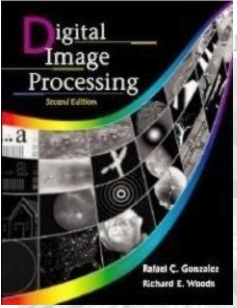



# Chapter 9
# Morphological Image Processing

# Some Basic Concepts from Set Theory

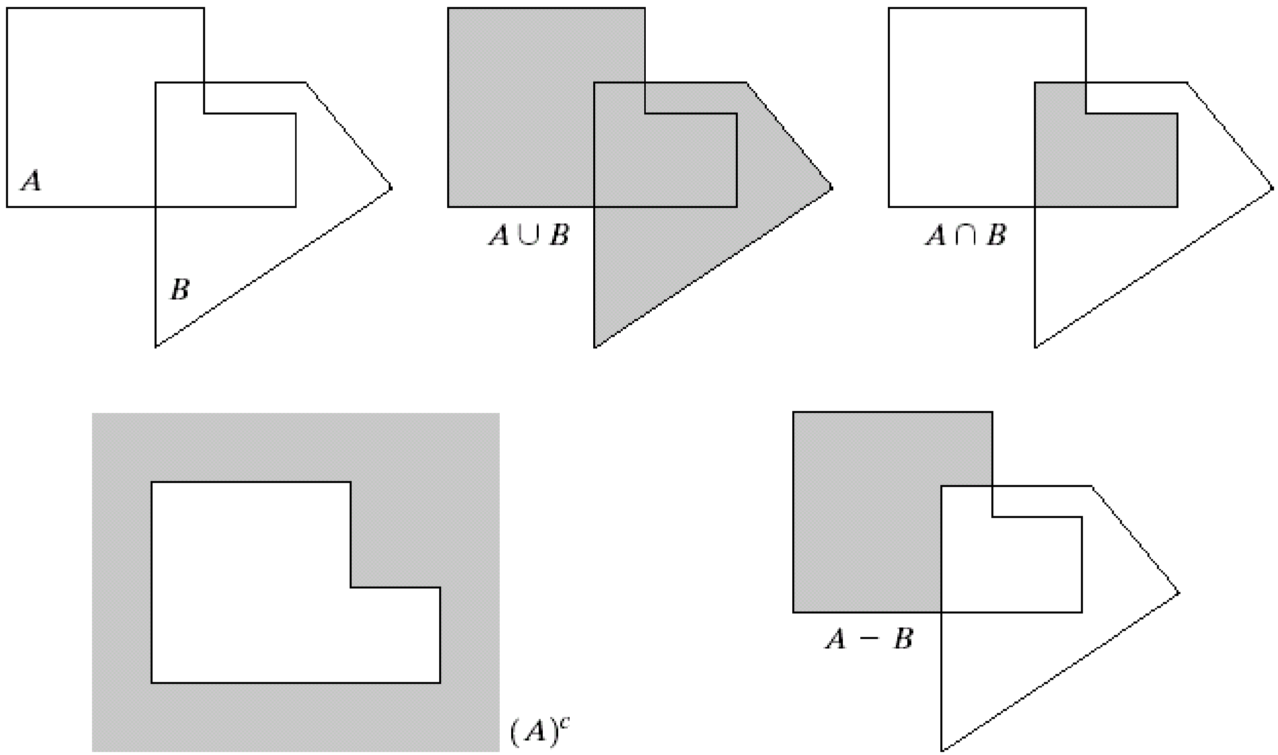
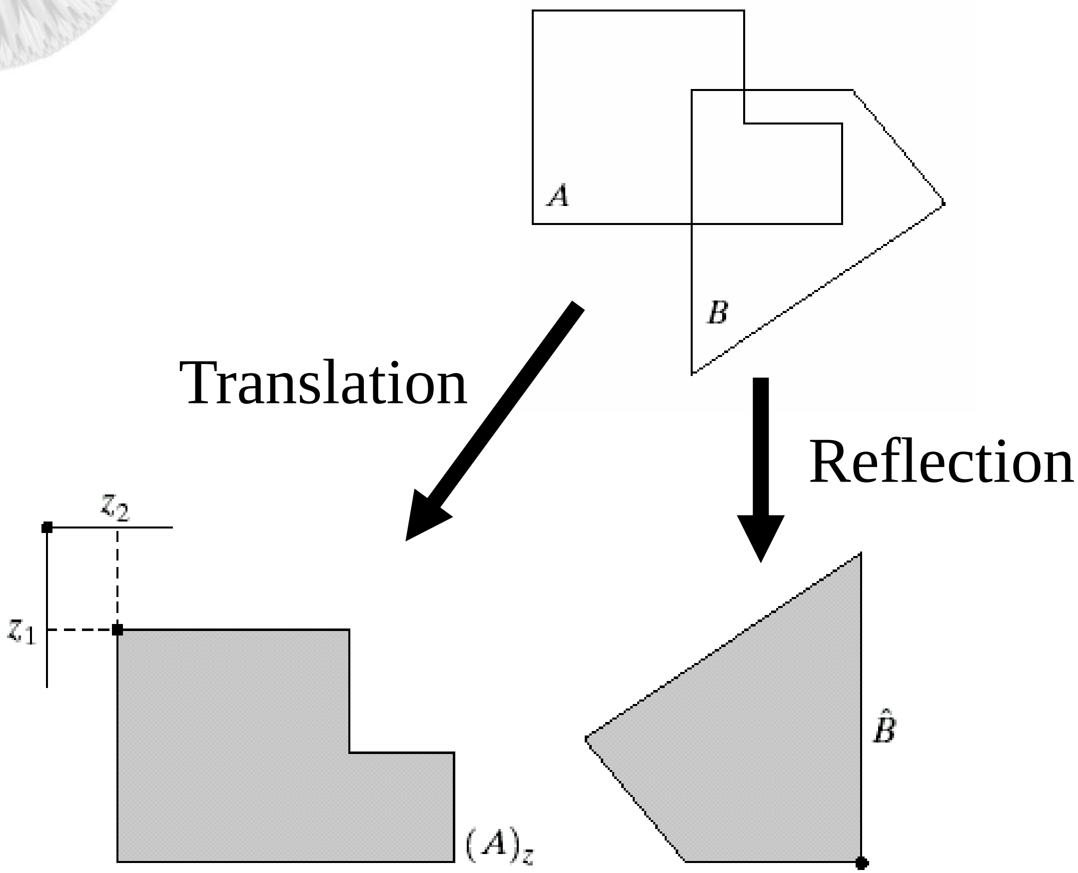



