HW# 1
Introduction to Neural Networks (CpE 520)
Due date: Sept. 7th , 2021

Q.1: Use MATLAB to find the rank of the following matrices, also decompose each matrix into its SVD components and comment on the SVD decomposition.

$$\mathbf{X}_1 = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \quad \mathbf{X}_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix} \quad \mathbf{X}_3 = \begin{bmatrix} 1 & 3 & 2 & 5 \\ 2 & 5 & 3 & 9 \\ 2 & 1 & -1 & 5 \\ 3 & 2 & -1 & 8 \\ 1 & 1 & 0 & 3 \end{bmatrix}$$

Q.2: Use MATLAB to find the SVD of the following matrix and comment on the SVD components

$$\mathrm{SVD}\{\mathbf{X}_1\mathbf{X}_2\}? \quad \mathrm{SVD}\{\mathbf{X}_2\mathbf{X}_1\}? \quad \mathrm{SVD}\{\mathbf{X}_3\mathbf{X}_3^{\mathrm{T}}\}? \quad \mathrm{SVD}\{\mathbf{X}_3^{\mathrm{T}}\mathbf{X}_3\}?$$

Q.3: Write the inverse expression of $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_1\mathbf{X}_2$, $\mathbf{X}_2\mathbf{X}_1$ and $\mathbf{X}_3$ using the SVD and use MATLAB to find their inverse .

HW# 2
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Q.1: Derive the expression for the Inverse for the linear matrix equations (systems):

$$\text{(a)} \qquad \mathbf{y}_{n\times 1} = \mathbf{A}_{n\times m}\mathbf{x}_{m\times 1} \quad \text{where n < m and rank=n}$$

$$\text{(b)} \qquad \mathbf{y}_{n\times 1} = \mathbf{A}_{n\times m}\mathbf{x}_{m\times 1} \quad \text{where n > m and rank=m}$$

Q.2: Use the following way to obtain the closed form solution (inverse) for **(a)** and **(b)**, respectively.

(a) $\mathbf{y}_{n\times 1} = \mathbf{A}_{n\times m}\mathbf{x}_{m\times 1}$ where $n < m$ and rank=n

we can find soultion by minimizing the follwing equation

$$\tilde{\mathbf{x}} = \min_{\mathbf{x}}\{\frac{1}{2}\|\mathbf{x}\|_2^2 + \boldsymbol{\lambda}^{\mathrm{T}}(\mathbf{y} - \mathbf{A}\mathbf{x})\}$$

(b) $\mathbf{y}_{n\times 1} = \mathbf{A}_{n\times m}\mathbf{x}_{m\times 1}$ where $n > m$ and rank=m

we can find the soultion by minimizing the following Eqs.

$$\tilde{\mathbf{x}} = \min_{\mathbf{x}}\{\frac{1}{2}(\mathbf{y} - \mathbf{A}\mathbf{x})^2\} \qquad \text{also}$$

$$\tilde{\mathbf{x}} = \min_{\mathbf{x}}\{\frac{1}{2}(\mathbf{y} - \mathbf{A}\mathbf{x})^2 + \frac{1}{2}\lambda\|\mathbf{x}\|_2^2\} \quad \text{solution is called Ridge Regression}$$