Problem 1 [BV 9.23]

(a)

$$\begin{aligned} g(t) = f(x + t\Delta x) &= -\sum_{i=1}^{m} \log(b_i - a_i^T x - t a_i^T \Delta x) \\ &= -\sum_{i=1}^{m} \log\left((b_i - a_i^T x)(1 - t\frac{a_i^T \Delta x}{b_i - a_i^T x})\right) \\ &= -\sum_{i=1}^{m} \log(b_i - a_i^T x) - \sum_{i=1}^{m} \log(1 - t\frac{a_i^T \Delta x}{b_i - a_i^T x}) \\ &= c - \sum_{i=1}^{m} \log(1 - t d_i) \end{aligned}$$

where $c = -\sum_{i=1}^{m} \log(b_i - a_i^T x)$ and $d_i = \frac{a_i^T \Delta x}{b_i - a_i^T x}$.

$$g'(t) = \sum_{i=1}^{m} \frac{d_i}{1 - t d_i}$$

Therefore, if k values of t are to be evaluated, the computation complexity of f is of order knm while with pre-computation it is $nm + km$. Also the the computation cost for $g'(t)$ is the same with or without the pre-computation is of order $nm + km$.

(b)

$$\begin{aligned} \tilde{f}(t) = f(x + t\Delta x) &= \log\left(\sum_{i=1}^{m} e^{a_i^T x + t a_i^T \Delta x + b_i}\right) \\ &= \log\left(\sum_{i=1}^{m} e^{a_i^T x + b_i} (e^{a_i^T \Delta x})^t\right) \\ &= \log\left(\sum_{i=1}^{m} e^{k_i + t h_i}\right) \end{aligned}$$

where $k_i = a_i^T x + b_i$ and $h_i = a_i^T \Delta x$.

$$\tilde{f}'(t) = \frac{\sum_{i=1}^m h_i e^{k_i + t h_i}}{\sum_{i=1}^m e^{k_i + t h_i}}$$

Therefore, if k values of t are to be evaluated, the computation complexity of f is of order kmn while with pre-computation it is of order $mn+km$. Also the the computation cost for $g'(t)$ is the same with or without the pre-computation, of order $mn + km$.

Problem 2 [BV-additional-exercises 9.10]

(a) False.

(b) True.

(c) False.

(d) True.

(e) False.

(f) True.

(g) False