**FIGURE 9.1** (a) Two sets $A$ and $B$. (b) The union of $A$ and $B$. (c) The intersection of $A$ and $B$. (d) The complement of $A$. (e) The difference between $A$ and $B$.

# Some Basic Concepts from Set Theory




a b

**FIGURE 9.2**
(a) Translation of $A$ by $z$.
(b) Reflection of $B$. The sets $A$ and $B$ are from Fig. 9.1.

# Logic Operations Involving Binary Images

**TABLE 9.1** The three basic logical operations.

| $p$ | $q$ | $p$ AND $q$ (also $p \cdot q$) | $p$ OR $q$ (also $p + q$) | NOT ($p$) (also $\bar{p}$) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 |

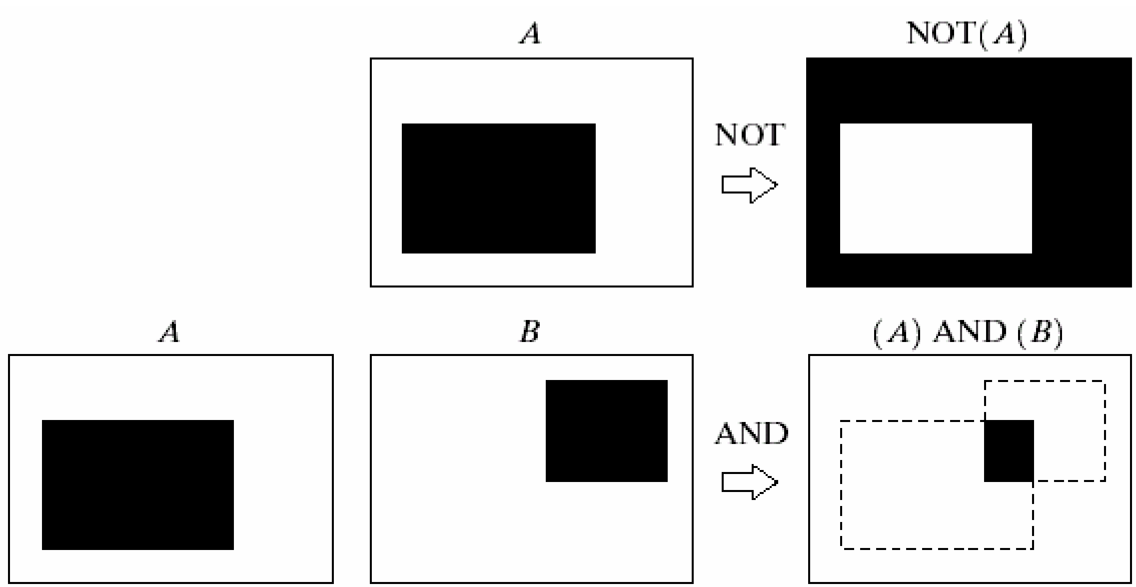
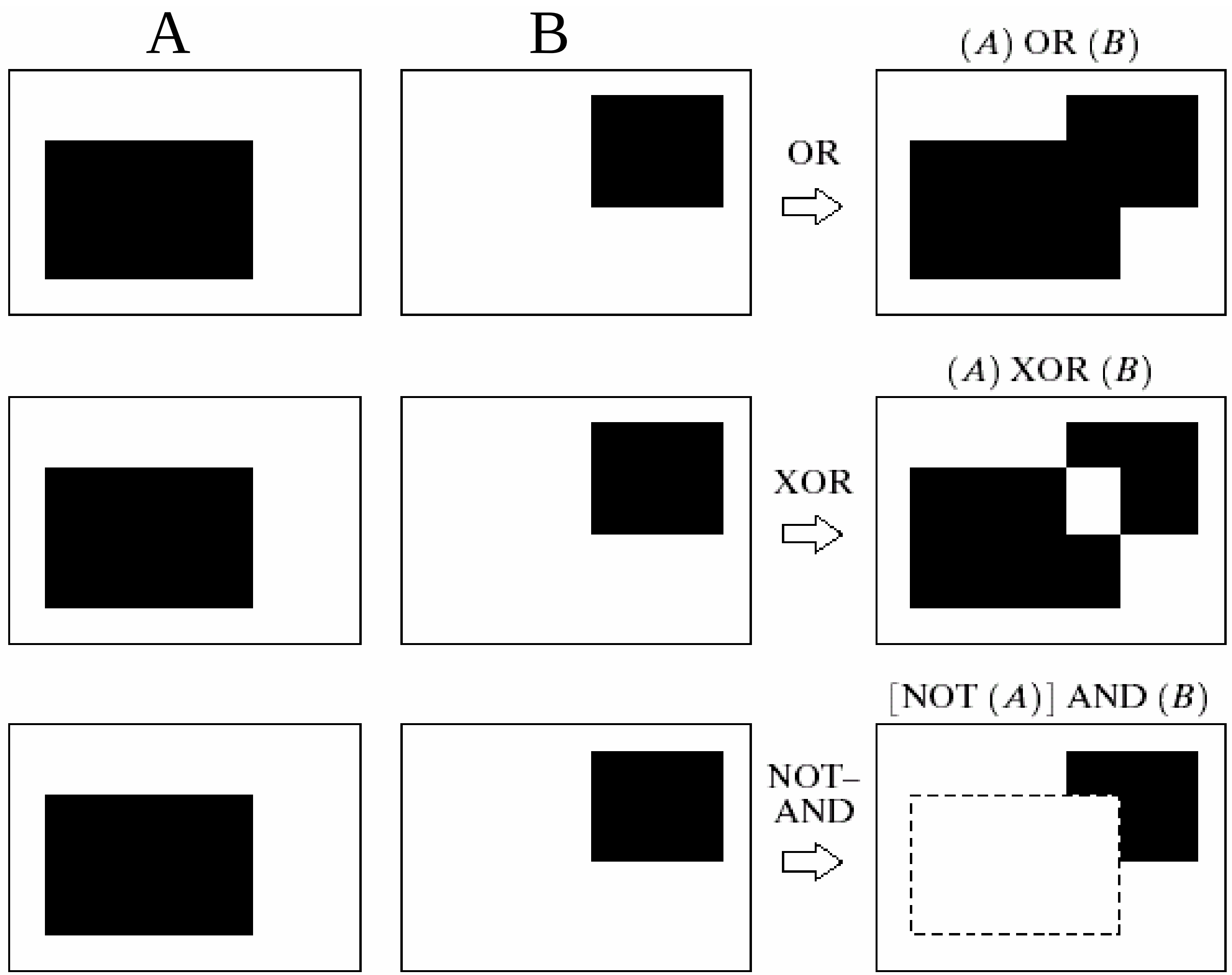



