MIDTERM 2

Mar 27 2023

1. Let

$$A = \begin{pmatrix} -1/2 & 0 & -3/2 \\ -3/2 & 0 & -3/2 \\ 0 & 0 & 1 \end{pmatrix}$$

a. *(40 points)* Find the eigenvalues and corresponding eigenspaces of A. Is A diagonalizable?
b. *(30 points)* Find $\lim_{n\to\infty} A^n$.

2. *(30 points)* Suppose all eigenvalues of $T \in \mathcal{L}(\mathbb{C}^n)$ are zero. Show that T is diagonalizable if and only if $T = 0$.

3. Let $U = \text{span}((-1, 0, 1), (0, 1, -1))$.
a. *(30 points)* Find an orthonormal basis of U.
b. *(30 points)* Find the point in U that is closest to the origin $(0, 0, 0)$. What is that minimum distance?

4. *(40 points)* Show that the diagonals of the quadrilateral MNPQ are perpendicular if an only if $MN^2 + PQ^2 = MQ^2 + NP^2$.