MAIA

Computer Aided Diagnosis (CAD)

Deformable Models Segmentation

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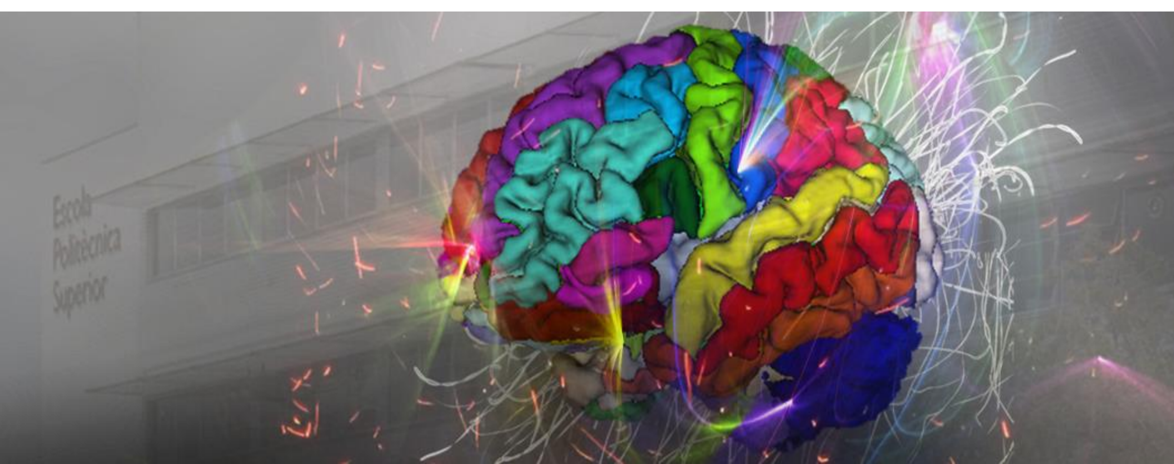


- Part 1: Template matching & deformable template matching

- Part 2: Active shape & Active appearance models

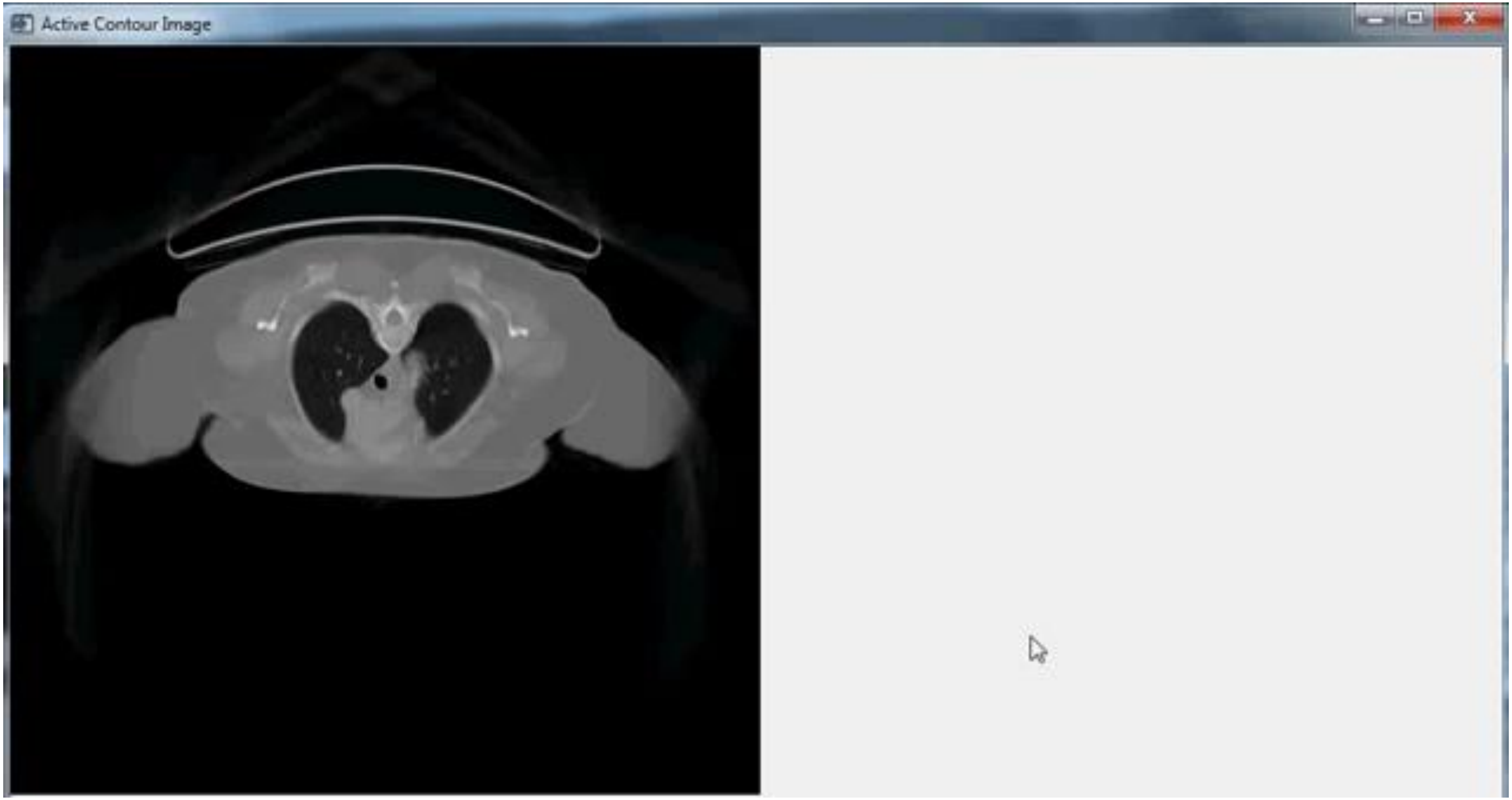
- Part 3: Active contours & level sets

Active contours

- Active contours (or snakes) segmentation.
- No prior information of the object to be segmented (although it can be added in posterior improvements).
- An initial boundary represented by points is iteratively modified by applying shrink/expansion operations.

