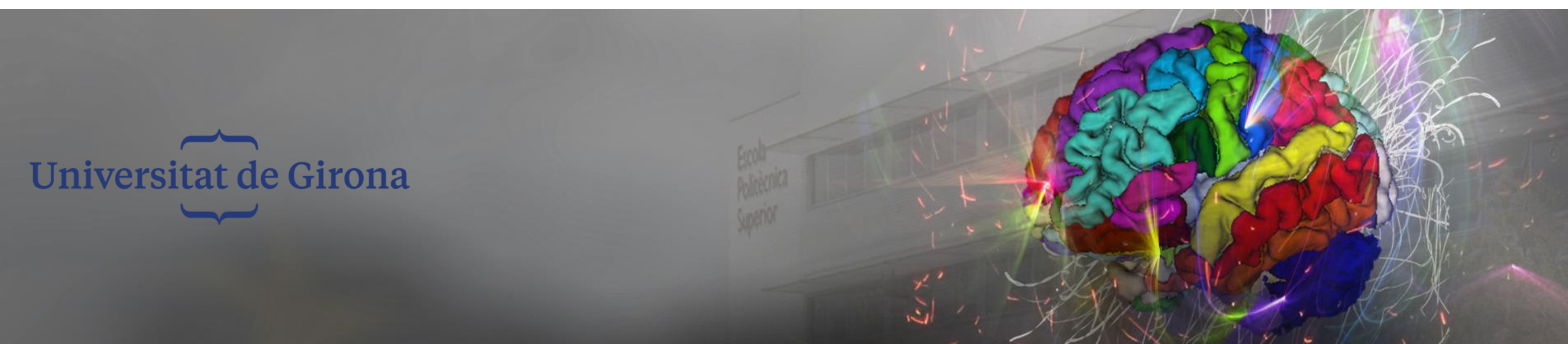
MAIA

Computer Aided Diagnosis (CAD)

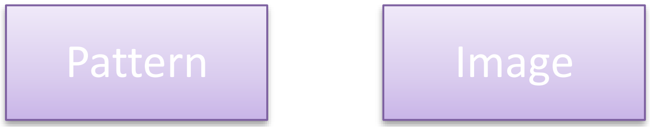
Deformable Models Segmentation



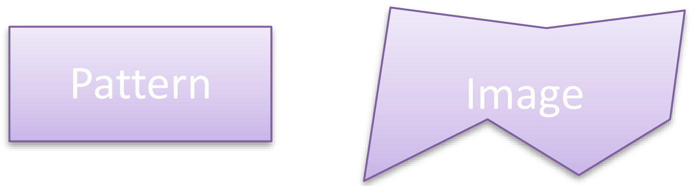
- Part 1: Template matching & deformable template matching
- Part 2: Active shape & Active appearance models
- Part 3: Active contours & level sets

Template Matching

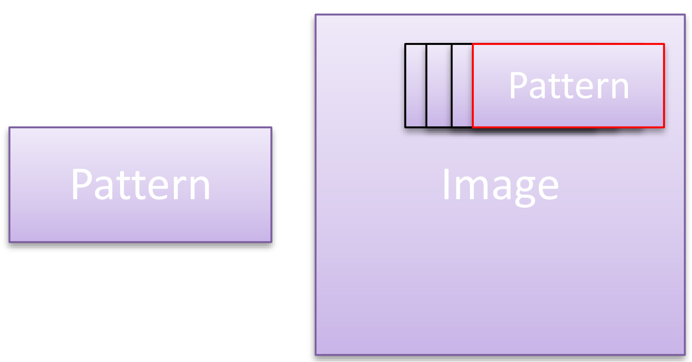
- Pattern matching is the act of checking a perceived sequence of tokens (or pixels, in computer vision) for the presence of the constituents of some pattern.



- In contrast to pattern recognition, the match usually has to be exact.



- Template matching is a technique in digital image processing for finding small parts of an image which match a template image.



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Template Matching

- Feature-based approach
  - If the template image **has strong features**
  - The approach may prove further useful if the match in the search image might be transformed in some fashion

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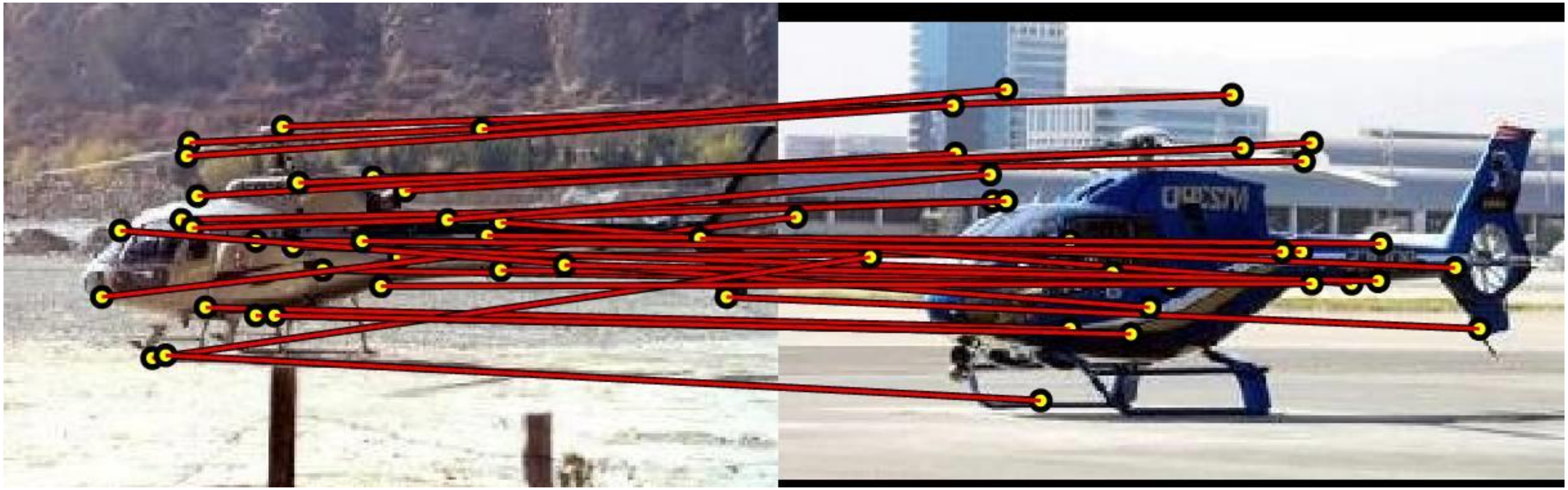
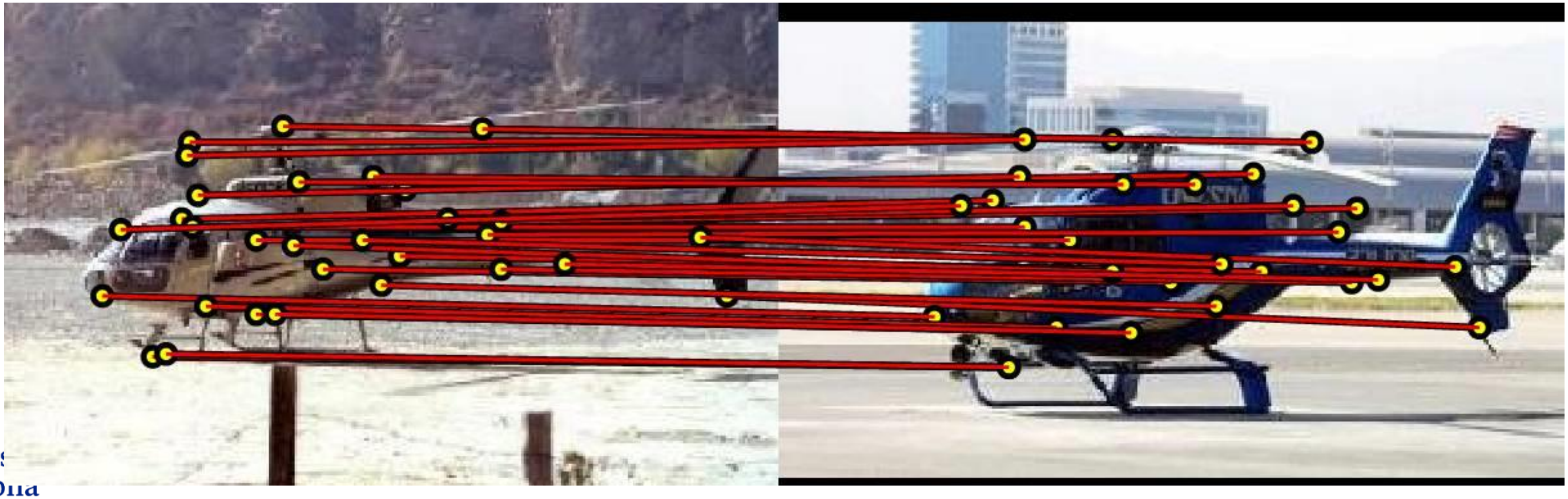
Template Matching



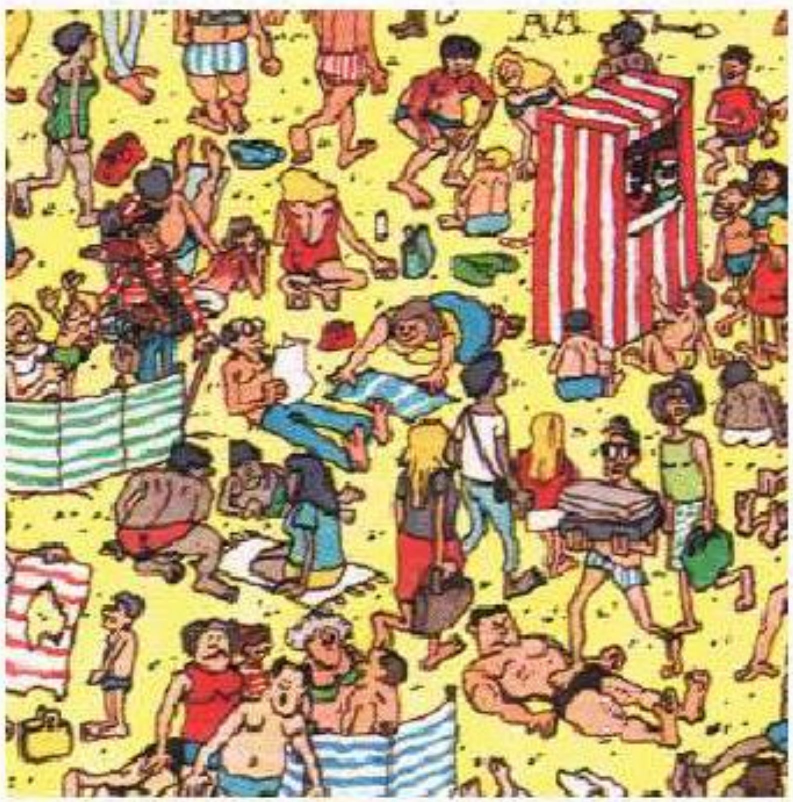
The deformable matching problem transforms to a correspondence problem



Template Matching

- Template-based approach

Template



If you find it, matching!

If you find it, matching!

Template Matching

- To do: implement a simple algorithm for Wally detection in the image.

Template Matching

- To do: implement a simple algorithm for Wally detection in the image.

- Sliding window (borders?)
- Similarity definition:
  - Difference between images
  - Correlation
  - Entropy
  - Mutual information

- … what problems do you see?

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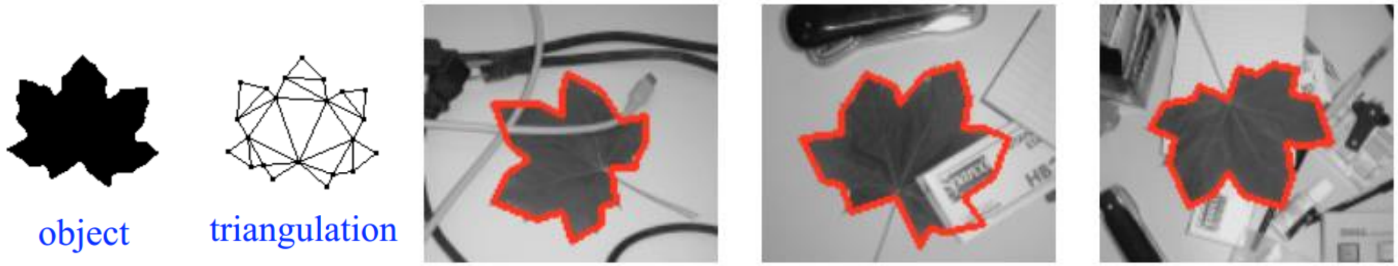
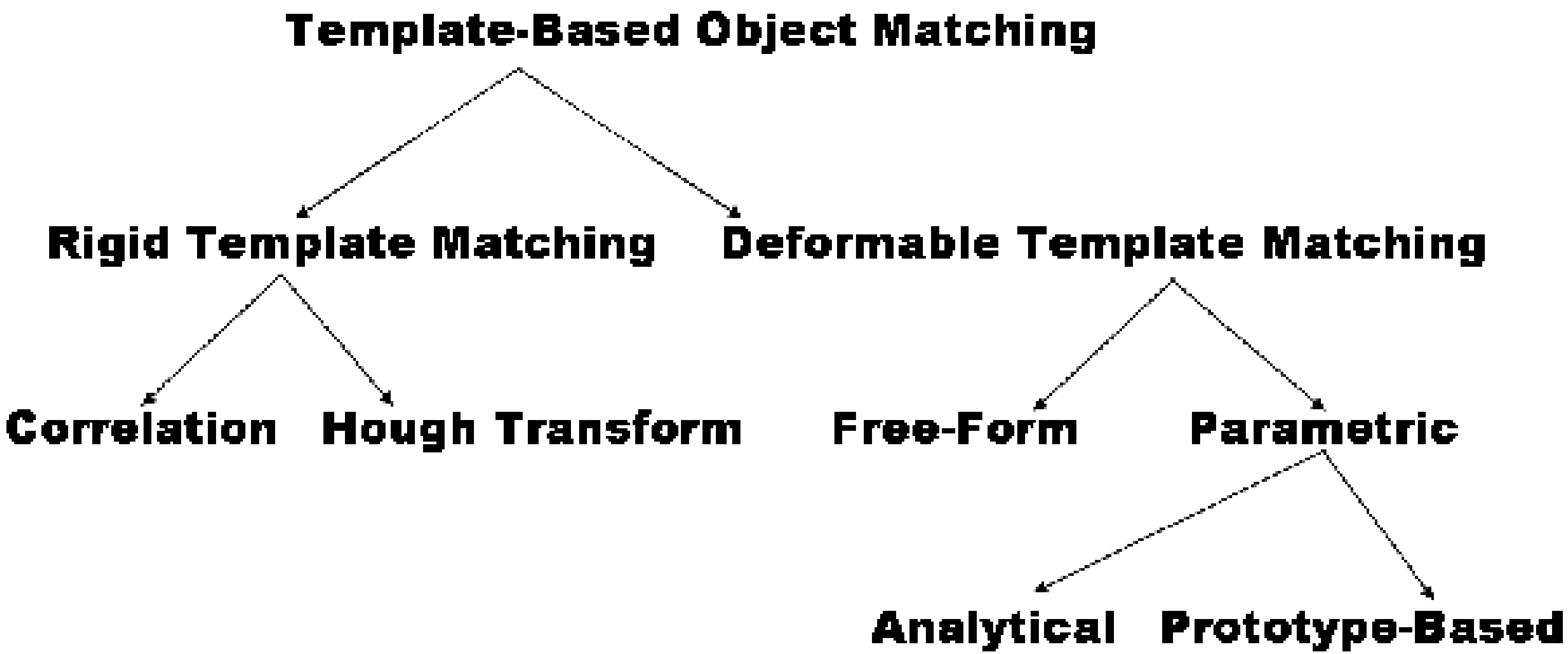
Template Matching

- Template matching from a template: which are the problems?
  - 2D rotation => add rotation as a new parameter
  - Scale => add scale as a new parameter
  - 3D rotation / perspective / orientation => we need a new view of the object or a 3D representation
  - Deformations => we need to know the ways that the object can deform
  - Occlusions => use a different similarity measure (per parts)