**FIGURE 9.3** Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

# Logic Operations Involving Binary Images



# Dilation and Erosion
## Dilation

- With $A$ and $B$ as sets in $Z^2$, the dilation of $A$ by $B$ is defined as

$$A \oplus B = \left\{ z \mid (\hat{B})_z \cap A \neq \phi \right\}$$

where $\hat{B}$ : the reflection of $B$ about its origin and shifting this reflection by $z$

- The dilation of $A$ by $B$ is the set of all displacements, $z$, such that and $\hat{B}$ $A$ overlap by at least one element. Thus,

$$A \oplus B = \left\{ z \mid [(\hat{B})_z \cap A] \subseteq A \right\}$$

- Set B is referred to as the structuring element in dilation.

# Dilation and Erosion
## Dilation

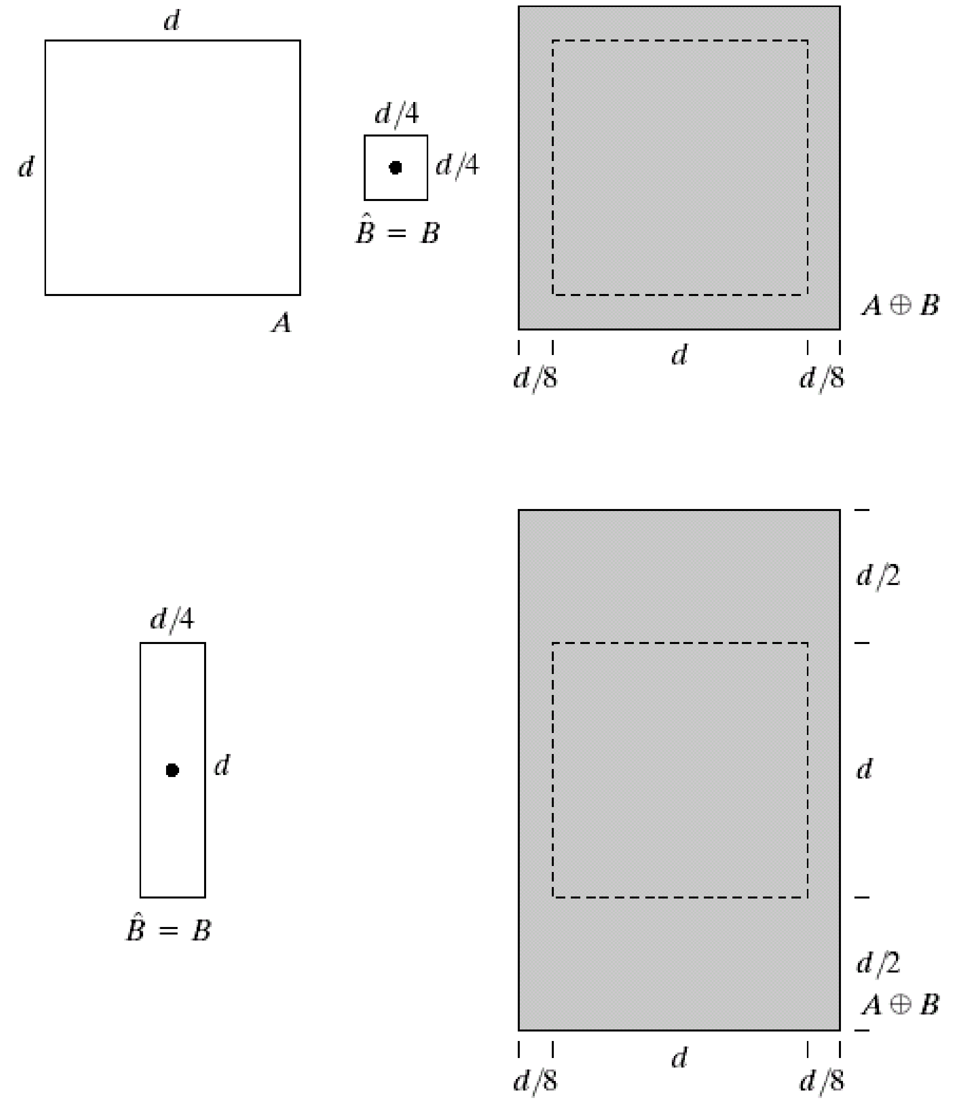
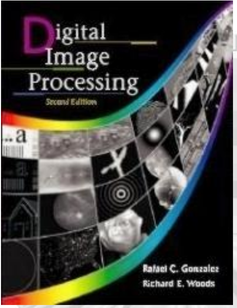
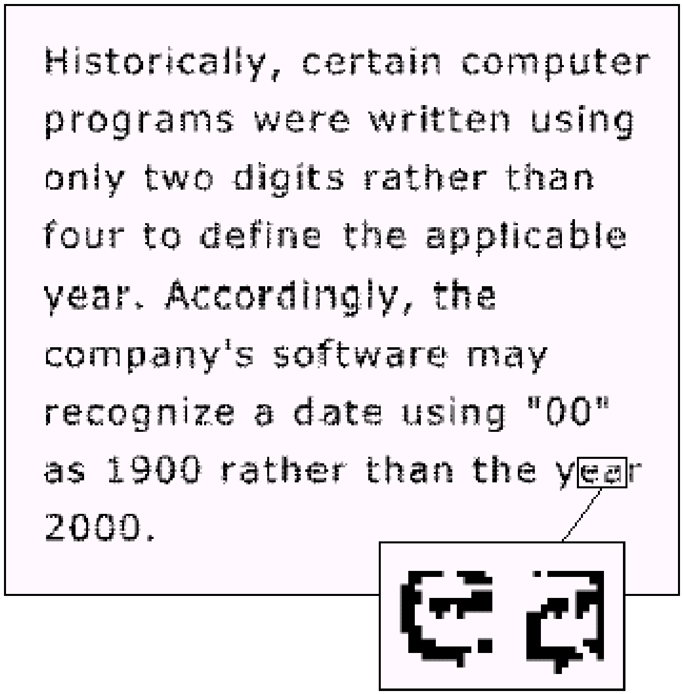