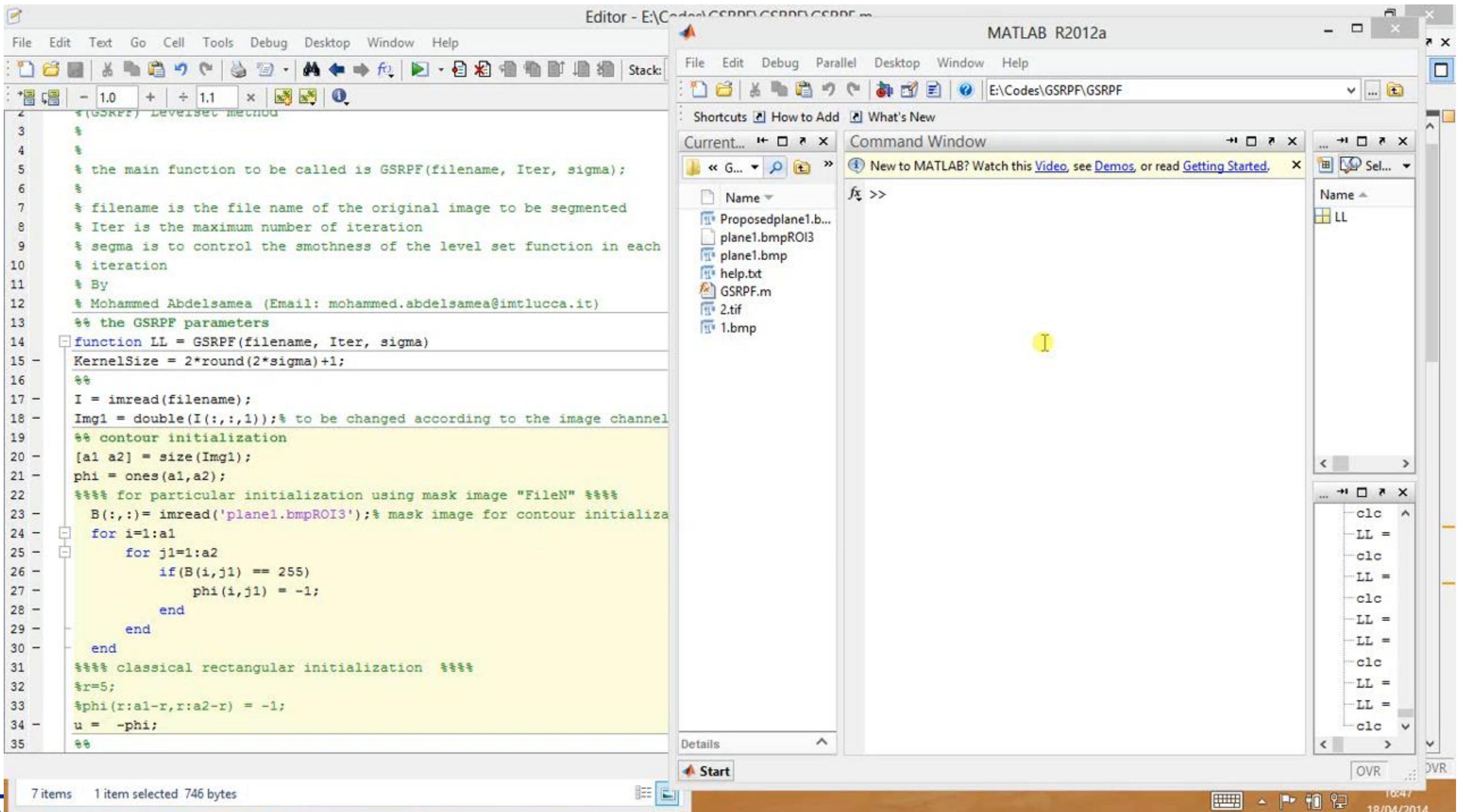
Active contours

- Active contours (or snakes) segmentation.



