EE 513: Stochastic Systems Theory
Fall 2022 Schmid

Homework Assignment 1
Distributed: Friday, August 26, 2022
Due: Friday, September 9, 2022

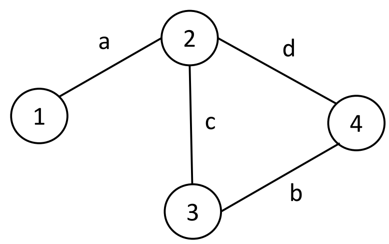
Reference Material:

1. class notes
2. Leon-Garcia, Ch. 1, Ch. 2 and Ch. 4
3. J. A. Gubner, Probability and Random Processes for Electrical and Computer Engineers, Cambridge University Press, 2006, Ch. 1, Ch. 2, and Ch. 4

General Comments: Solutions for this homework assignment are due in **2 weeks**.

Two of the following problems will be selected at random and graded.

Problem 1.1



A communication network has four terminal nodes, labeled 1, 2, 3, and 4 in the above figure, connected via four links, labeled a, b, c, d. Not all the links are available, however. Let p denote the probability that any particular link is available and assume that the availability of each link is statistically independent of the state of all other links. Two nodes can communicate with one another if and only if they are connected by at least one chain of available links.

a. Construct an appropriate probability model (S, F, P) with sixteen sample points ξ, each representing a state of the network. Specify S, F, and P.
b. Let $A = \{\xi$: node-1 and node-4 can communicate$\}$. Calculate $P(A)$.
c. Let $B = \{\xi$: node-2 and node-3 can communicate$\}$. Calculate $P(B)$.
d. Calculate $P(A \cap B)$. How many sample points of S does the event $\{\xi : A \cap B)\}$ contain? Are the events A and B statistically independent?
e. Show that $P(A) = p\ P(A|$ link-c available$) + (1-p)P(A|$ linc-c not available$)$. Use this formula to recalculate $P(A)$ by inspection.
f. Prove that $P(A)$ would be increased by moving link-c between node-1 and node-2 rather than between node-2 and node-3.

Problem 1.2
A homogeneous stick of unit length is broken at two points at random. What is the probability that a triangle can be formed from the three pieces?

Problem 1.3

A random experiment A has N mutually exclusive outcomes $A_1, A_2, \ldots, A_N$, and random experiment B has M mutually exclusive outcomes $B_1, B_2, \ldots, B_M$. Establish **Bayes' rule**,

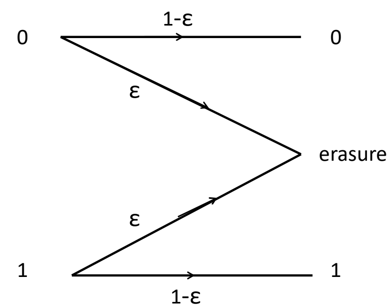
$$\Pr(B_m \mid A_n) = \frac{\Pr(A_n \mid B_m)\Pr(B_m)}{\sum_{i=1}^{M} \Pr(A_n \mid B_i)\Pr(B_i)}.$$

Problem 1.4

A communication channel called erasure channel is shown in the figure below. Suppose that the input symbols 0 and 1 occur with probabilities 0.4 and 0.6, respectively.

(a) Find the probabilities of the output symbols.

(b) Suppose that a 1 is observed on the output. What is the probability that the input is 0? 1?



Comment: The probabilities ε and $(1-\varepsilon)$ are known as transition probabilities. These are conditional probabilities of the outputs given the values of the inputs.

Problem 1.5 (based Problem 2.4, Leon-Garcia)

A binary communication system transmits a signal X that is either a +2-voltage signal or a -2-voltage signal. A malicious channel reduces the **magnitude** of the received signal by the number of heads it counts in two tosses of a coin. Let Y be the resulting signal.

(a) Find the sample space.

(b) Find the set of outcomes corresponding to the event "transmitted signal was definitely +2."

(c) Describe in words the event corresponding to the outcome $Y = 0$.

Problem 1.6 (based on Problem 2.97 on page 91, Leon-Garcia'06)

A block of 100 bits is transmitted over a binary communication channel with probability of bit error $p = 10^{-2}$.

(a) If the block has 2 or fewer errors, then the receiver accepts the block. Find the probability that the block is accepted?

(b) If the block has more than 2 errors, then the block is retransmitted. Find the probability that *M* retransmissions are required?