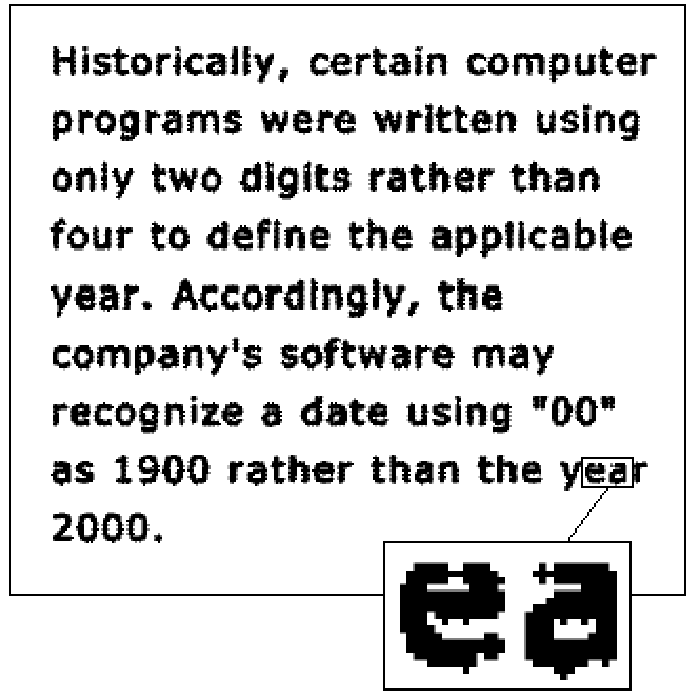

**FIGURE 9.4** (a) Set $A$. (b) Square structuring element (dot is the center). (c) Dilation of $A$ by $B$, shown shaded. (d) Elongated structuring element. (e) Dilation of $A$ using this element.




# Dilation and Erosion
## Dilation Example



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

a c
b

**FIGURE 9.5** (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Structuring element

# Dilation and Erosion
## Erosion

- For sets $A$ and $B$ in $Z^2$, the erosion of $A$ by $B$ is defined as

$$A \ominus B = \left\{ z \mid (\hat{B})_z \subseteq A \right\}$$

where $\hat{B}$ : the reflection of $B$ about its origin and shifting this reflection by $z$

- The erosion of $A$ by $B$ is the set of all points $z$, such that $B$, translated by $z$ is contained in $A$.

