CAD: Image Characterization

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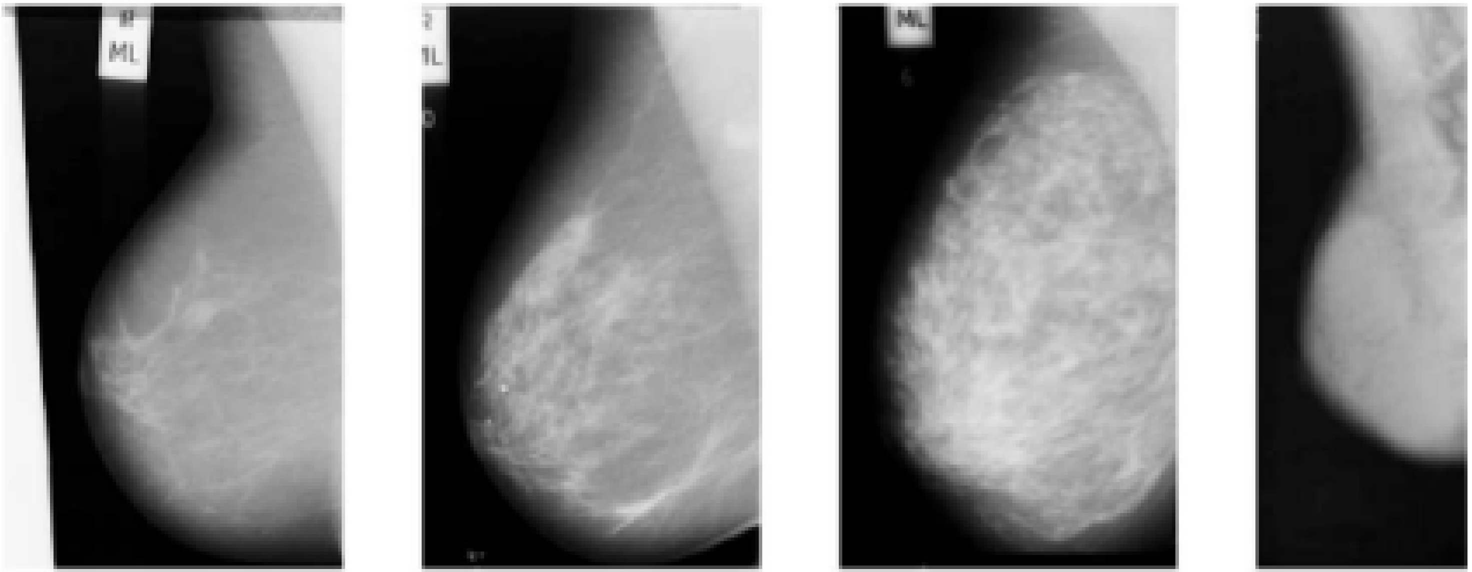
Characterization

- How can we model breast density?



(a) (b) (c) (d)

  - Texture information is used

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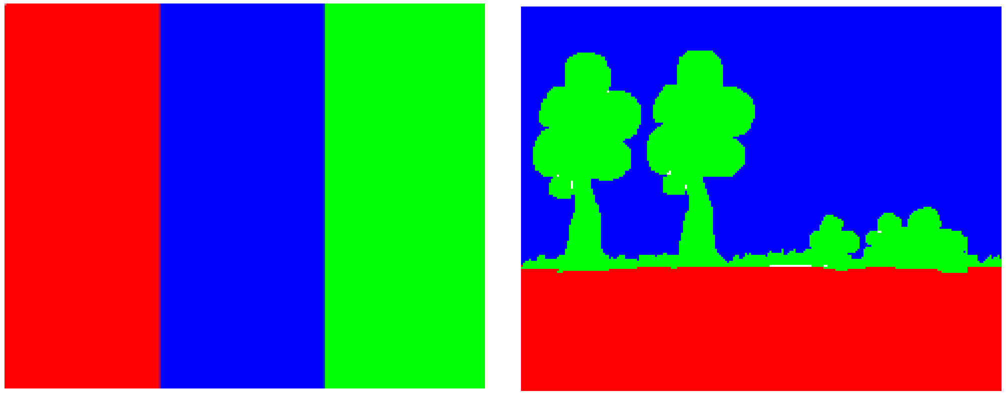
Characterization

- Sometimes a single characteristic is not enough to characterise a region…



They are the same images based on just colour characteristics!!

  - Shape and Texture information is needed

Content

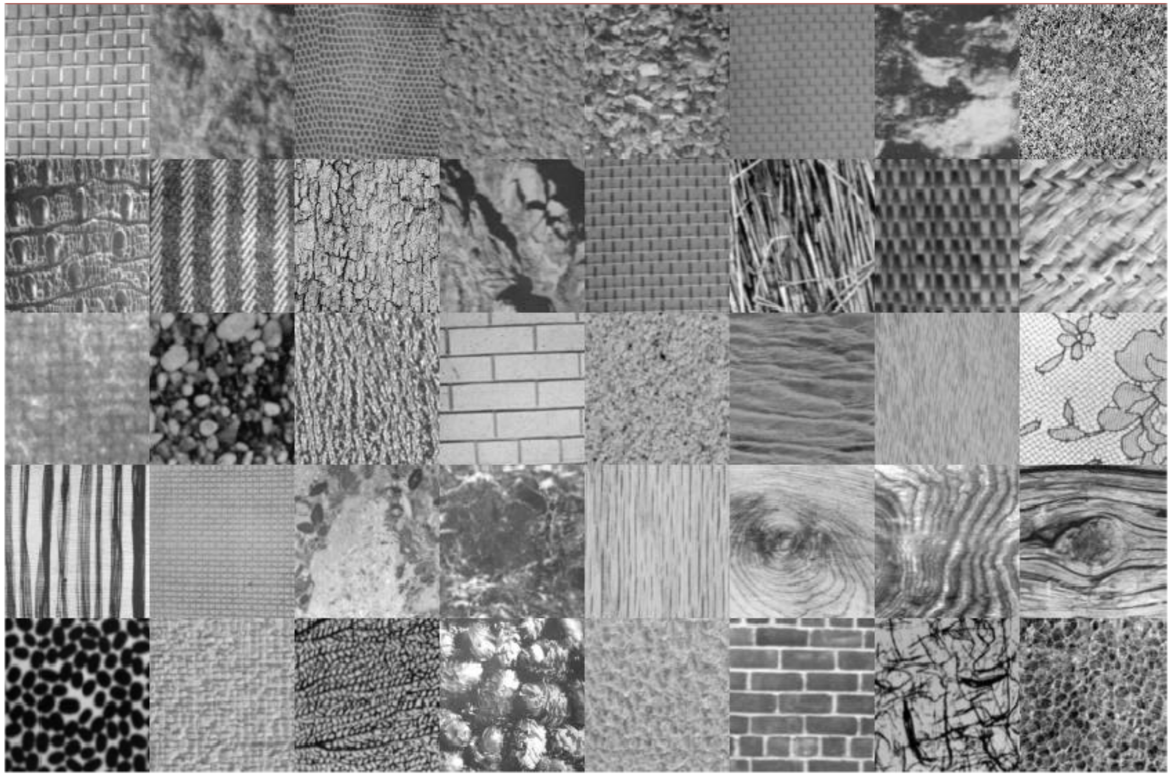
- Texture
  - Co-ocurrence matrices
  - Law Masks
  - Local Binary Patterns
- Colour
  - Physical properties
  - Colour Spaces
- Shape
  - Statistical
  - Shape context
  - Key point descriptors

Texture

- What is texture?

Texture

There is not **an accepted definition** for this visual cue



It is the property of some surfaces

Texture. Introduction

What is texture?

- Is it something related to the touch?
- Can we extract information from images? has it got color?
- Is a property of the surface of an object, and can not be defined by a single point
- A given texture depends on the scale at which is regarded

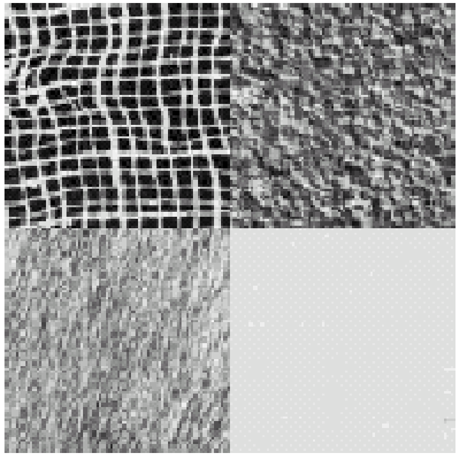


Image with 4 textures

Texture. Introduction

Everyday texture terms - *rough, silky, bumpy* - refer to touch

A texture that is *rough* to touch has:

- a large difference between high and low points, and
- a space between highs and lows approximately the same size as a finger

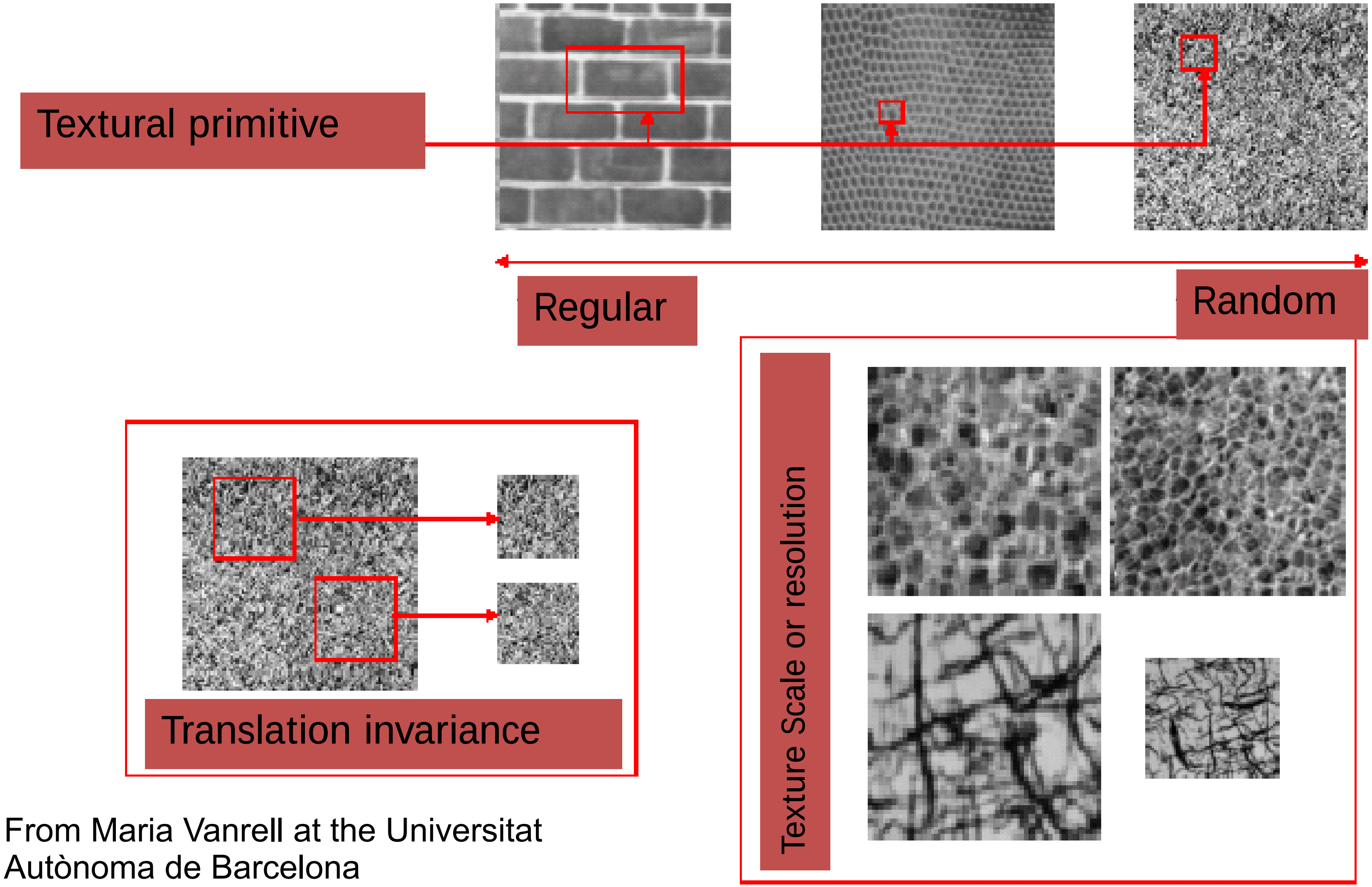
Silky would have

- little difference between high and low points, and
- the differences would be spaced very close together relative to finger size

Image texture works in the same way, except the highs and lows are brightness values instead of elevation changes. Instead of probing a finger over the surface, a "window" box is used

Texture. Introduction

Types of texture



From Maria Vanrell at the Universitat Autònoma de Barcelona

Texture operators

- Statistical methods
  - Co-occurrence matrices
  - Energy (Laws masks)
  - Parametric masks
  - Local Binary Patterns (LPB)
- Structural methods
- Modelization methods
  - Markov Random Fields
- Space-frequency filtering methods
  - Gabor filters
  - Wavelets

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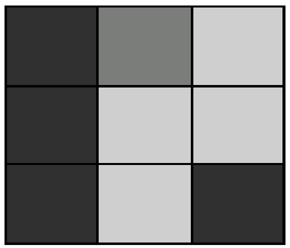
Co-occurrence matrices