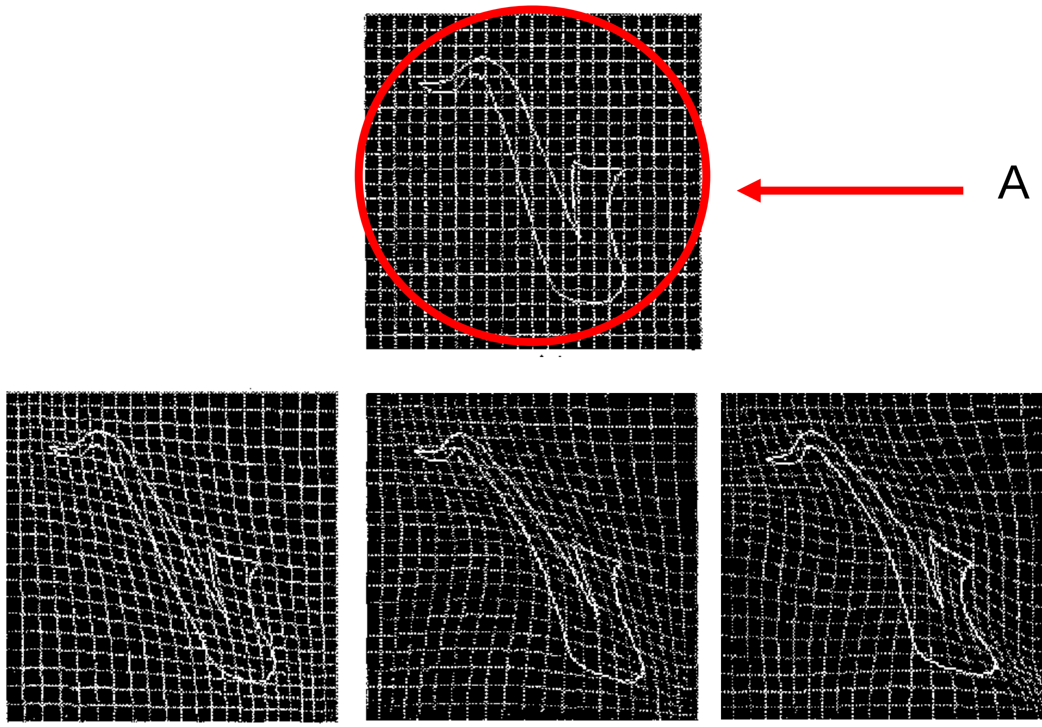


Template Matching



Deformable Template Matching

- Prototype-based segmentation [Jain et al., 1996]



A model (a sketch)

Template

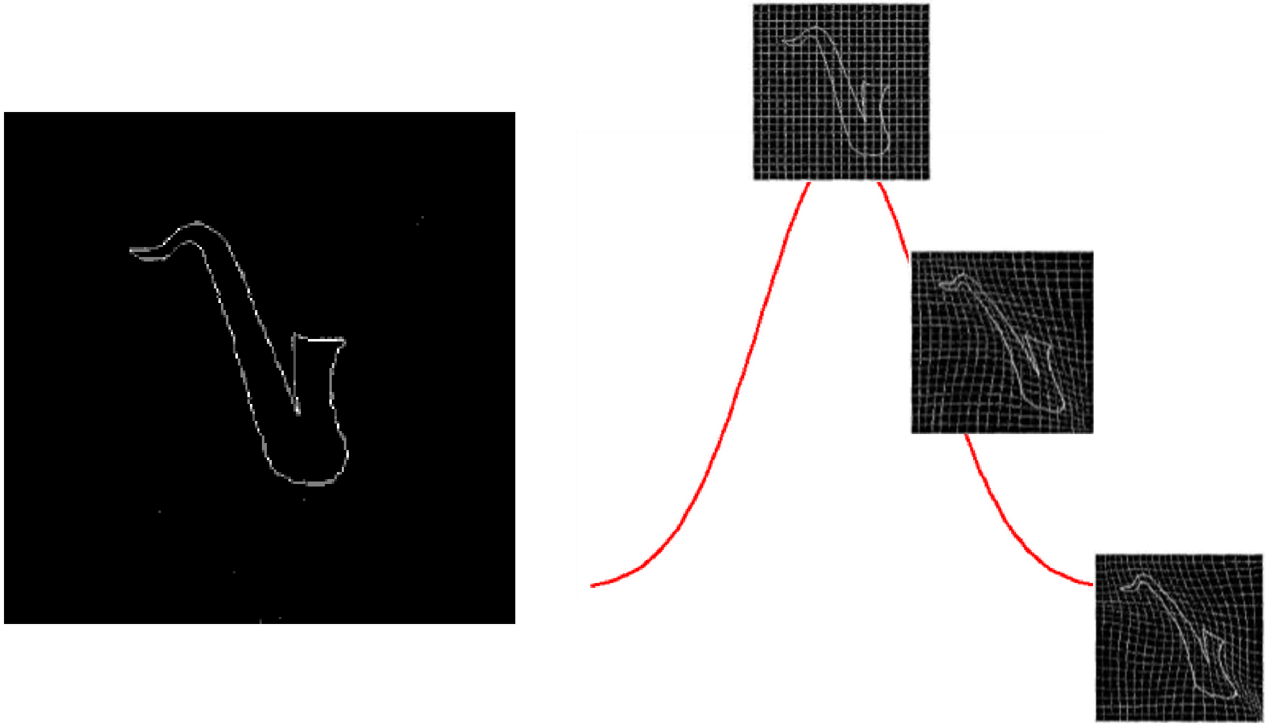
The ways the model can be deformed

Deformations

[Jain et al, 1996, Object Matching Using Deformable Templates, PAMI

Deformable Template Matching

- Uses prototype template, a set of deformation transformation, and a probabilistic model (Bayesian scheme) to find a match between the deformed template and objects in the image



$$P(T|I) = \frac{P(I|T)P(T)}{P(I)}$$

Deformable Template Matching

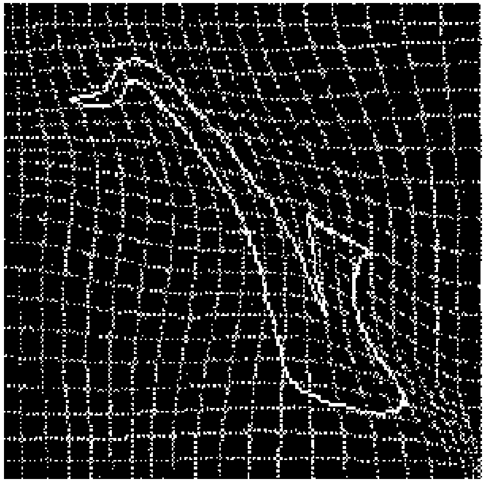
Main Idea: $(x, y) \rightarrow (x, y) + D_{\xi}(x, y)$

$$\boxed{D_{\underline{\xi}}(x, y) = \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{\xi_{mn}^{x} \cdot e_{mn}^{x} + \xi_{mn}^{y} \cdot e_{mn}^{y}}{\lambda_{mn}}}$$

$$e_{mn}^{x}(x, y) = (2 \sin(\pi n x) \cos(\pi m y), 0)$$

$$e_{mn}^{y}(x, y) = (0, 2 \cos(\pi m x) \sin(\pi n y))$$

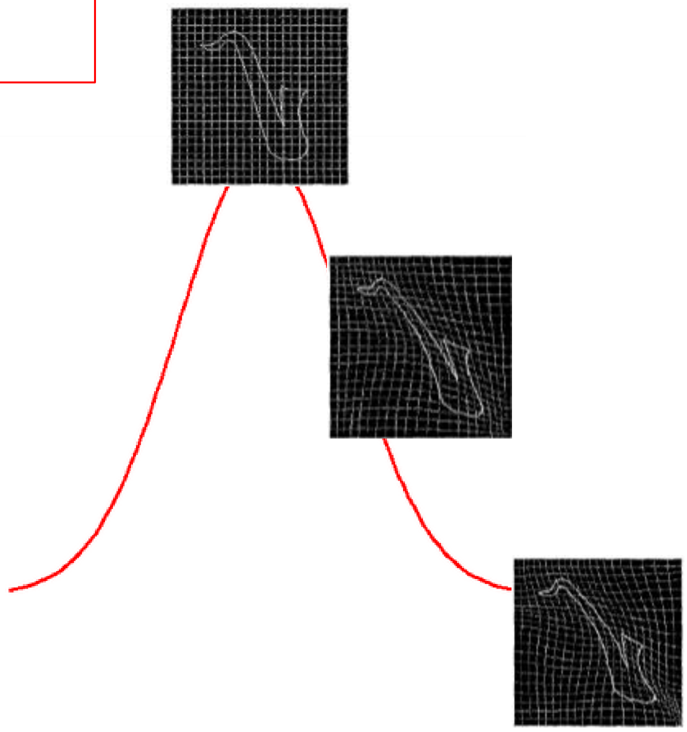
$$\underline{\xi} = \{\xi_{mn}^{x}, \xi_{mn}^{y}, m, n\}$$



Deformable Template Matching

- Gaussian distribution to biased the possible deformed templates.

$$P_r\left(\underline{\xi}\right) = \frac{1}{(2\pi\sigma^2)^{MN}} e^{-\frac{1}{2\sigma^2}\sum_{m,n}\left({\xi_{mn}^{x}}^2 + {\xi_{mn}^{y}}^2\right)}$$

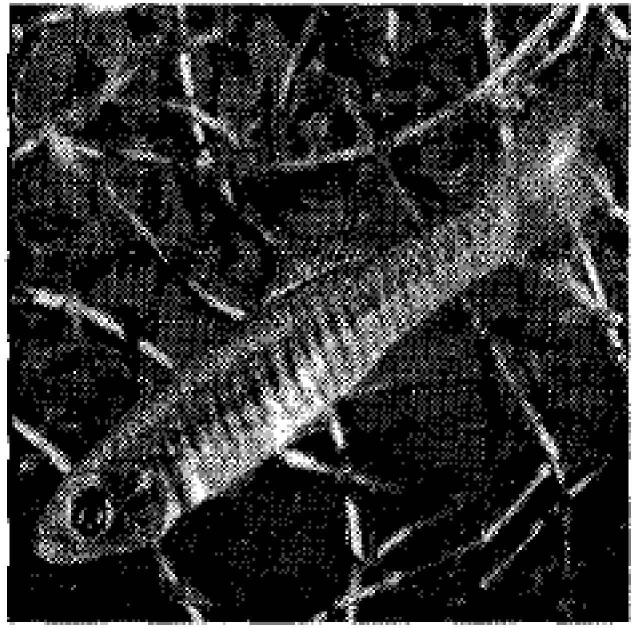


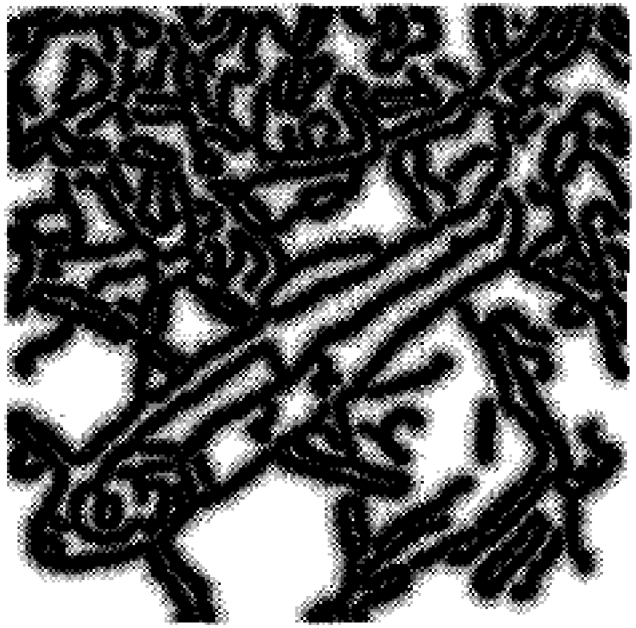
Deformable Template Matching

- The likelihood has the image information:

$$P_r\left(I \middle| s,\theta,\underline{\xi},\underline{d}\right) = \alpha e^{-\frac{1}{n_t}\sum(1+\Phi(x,y)|\cos(\beta(x,y)))}$$

$$\Phi(x,y) = -\exp\left\{-\rho\left(\delta_x^2+\delta_y^2\right)^{1/2}\right\},$$





Deformable Template Matching

- The Bayesian schema is employed to integrate the prior knowledge of the template and the observed object in the input image.

- We want to find the probability of observing the template (T) given the image (I):

$$\boxed{P(T|I) = \frac{P(I|T)P(T)}{P(I)}}$$

Maximising P(T|I) is equivalent to the minimise:

$$\mathcal{L}\left(\mathcal{T}_{s,\Theta,\xi,d}, Y\right) =$$

$$\mathcal{E}\left(\mathcal{T}_{s,\Theta,\xi,d}, Y\right) + \gamma \sum_{m=1}^{M} \sum_{n=1}^{N} \left({\xi^{x}_{mn}}^{2} + {\xi^{y}_{mn}}^{2} \right)$$

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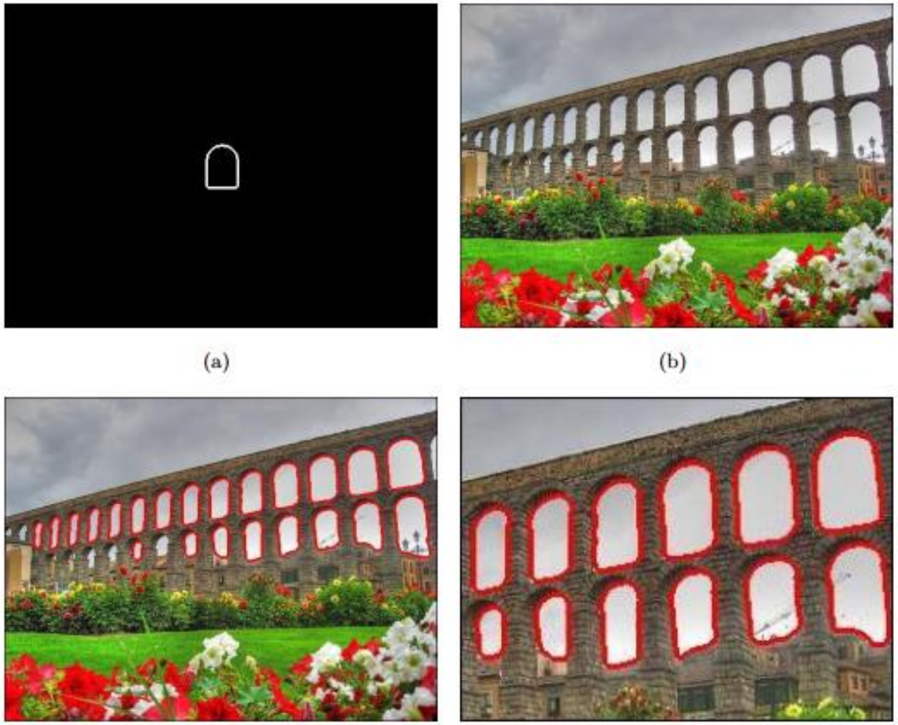
Deformable Template Matching

- Consist of a coarse-to-fine matching:
- First: roughly locate the global optima efficiently without regard to accuracy.
- The finer stage levels initialized using the good candidates screened from the previous stage.



(a) (b) (c) (d)

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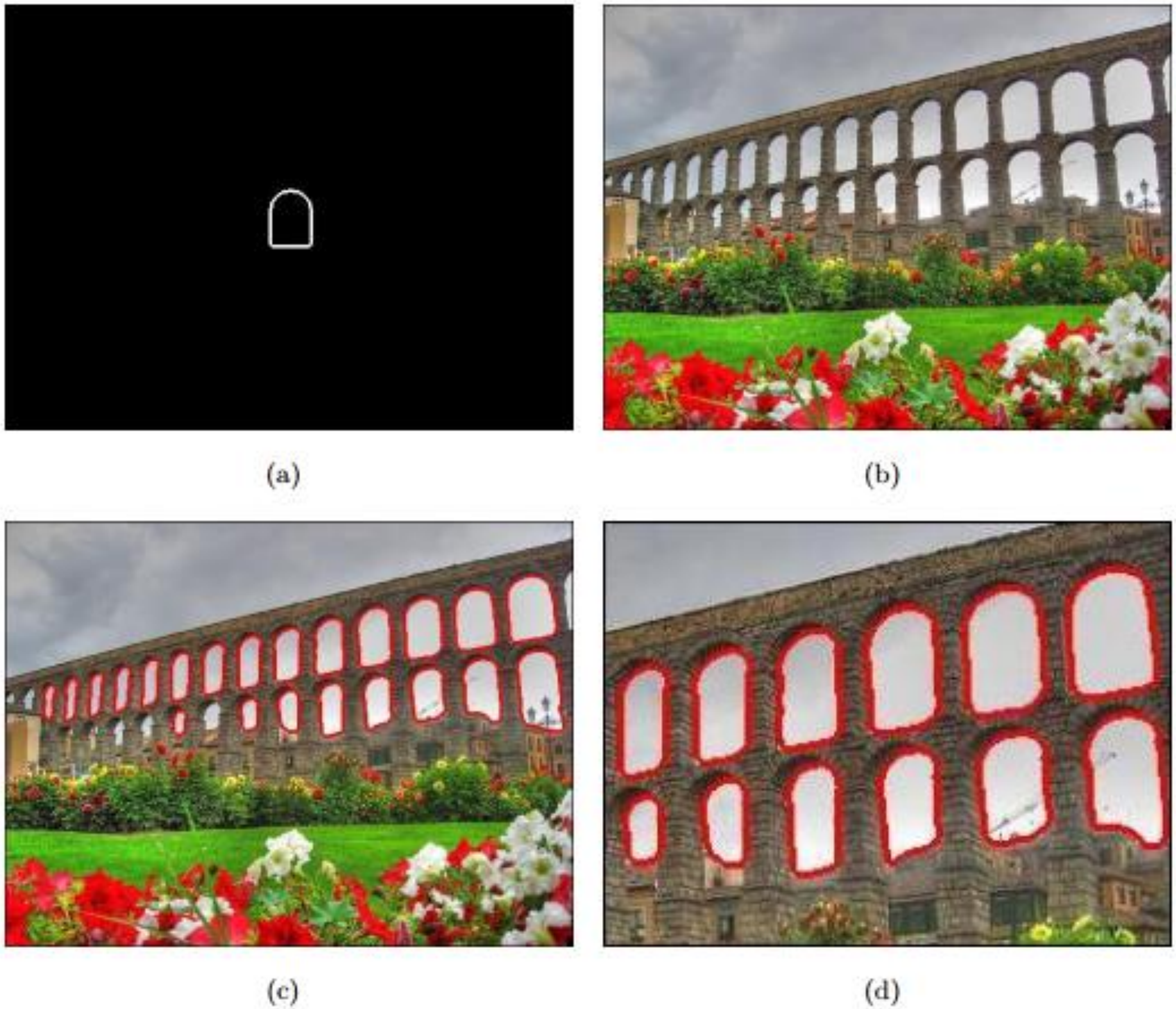
Deformable Template Matching

