_2 \partial_\mu \varphi_1\right].$$

$\mu = 0, 1, 2, 3$

$t \quad x \quad y \quad z$

$$\frac{d}{dt} x = \dot{x}$$

$$Q = \omega \int dx^3 \left[\varphi_1 \partial_0 \varphi_2 - \varphi_2 \partial_0 \varphi_1\right].$$

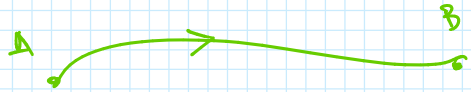
$$\boxed{Q = \omega \int dx^3 \left[\varphi_1 \dot{\varphi}_2 - \varphi_2 \dot{\varphi}_1\right].} \rightarrow \text{Conserved charge.}$$

Physics.

Classical Mechanics

$\downarrow$

Newton :-



$F = ma$

$F = m\frac{dv}{dt}$

$F = m\frac{d}{dt}\left[\frac{dx}{dt}\right]$

$$\boxed{F = m\frac{d^2x}{dt^2}}$$

$x(t)$

$\left.\begin{matrix} x \\ t \end{matrix}\right\}$ $\left.\begin{matrix} \text{space} \\ \text{time} \end{matrix}\right\} \rightarrow$ constant

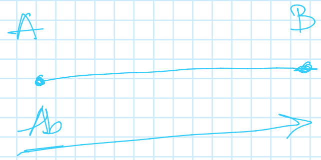
= 10 s

4m

Modern Physics

Einstein :-

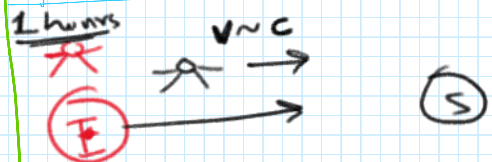
$\left.\begin{matrix} \text{time} \\ \text{space} \end{matrix}\right\} \rightarrow$ variables



10 s

2.9999×10^8

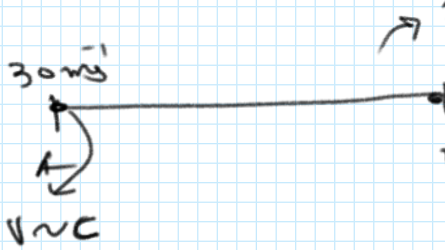
$$\boxed{c = 3.0 \times 10^8 \, ms^{-1}}$$



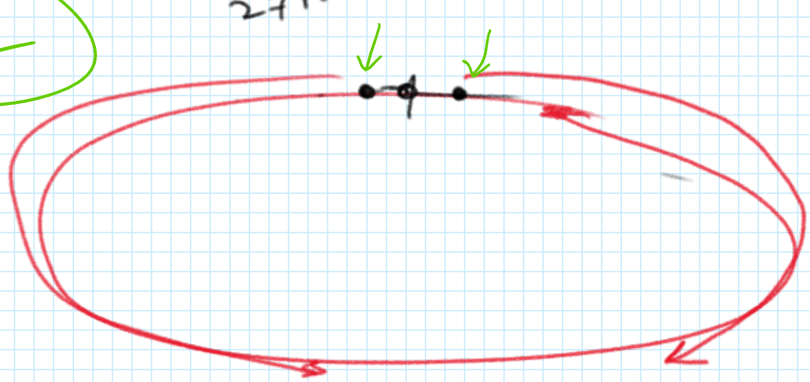
23 years

Length Contraction

1 km ~ 100m

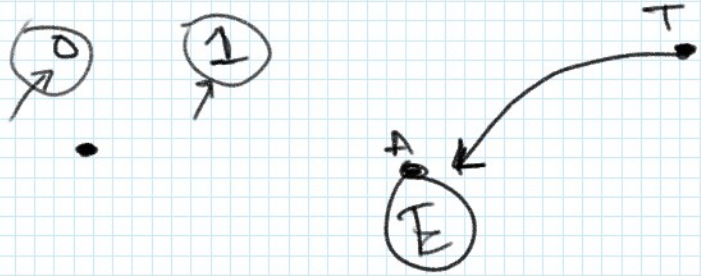


27 km



Quantum Mechanics

Quantum Mechanics :-

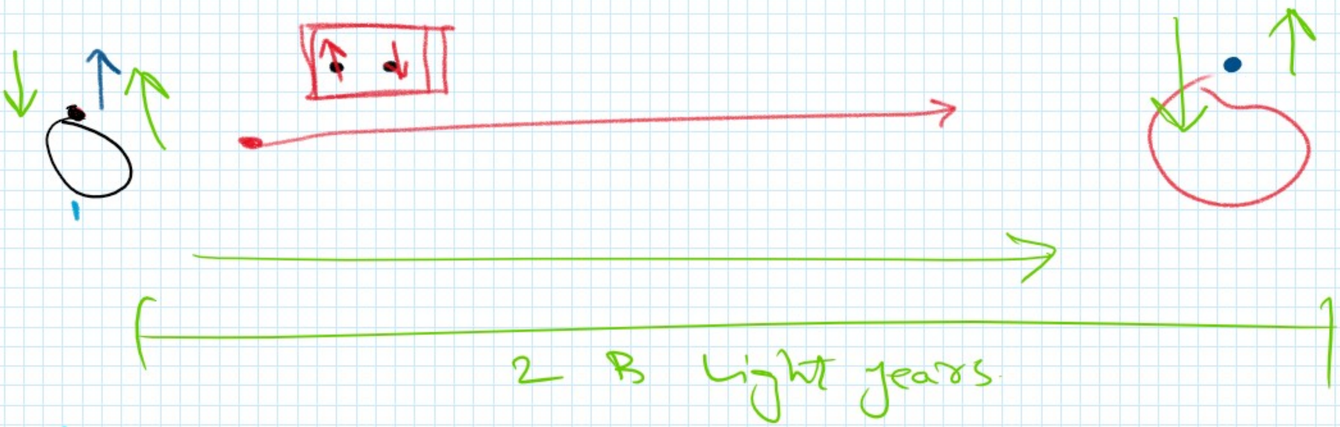


1) Superposition:

$0 \leftrightarrow 1$

Measurement :-

Entanglement :-



Needs of Quantum:

CM $\longrightarrow$ Modern Physics

↳ for $v \approx c$

✓ $v \ll c$

$1 - \frac{v^2}{c^2}$ → less, large

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

v^2 → object

c^2 → Speed of light

$\gamma = 1$

$3.0 \times 10^8 \ ms^{-1}$

$$t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\, t_{actual} \qquad t = 5\,s.$$

$v << c$

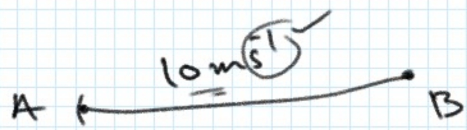
$$\boxed{t = t_{actual}}$$

$\boxed{c \rightarrow \text{speed of light}}$ $3.0 \times 10^{8}\ m s^{-1}$

Similarly, in QM,

$\boxed{h \Rightarrow \text{plank's constant} = 6.62 \times 10^{-34}\ J s}$

$10\,m$

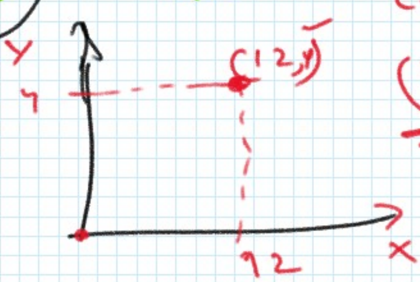


Classical Mechanics :-

$x \rightarrow$ position $\qquad x,\ \frac{dx}{dt} \Rightarrow \frac{d^2x}{dt^2}$ $\quad a$

certain

$(12, 4)$

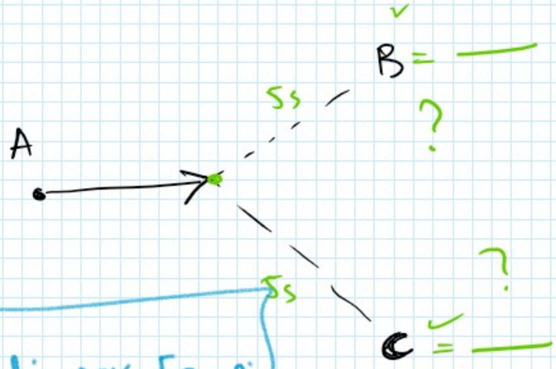


$$F = ma$$

x

$F = ma \rightarrow$ evolution of a particle:

$$\boxed{F = m\frac{d^2x}{dt^2}}$$



particle

Schrodinger Equation $\rightarrow$ DE

Schrodinger Equation

ve, +ve, comple

Schrödinger Equation → DE

Maxwell- Equations

↳ light

wavefunction $\Psi(r)$ → $-ve$, $\cancel{+ve}$, complex

$\Psi(r,t)$

$\Psi(r) \times \Psi^*(r)$

$(1+i) \times (1-i)$

$1^2 - i^2 = 1+1 = 2$

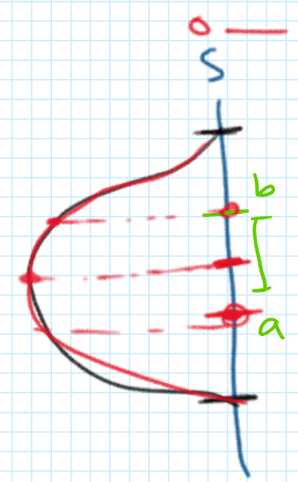
$(-4) \times (-4) = 16$

$\rho \to$ P.P.

$$\boxed{\rho = \Psi \times \Psi^*}$$

$$\int_a^b \rho \, dx .$$

SE



- Choose Equation
- Ψ
- $\rho = \Psi * \Psi^*$
- prob. $= \int_a^b \rho \, dx$

→ set of Rules.

Postulates of Quantum Mechanics :-

1) full state of system : $\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix} \to |\Psi\rangle$

ket/ ket vector.

$|\Psi\rangle$

$|\psi\rangle$ ket / ket vector.

2) Observables & operators

Observable $\longrightarrow$ operator.

Eigenvalue $\leftarrow$ $\boxed{\hat{O}|\psi\rangle = \lambda|\psi\rangle}$

momentum operator

3) $\lambda \rightarrow$ Measurement

$$\hat{A}|\psi\rangle = \lambda|\psi\rangle$$

momentum

4) SE

3D $\rightarrow$ Time Dependent.

$$\boxed{i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi}$$

$$\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$$

$$\hbar = \frac{h}{2\pi}$$

$$i\hbar\,\partial\vec{\Psi} = -\hbar^2\, d^2\Psi + V\Psi$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi$$

time

mass.

P.E

$\nabla^2 \to$ Laplacian op.

$$\boxed{\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}}$$