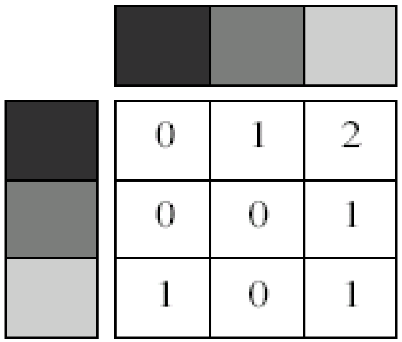
What is it? **The GLCM is a tabulation of how often different combinations of pixel brightness values (grey levels) occur in an image**

A position in the matrix (i,j) corresponds to the number of pixels of intensity i and j separated by a given distance and angle. They are always square matrices with the same dimensionality as the number of grey-levels

GLCM:

Input image:





Co-occurrence matrices

- The GLCM is used for a series of **"second order"** texture calculations
- **First order** texture measures are statistics calculated from the original image values, like variance, and do not consider pixel neighbour relationships
- **Second order** measures consider the relationship between groups of two (usually neighbouring) pixels in the original image
- **Third and higher order** textures (considering the relationships among three or more pixels) are theoretically possible but not commonly implemented due to calculation time and complexity

Co-occurrence matrices

The matrix (when normalized) represents the probability that two points with grey-levels ***i*** and ***j*** exist in a distance ***d*** and orientation θ

(orientations: 0º, 45º, 90º and 135º)

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

$$M_V = M(1, 90^{\circ}) = \begin{pmatrix} 6 & 0 & 2 & 0 \\ 0 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

Co-occurrence matrices

Energy $= \left(\sum_{i,j=0}^{N-1} m_{ij}^2\right)^{1/2}$

Entropy $= \sum_{i,j=0}^{N-1} m_{ij} \times \log(m_{ij})$

Probability $= \max_{i,j=0}^{N-1} (m_{ij})$

Contrast $= \sum_{i,j=0}^{N-1} (i-j)^2 \times m_{ij}$

Measures related to orderliness (how regular the pixel values are within the window)

Measures related to the distance from the GLCM diagonal

Inverse $= \sum_{i,j=0}^{N-1} \frac{m_{ij}}{(i-j)^2} \quad i \neq j$

Correlation $= \sum_{i,j=0}^{N-1} \frac{(i-\mu)\times(j-\mu)\times m_{ij}}{\sigma^2}$

$$\mu = \sum_{i,j=0}^{N-1} i \times m_{ij}$$

Homogeneity $= \sum_{i,j=0}^{N-1} \frac{m_{ij}}{1+|i-j|^2}$

$$\sigma = \sqrt{\sum_{i,j=0}^{N-1} (i-\mu)^2 \times m_{ij}}$$

Statistics derived from the GLCM

Co-occurrence matrices

- **Contrast:** indication of the local variation in the image (big value → large variation, small value → uniform)

- **Homogeneity**: values concentrated/located in the diagonal of the matrix (big value → homogeneity)

- **Energy ($\sum m_{ij}^2)^{1/2}$**: indication of the uniformity of the image (big values → few entries in the matrix with large values, small values → entries in the matrix with similar values)

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Co-occurrence matrices

- Global / Local texture descriptors

Co-occurrence matrices

Activity

- Given the following 2 images (2bits), compute the co-occurrence matrix **M(1,0º)**, and extract the **contrast feature** for both images

0	2	0	2	0
0	2	0	2	0
0	2	0	2	0
1	1	1	3	1
1	1	1	3	1

Image 1

2	2	2	2	2
2	2	2	2	2
2	2	2	2	2
1	1	1	1	1
1	1	1	1	1

Image 2

$$\text{Contrast} = \sum_{i,j=0}^{N-1} (i-j)^2 \times m_{ij}$$

Co-occurrence matrices

Activity

Image 1

0	2	0	2	0
0	2	0	2	0
0	2	0	2	0
1	1	1	3	1
1	1	1	3	1

Image 1

2	2	2	2	2
2	2	2	2	2
2	2	2	2	2
1	1	1	1	1
1	1	1	1	1

Image 2

Contrast $= \sum_{i,j=0}^{N-1} (i-j)^2 \times m_{ij}$

0	0	12	0
0	8	0	4
12	0	0	0
0	4	0	0

M(1,0º)

0	0	0	0
0	16	0	0
0	0	24	0
0	0	0	0

M(1,0º)

Contrast Image 1 = 2^2*12 + 2^2*4+ 2^2*12 + 2^2*4 = **128**

Contrast Image 2 = **0**

Co-occurrence matrices

Questions:

- What is the texture of a pixel?
- What is the texture of a region?

- Variables...
  - Matrix size
  - Computational cost
  - Parameters
  - Distance?
  - Orientation?
  - Statistic?

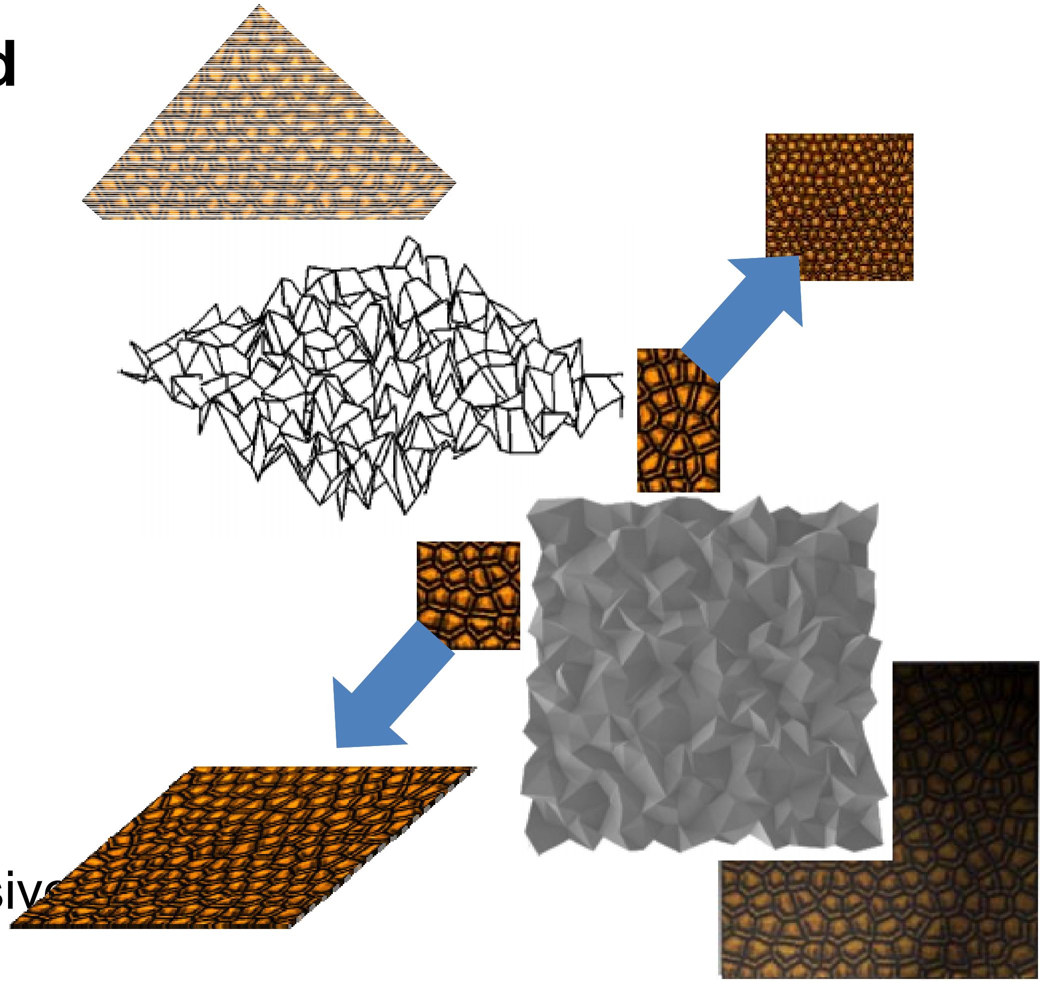
Texture operators

- Statistical methods
  - Co-occurrence matrices
  - Energy (Laws masks)
  - Parametric masks
  - Local Binary Patterns (LPB)
- Structural methods
- Modelization methods
  - Markov Random Fields
- Space-frequency filtering methods
  - Gabor filters
  - Wavelets

LBP (Local Binary Patterns)

Characteristics of a good texture classifier

- Invariant to:
  - Scale
  - Rotation
  - Illumination
  - Other homographies
- Unique
- Computationally inexpensive
- Multi-dimensional

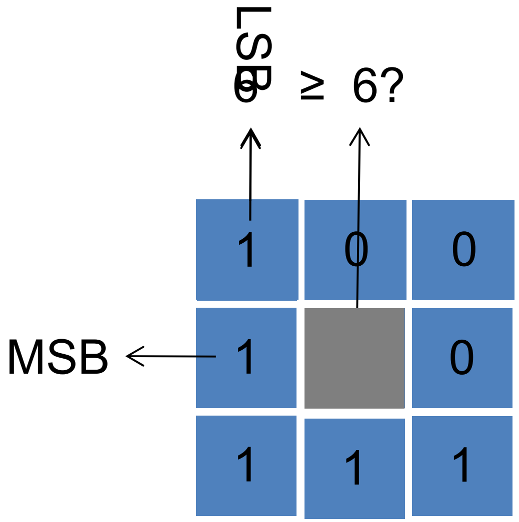


LBP (Local Binary Patterns)

- A texture measure
- Originally developed by Timo Ojala and Matti Pietikäinen
- Based on simple ideas:
  - Each texture is composed of micro-structures
  - These micro-structure repeat themselves
  - They can be represented with binary patterns easily computed
- …how does it work?

Binary patterns derivation

The LBP operator:



PATTERN = 11110001b = 241d

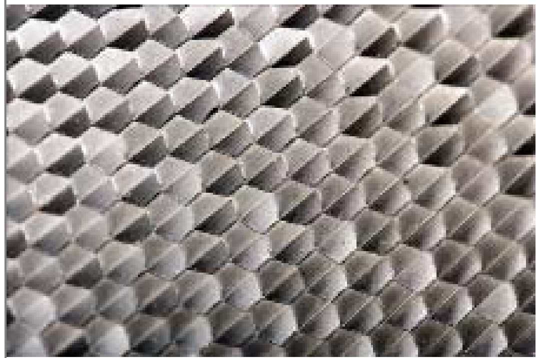
LBP (Local Binary Patterns)

Textures Classification

- Each texture is classified using the LBP codes derived for each pixel
- A LBP histogram is employed as a measure

LBP (Local Binary Patterns)

Textures Classification



Occurrence	LBP Codes	Occurrence
0	0000000	0
0	0000001	0
0	0000011	0
0	0000111	0
0	0001111	0
0	0011111	0
0	0111111	0
0	1111111	0

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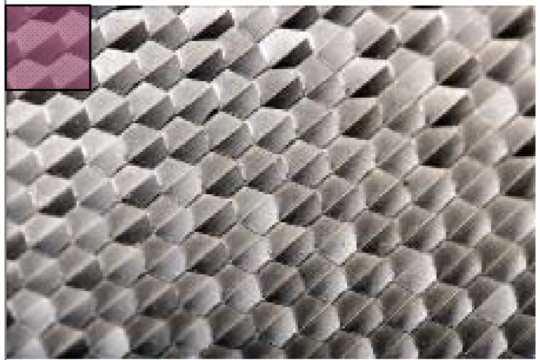
LBP (Local Binary Patterns)

Textures Classification



Occurrence	LBP Codes	Occurrence
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0	0000001	1
1	0000011	0
0	0000111	0
0	0001111	0
0	0011111	0
0	0111111	0
0	1111111	0

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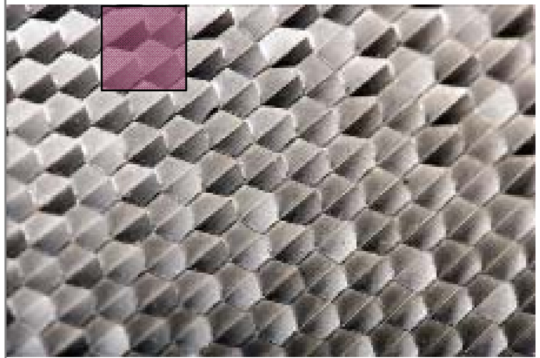
LBP (Local Binary Patterns)

Textures Classification



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1	0001111	0
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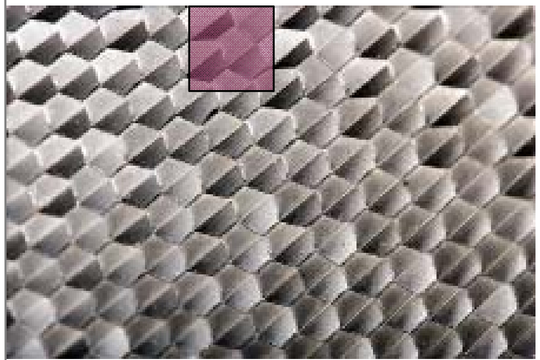
0	0111111	0
0	1111111	0

LBP (Local Binary Patterns)