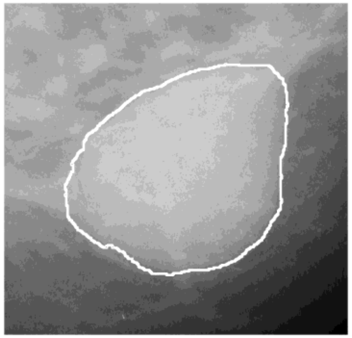
- Consist of a coarse to fine matching:
- Fir… …ut reg…
- Th… …ates scr…

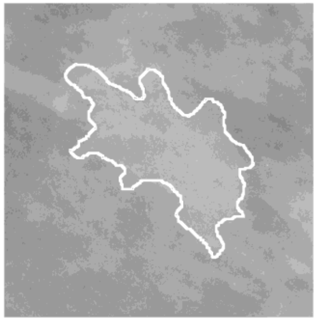


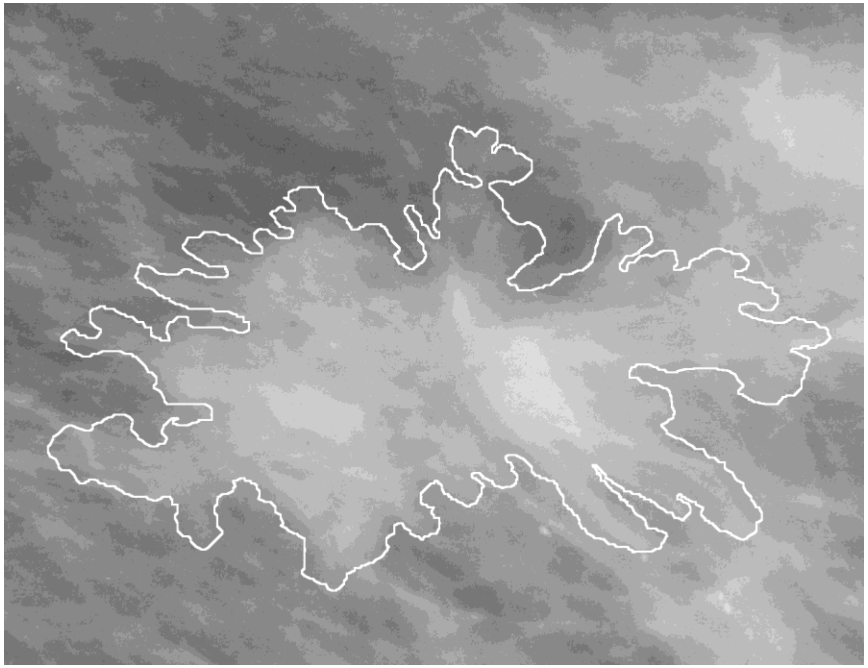
(a) (b) (c) (d)

Deformable Template Matching

- How can we do this for masses?
  - There is a large range of mass shapes and sizes







[Oliver PhD, 2007]

[Freixenet et al. 2008] Eigendetection of masses considering false positive reduction and breast density information. Medical Physics

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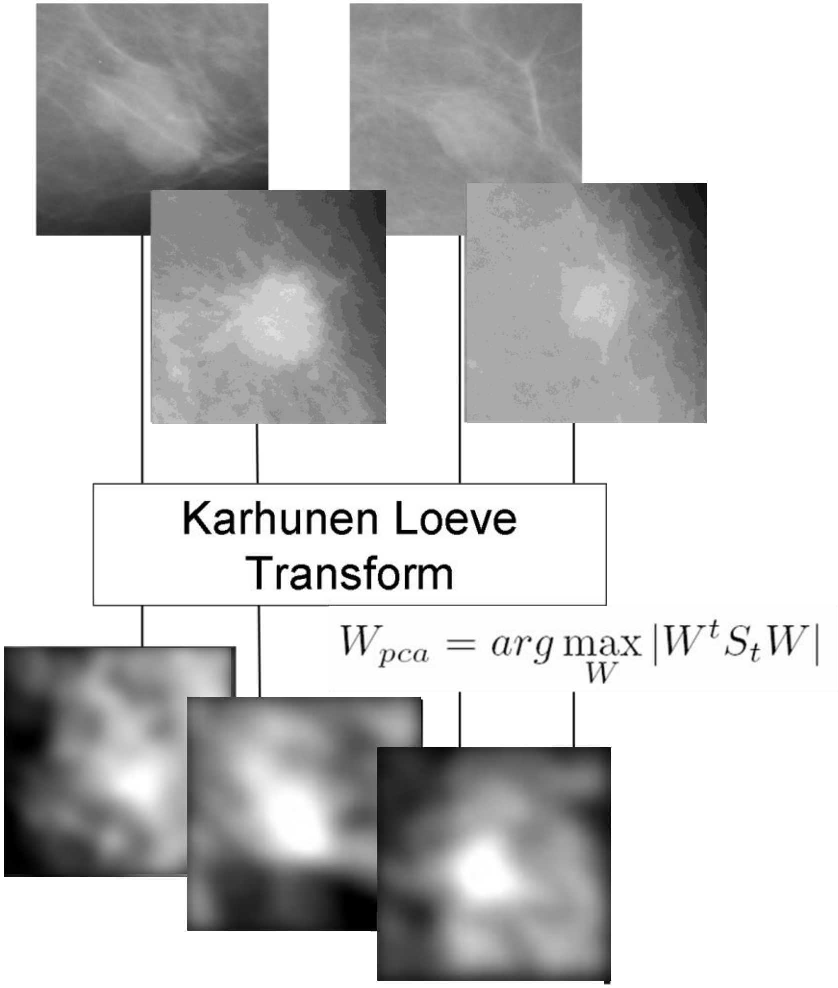
Deformable Template Matching

- We adapted the eigenfaces approach*



Initial mass database

PCA transform

Eigenmasses

Deformable Template Matching

- Eigenfaces are calculated following these steps:
  - 1) Choose a (representative) training set
  - 2) Transform the images in a single column-vector
  - 3) Calculate the mean vector and subtract it to all vectors
  - 4) Calculate the covariance matrix
  - 5) Calculate its eigenvectors and eigenvalues
  - 6) Choose the principal components
  - 7) Optionally, re-arrange the principal vectors in images

Eigenmasses

Deformable Template Matching

- Mathematically, the template is defined as:

$$\psi^0(x,y) = \frac{1}{N}\sum_{k=1}^{N} w_k W_k(x,y)$$

- But we use only the contours of the eigenmasses

$$\nabla\psi^0(x,y) = \nabla\left\{\frac{1}{N}\sum_{k=1}^{N} w_k W_k(x,y)\right\} =$$

$$= \frac{1}{N}\sum_{k=1}^{N} w_k \nabla W_k(x,y)$$

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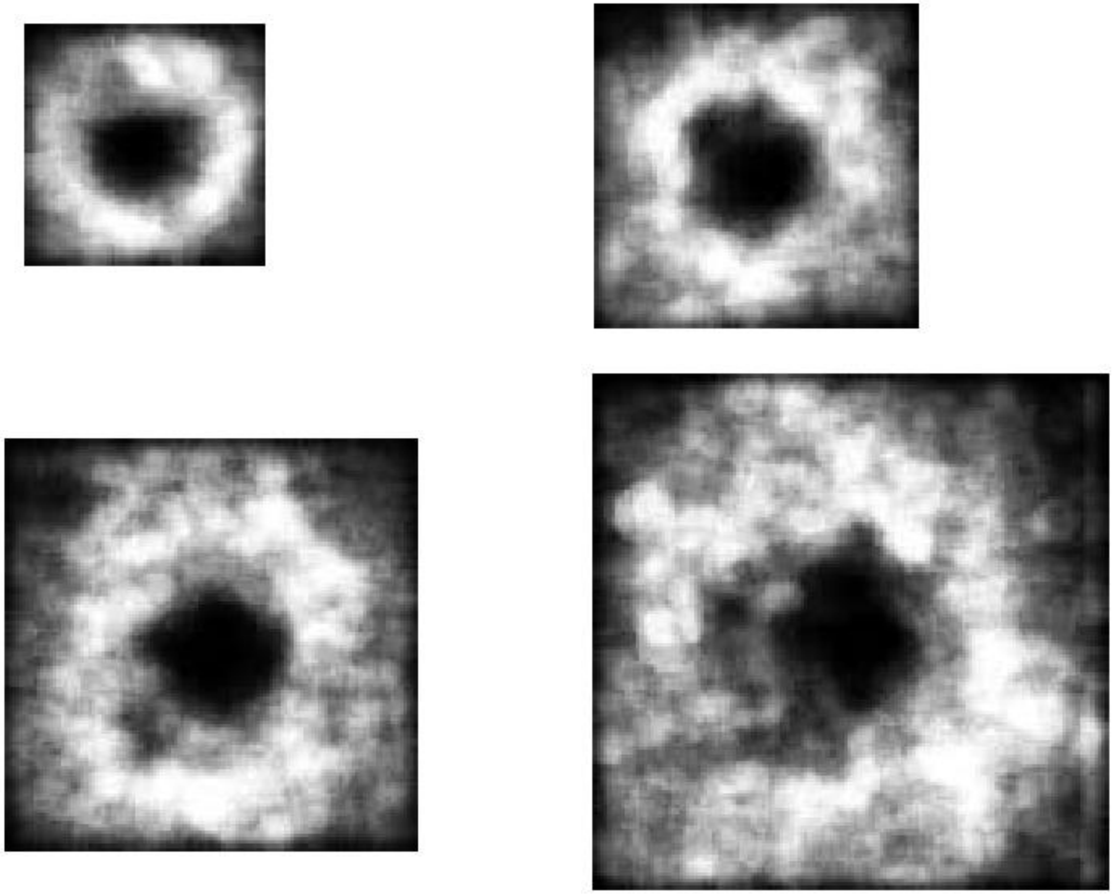
Deformable Template Matching

- We cluster the database according to a set of sizes. Thus, we have a model per size



Deformable Template Matching

- The template is the model. But, what about the deformations?

$$\nabla \psi^d(x,y) = \kappa \sum_{k=1}^{N} \xi_k w_k \nabla W_k(x,y)$$

- We assume that masses are linear combinations of the eigenmasses. Hence, we introduced a vector of coefficients which modifies the template

- Once we have defined the template and its plausible deformations, we have to perform the matching step

Deformable Template Matching

- To match the model into new mammograms we follow a probabilistic Bayesian strategy

$$posterior = \frac{prior * likelihood}{evidence}$$

- The prior distribution is used to bias the global and local deformations

$$Pr(s,d,\xi) = K \exp\{-\frac{1}{2\sigma^2}\sum_{k=1}^{N}(\xi_k - 1)^2\}$$

- A deformed template with a geometric shape similar to the prototype template is favoured

Deformable Template Matching

- The likelihood is a measurement of the similarity between the deformed template and the object(s) present in the image

- As we have defined a contour-based template, we have to obtain a contour-enhanced mammogram representation

$$\phi_{\mathrm{Y}}(x, y) = -\exp(-\rho\sqrt{\delta_x^2 + \delta_y^2})$$

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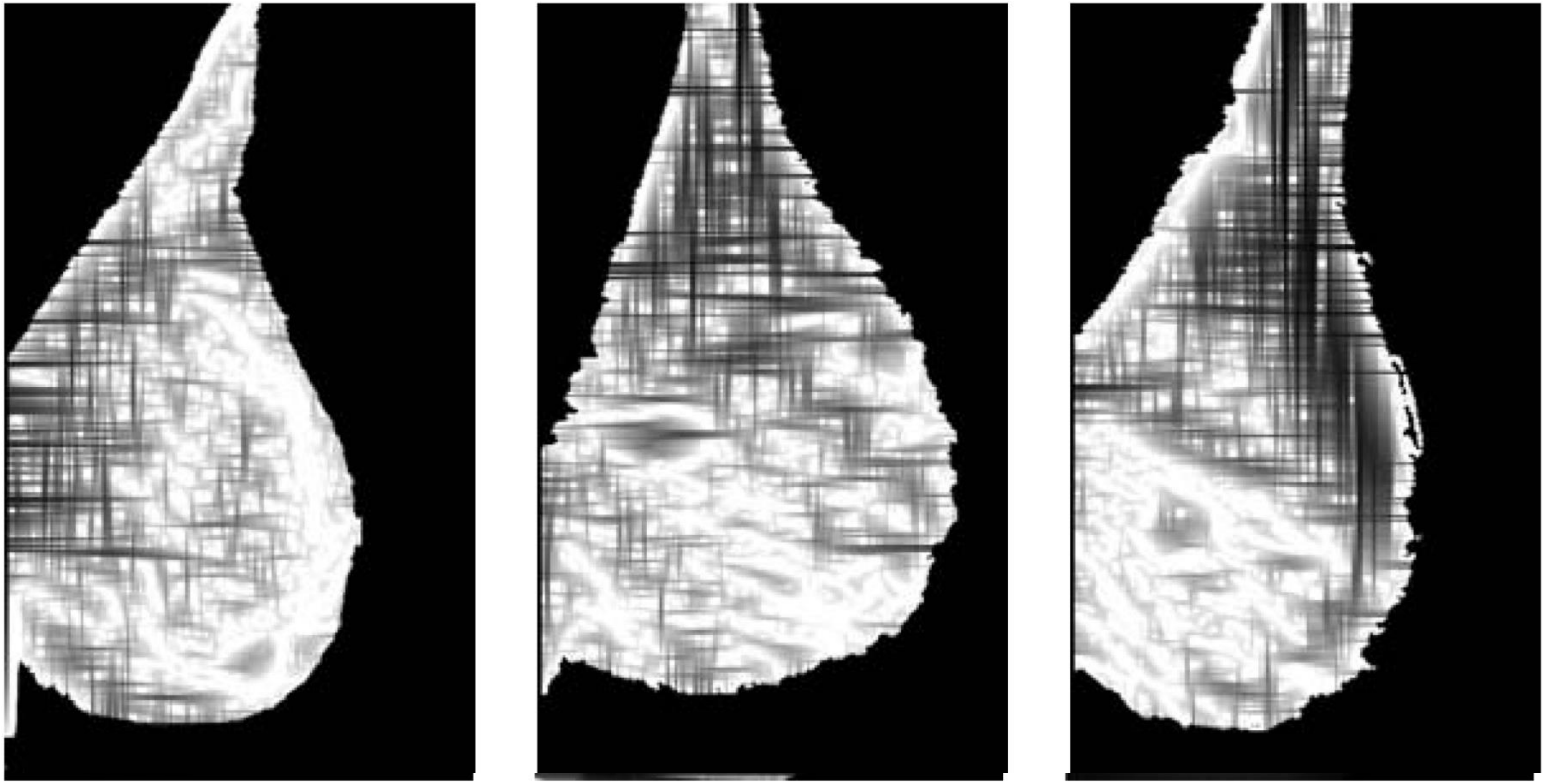
Deformable Template Matching

- Potential images



Deformable Template Matching

- Likelihood probability

$$Pr(Y|s, d, \xi) = \alpha \exp\{-\Upsilon(\psi^{s,\xi,d}, Y)\}$$

- Where

$$\Upsilon(\psi^{s,\xi,d}, Y) = \frac{1}{T} \sum_{x,y \in \psi^{s,\xi,d}} (1 + \phi_Y(x, y)|cos(\beta(x, y))|)$$

- This definition requires that the template boundary agrees with the image edges not only in position, but also in the tangent direction

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Deformable Template Matching

- The posterior probability is what we want

$$Pr(s,d,\xi|Y) = \frac{Pr(s,d,\xi)\ Pr(Y|s,d,\xi)}{Pr(Y)}$$

- Doing the operations we found that maximising the posterior was equivalent to minimise the following equation

$$\Lambda(\psi^{s,\xi,d}, Y) = \boxed{\sum_{k=1}^{N} (\xi_k - 1)^2} + \boxed{\Upsilon(\psi^{s,\xi,d}, Y)}$$