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$\begin{aligned}(A \ominus B)^c &= \left\{ z \mid (B)_z \subseteq A \right\}^c \\ &= \left\{ z \mid (B)_z \subseteq A^c = \phi \right\}^c \\ &= \left\{ z \mid (B)_z \subseteq A^c \neq \phi \right\} \quad = A^c \oplus \hat{B}\end{aligned}$$

# Dilation and Erosion
## Erosion

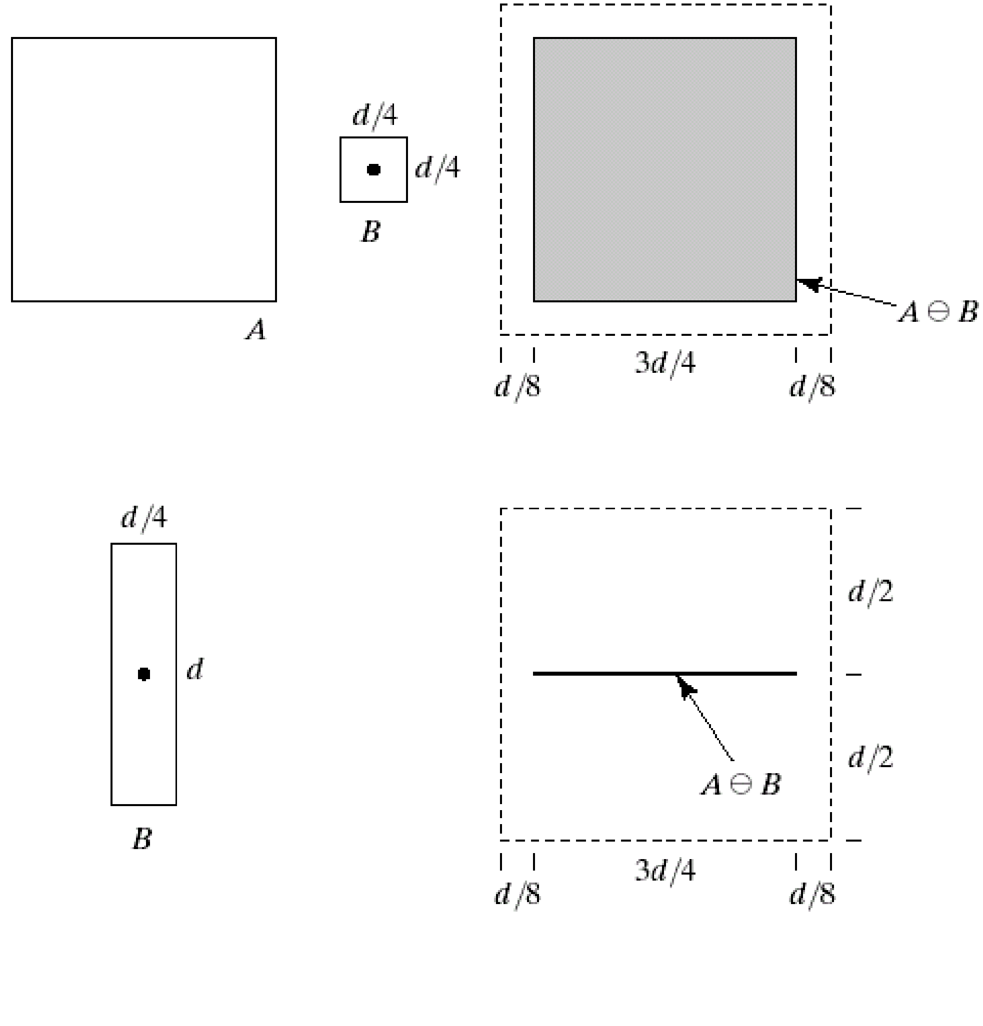
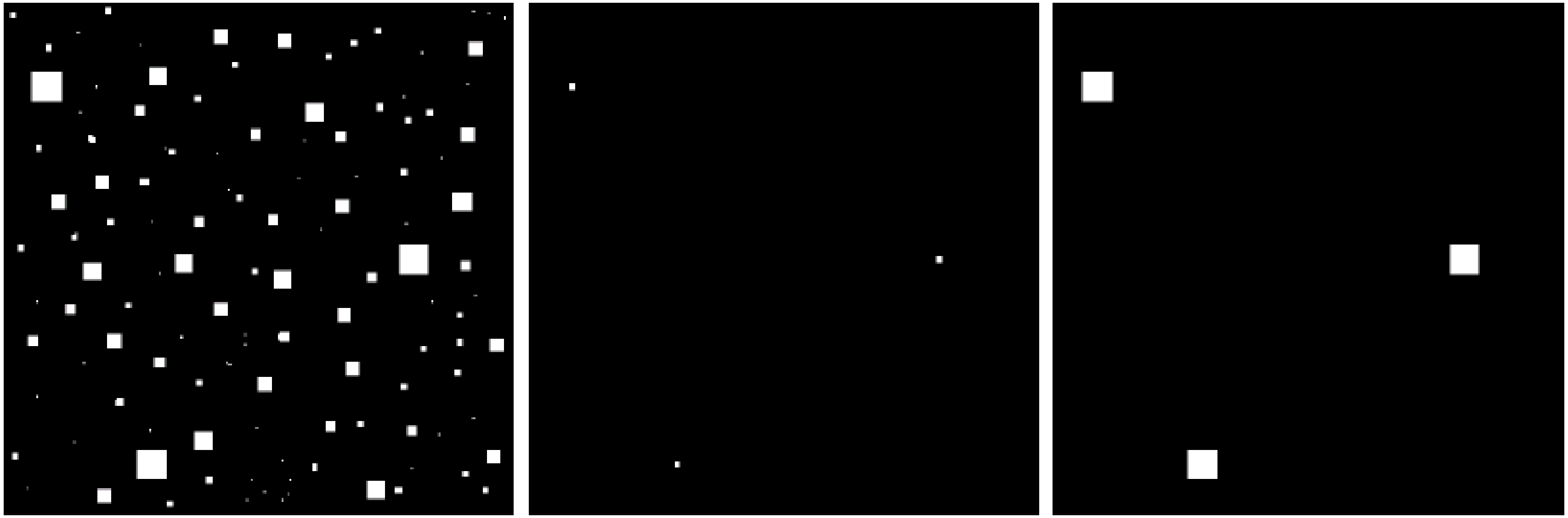