Textures Classification



Occurrence	LBP Codes	Occurrence
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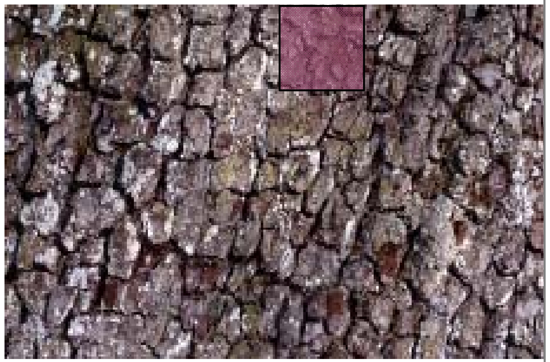
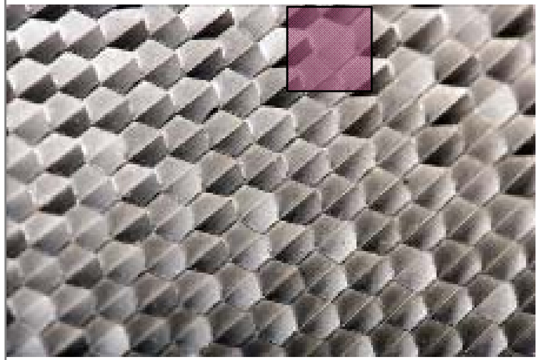
0	0000001	1
1	0000011	1
0	0000111	0
2	0001111	1
0	0011111	0
0	0111111	0
0	1111111	0

LBP (Local Binary Patterns)

Textures Classification





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0	0000111	0
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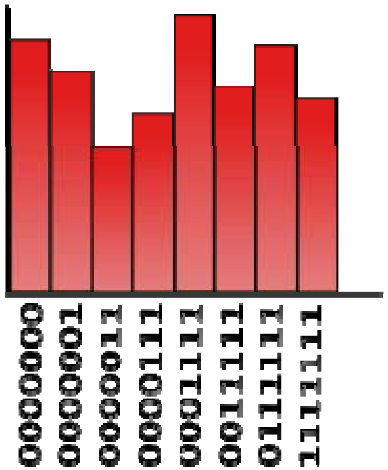
1	0011111	1
0	0111111	0
0	1111111	0

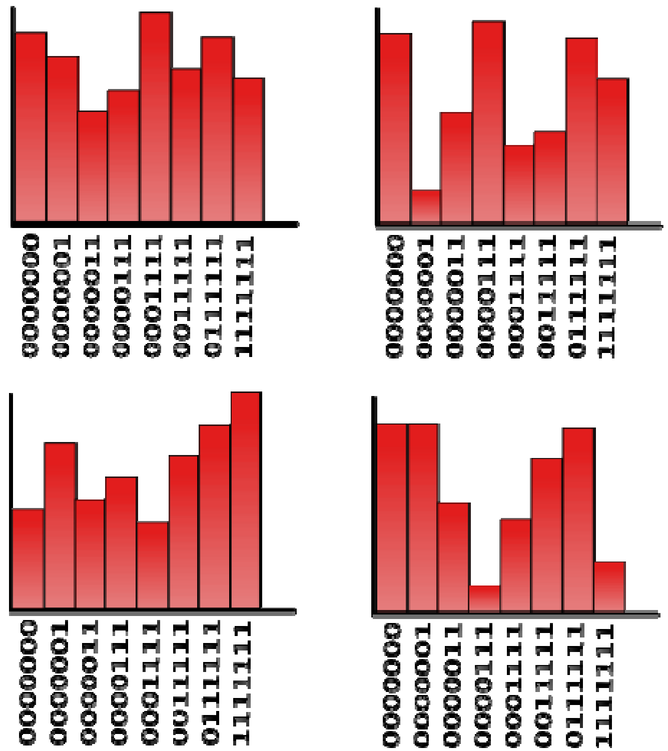
LBP (Local Binary Patterns)

Textures Classification

New LBP Histogram



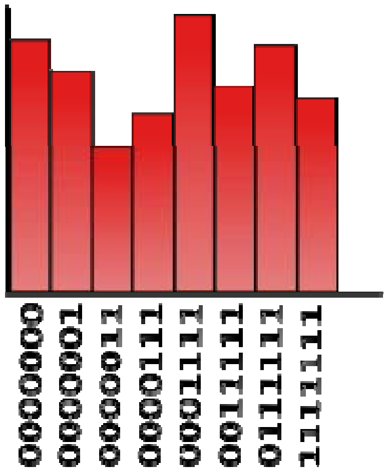
Textures Database



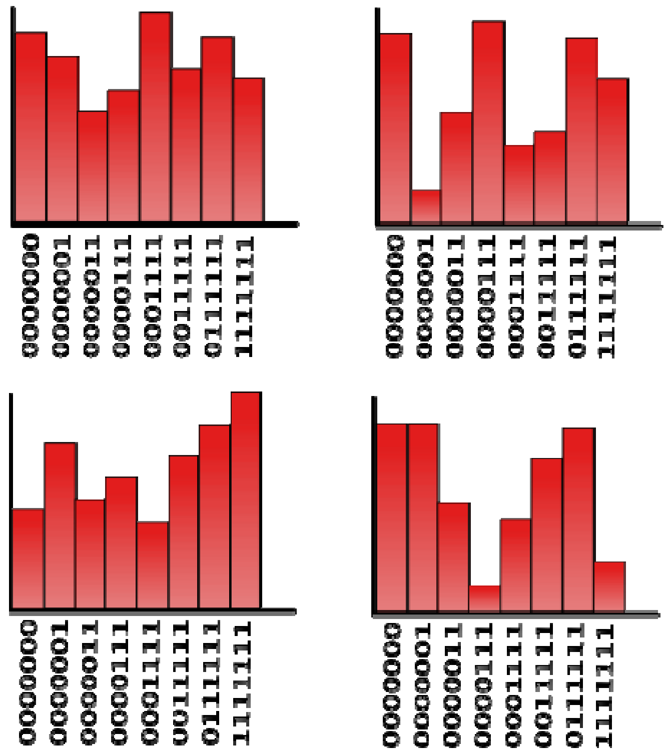
LBP (Local Binary Patterns)

Textures Classification

New LBP Histogram

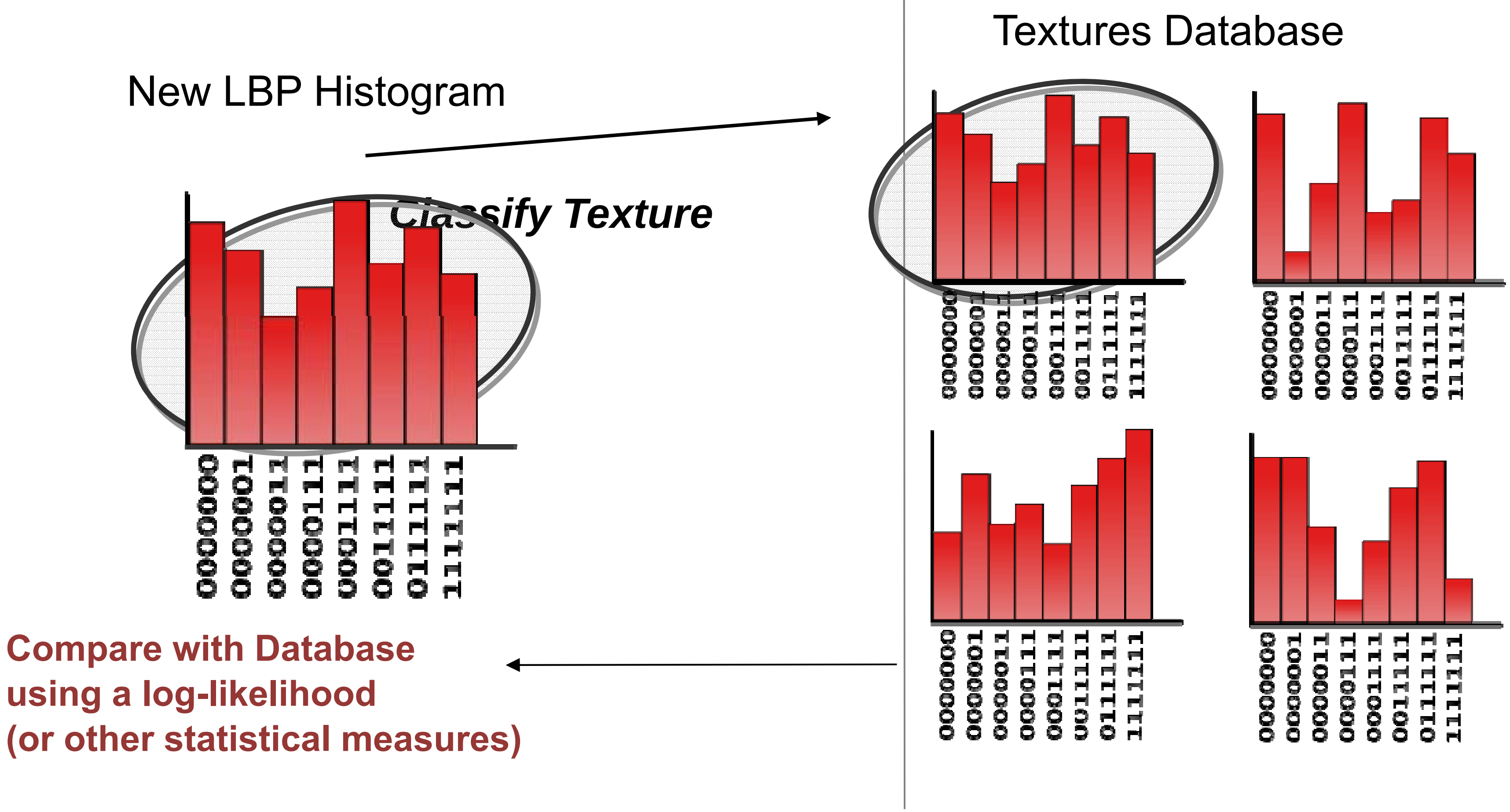


Textures Database



Compare with Database using a log-likelihood (or other statistical measures)

Textures Classification



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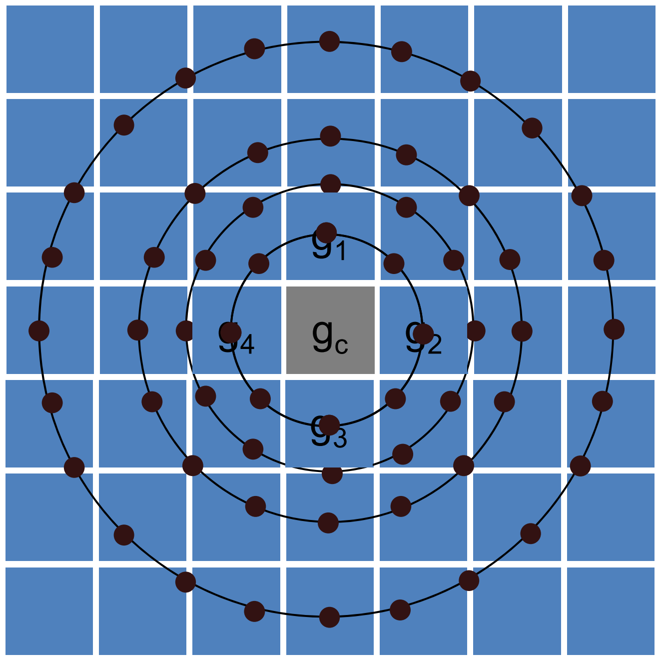
LBP (Local Binary Patterns)

The Circular Extension

- LBP codes can be derived considering a circular neighbourhood

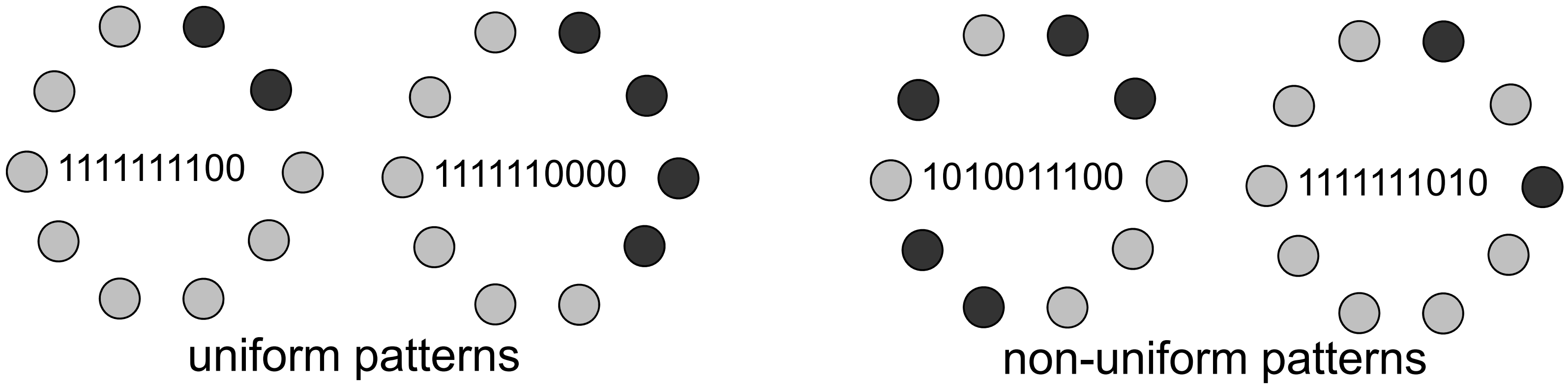
$$LBP_{P,R} = \sum_{p=0}^{P-1} sign(g_p - g_c)2^p$$



P=8 R=1.5

LBP (Local Binary Patterns)