Related to the deviation of the templates from the prototype

Related to the fitness of the deformed template to the boundaries

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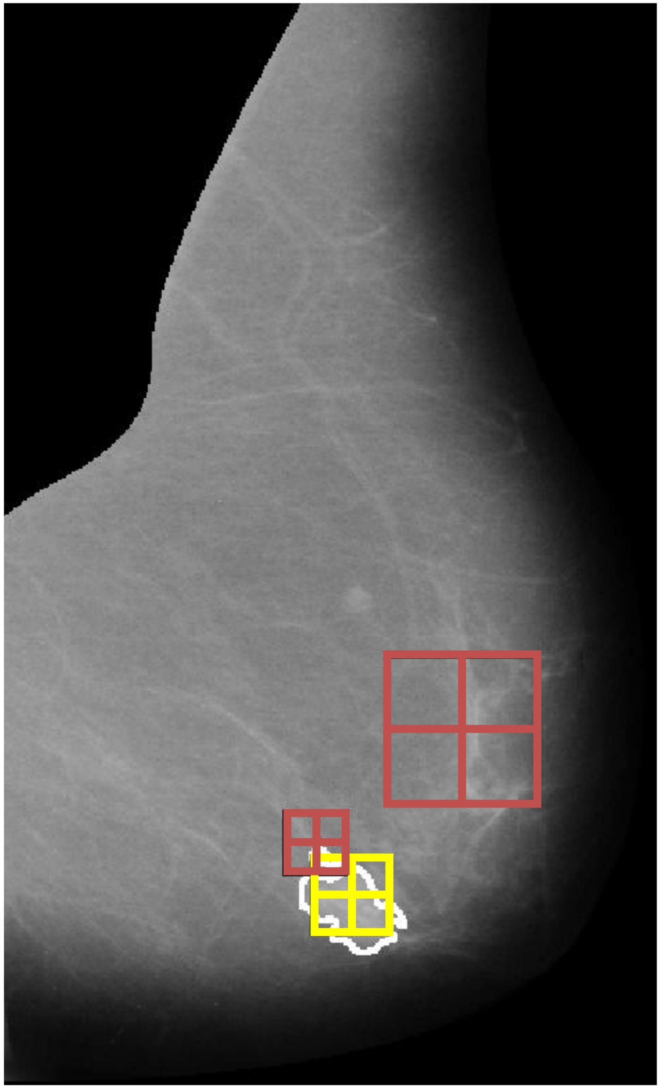
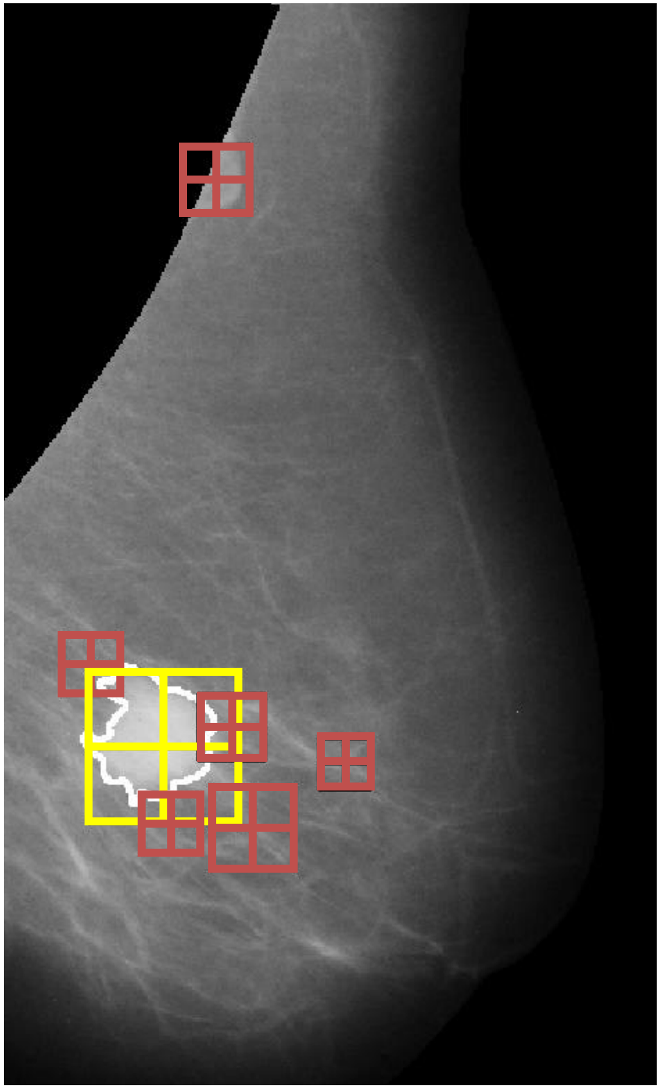
Deformable Template Matching

- Examples of the algorithm (put a dense mammo):





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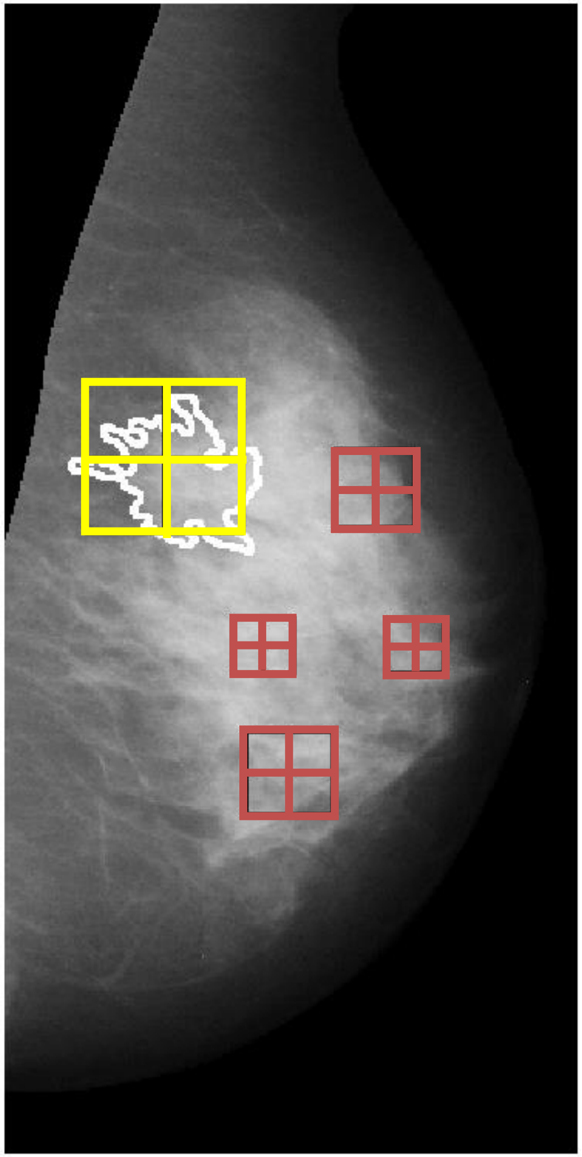
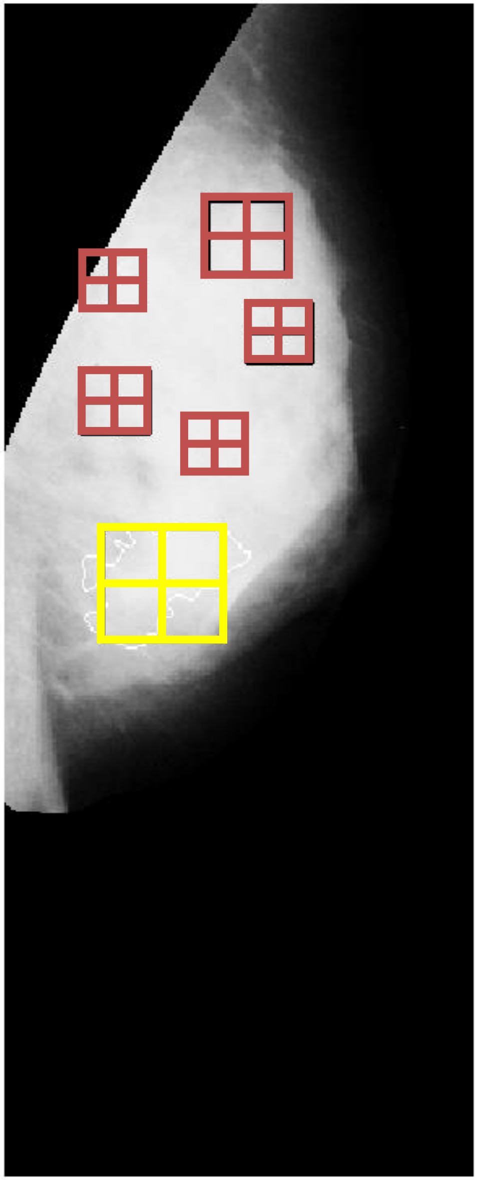
Deformable Template Matching

- Examples of the algorithm:





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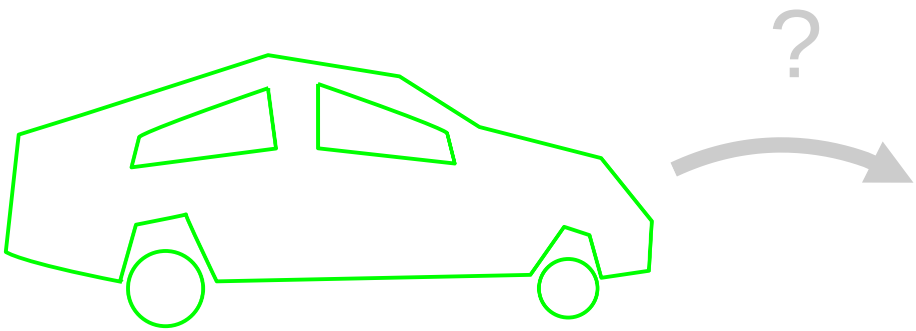
Template Matching by Part Models

- The modern approach is dividing the model by parts:

Image

Model of Car



Template Matching by Part Models

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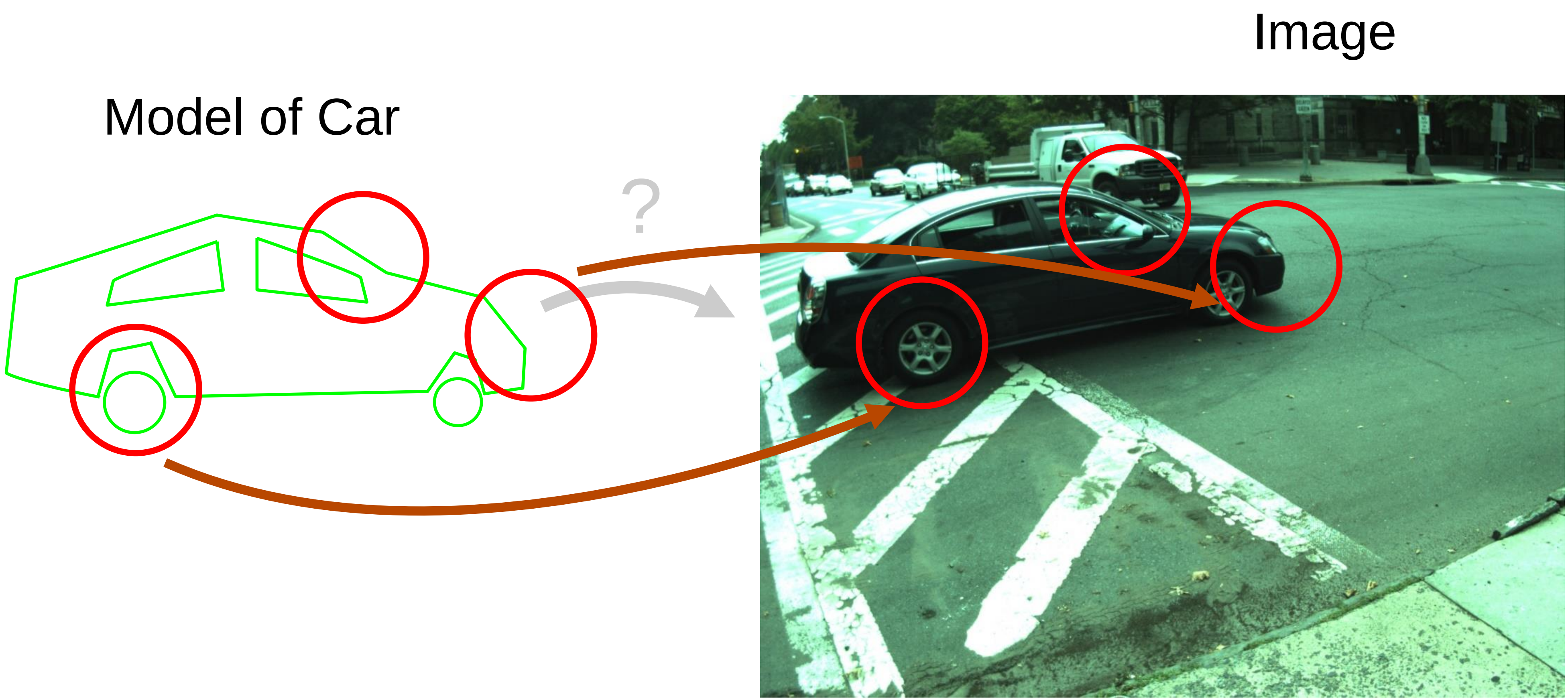

Template Matching by Part Models

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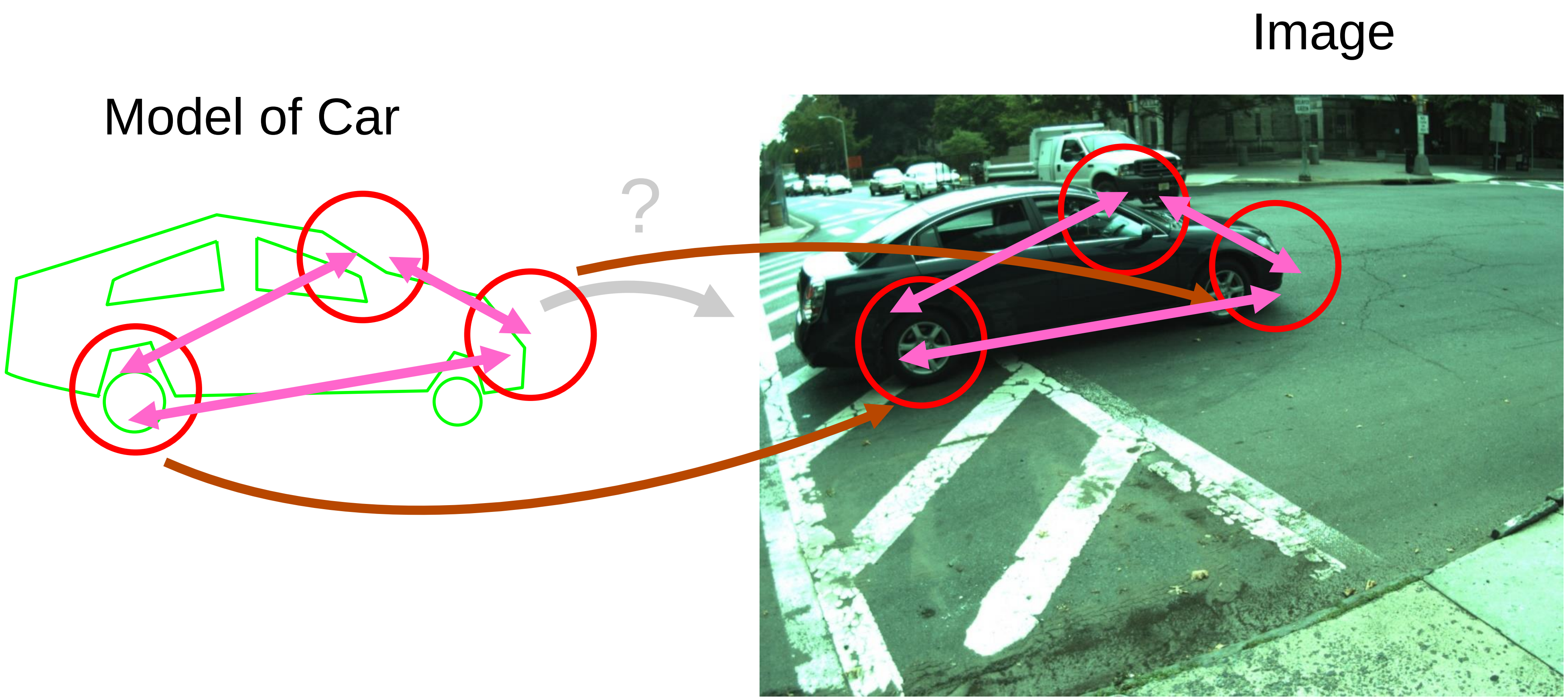

