EE 668: Information Theory

Fall 2022 Schmid

Homework Assignment 1

Distributed: Thursday, August 25, 2022

Due: Thursday, September 1, 2022

Reference Material:

1. Class notes.
2. T. M. Cover and J.A. Thomas, (Ed. 2006) *Elements of Information Theory*, Ch. 2. Entropy, Relative Entropy and Mutual Information.
3. R. E. Blahut, *Principles and Practice of Information Theory,* Sec. 3.2, 4.3, and 5.2.
4. R. G. Gallager, *Information Theory and Reliable Communication*, John Wiley & Sons, 1968.

General Comments:

Solutions for this homework assignment are due in 2 weeks.

Problems:

Problem 1.1 (Based on Problem 2.1, page 43, CT'06)
A fair coin is flipped until the first head occurs. Let X denote the number of flips required.

(a) Find the entropy $H(X)$ in bits.

Hint 1: The occurrence of the first head in a sequence of coin flips follows a geometric distribution. The probability that k heads occur in n trials follow Binomial distribution $C_n^k p^k (1-p)^{(n-k)}$, where $p = 1/2$.

Hint 2: The following expressions may be useful: $\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$, $\sum_{n=1}^{\infty} nr^n = \frac{r}{(1-r)^2}$.

(b) A random variable X is drawn according to this distribution. Find an "efficient" sequence of yes-no questions of the form, "Is X contained in the set S?" Compare $H(X)$ to the expected number of questions required to determine X.

Problem 1.2 (based on Problem 2.12, page 46, CT'06) *Example of joint entropy.*
Let $p(x, y)$ be given by

joint probability of (x,y)	Y = 0	Y = 1
X = 0	1/4	1/4
X = 1	1/6	1/3

Find

(a) $H(X), H(Y)$

(b) $H(X \mid Y), H(Y \mid X)$

(c) $H(X,Y)$

(d) $H(Y) - H(Y \mid X)$

(e) $I(X;Y)$

(f) Draw a Venn diagram for the quantities in (a) through (e).

Problem 1.3 (based on Problem 2.35, page 51, CT'06)
Relative Entropy is not symmetric. Let the random variable X has three possible outcomes $\{a, b, c\}$. Consider two distributions on this random variable:

Symbol	$p(x)$	$q(x)$
a	1/2	1/3
b	1/4	1/3
c	1/4	1/3

Calculate $H(p)$, $H(q)$, $D(p \| q)$, and $D(q \| p)$. Verify that in this case, $D(p \| q) \neq D(q \| p)$.

Problem 1.4 (Problem 2.13, page 46, CT'06)

Show that $\ln(x) \geq 1 - 1/x$, for $x \geq 0$.