Uniform LBP Patterns

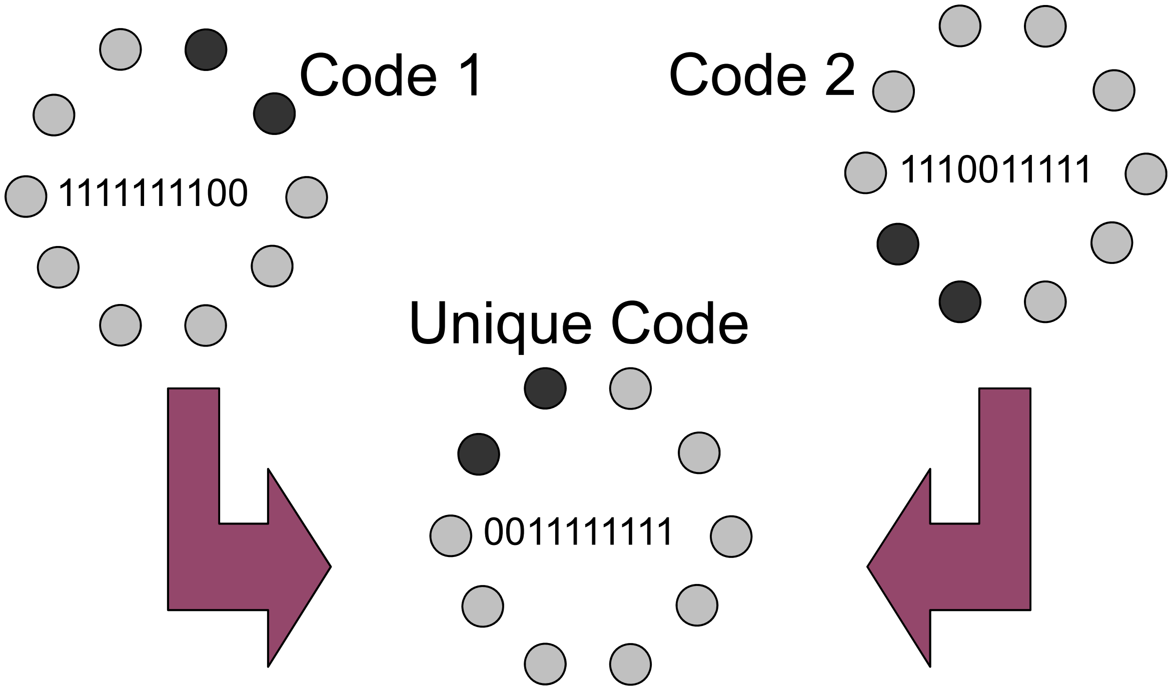


- Through empirical experiments it was discovered that *uniform patterns* are:
  - More stable to transformations
  - So common that can be used as the only LBP descriptors for textures

LBP (Local Binary Patterns)

Rotation Invariance

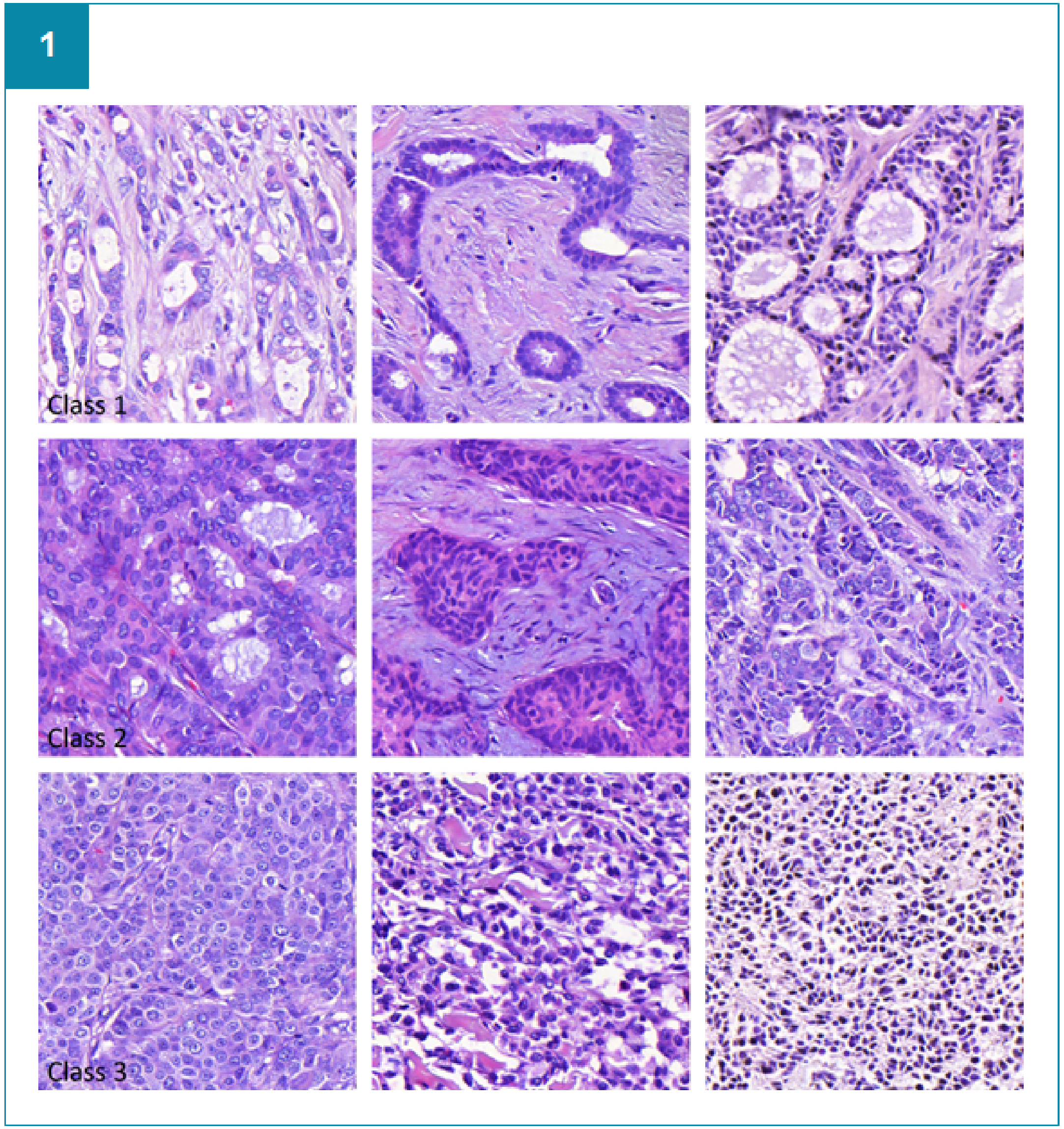
- Each LBP code can be binary shifted to achieve rotation invariance



- All the possible transformations are stored in a look-up table

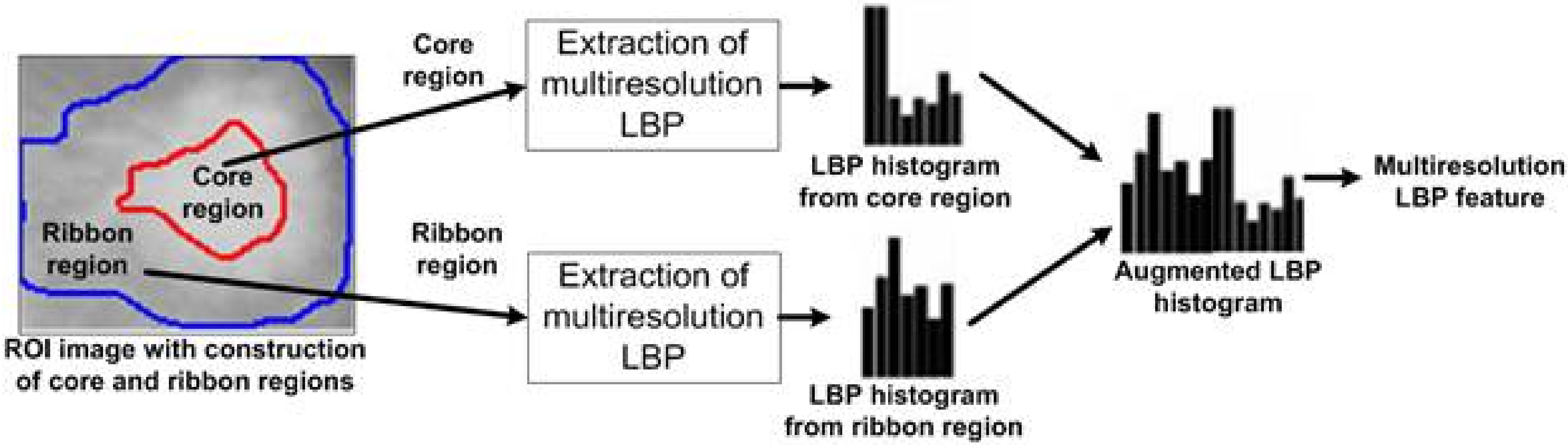
LBP (Local Binary Patterns)

Classification of Microscopic Tissue



LBP (Local Binary Patterns)

Breast Classification



LBP (Local Binary Patterns)

- Timo Ojala, Matti Pietikäinen “A comparative study of texture measures with classification based on feature distribution ” *Pattern Recognition*, vol. 29 (1), 1996, pp. 51-59.
- Timo Ojala, Matti Pietikäinen and Topi Mäenpää “Texture Analysis with Local Binary Patterns” in *Chen and Wang (eds) Handbook of Pattern Recognition and Computer Vision*, 3rd ed., Singapore: Wold Scientific Press, 2005.
- Timo Ojala, Matti Pietikäinen and Topi Mäenpää “Multiresolution Gray-Scale and Rotation Invariant Texture Classification with Local Binary Patterns” *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 24 (7), Jul. 2002, pp. 971-987.
- Machine Vision Group. “Local Binary Pattern” [online]. Finland: University of Oulu. Available at:

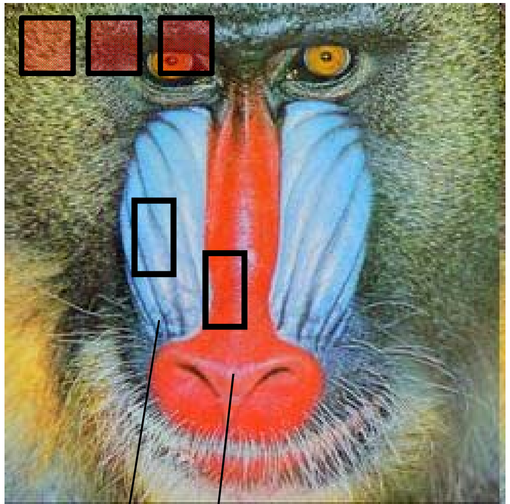
http://www.ee.oulu.fi/research/imag/texture/lbp/about/Texture Analysis with Local Binary Patterns.pdf (accessed 18/12/2015)

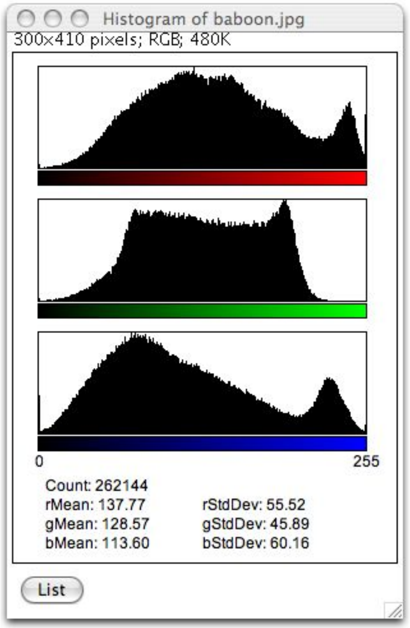
Content

- Texture
  - Co-ocurrence matrices
  - Law Masks
  - Local Binary Patterns
- Colour
  - Physical properties
  - Colour Spaces
- Shape
  - Statistical
  - Shape context
  - Key point descriptors

Color / Intensity characterization

Patch information (for detection and classification strategies)





Feature vector 1=[mean R, mean G, mean B, mean H, mean S, mean V, …]

Feature vector 2=[mean R, mean G, mean B, mean H, mean S, mean V, …]

Content

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Shape analysis

- Statistical description
  - Topological
  - Geometrical
- Structural description
- Other methods

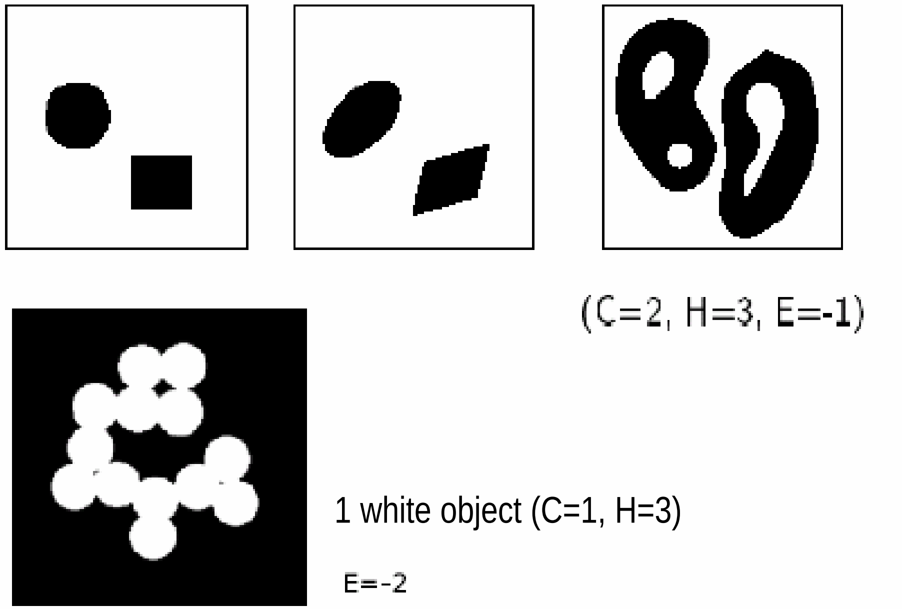
Shape. Statistical description

- Statistical Description. Different descriptors:
  - Topological
    - Number of objects
    - Euler number
    - Bounding box
  - Geometric
    - Area, width, perimeters
    - Elongation (excentricity)
    - Compacity
    - Inertia moments
    - Vertical, horizontal or diagonal projections