Template Matching by Part Models

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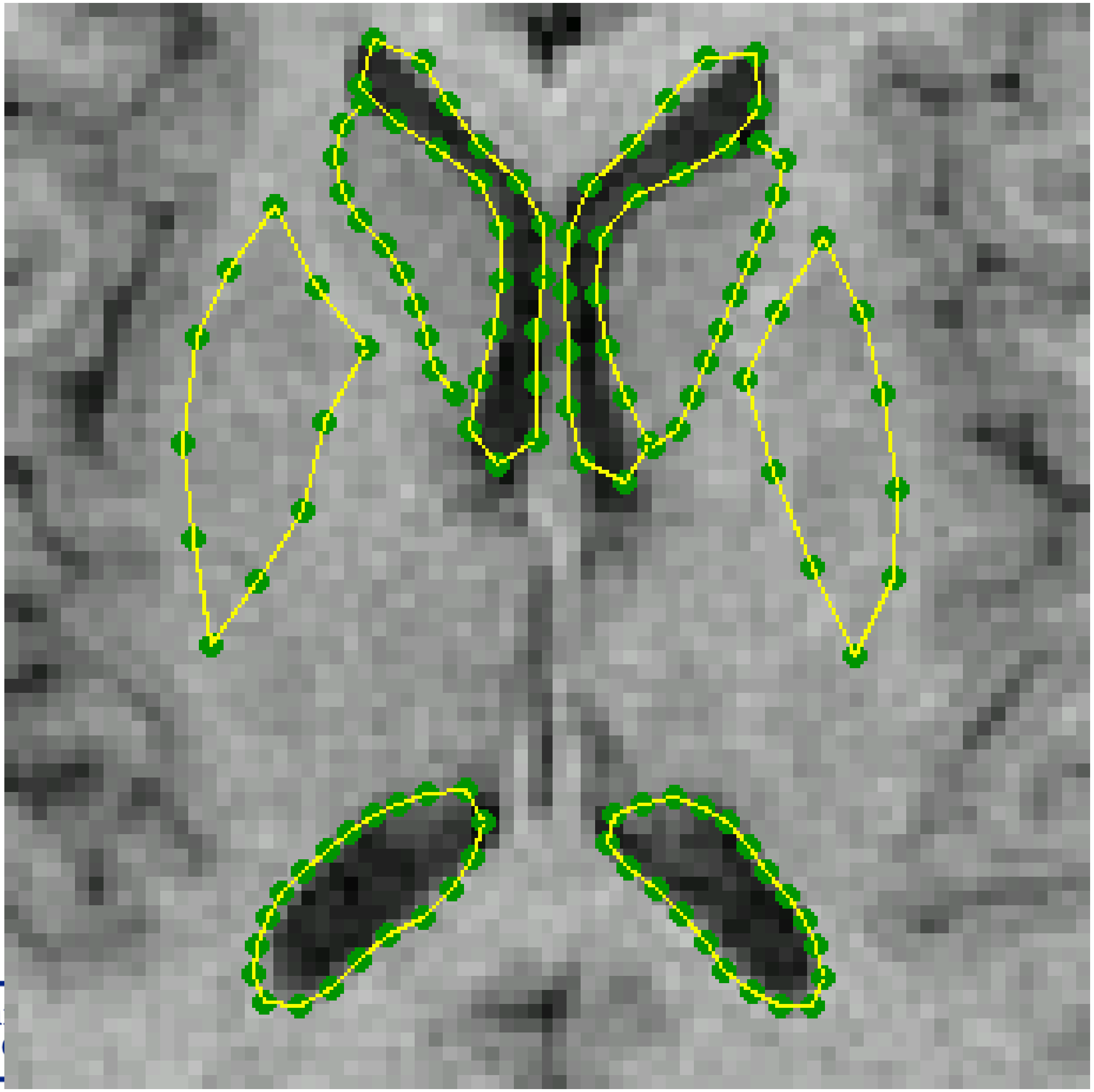
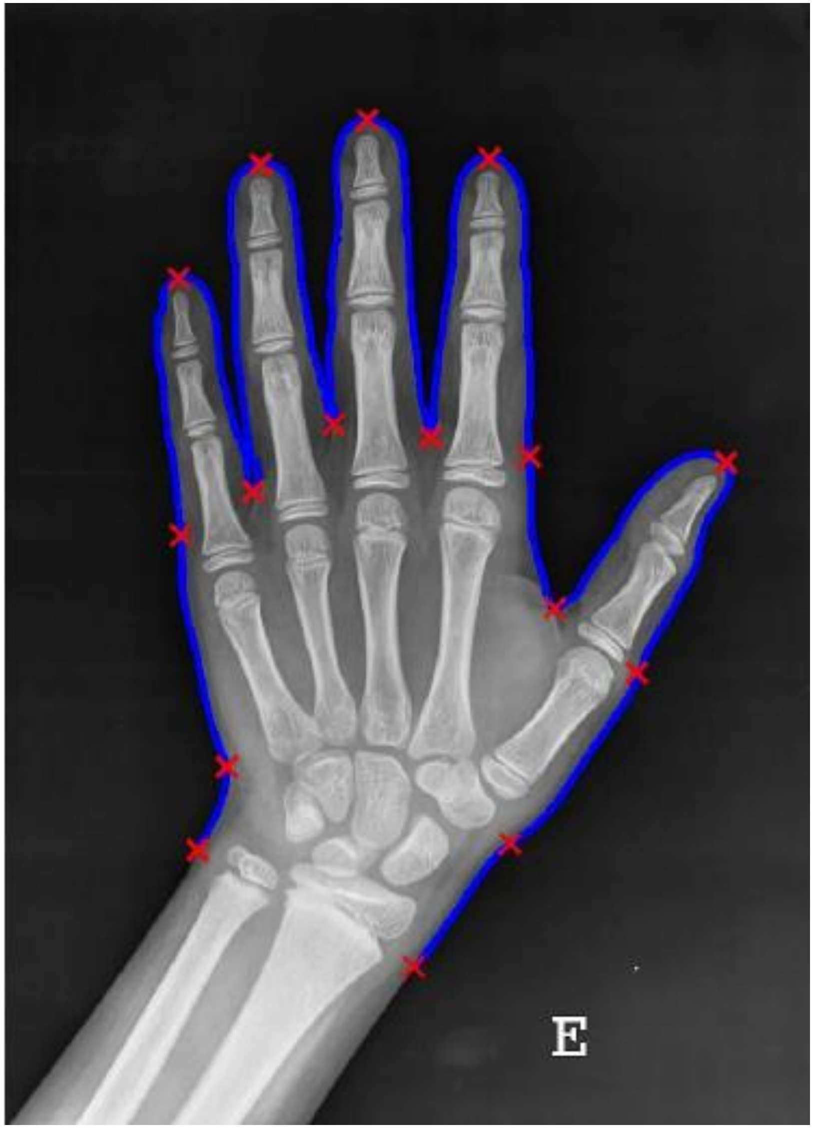
Active Shapes Models

- A deformable template matching approach but using a different way for creating the model
- Developed by Tim Cootes, Active Shape Models-Their Training and Application, CVIU, 1995
- Posterior improvement: Active appearance models, PAMI, 2001
- Widely used in medical imaging

Active Shapes Models

- Widely used in medical imaging





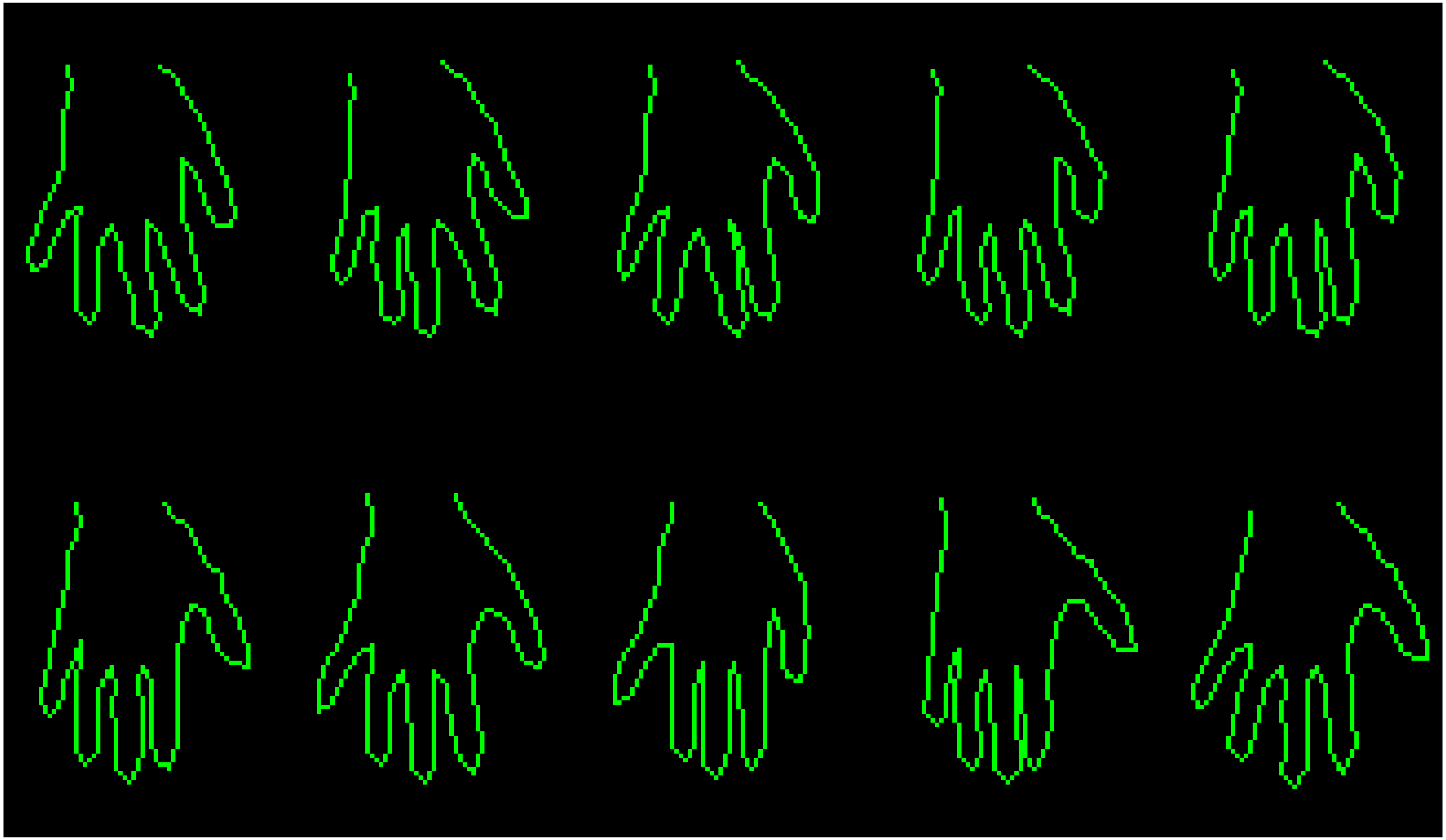
Active Shapes Models

- Active Shape models are based in mainly two steps:
  - Build a statistical shape model (PDM: point distribution model)
  - Adjust it to the image

- The shape is represented by keypoints
- In all the examples the keypoints are placed in the same exact position

Active Shapes Models

- Consider the outline of a hand, represented by 72 labelled points.
- Here are some examples from a training set:



Active Shapes Models

- Mean Model alignment: Procrustes analysis
  - The Procrustes distance is a least-squares type shape metric that requires two aligned shapes with one-to-one point correspondence
  - The alignment part involves four steps:
    - Compute the centroid of each shape.
    - Re-scale each shape to have equal size.
    - Align w.r.t. position the two shapes at their centroids.
    - Align w.r.t. orientation by rotation.

Active Shapes Models

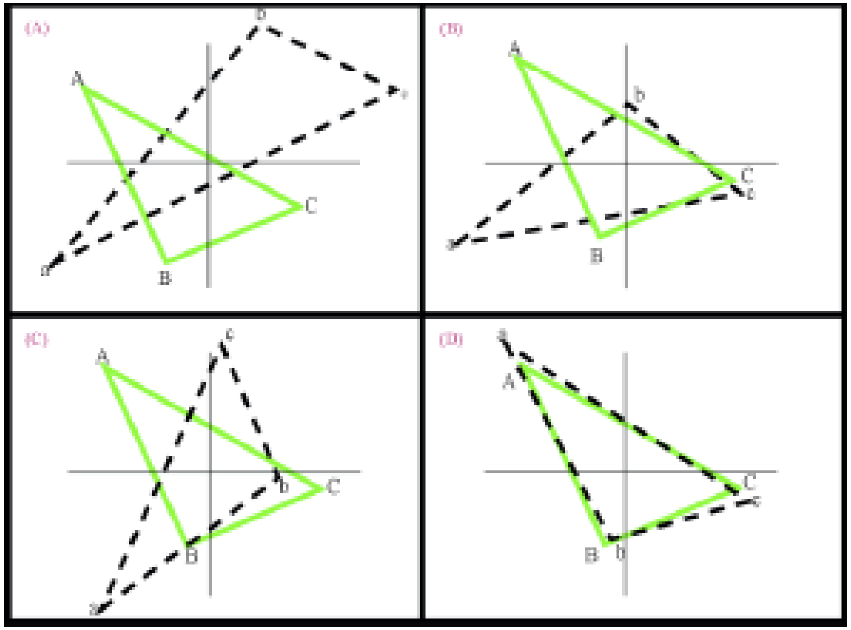
- Statistical Models: initial image processing
  - Procrustes analysis for 2 vectors: find transformation which minimizes

$$|x_1 - T(x_2)|^2$$

  - for n vectors, minimize:

$$\sum |m - Ti(x_i)|^2$$

$$\mathbf{m} = \frac{1}{n} \sum T_i(\mathbf{x}_i) \quad |\mathbf{m}| = 1$$



  - Resulting shapes have
    - Identical centre of gravity
    - Approximately the same scale and orientation

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Active Shapes Models

- Each hand is represented by a 2n element vector

  $x = (x_1, \ldots, x_n, y_1, \ldots, y_n)$

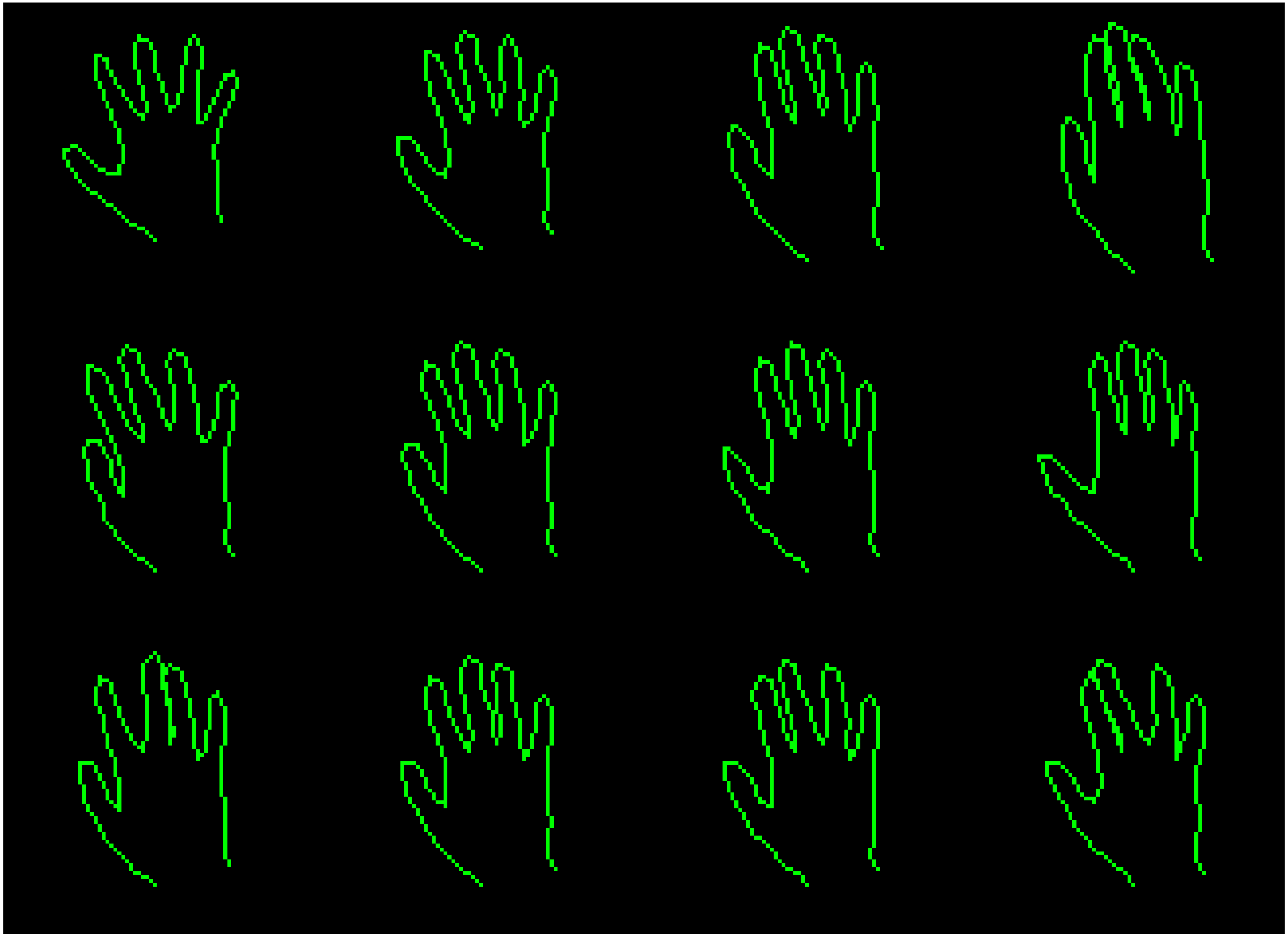
- We use Principal Component Analysis (PCA) to pick out the main axes of the cloud, and model only the first few, which account for the majority of the variation.
- The shape model is then

  $x = x_{mean} + Pb$

- where
  - x_{mean} is the mean of the aligned training examples,
  - P is a 2n x t matrix whose columns are unit vectors along the principal axes of the cloud (the t eigenvectors), and
  - b is a t element vector of shape parameters.