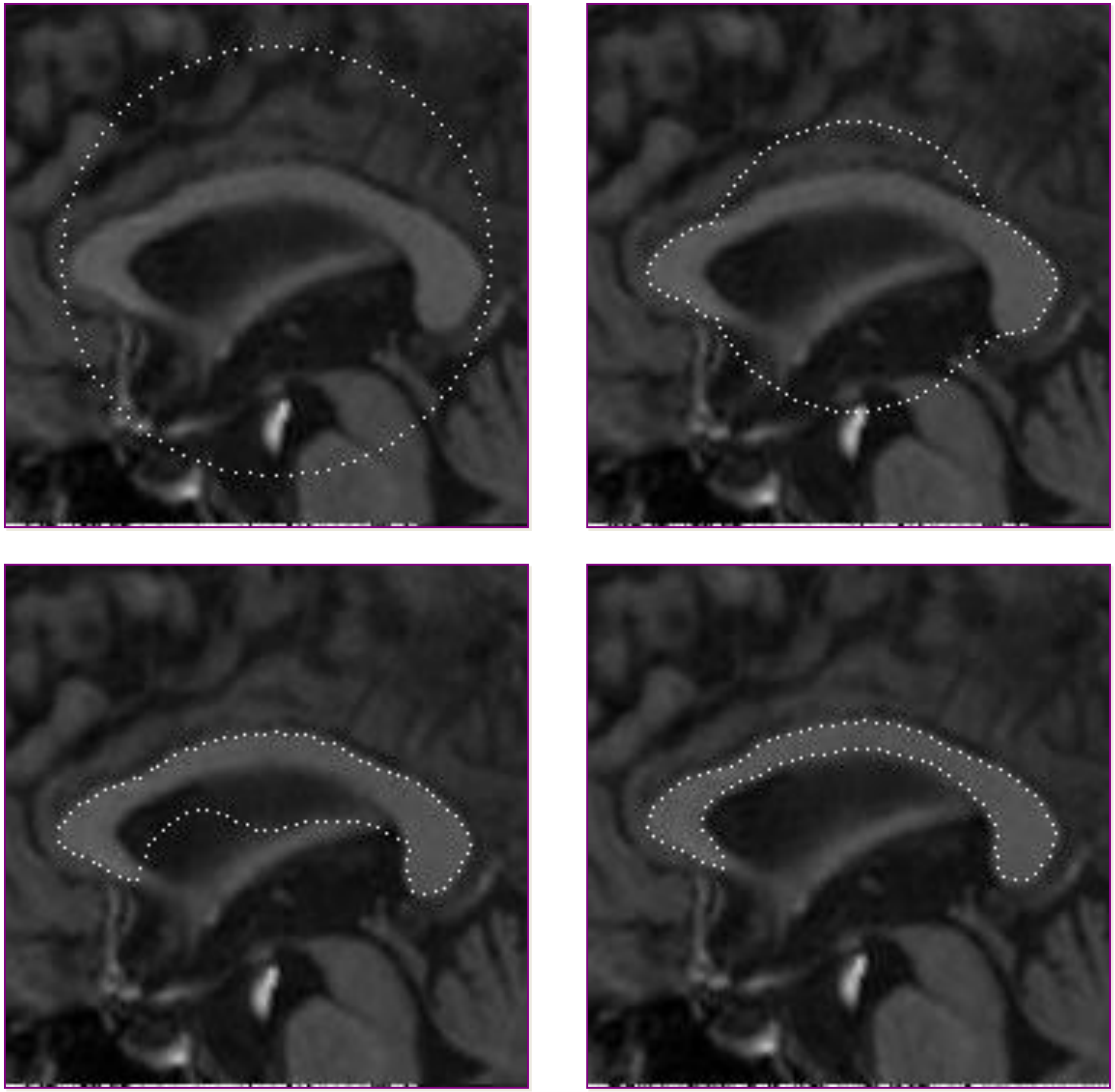
Active contours

- Active contours (or snakes) segmentation. Actually, a level set.

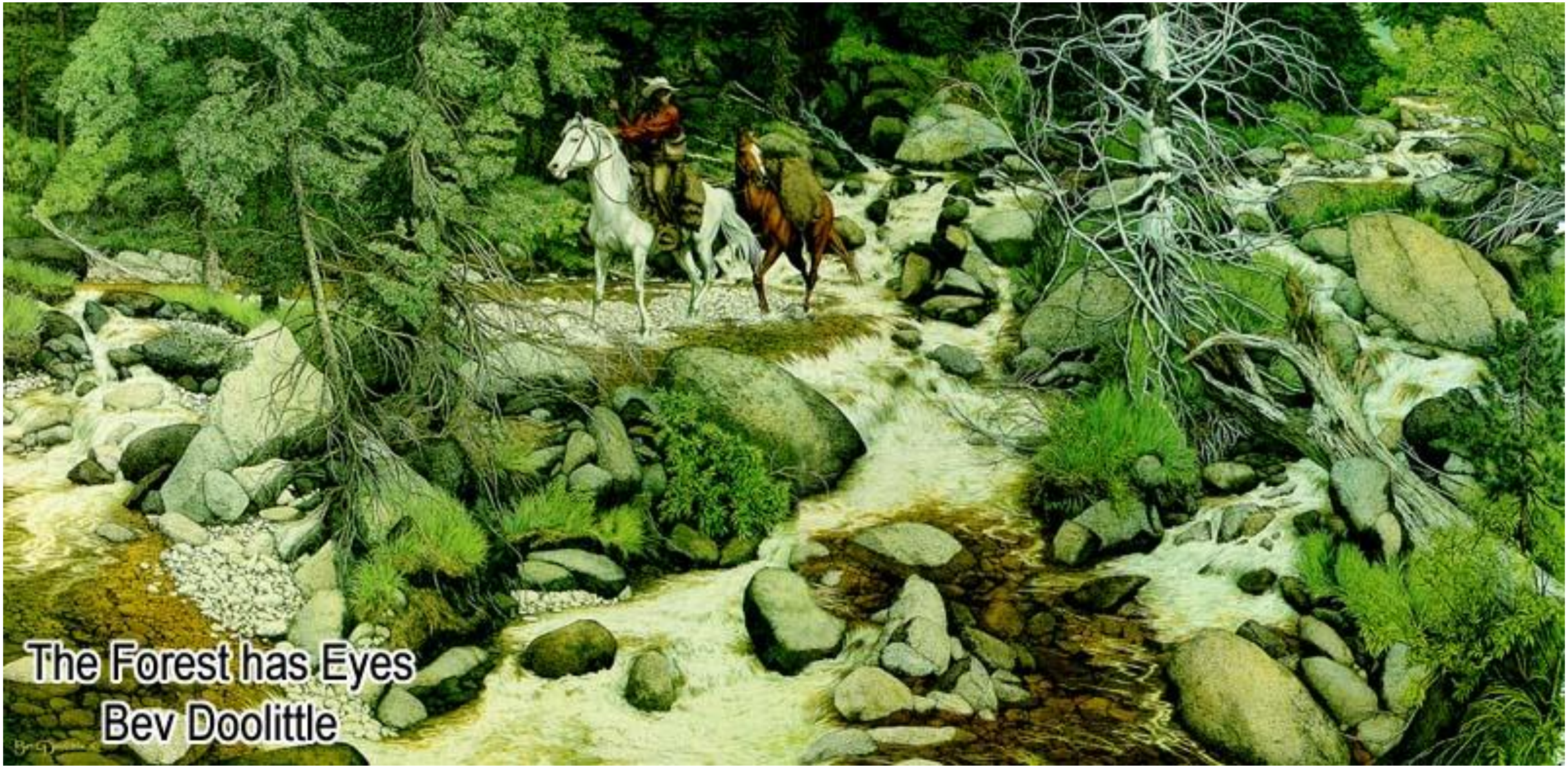


Active contours

- Segmentation of the corpus callosum (brain MRI)