Shape. Statistical description

Topological descriptors (Binary images)

- Number of objects (C)
- Number of holes (H)
- Euler Number (E)

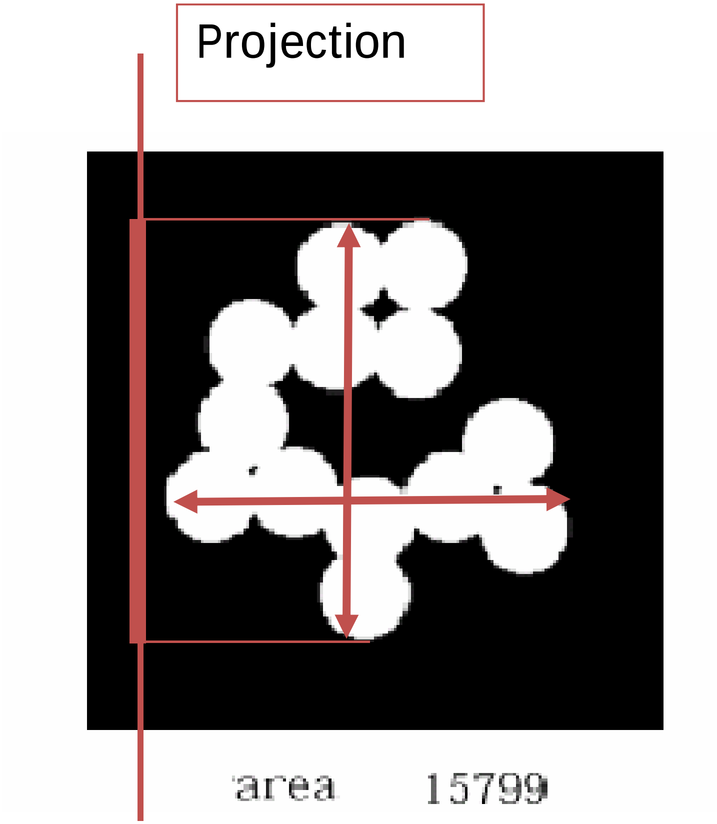


From Jordi Vitrià at the Universitat Autònoma de Barcelona

Shape. Statistical description

Geometric descriptors

- Area, Perimeter
- Width, Height
- Compacity $\frac{P^2}{(4\pi A)}$
- Elongation: W/H



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Shape. Statistical description

Inertia Moments

$$m_{ij} = \sum_{x=1}^{N}\sum_{y=1}^{N} f(x,y)\ x^i\ y^j$$

- Area = m_{00}
- $\mu_x = m_{10}/m_{00},\ \mu_y = m_{01}/m_{00}$

- With this definition, the same shape in a different position will have different moments

$$\mu_{ij} = \sum_{x=1}^{N}\sum_{y=1}^{N} f(x,y)\ (x-\mu_x)^i\ (y-\mu_y)^j$$

⇩

Central Moments

Where μ_x i μ_y are the coordinates of the centroid of the shape

Shape. Statistical description

Central Moments

Center

(μ_x, μ_y)

Variance

μ_{20}/μ_{00} and μ_{02}/μ_{00}

Covariance

$$\frac{1}{\mu_{00}}\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix}$$

Eccentricity

$$\frac{\lambda_1}{\lambda_2}$$

Orientation

$$\frac{v_1}{||v_1||}$$

or $\theta = \frac{1}{2}\tan^{-1}\frac{2\mu_{11}}{\mu_{20}-\mu_{02}}$

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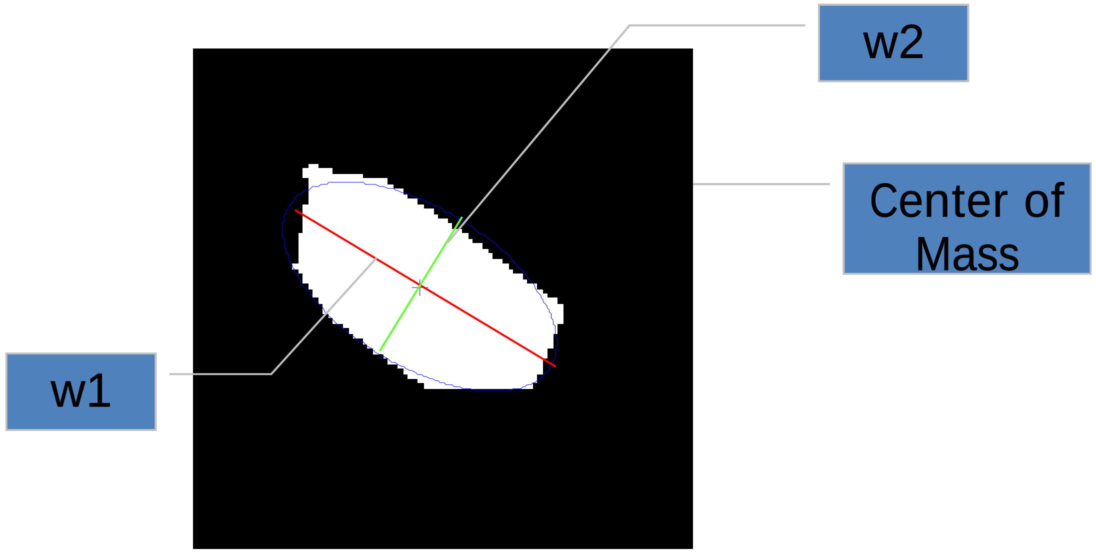
Shape. Statistical description

Example of Central Moments

- Principal axes are related to the eigenvectors and eigenvalues of the covariance matrix



Invariant moments are not affected by specific transformations

- Translation: Central moments
- Rotation: eigenvalues of the covariance matrix
- Scale: ratio between eigenvalues

Shape analysis

- Statistical description
  - Topological
  - Geometrical
- **Structural description**
- Other methods

Shape analysis

Statistical description: shape representation using a vector of numerical characteristics. Similarity between shapes is defined as distance metric in the feature space

Structural description: explicit or implicit representation of the structure of an object, where structure is the hierarchical and relational organisation of lower level characteristics

Example

Shape $\boldsymbol{F}$

Statistical Desc.: $\boldsymbol{F}$ = (#components, height, width)

Structural Desc:

LeftLeg, RightLeg: VERTICAL RECTANGLE;

LeftArm, RightArm: HORITZONTAL RECTANGLE;

body: SQUARE;

head: CERCLE;

$\boldsymbol{F} = \text{head} \Uparrow (\text{LeftArm} \Leftrightarrow (\text{body} \Uparrow (\text{LeftLeg} \Leftrightarrow \text{RightLeg})) \Leftrightarrow \text{RightArm})$

Structural description

Structural shape description can be split into two categories based on the model used for description:

- Syntactical description:
  - Representation using formal grammar
  - Recognition is performed using a parser

- Structural prototypes:
  - Explicit Representation using structural definitions such as strings and graphs
  - Recognition is performed by stating the matching using an implicit distance (or similarity) function

Structural description

Syntactial Description

A set of graphic primitives and a grammar are used in order to explain a given shape

Recognition (parsing). Given a shape x the parser decides if it belongs to the grammar

$G=(N,T,P,S)$

$N=\{S,A,B\}$

$T=\{a,b,c\}$

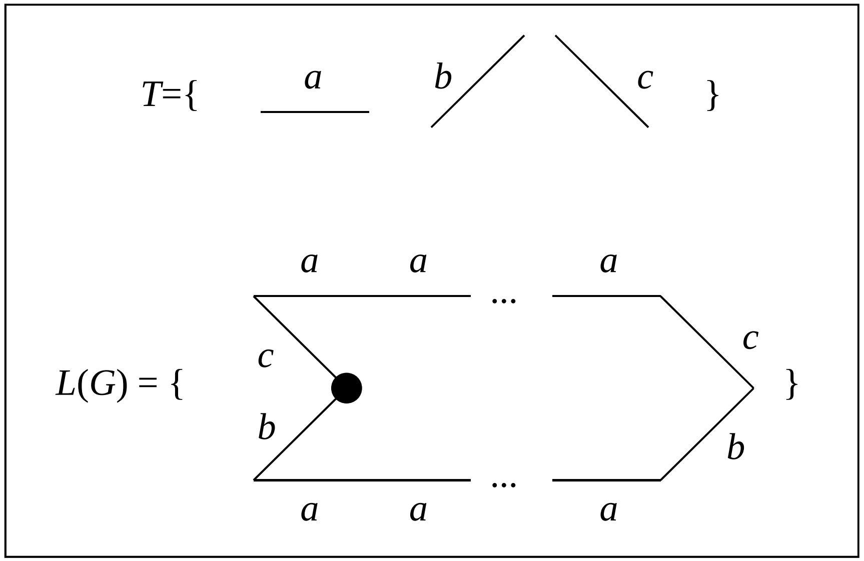
$P = \{\ S \rightarrow cAb,$

$\quad A \rightarrow aBa,$

$\quad B \rightarrow aBa \mid cb\ \}$

$L(G) = \{ca^n cba^n b\}$

Graphical interpretation



Structural description

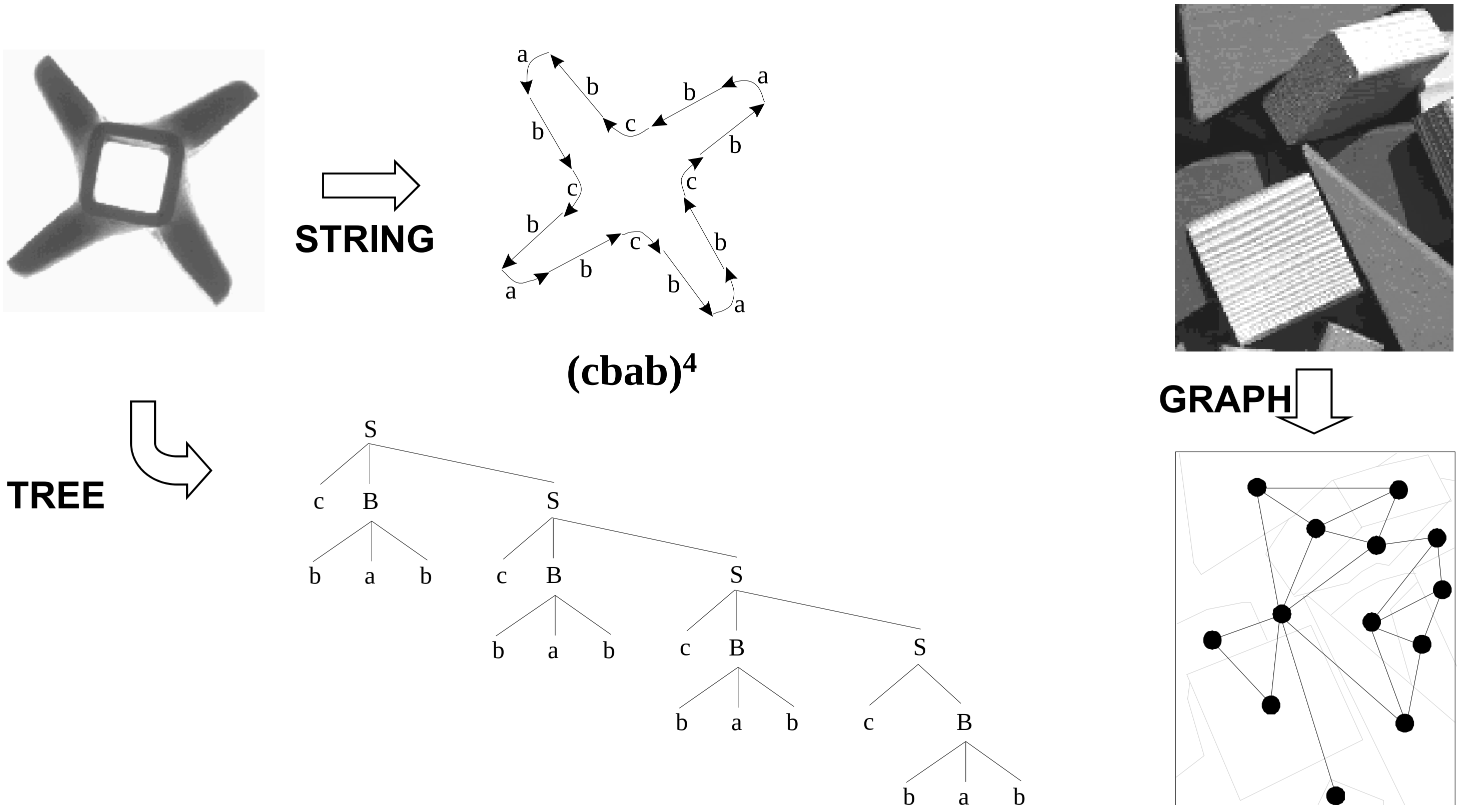
Structural Prototypes

The same data structure is used to represent models and unknown shapes

- Recognition is performed by directly comparing the models with the new instances using a distance (similarity) function
- Basically, two types of structures are used for representing shapes
  - One dimensional: Strings (eg. Chain Codes)
  - Multi dimensional: Graphs (eg. RAG)

