EE 513: Stochastic Systems Theory

Fall 2022 Schmid

Homework Assignment 6

Distributed: Friday, November 11, 2022

Due: Wednesday, November 30, 2022 (1.5 weeks)

Reference Material:

1. class notes
2. Leon-Garcia'08, Ch. 8
3. H. L. Van Trees, *Detection, Estimation, and Modulation Theory*, *Part I*, John Wiley & Sons, New York, 2001, Ch. 2.

Below are required problems. Three problems will be selected at random and graded.

Problem 6.1

Suppose you have a coin, and you know that either H_1: the coin is biased showing heads on each flip with probability $2/3$ or H_0: the coin is fair. Suppose you flip the coin five times. Let X be the number of times heads shows. Describe the ML and MAP decision rules, and find the probability of false alarm, probability of miss, and the total probability of error for both of them, using the prior $(\pi_1, \pi_0) = (0.2, 0.8)$ for the MAP rule.

Problem 6.2

The number of attempts Y required for a certain basketball player to make a 25-foot shot, is observed, in order to choose one of the following two hypotheses:

H_1 (otstanding player): Y has the geometric distribution with parameter $p = 0.5$.
H_0 (average player): Y has the geometric ditribution with parameter $p = 0.2$.

(a) Describe the ML decision rule. Express it as directly in terms of Y as possible.
(b) Find the probability of false alarm and the probability of miss for the ML rule.
(c) Describe the MAP decision rule under the assumption that H_0 is a priori twice as likely as H_1. Express the rule as directly in terms of Y as possible.
(d) Find the total probability if error for both the ML rule and the MAP rule, using the same prior distribution given in part (c). For which rule is the average error probability smaller?

Problem 6.3

Consider the hypothesis pair $H_0 : Y = N$ versus $H_1 : Y = N + S$, where N and S are independent random variables with pdfs

$$f_S(x) = f_N(x) = \begin{cases} \exp(-x), & x \geq 0 \\ 0, & x < 0. \end{cases}$$

(a) Find the likelihood ratio between H_0 and H_1.
(b) Assume a Bayes' test with equal priors. Find the probability of false alarm, probability of miss, and the total probability of error.

Problem 6.4

Given are n independent measurements $v_1, v_2, \ldots, v_n$ of the noise voltage v at a certain point in a receiver. The noise v is a Gaussian random variable with unknown mean and unknown variance. Work out the maximum-likelihood estimators for the unknown parameters based on n measurements. Calculate the expected values of these estimates as functions of true parameters.

Problem 6.5 (Based on Problem 2.4.5 on page 144 of HLVT-I)
Let R_k , $k = 1,2,\ldots,K$ be K noisy measurements of a Gaussian random-variable A having zero mean and variance σ_A^2, where

$$R_k = A + N_k .$$

The noise variables N_k , $k = 1,2,\ldots,K$ are mutually independent, independent of A, zero mean Gaussian random-variables with variance σ_N^2. Determine the maximum a posteriori (MAP) estimate $\hat{A}_{MAP}$ of A.