Active contours



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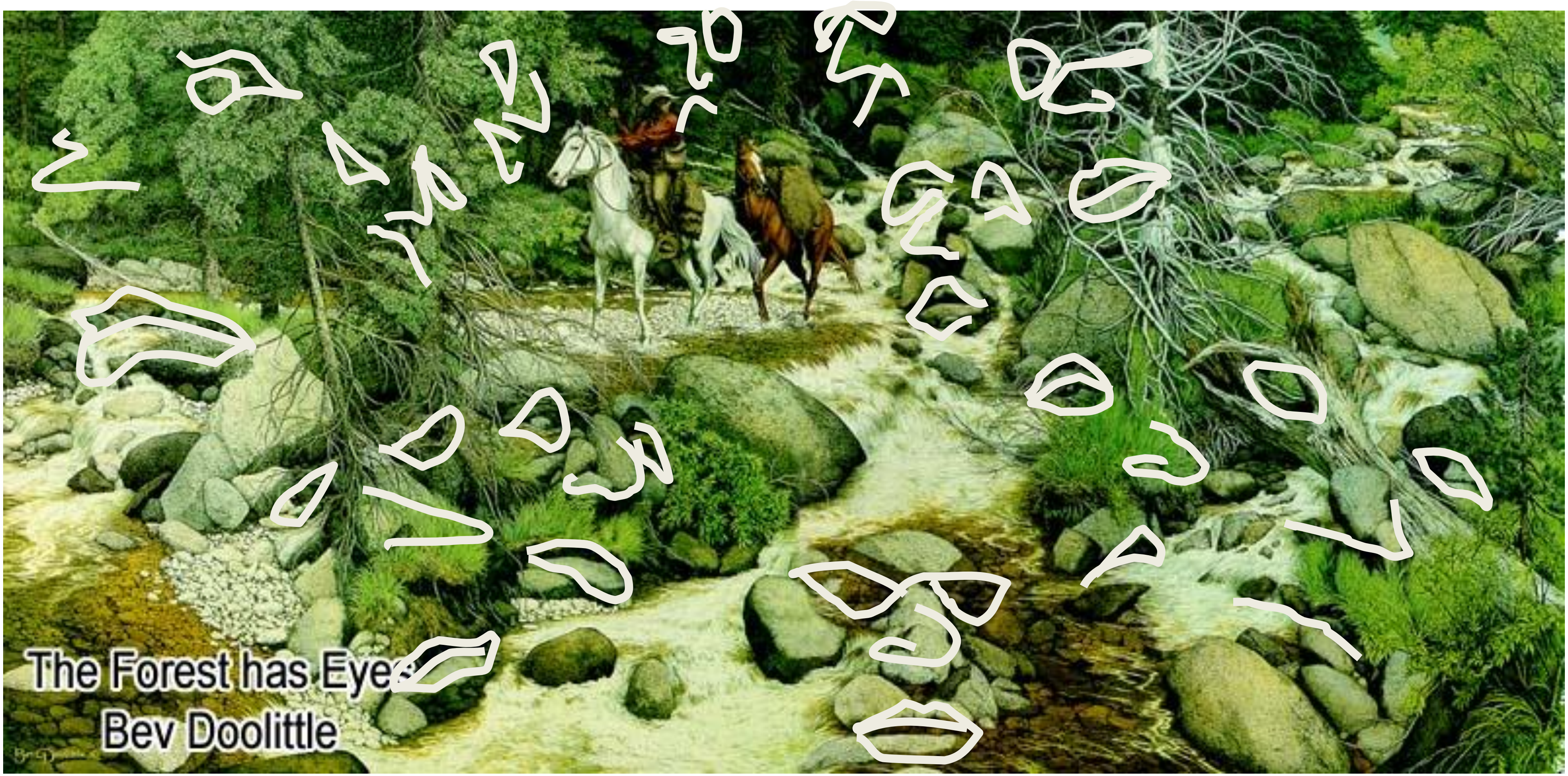
Active contours



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Active contours

- It is an approach based on “energies”

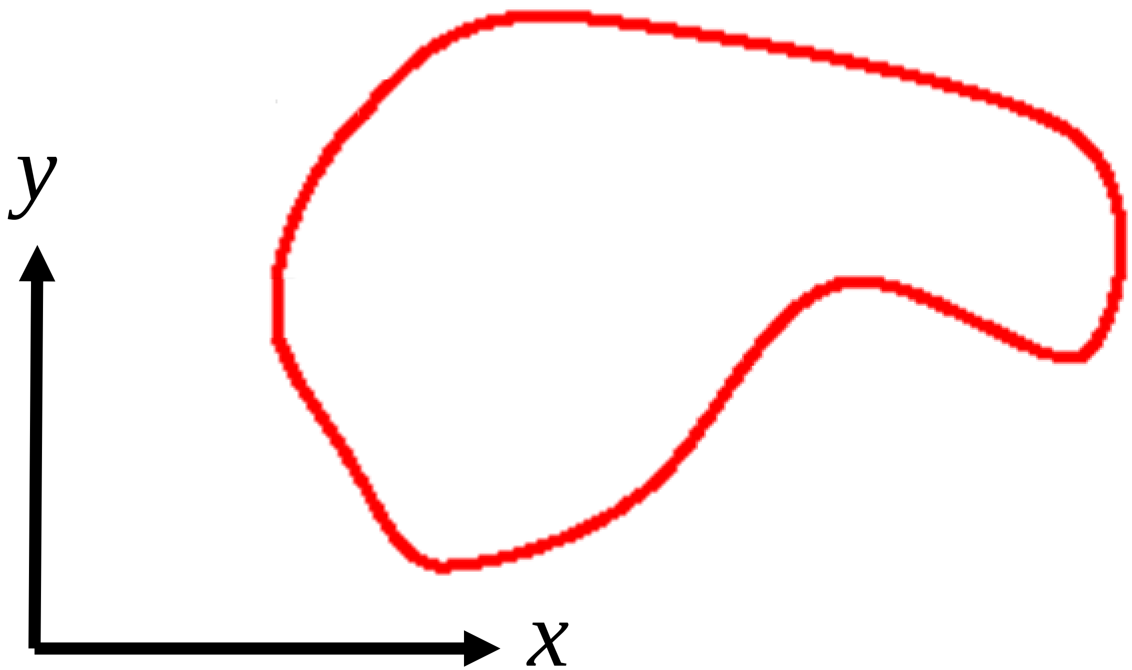
- The objective is to find a parameterised curve that minimises the weighted sum of internal energy and external energy.

  - Internal forces: hold the curve together (elasticity forces) and keep it from bending too much (bending forces).
  - External energy (potential energy) attract the curve towards the desired object boundaries.

- Parametric curves are initialised within the image domain and forced to move toward the energy minima.