Structural description

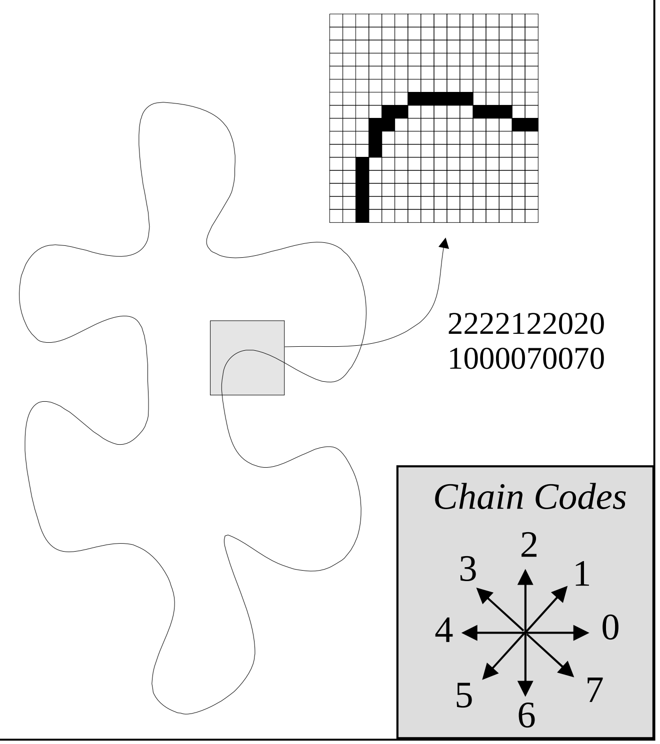
- Use of strings, trees, graph, tables, etc to describe objects
- Spatial, temporal and conceptual relationships are also described



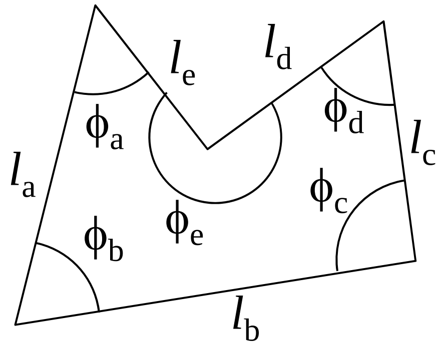
Structural description

Strings

Finite number of Symbols



Strings with attributes



F =

a	b	c	d	e
l_a	l_b	l_c	l_d	l_e
ϕ_a	ϕ_b	ϕ_c	ϕ_d	ϕ_e

Shape analysis

- Statistical description
  - Topological
  - Geometrical
- Structural description
- **Other methods**

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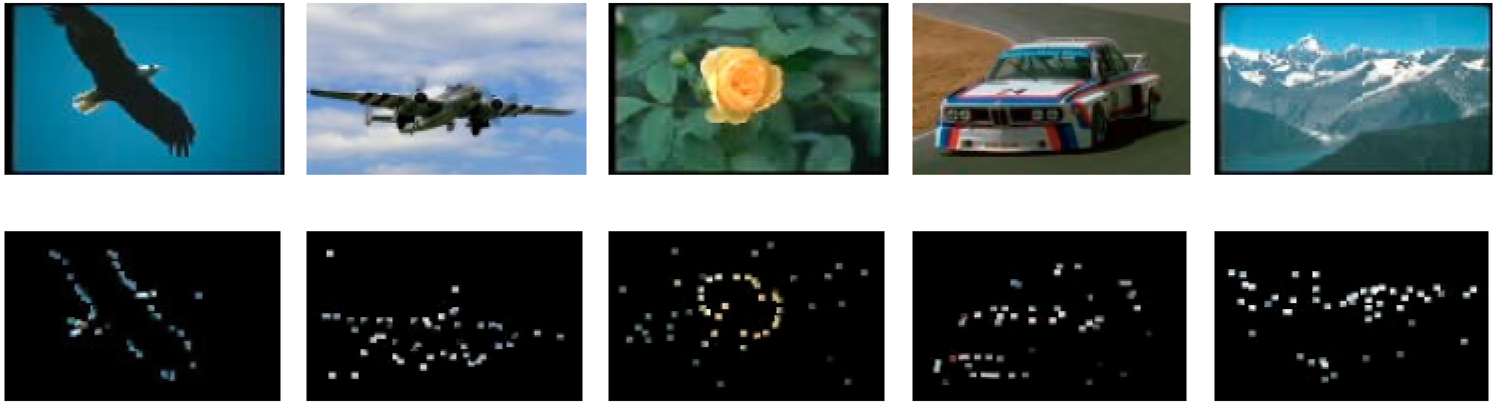
Salient points

Salient Points

A shape can also be described by a set of salient points

- Methods
  - Edge Corners
  - Wavelets
  - Edge Curvature
  - Etc..

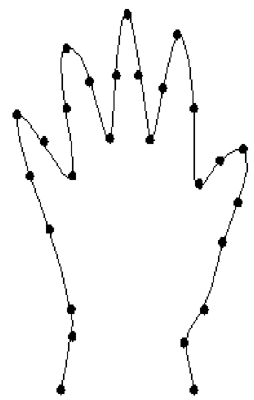


Extracted from Michael S. Lew

PDM

PDM (Point Distribution model / Statistical Shape model)

- Describe a shape from studying the statistical variation of characteristic points from a set of training images of this shape



- Overview
  - Align the training set
  - Obtain a mean shape
  - Compute the variance of the training set
  - Extract eigenvectors and eigenvalues of the covariance
  - The shape can be modeled by taking only the eigenvectors with the larger eigevalues (account for most of the variation)

Examples extracted from Tim Cootes

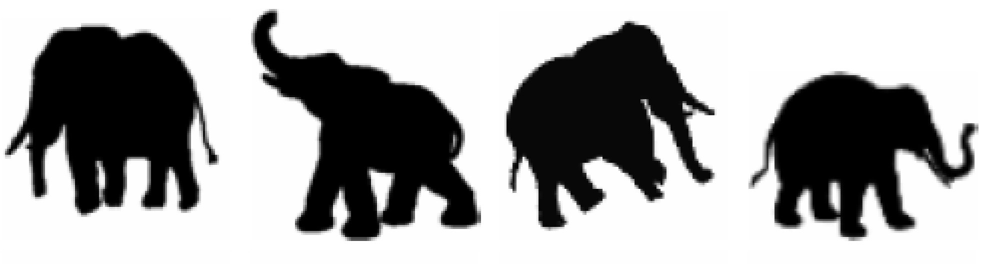
Shape Context

By: Carlos J. Becker, Sophia Bano

Motivation

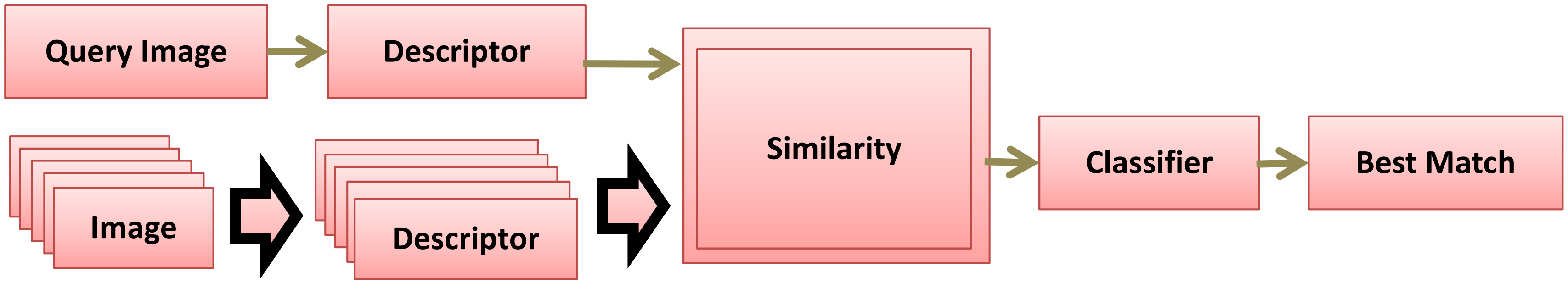
- Shape matching

- Object Recognition based on shapes



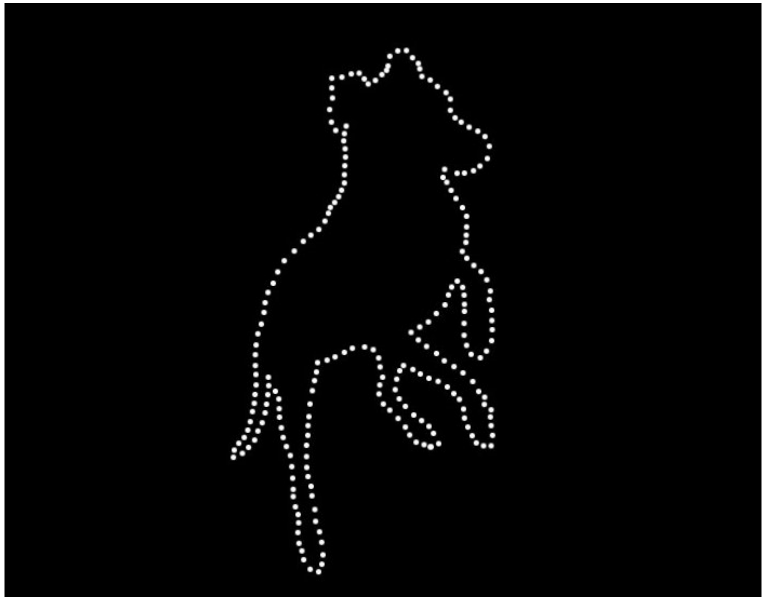
Motivation

- Classification Problem, we need:
  - Descriptor
    - Characterization
  - Similarity between two descriptors (i.e Distance)
    - Matching



Shape representation

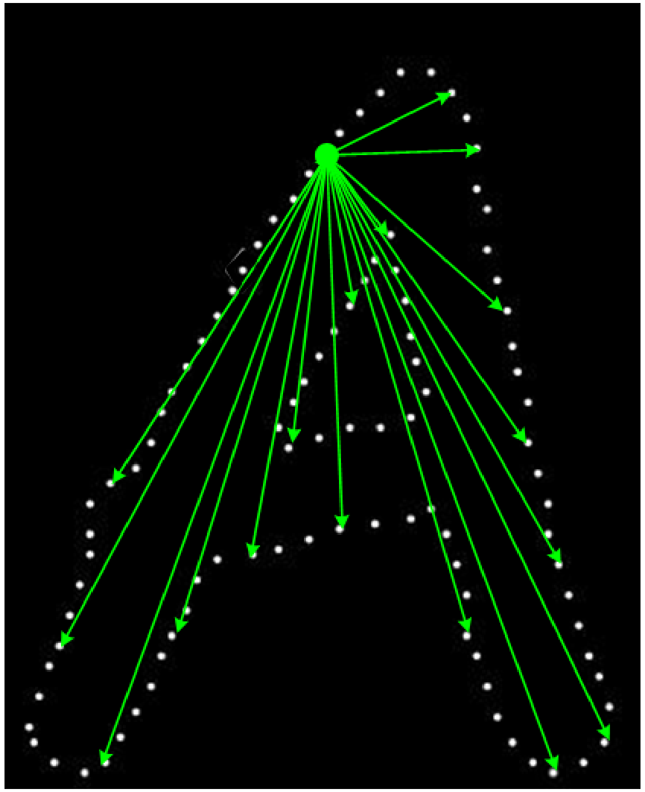
– Set of points taken from external and internal contour of the object



– Points could or could not represent key points in the image (corners/edges/etc)

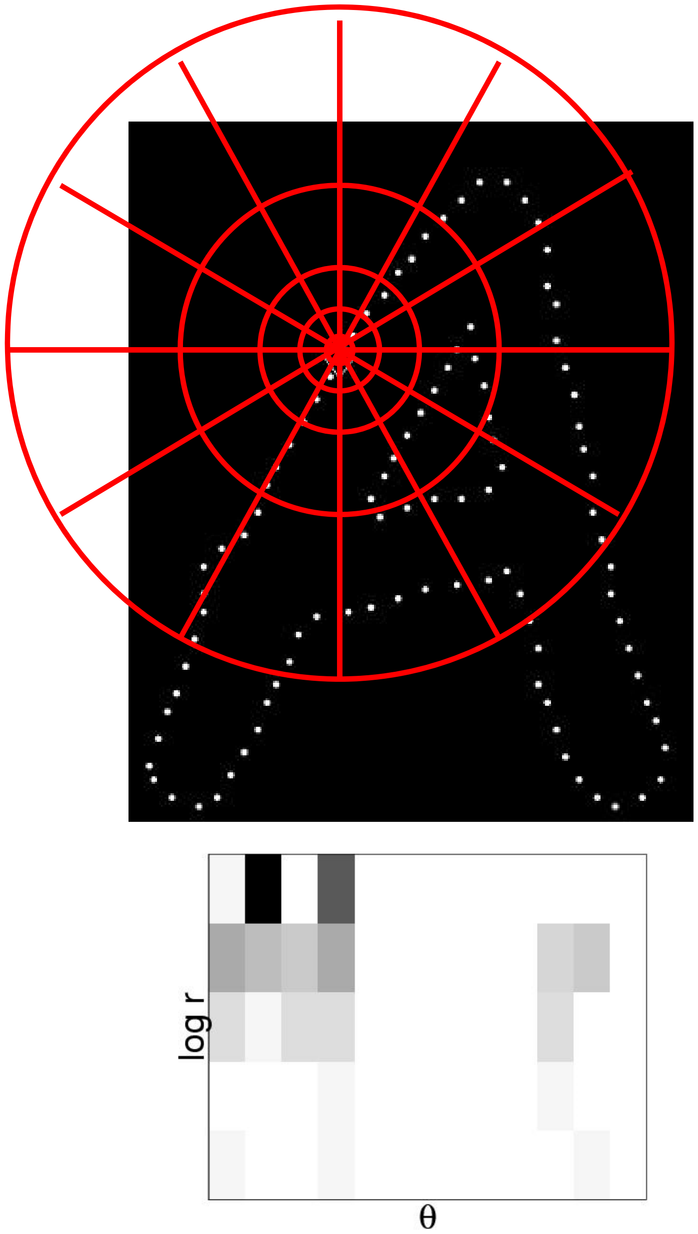
Shape context

- Novel shape Descriptor
  - Describes the relationship between each point of shape contour with rest of points
    - Histogram Representation
  - All the information is relative to the contour points.
  - Rich Descriptor