**FIGURE 9.6** (a) Set A. (b) Square structuring element. (c) Erosion of $A$ by $B$, shown shaded. (d) Elongated structuring element. (e) Erosion of $A$ using this element.

# Dilation and Erosion
## Erosion Example



a b c

**FIGURE 9.7** (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

# Opening and Closing

- Opening:
  - smooth the contour of an object, break narrow isthmuses, and eliminate thin protrusions.
  - The opening *A* by *B* is the erosion of *A* by *B*, followed by a dilation of the result by *B*

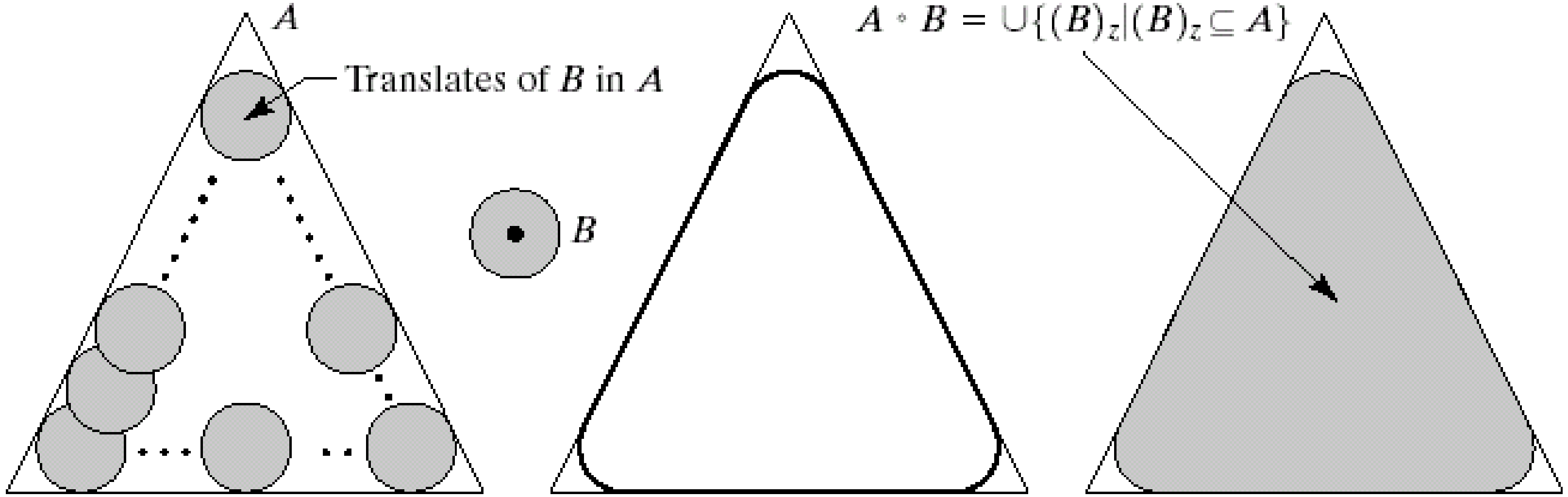
$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \cup \{(B)_z \mid (B)_z \subseteq A\}$$