Active contours: curves

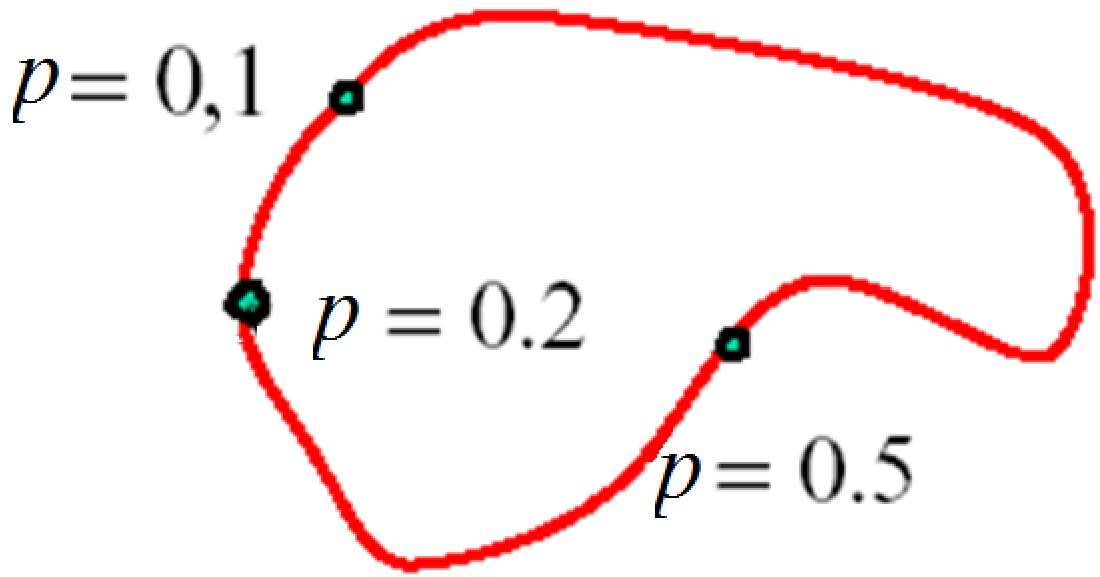
- Curves are a generalisation of functions (functions have a single *y* for each *x*).



Active contours: curves

- Curves. Parametric representation:

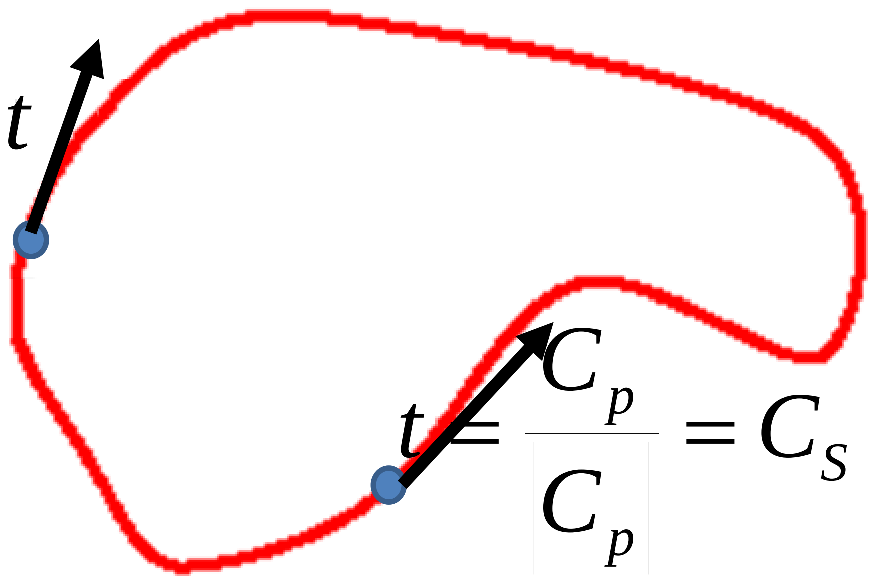
$$C(p) = (x(p), y(p)) \qquad 0 \le s \le 1$$



- Closed contour: $C(0) = C(1)$

Active contours: curves

- The tangent is the derivative of the curve in that point.
- We normalise it to unit length.



$$t = \frac{C_p}{\left|C_p\right|} = C_S$$

$$C_p = \frac{\partial C}{\partial p} = \left(\frac{\partial x}{\partial p}, \frac{\partial y}{\partial p} \right) = \left(x_p, y_p \right)$$

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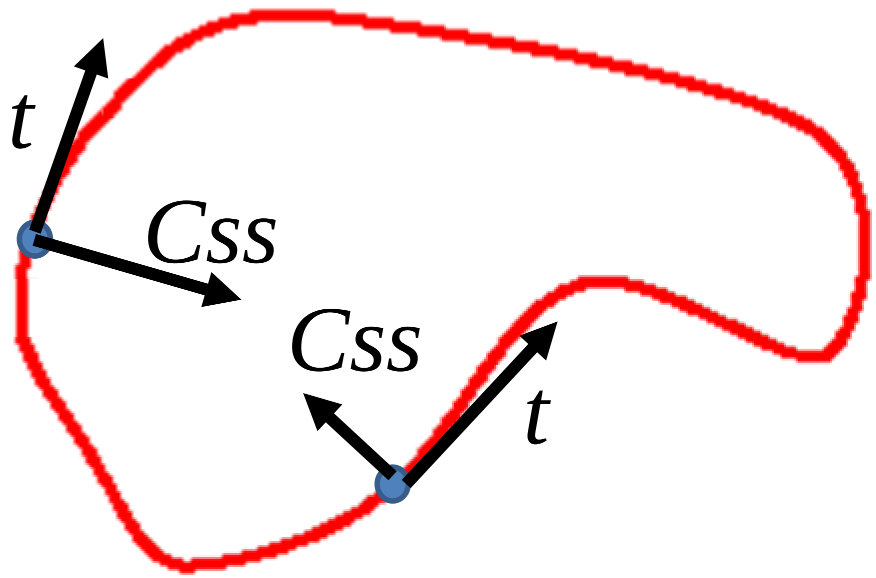
Active contours: curves

- The normal is orthogonal to the tangent.
- The derivative of the tangent is orthogonal.



$$\left|C_s\right| = 1$$

$$\langle C_s, C_s \rangle = 1$$

$$\frac{\partial}{\partial s}\langle C_s, C_s \rangle = \frac{\partial}{\partial s} 1$$

$$2\langle C_s, C_{ss} \rangle = 0$$

$$C_s \perp C_{ss}$$

Active contours: curves

- The curvature relates the derivative of the tangent with the normal:

$$C_{SS} = \kappa N$$

- The curvature of a straight line is 0 (tangent do not modify).