Active Shapes Models

- By varying the shape parameters (b) within limits learnt from the training set ($-3\sqrt{\lambda_k} \leq b_k \leq 3\sqrt{\lambda_k}$), we can generate new examples.



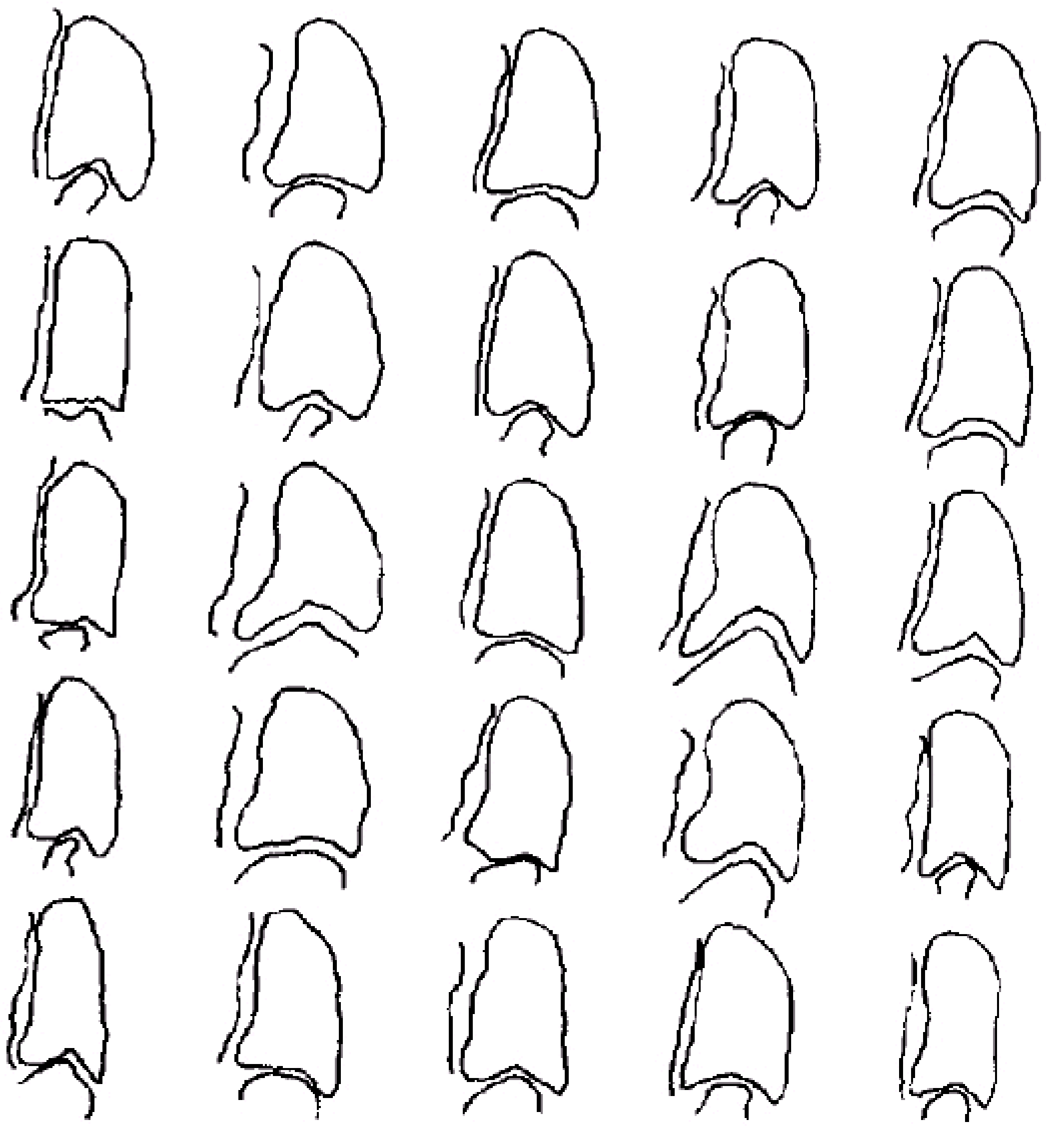
Active Shapes Models

Heart Example

- 66 examples
- 96 points
  - Left ventricle
  - Right ventricle
  - Left atrium
- Traced by cardiologists

Eigenvalues of the Covariance Matrix Derived from a Set of Heart Ventricle Shapes

Eigenvalue	$\frac{\lambda_i}{\lambda_T} \times 100\%$
λ_1	37%
λ_2	17%
λ_3	13%
λ_4	7%
λ_5	6%
λ_6	4%



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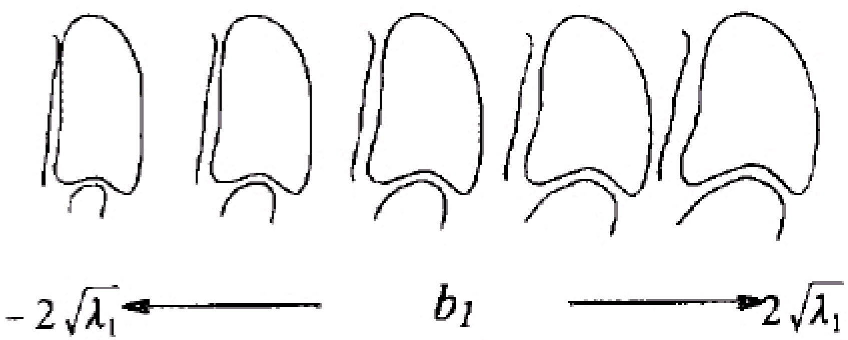
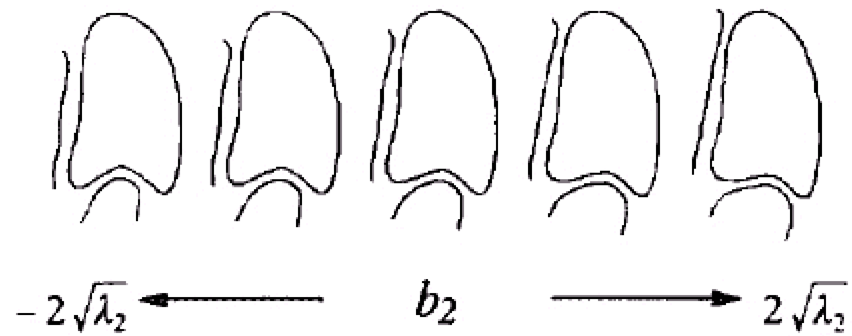
Active Shapes Models

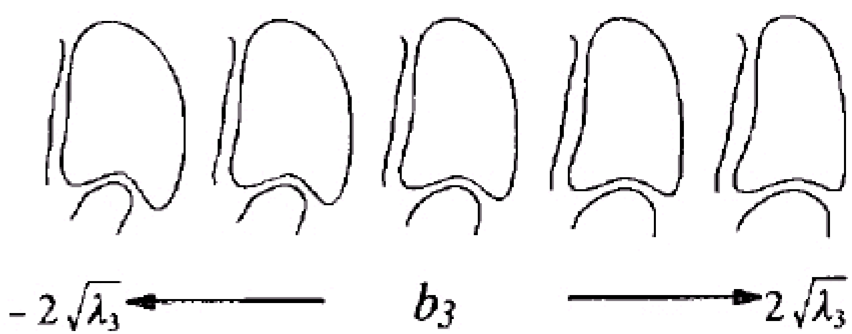
Heart Example



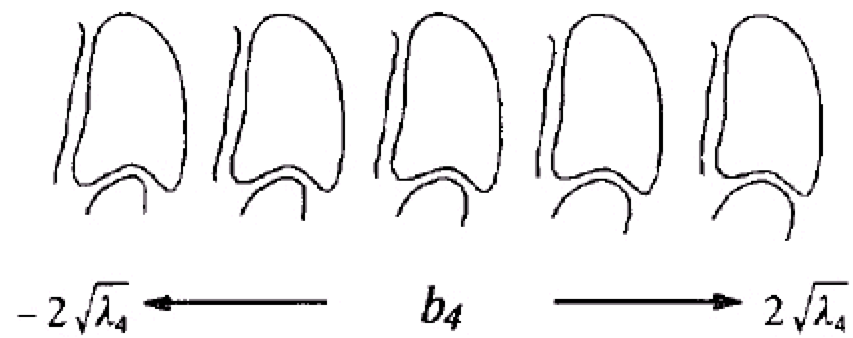
Varies Width



Varies Septum



Varies LV



Varies Atrium

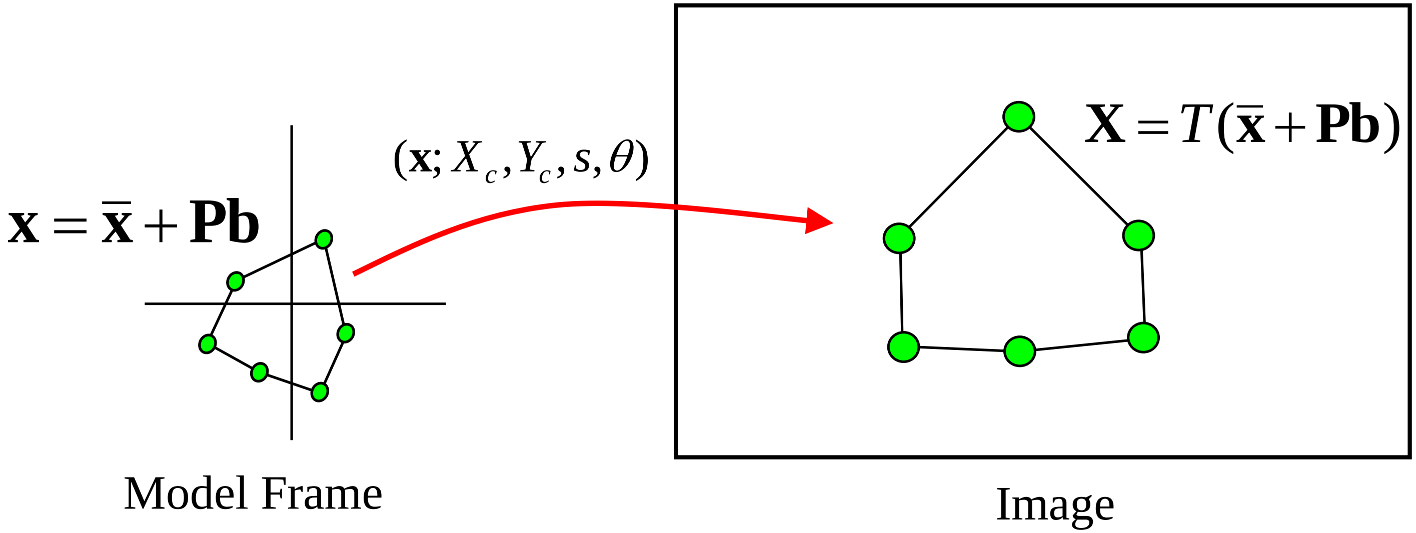
Active Shapes Models

- How to adjust to a new image?
  - Put the mean model over the image
  - Look along normals through each model point to find the best local match for the model of the image appearance at that point (eg strongest nearby edge)
  - Update the pose and shape parameters to best fit the model instance to the found points
  - Repeat steps 2 & 3 until convergence

- The performance can be significantly improved using a multi-resolution implementation

Active Shapes Models

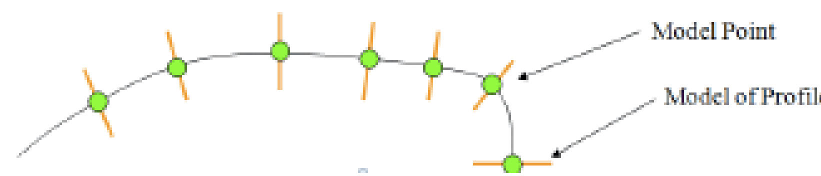
- Active Shape Models (ASMs)
  - Must apply global transformation $\mathbf{T}$ to place in image.

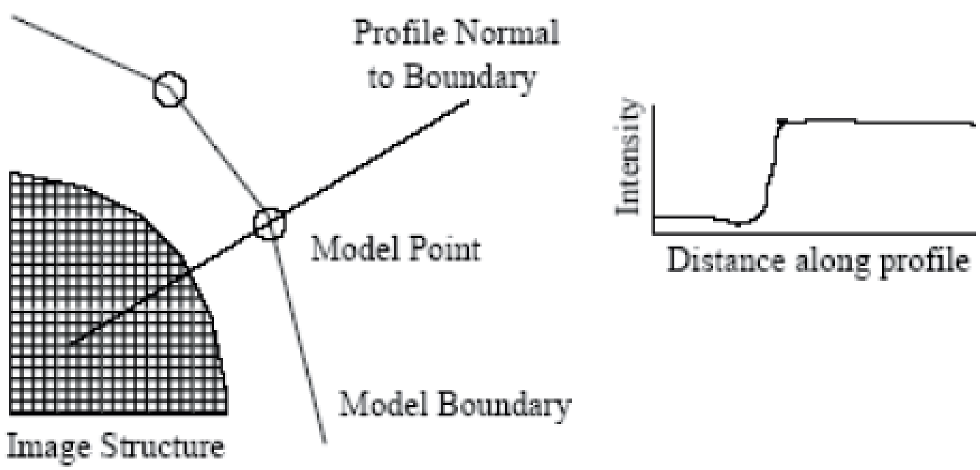


where $(\mathbf{X_c},\mathbf{Y_c})$ is the position of the centre of the model in the image frame, rotated by $\boldsymbol{\theta}$ and scaled by $\mathbf{s}$.

Active Shapes Models

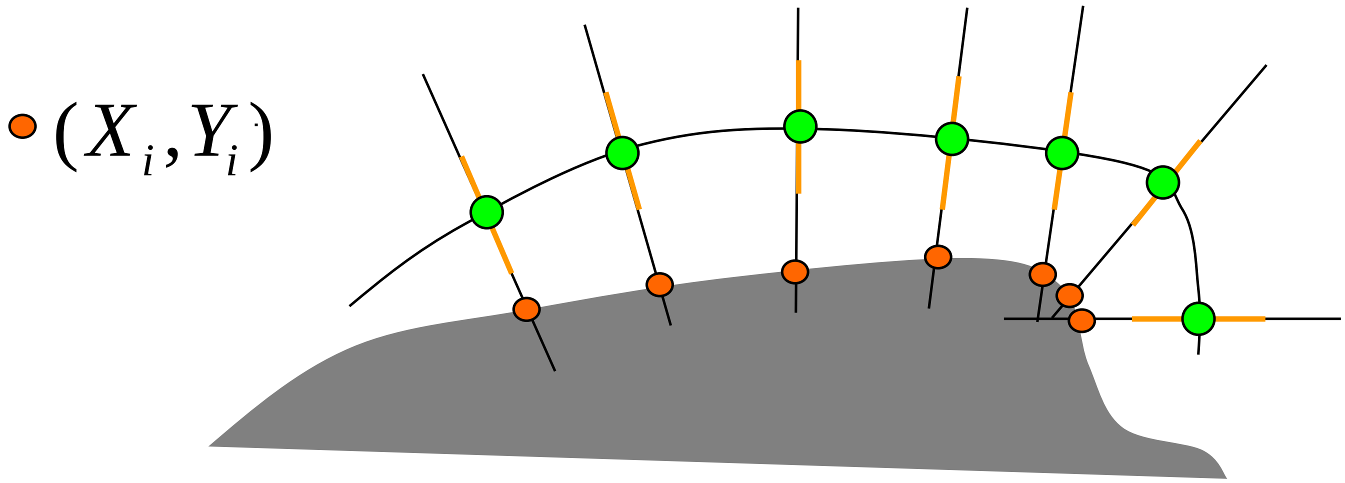
- Active Shape Models (ASMs): how it works
  - Match shape model to new image
  - Require:
    - Statistical shape model
    - Model of image structure at each po





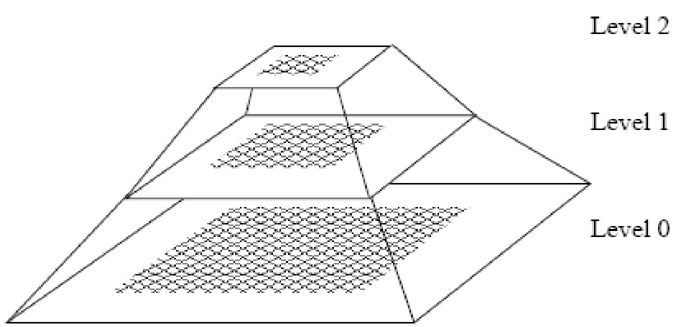
Active Shapes Models

- Active Shape Models (ASMs): how it works
  - Algorithm
    - Examine a region of the image around each point (X_i, Yi) to find the best nearby match.
    - Update parameters (X_t, Y_t, s, θ, b) to best fit the new found points.
    - Repeat until convergence.