Shape context

- Relationship between points is summarized in a Histogram
  - Polar coordinates
    - Angle
    - Distance
  - Distance taken as log r
    - More sensitive to positions closer to the point than farther



Shape context

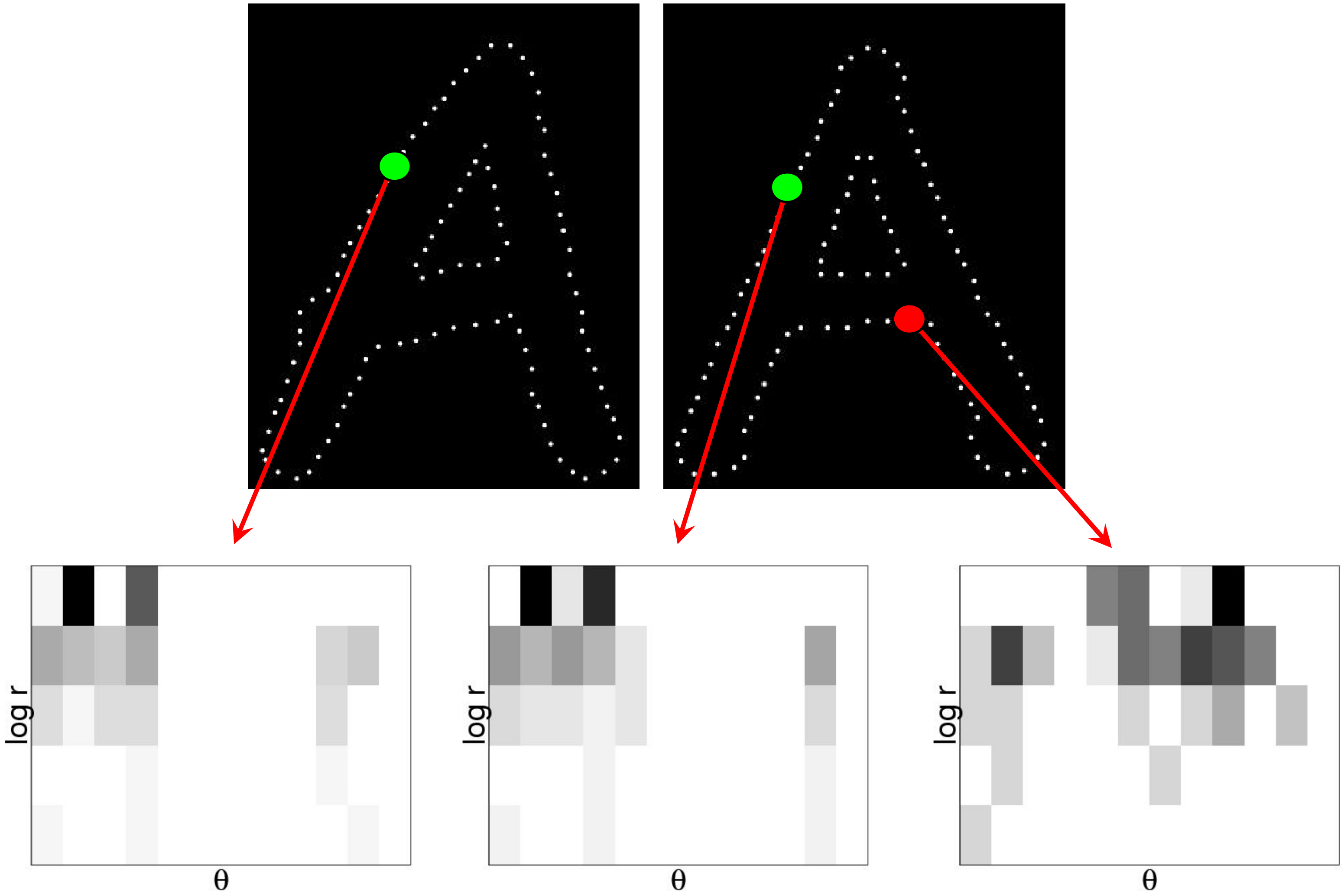
Shape Context Properties

- Invariant to Translation
- Invariant to Scaling
- Can be made invariant to Rotation
- Robust to
  - Small nonlinear transformations
  - Occlusions
  - Presences of outliers

- A blurry or noisy image could lead to a wrong representation of its shape

Shape context

Two shapes: which is the best match?



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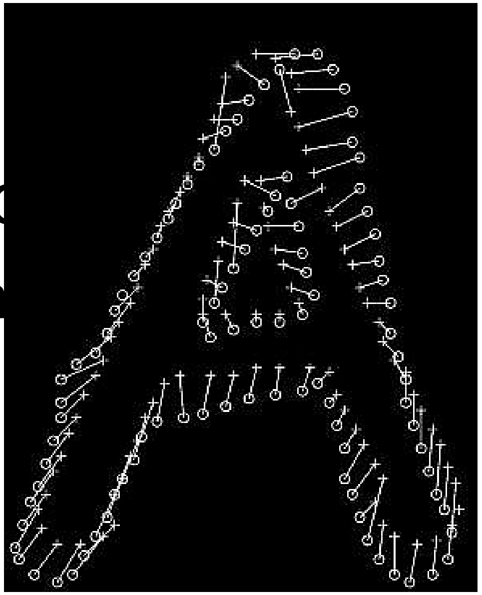
Two shapes: which is the best match?

- We are comparing histograms
  - **Chi-Square Test** to determine the matching cost:

$$C_{ij} = C(p_i, q_j) = \frac{1}{2}\sum_{k=1}^{K}\frac{[h_i(k) - h_j(k)]^2}{h_i(k) + h_j(k)}$$

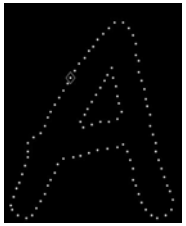
- But we want to minimize the total c
  - **Bipartite Matching** using Hungarian method is applied



$$H(\pi) = \sum_i C(p_i, q_{\pi(i)})$$

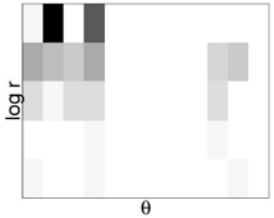
Matching with shape context

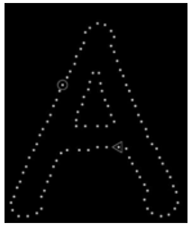
Query Image



Shape
(N points)

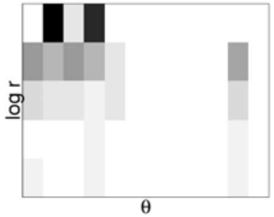
Shape Context
(N histograms)





Reference Image

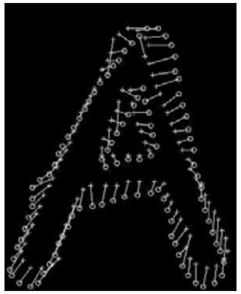
Shape
(N points)



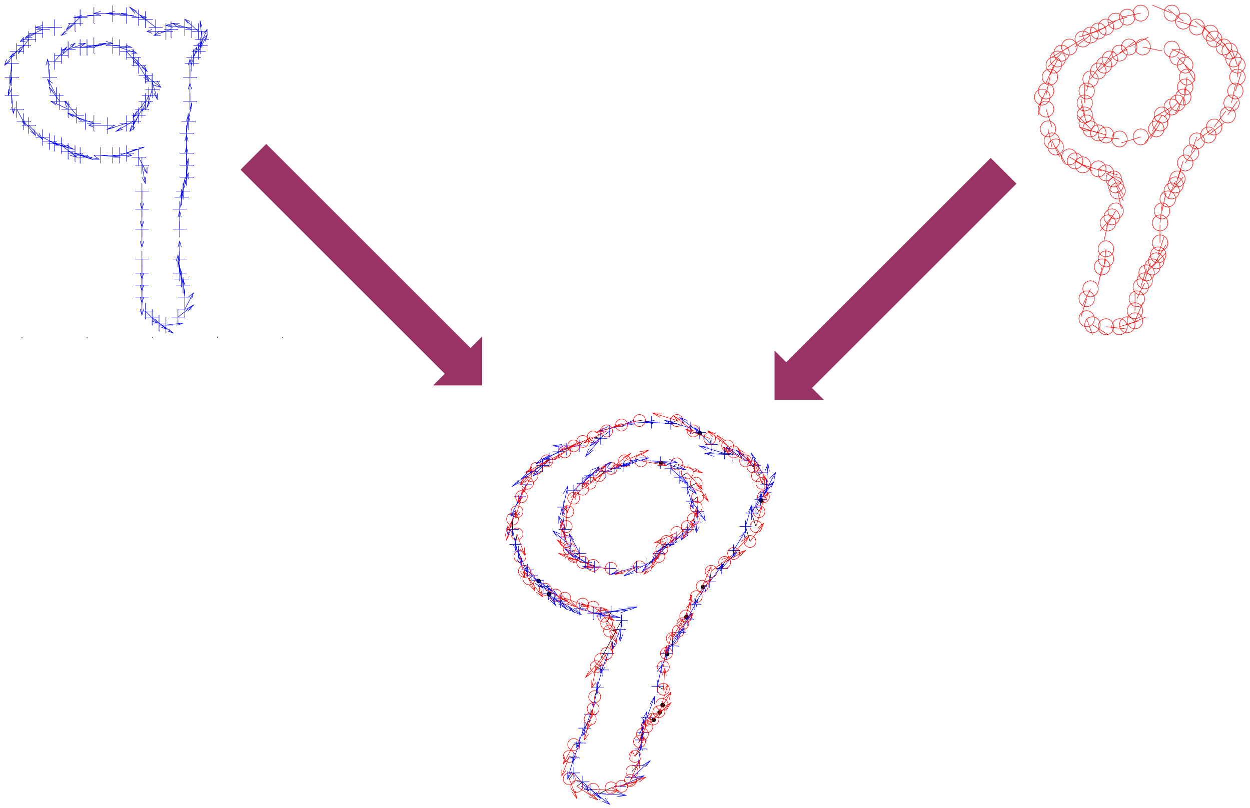
Shape Context
(N histograms)

Correspondence
(Matching cost)

Matching



Matching example



Matching

So far …

- We have our Shape Descriptor
- We know how to solve the correspondence between two Shape Descriptors:

- This will minimize the total matching cost, regardless of how similar or not the shape descriptors are:

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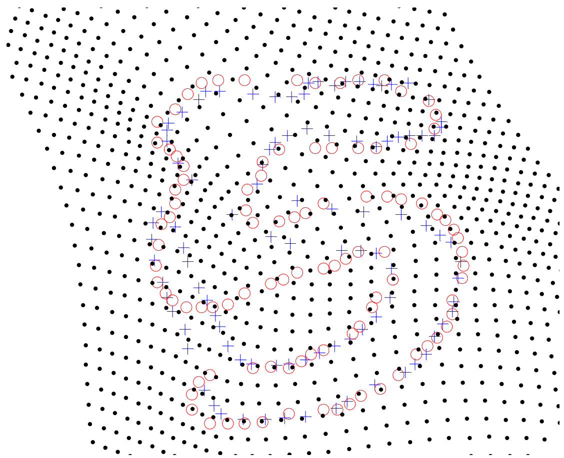
Similarity measure

- We need a measure of similarity
  - How much can we deform a shape to match it with reference shape?
  - Spatial transformation of points

$$(x', y') = T(x, y)$$



  - Thin-Plate Spline chosen
    - Affine transformation is a special case
    - Able to express organic growth in nature
    - Non-linear transformation