- The curvature of a circle is constant (actually, for a circle of radius r , $\kappa = 1/r$

- As sharper the boundary, larger the curvature

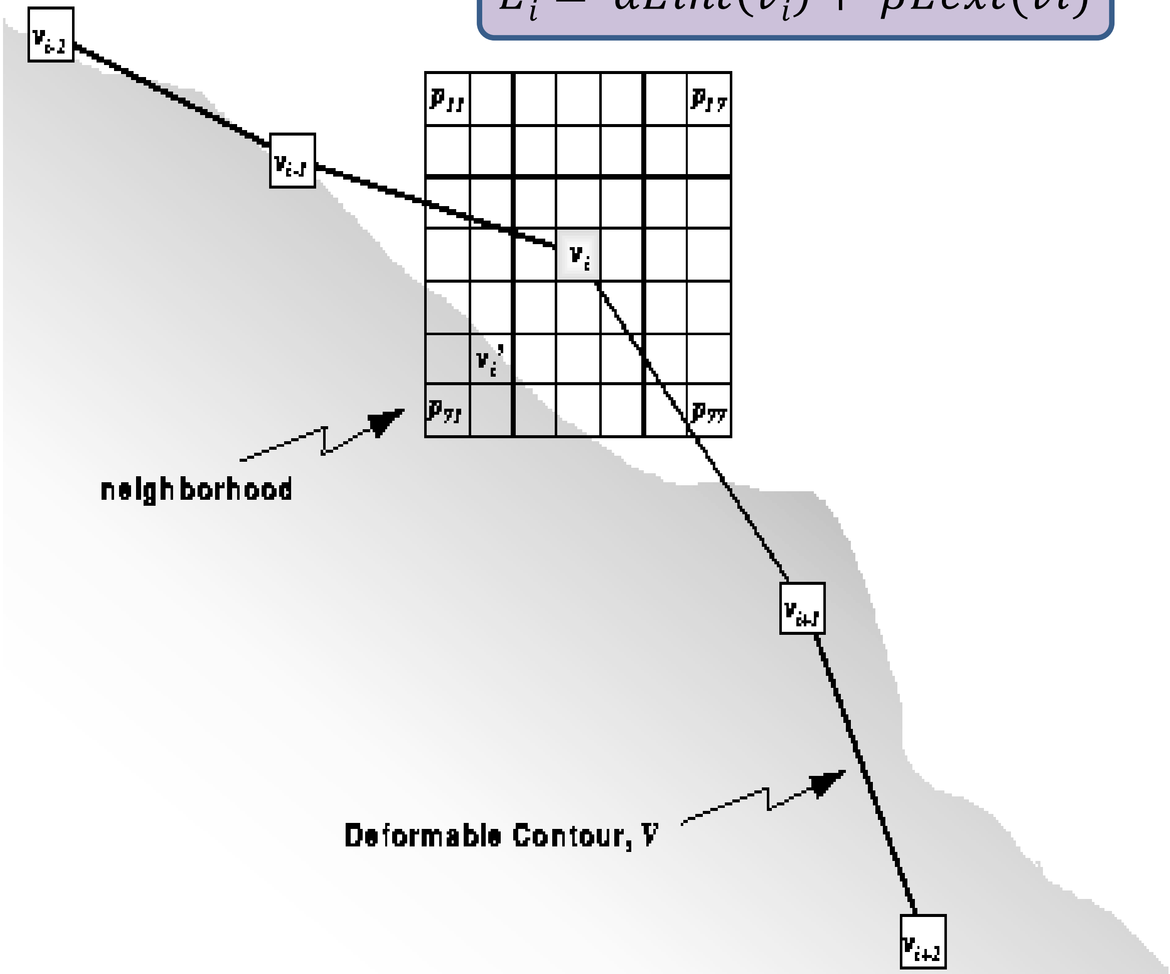
Active contours

- Active contour: compute the energy at each point of the curve

Each point v_i is moved to the point v_i' corresponding to the location of the minimum value in E_i.

If the energy functions are chosen correctly, the contour V should approach and stop at the object boundary.

$$E_i = \alpha Eint(v_i) + \beta Eext(vi)$$



Active contours: the energies

- Original formulation (Kass, Witkin, Terzopoulos, 1987):

$$E[(C)(p)] = \alpha \int_0^1 E_{int}(C(p))dp \; + \; \beta \int_0^1 E_{img}(C(p))dp \left[+ \; \gamma \int_0^1 E_{con}(C(p))dp \right]$$

- Internal forces: hold the curve together (elasticity forces) and keep it from bending too much (bending forces).
- External energy (potential energy) attract the curve towards the desired object boundaries.
- The last term is optional, and can be used to account for user-defined constraints, or prior knowledge on the structure to be recovered

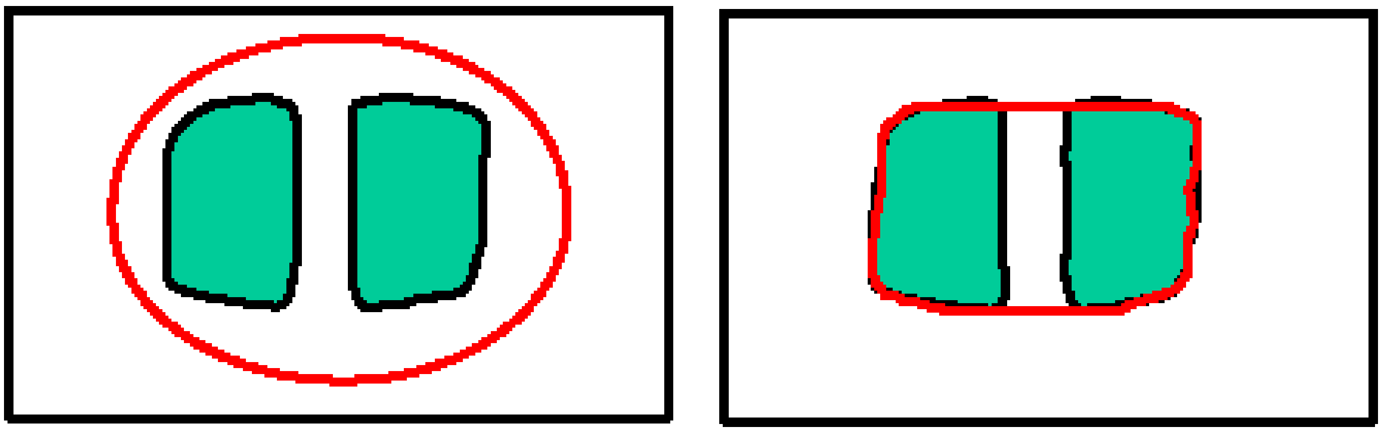
Active contours: internal energy

- In principle, we want to favor smooth shapes, contours with low curvature, contours similar to a known shape, etc. to balance what is actually observed (i.e., in the gradient image).