Active Shapes Models

- Active Shape Models (ASMs): how it works
  - Improved efficiency
    - Multi-resolution framework: gaussian image pyramid formed by repeated smoothing and sub-sampling.



Each level half the size of the one below

    - Rapid location of the boundary of objects with similar shape
    - Useful to classify objects based on shape or appearance
    - Useful when approximate location of target object is known

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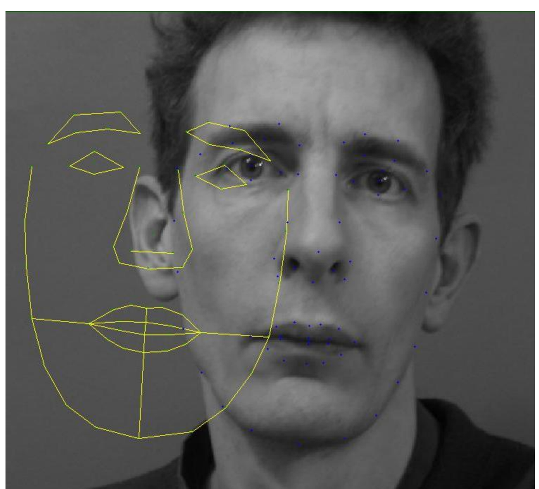
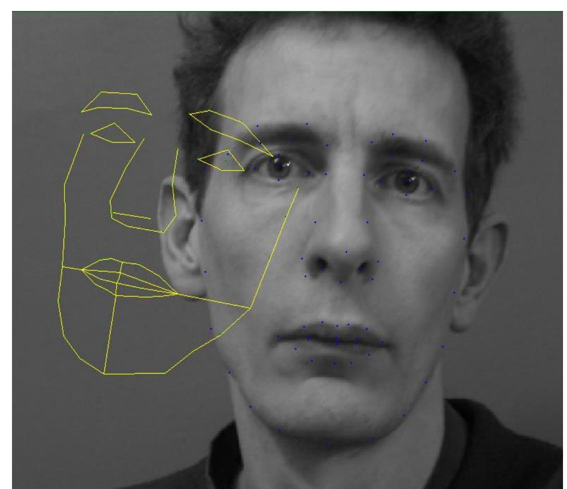
Active Shapes Models

- Active Shape Models (ASMs): Limitations
  - Can fail if the initial guess is too far from the target.
  - Problems when position/size/orientation of targets is not known approximately.
  - Doesn’t work with widely varying shapes.
  - The model can only deform in ways observed in the training set. If is not there, the model will not fit to it.



Initial guess



Search result

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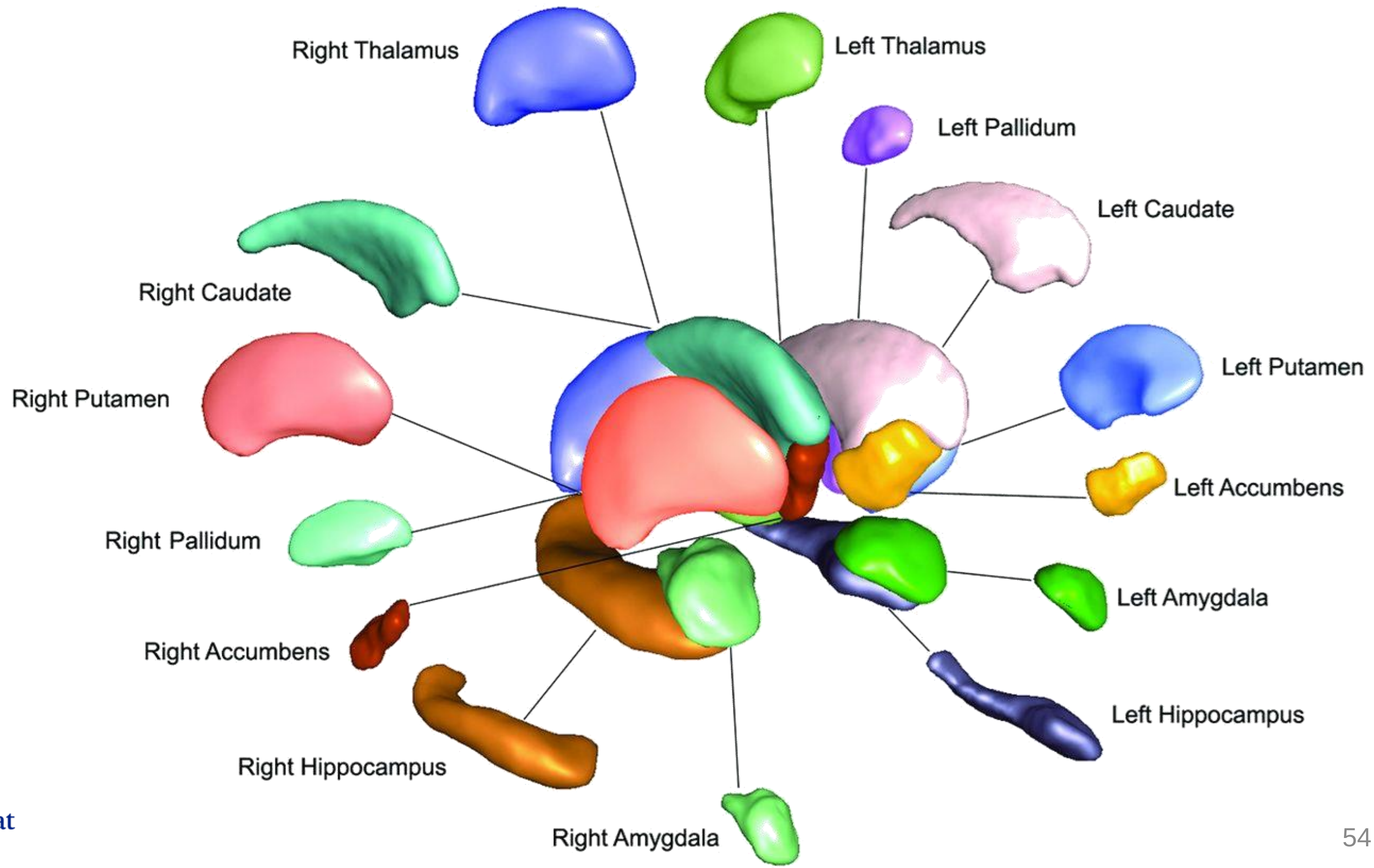
Active Appearance Models

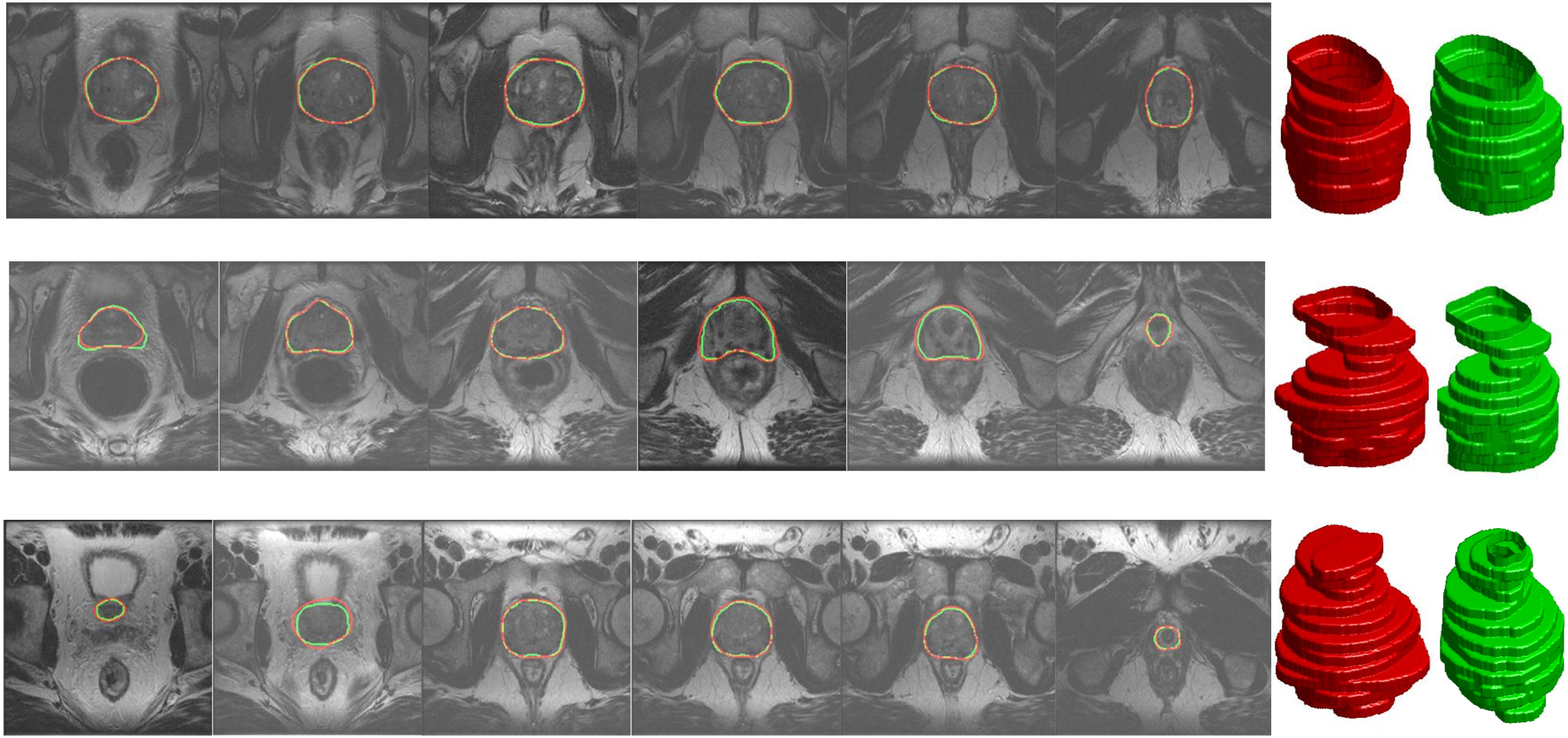
- The Active Appearance Model (AAM) is a generalization of the Active Shape Model approach.
- AAM uses all the information in the image region covered by the target object, rather than just near modelled edges.
- Widely used in medical imaging:
- Examples: brain structure segmentation, prostate segmentation, …

- FIRST: brain structures segmentation



Active Appearance Models

- Ghose PhD, 2013: active appearance model for prostate segmentation



Active Appearance Models

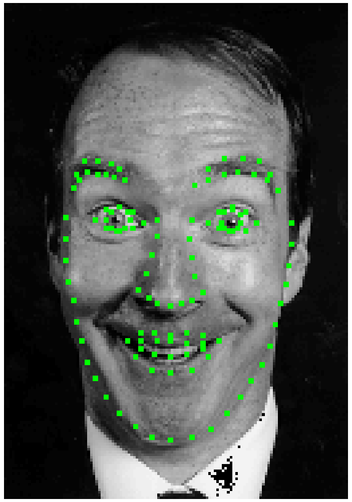
- Method: given a set of training images, labelled with landmark points, we can use image warping to deform each image so that the object has the mean shape, then build a statistical model of the grey-levels across the object.

- Mainly:
  - Shape models represent shape variation
  - Eigen-models can represent texture variation

- But… combined appearance models represent both!

Active Appearance Models

- For each example extract shape vector



Shape, $\mathbf{x} = (x_1, y_1, \ldots, x_n, y_n)^T$

- Build statistical shape model,

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}_s \mathbf{b}_s$$

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Active Appearance Models

- For each example, extract texture vector

Shape, $\mathbf{x} = (x_1, y_1, \ldots, x_n, y_n)^T$

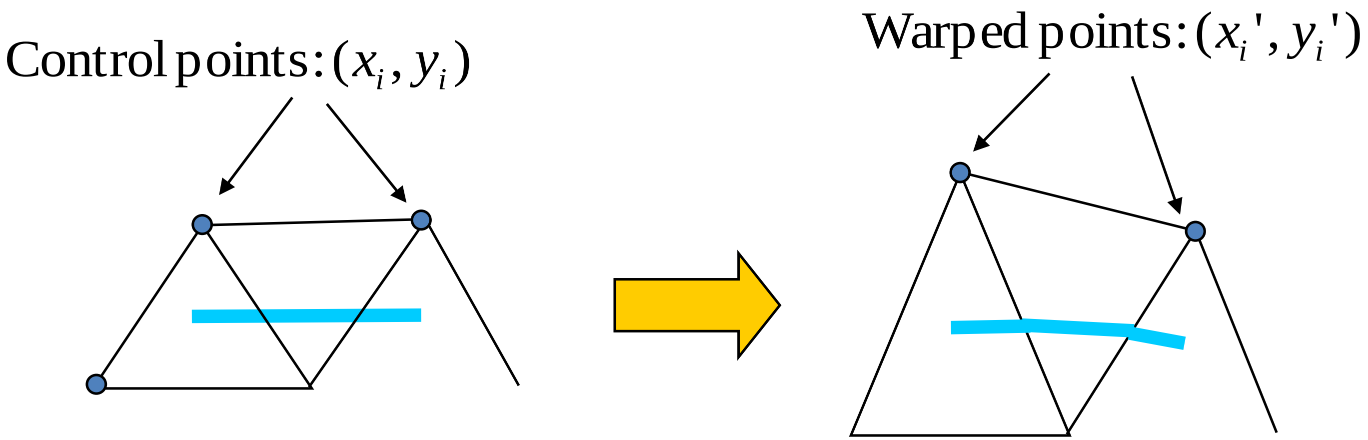
Warp to mean shape

Texture, $\mathbf{g}$

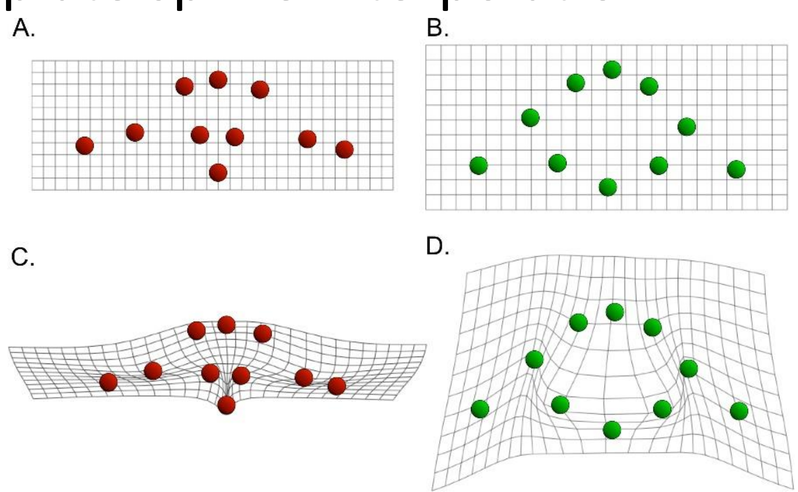
- Given corresponding points in two images, how do we warp one into the other?