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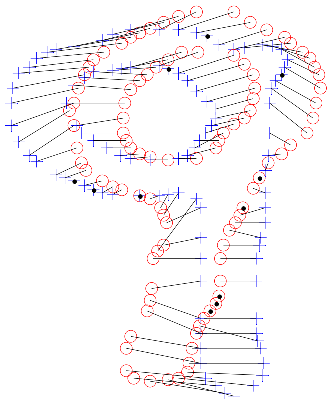
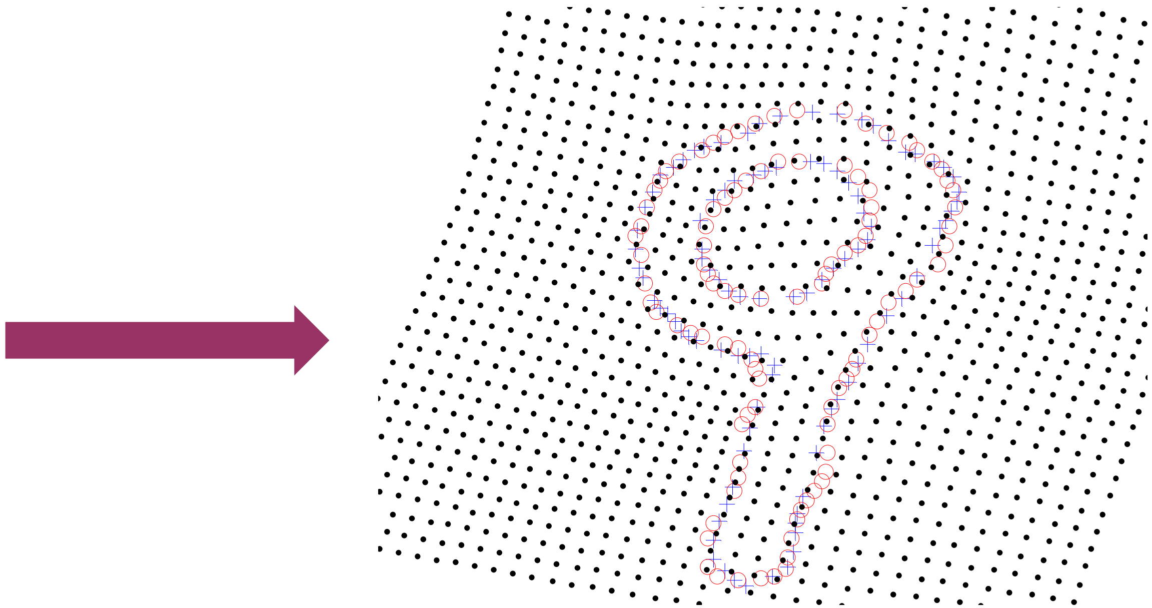
Thin plate spline (TPS)

- TPS parameter estimation is fairly easy to do
- Physical analogy to bending a thin sheet of metal
- Bending Energy expresses the 'effort' needed to 'bend' the space during the transformation
  - GREAT AS DISIMILARITY MEASURE

TPS results

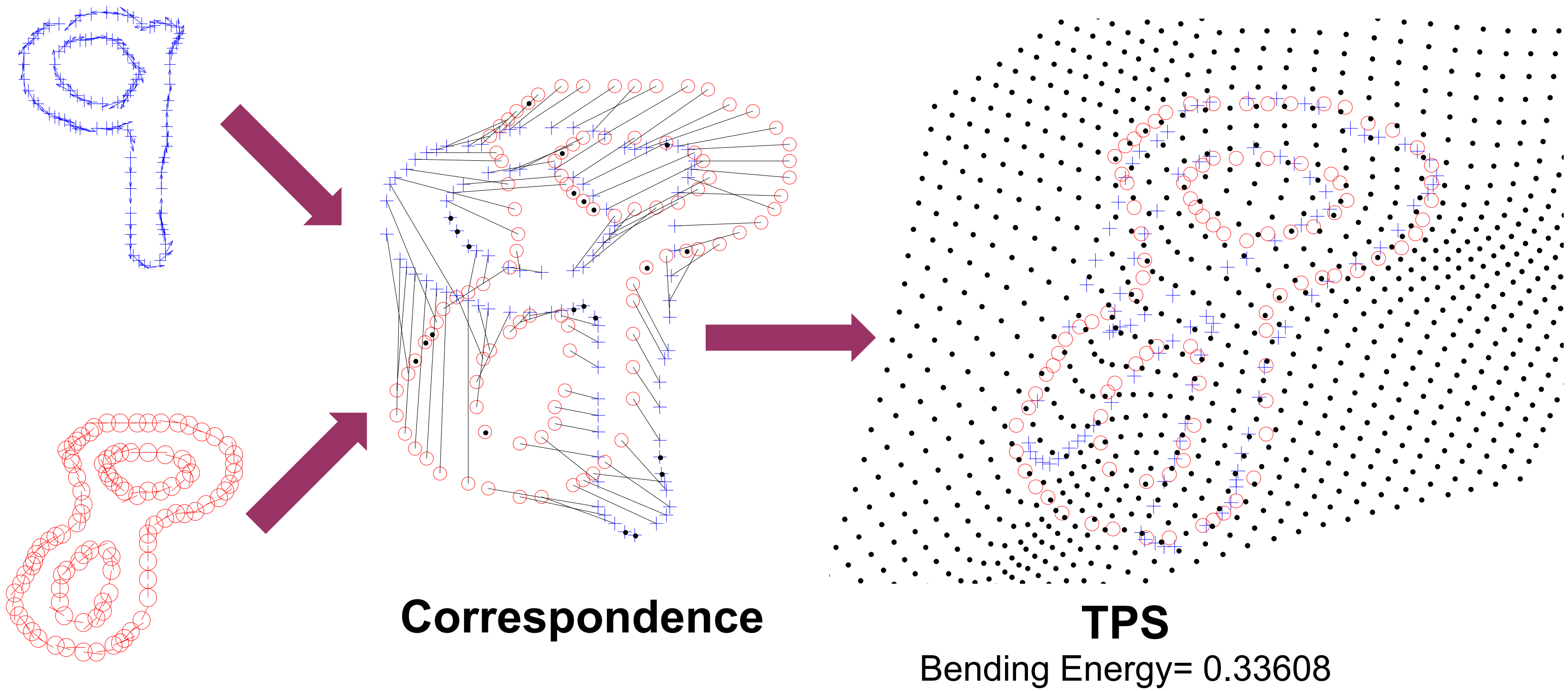


Correspondence



TPS
Bending Energy= 0.070513

TPS results

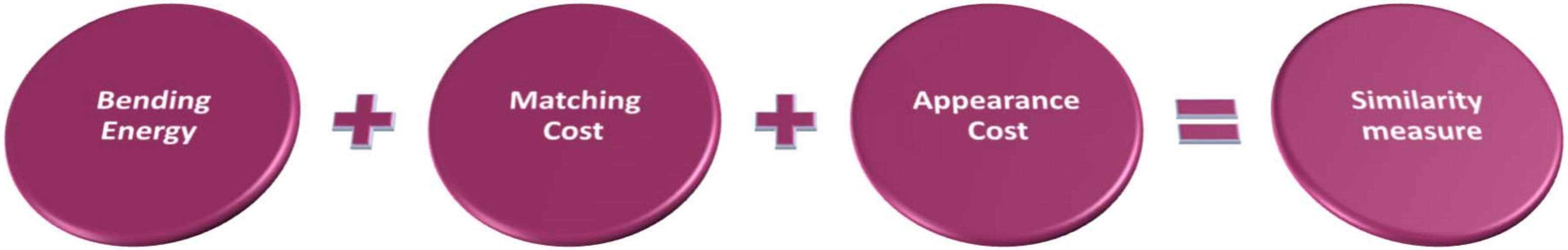


Similarity measure

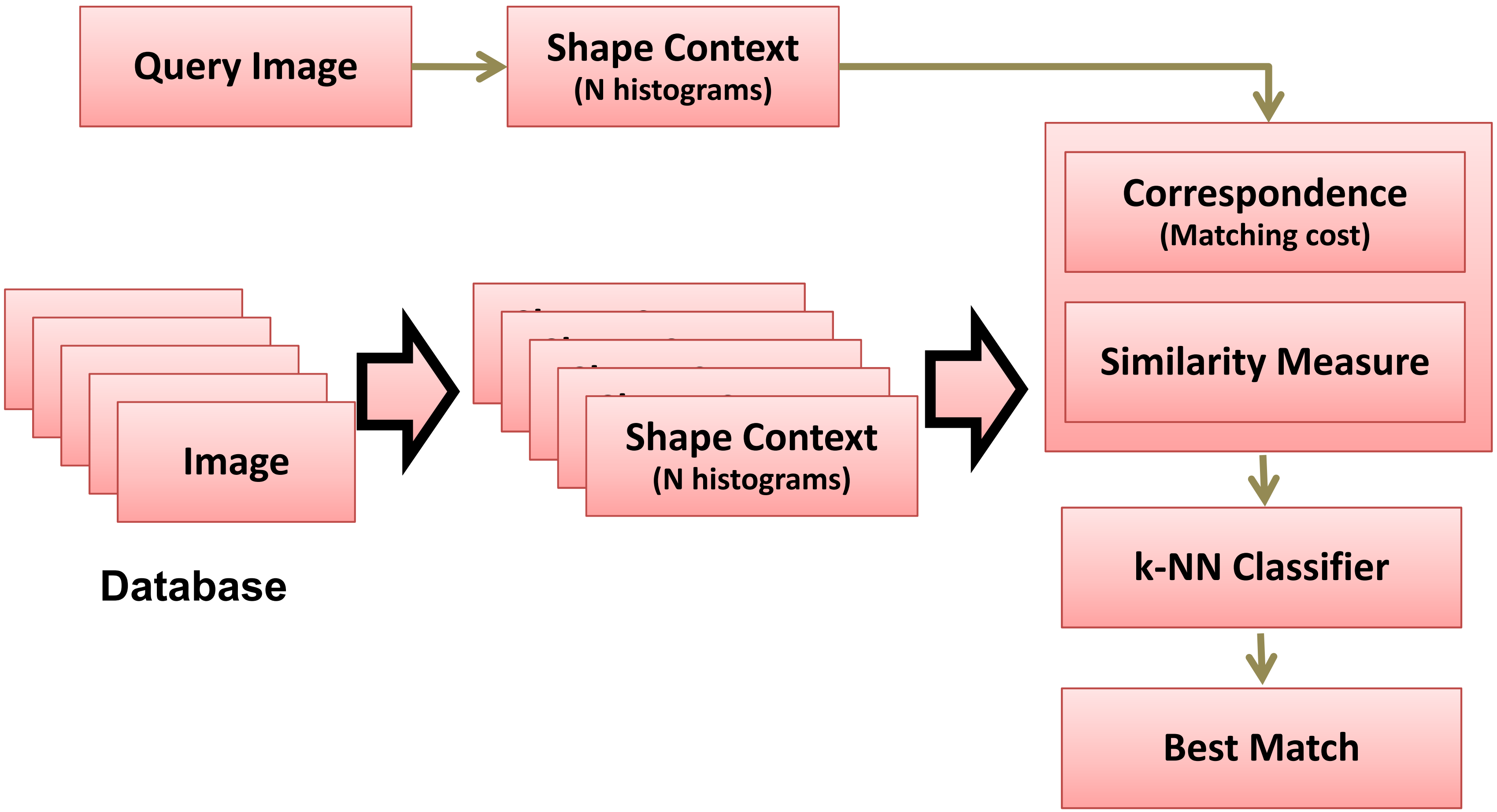
- In order to provide a consistent distance measure, three similarity measures are merged:
  - **TPS Bending Energy**
  - **Matching Cost**

    How well TPS matches one shape into the other
  - **Appearance Cost**

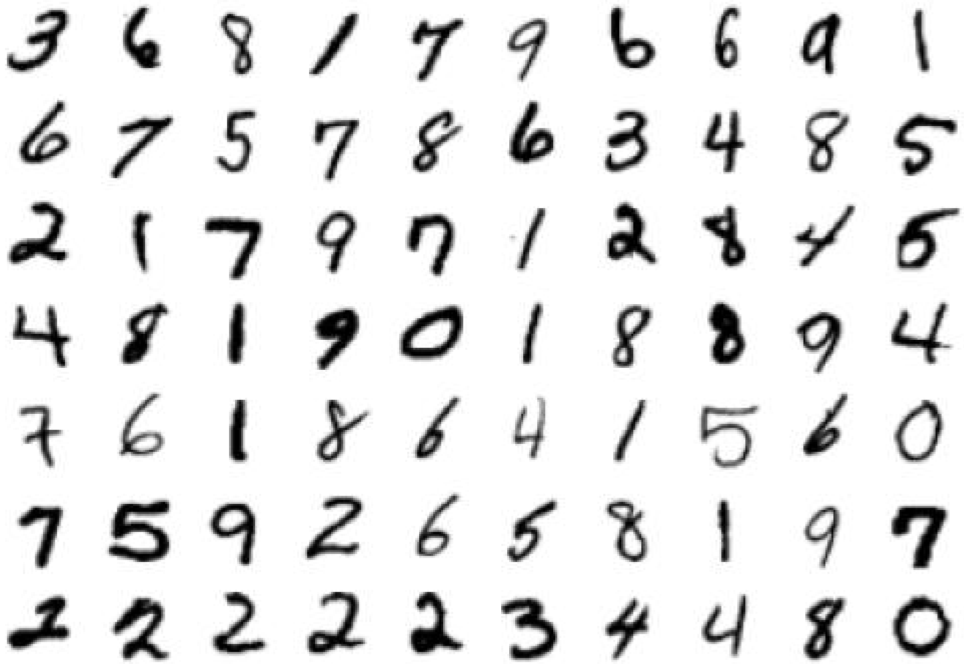
    Taking into account brightness difference around shape points



Classification process



Results (MNIST database)



- Shape Representation
  - 100 points sampled from canny edge detector
- Error Rate 0.63% with a database of 20,000 images

Conclusions

- Estimation of shape similarity and correspondence based on a novel descriptor **THE SHAPE CONTEXT**

- Shape Context is
  - Simple and intuitive shape descriptor which is easy to apply
  - Rich descriptor, greatly improving point set registration, shape matching, shape recognition
  - Invariance to several common image transformations (translation, scale, rotation, occlusions)

- Main disadvantage: a blurry, diffuse or extremely noisy image may lead to an incorrect representation of its shape (i.e.: canny)
- Problems in cluttered background