Active contours: internal energy

- There are many different approaches according the ways the energies are defined.
- Internal energies define the “bending” energy:



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Active contours: internal energy

- Internal energies usually have two terms:

$$E_{internal}(v(s)) = \alpha \boxed{\left|\frac{dv}{ds}\right|^2} + \beta \boxed{\left|\frac{d^2v}{d^2s}\right|^2}$$

Elasticity, Tension

Stiffness Curvature

The more the curve bends → the larger this energy value is.

The weights α and β dictate how much influence each component has.

- Internal energy for whole curve :

$$E_{internal} = \int_0^1 E_{internal}(v(s))\,ds$$

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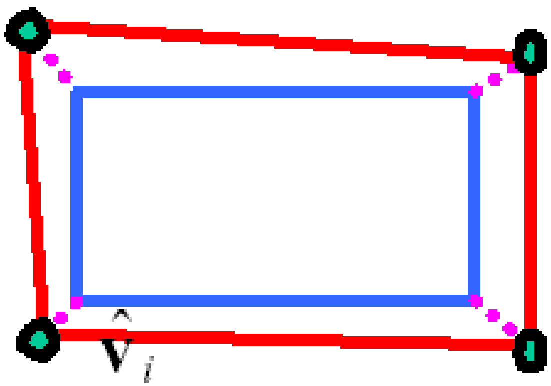
Active contours: adding priors

- If the object to segment is some smooth variation on a known shape, we can use a term that will penalize deviation from that shape:



$$E_{con} = \alpha \cdot \sum_{i=0}^{n-1} (v_i - \hat{v}_i)^2$$

where $\{\hat{v}_i\}$ are the points of the known shape.

Fig from Y. Boykov

Active contours: external energy

- The external energy measures how well the curve matches the image data.
- We want to "attract" the curve toward the image features: an energy expression proportional to the gradient magnitude will attract the contour to any edge.

$$E_{external}(v(s)) = -\nabla(I(v(s))^2) = -(|G_x(v(s))|^2 + |G_y(v(s))|^2)$$

$$E_{external}(v(s)) = -\nabla(G_\sigma(v(s)) * I(v(s)))$$

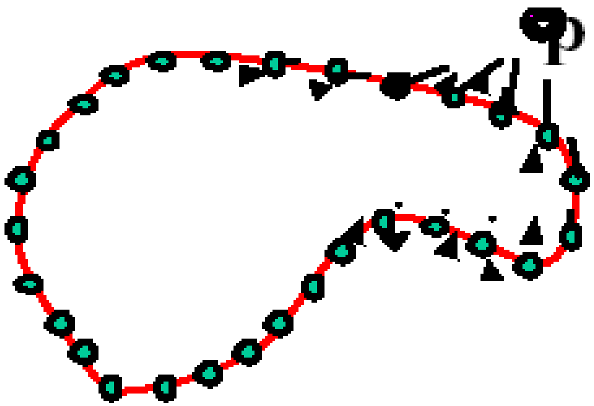
$$E_{external}(v(s)) = \frac{1}{1+\nabla(G_\sigma * I)}$$

- External energy for the whole curve:

$$E_{external} = \int_0^1 E_{external}(v(s))\, ds$$

Active contours: interactive energy

- The energy function can be altered online based on user input (interactive segmentation).
- The cursor allows to push/pull the snake away from a point.



$$E_{push} = +\sum_{i=0}^{n-1} \frac{r^2}{|v_i - p|^2}$$

Nearby points get pushed hardest

What expression could we use to pull points towards the cursor position?

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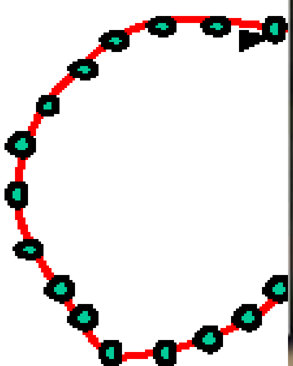