Active Appearance Models

- Warping texture:
- Two common solutions
  - Piece-wise linear using triangle mesh

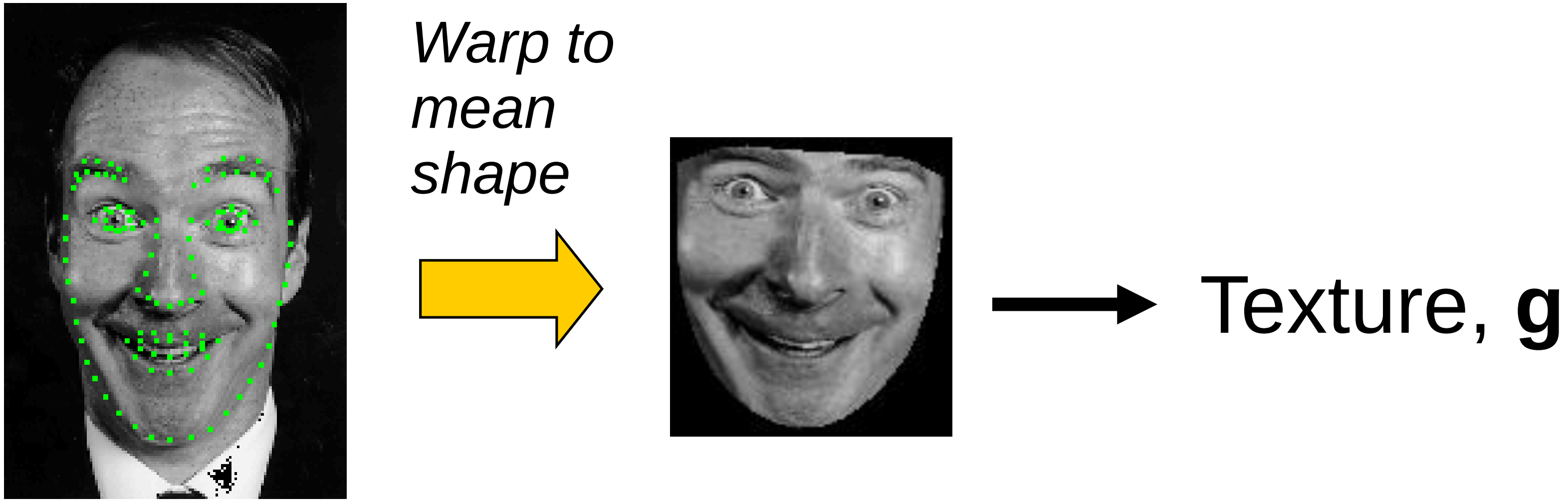


  - Thin-plate spline interpolation



Active Appearance Models

- For each example, extract texture vector

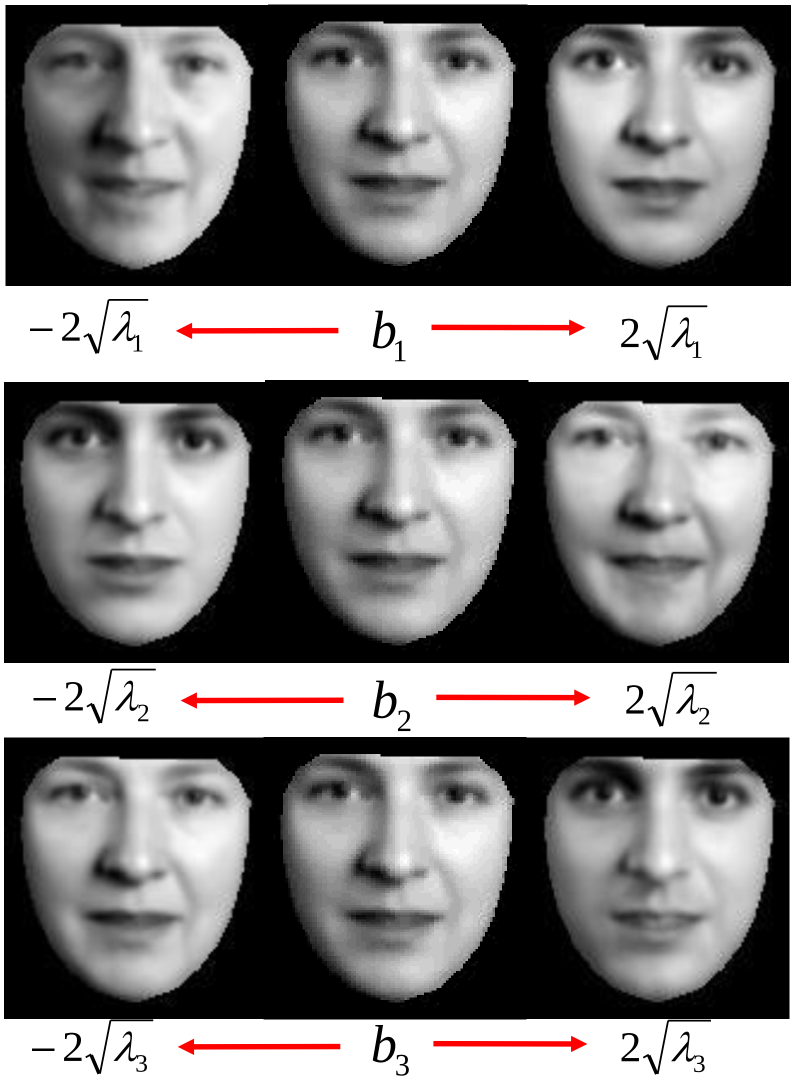


- Normalise vectors (as for eigenfaces)
- Build eigen-model

$$\mathbf{g} = \bar{\mathbf{g}} + \mathbf{P}_g \mathbf{b}_g$$

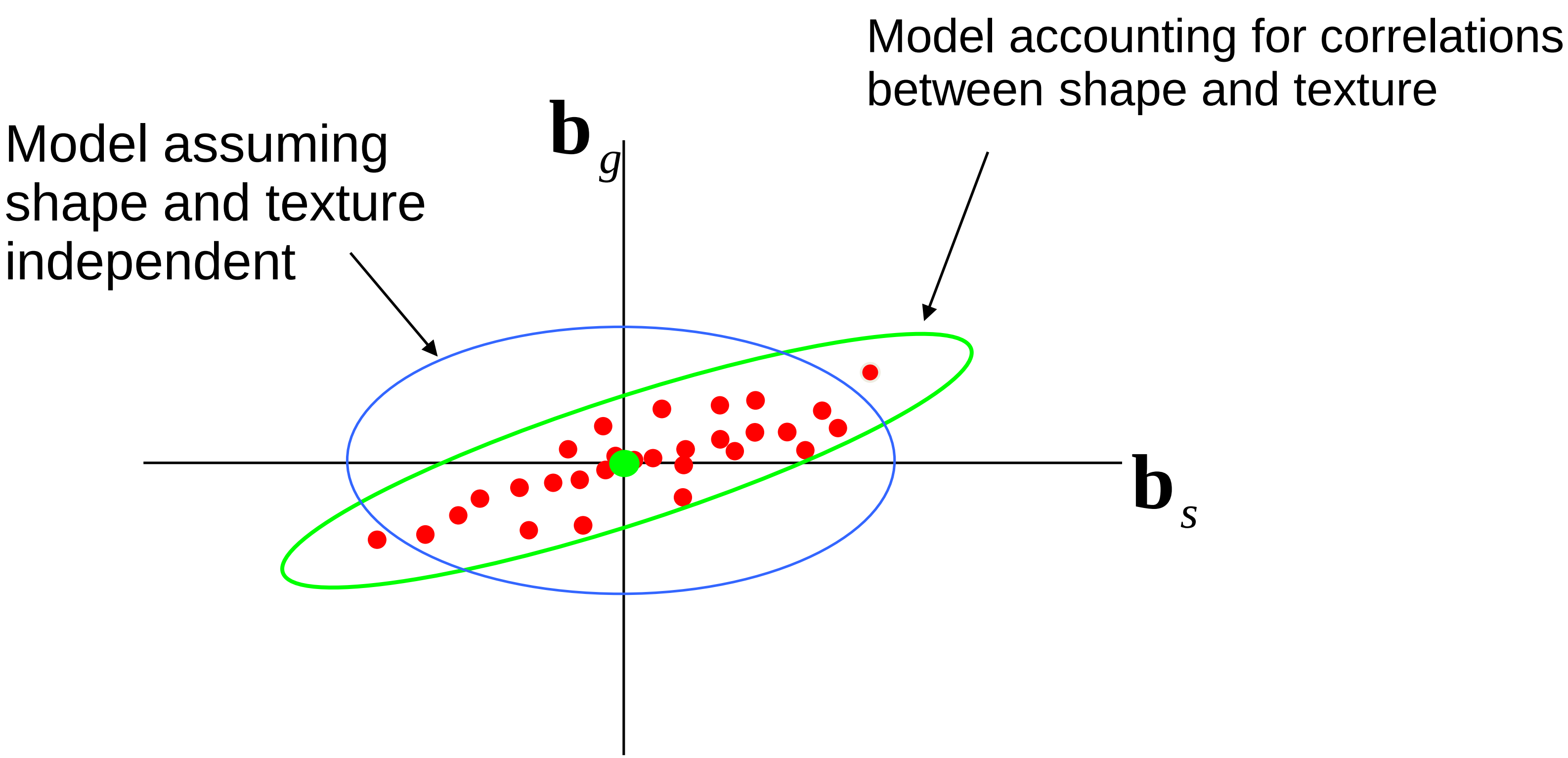
Active Appearance Models

- Face Texture Models



Active Appearance Models

- Shape and texture often correlated:
  - When smile, shadows change (texture) and shape changes
- Learning this correlation leads to more compact (and specific) model



Active Appearance Models

- For each image in training set we have best fitting shape and texture parameters $\mathbf{b}_s, \mathbf{b}_g$
- How to decorrelate the parameters? PCA again!
- Given the following vector:

$$\mathbf{b}_c = \begin{pmatrix} \mathbf{W}\mathbf{b}_s \\ \mathbf{b}_g \end{pmatrix} \qquad \begin{aligned} \mathbf{x} &= \bar{\mathbf{x}} + \mathbf{P}_s \mathbf{b}_s \\ \mathbf{g} &= \bar{\mathbf{g}} + \mathbf{P}_g \mathbf{b}_g \end{aligned}$$

W is a diagonal matrix to compensate the difference in units between shape and grey-level parameters

- apply PCA:

$$\mathbf{b}_c = \mathbf{Q}\mathbf{c} = \begin{pmatrix} \mathbf{Q}_s \\ \mathbf{Q}_g \end{pmatrix} \mathbf{c} \qquad \begin{aligned} \mathbf{x} &= \bar{\mathbf{x}} + \mathbf{Q}_s \mathbf{c} \\ \mathbf{g} &= \bar{\mathbf{g}} + \mathbf{Q}_g \mathbf{c} \end{aligned}$$

- Hence, just varying **c** changes both shape and texture

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Active Appearance Models

$$-2\sqrt{\lambda_1} \longleftarrow b_1 \longrightarrow 2\sqrt{\lambda_1}$$

$$-2\sqrt{\lambda_2} \longleftarrow b_2 \longrightarrow 2\sqrt{\lambda_2}$$

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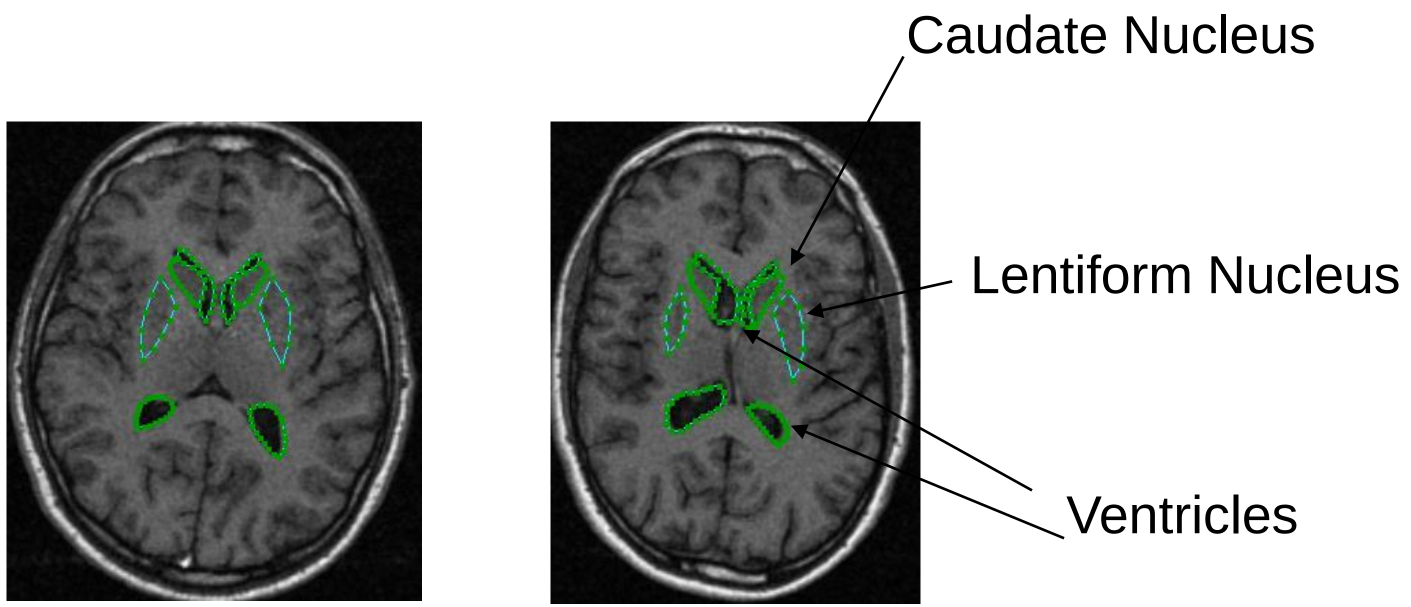
Active Appearance Models

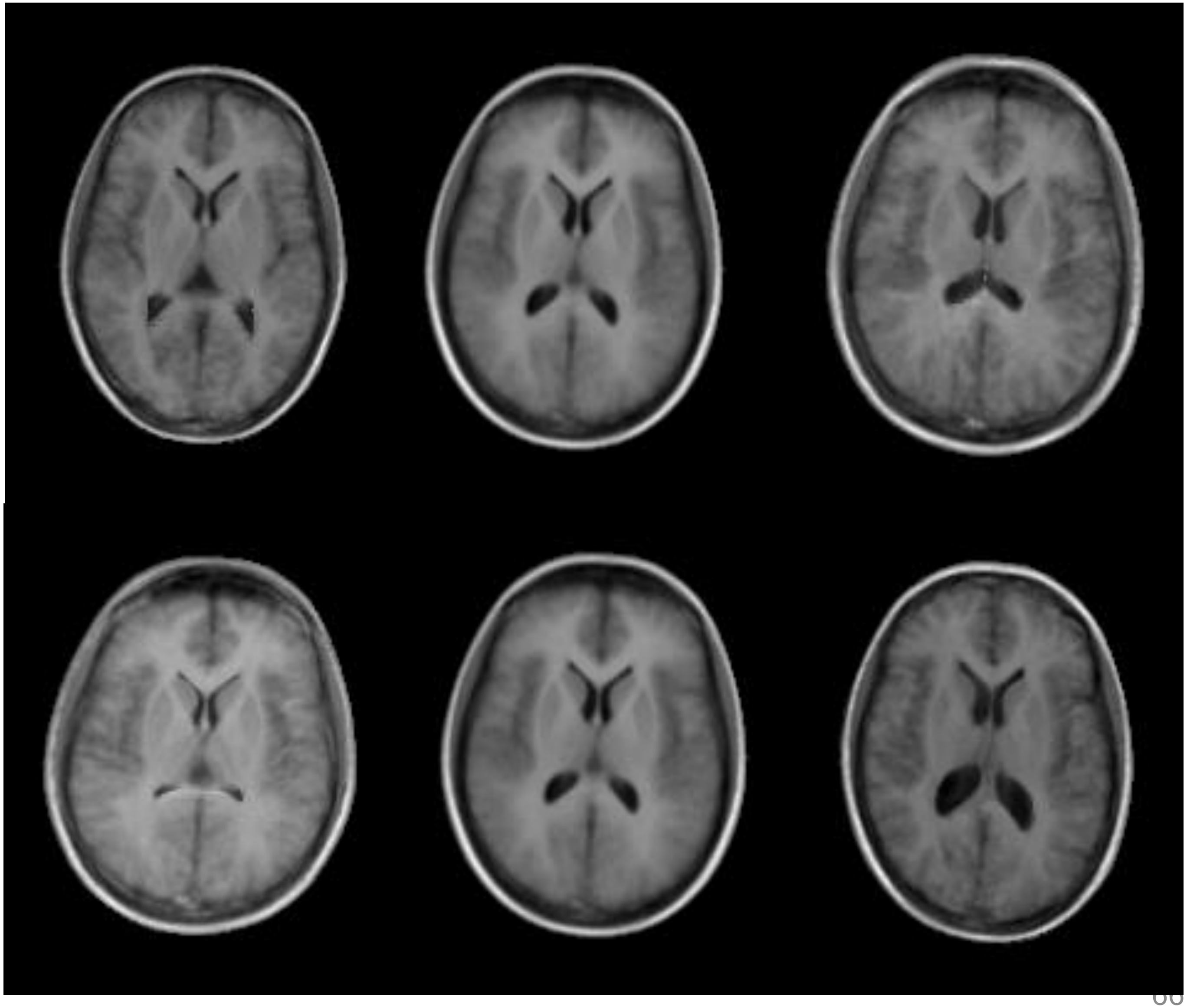
- Sub-cortical structures:
  - 72 examples
  - 123 points
  - 5000 pixel model



Active Appearance Models

Combined
Mode 1

Combined
Mode 2



To know more…

- "Deformable template models: A review", A.K. Jain et al. Signal Processing, Vol 71, 109-129, 1996
- "Object Matching Using Deformable Templates", A.K. Jain et al. IEEE Pattern Analysis and Applications, Vol 18(2), 267-278, 1996
- "Active Shape Models – Their Training and Application". T.F. Cootes et al. Computer Vision and Image Understanding Vol. 61(1), 38-59, 1995.
- "Active Appearance Models". T.F. Cootes et al. Proc. European Conference on Computer Vision 1998(2), pp. 484-498, 1998.
- "3-D Active Appearance Models: Segmentation of Cardiac MR and Ultrasound Images". S.C. Mitchell et al. IEEE Transactions on Medical Imaging, vol. 21(9), 1167-1178, 2002.

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