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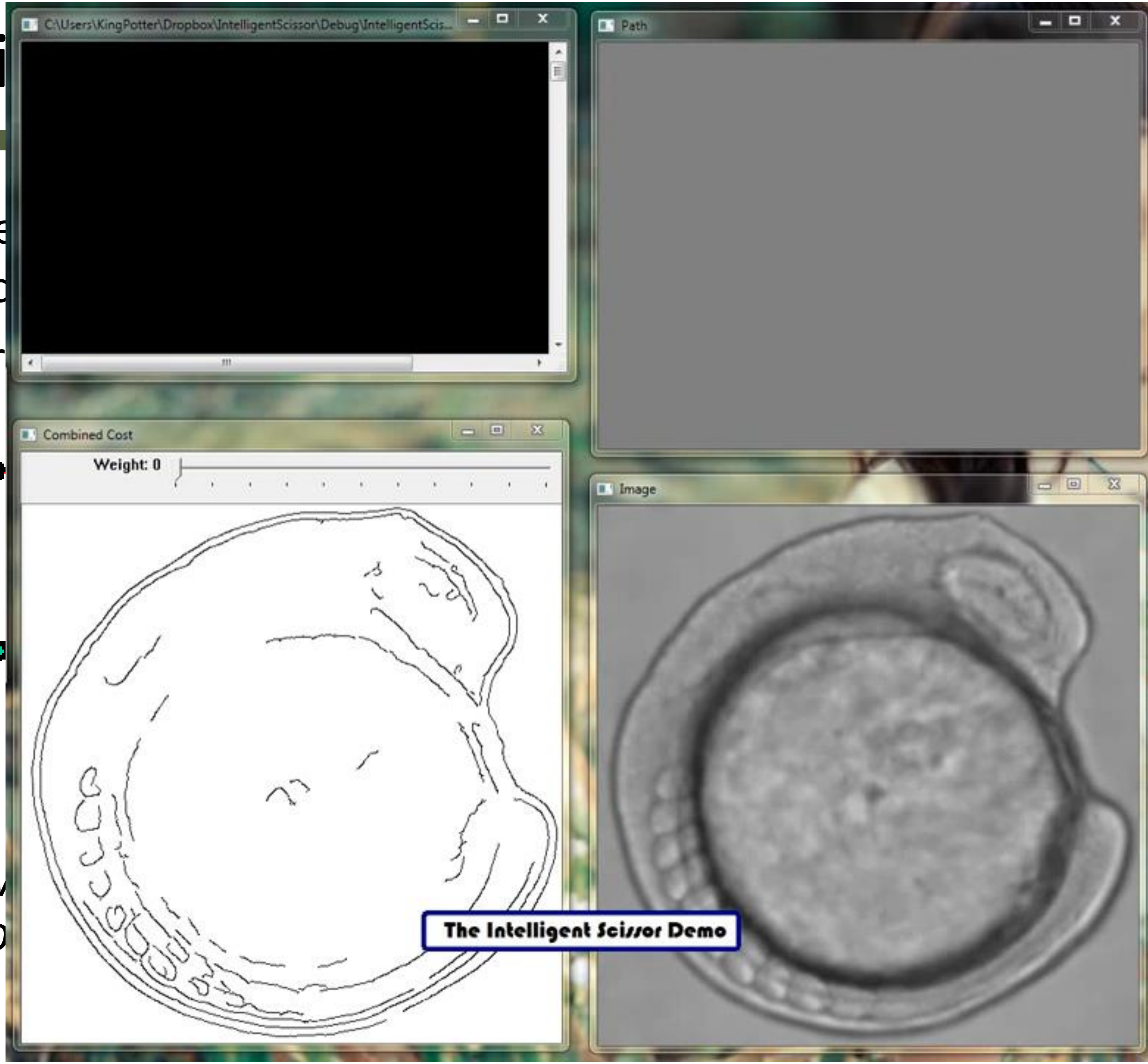

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Active contours: energy minimisation

$$E[(C)(p)] = \alpha \int_0^1 E_{int}(C(p))dp \; + \; \beta \int_0^1 E_{img}(C(p))dp \; + \; \gamma \int_0^1 E_{con}(C(p))dp$$

- There are different ways to minimise the energy:
  - Greedy search
  - Dynamic programming
  - Gradient descent

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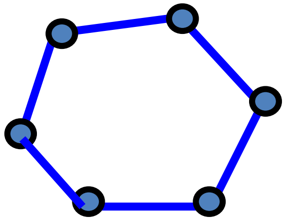
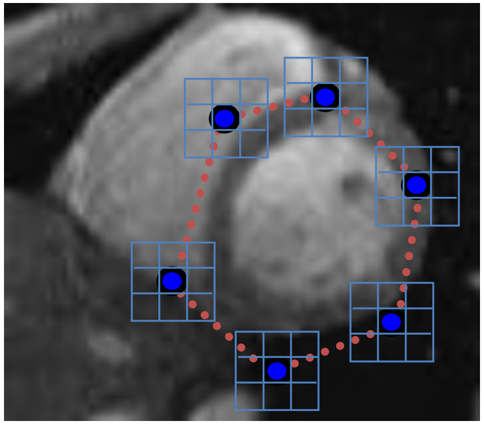
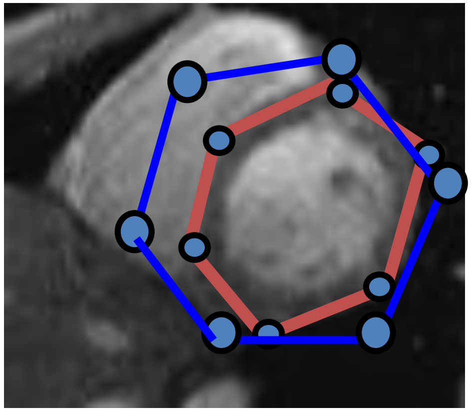
Active contours: summary

- A simple elastic snake is defined by
  - A set of *n* points,
  - An internal elastic energy term
  - An external edge based energy term



- To use this to locate the outline of an object
  - Initialize in the vicinity of the object
  - Modify the points to minimize the total energy: for each point, search window around it and move to where energy function is minimal
  - Stop when predefined number of points have not changed in last iteration, or after max number of iterations





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Active contours

- Pros
  - Low complexity.
  - Easy to introduce prior knowledge.
  - Can account for open as well as closed structures.
  - A well established technique, numerous publications it works.
  - User Interactivity.
- Cons
  - Selection on the parameter space and the sampling rule affects the final segmentation result.
  - Estimation of the internal geometric properties of the curve in particular higher order derivatives.
  - Quite sensitive to the initial conditions.
  - Changes of topology.

Active contours

- Let's implement two examples:
  - Follow the tutorial of Cris Luengo: http://www.crisluengo.net/index.php/archives/217
  - Follow the procedure of Emmanuel Trucco:

    (pdf in the moodle webpage)

To know more…

- The origins of snakes is:
  - *Kass, M.; Witkin, A.; Terzopoulos, D. (1988). "Snakes: Active contour models" (PDF). International Journal of Computer Vision.*
- Many improvements:
  - Laurent D. Cohen, On active contour models and balloons, CVGIP: Image Understanding, Volume 53, Issue 2, March 1991
  - Chenyang Xu and Jerry L. Prince. “Snakes, Shape, and Gradient Vector Flow”. IEEE Transactions on Image Processing, vol. 7, no. 3, pp. 359-368, March1998
  - Cremers, D.; Schnorr, C.; Weickert, J. "Diffusion-snakes: combining statistical shape knowledge and image information in a variational framework". Proceedings. IEEE Workshop on. 50: 137–144.
  - Blake & Isard, Active contours.

To know more…

- The origins of level sets are:
  - *Osher, S.; Sethian, J. A. (1988),* "Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton–Jacobi formulations”
  - V. Caselles, R. Kimmel, G. Sapiro, *Geodesic Active Contours*,