- Closing:
  - smooth sections of contours but it generally fuses narrow breaks and long thing gulfs, eliminates small holes, and fills gaps in the contour.

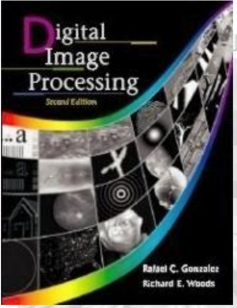
$$A \bullet B = (A \oplus B) \ominus B$$

# Opening and Closing
## Opening



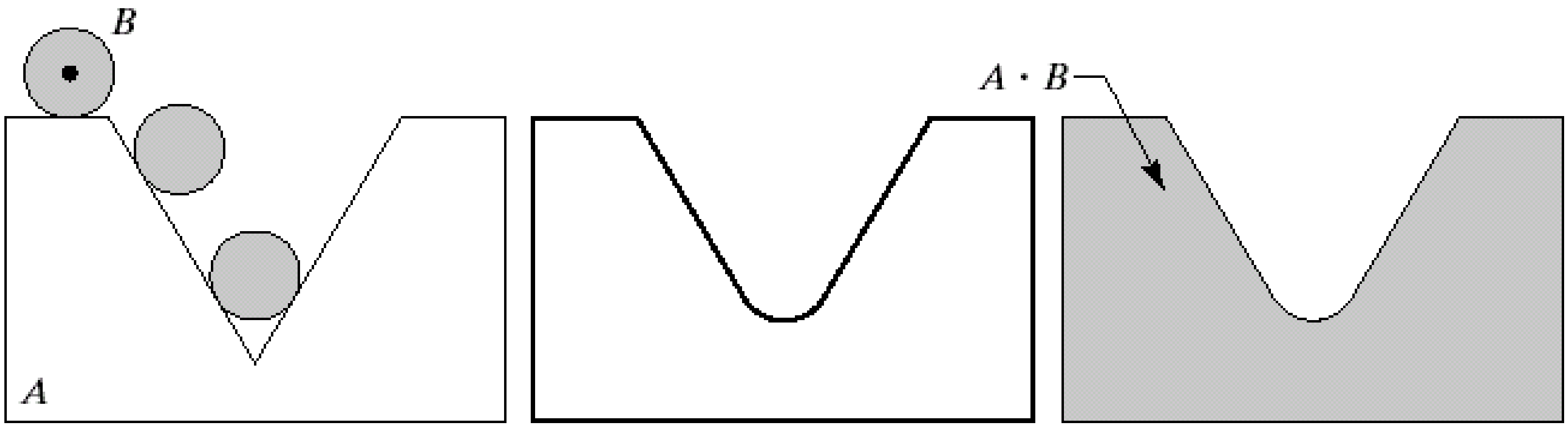

a b c d

**FIGURE 9.8** (a) Structuring element $B$ "rolling" along the inner boundary of $A$ (the dot indicates the origin of $B$). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).






# Opening and Closing
# Closing




a b c

**FIGURE 9.9** (a) Structuring element $B$ "rolling" on the outer boundary of set $A$. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

# Opening and Closing
# Opening Example

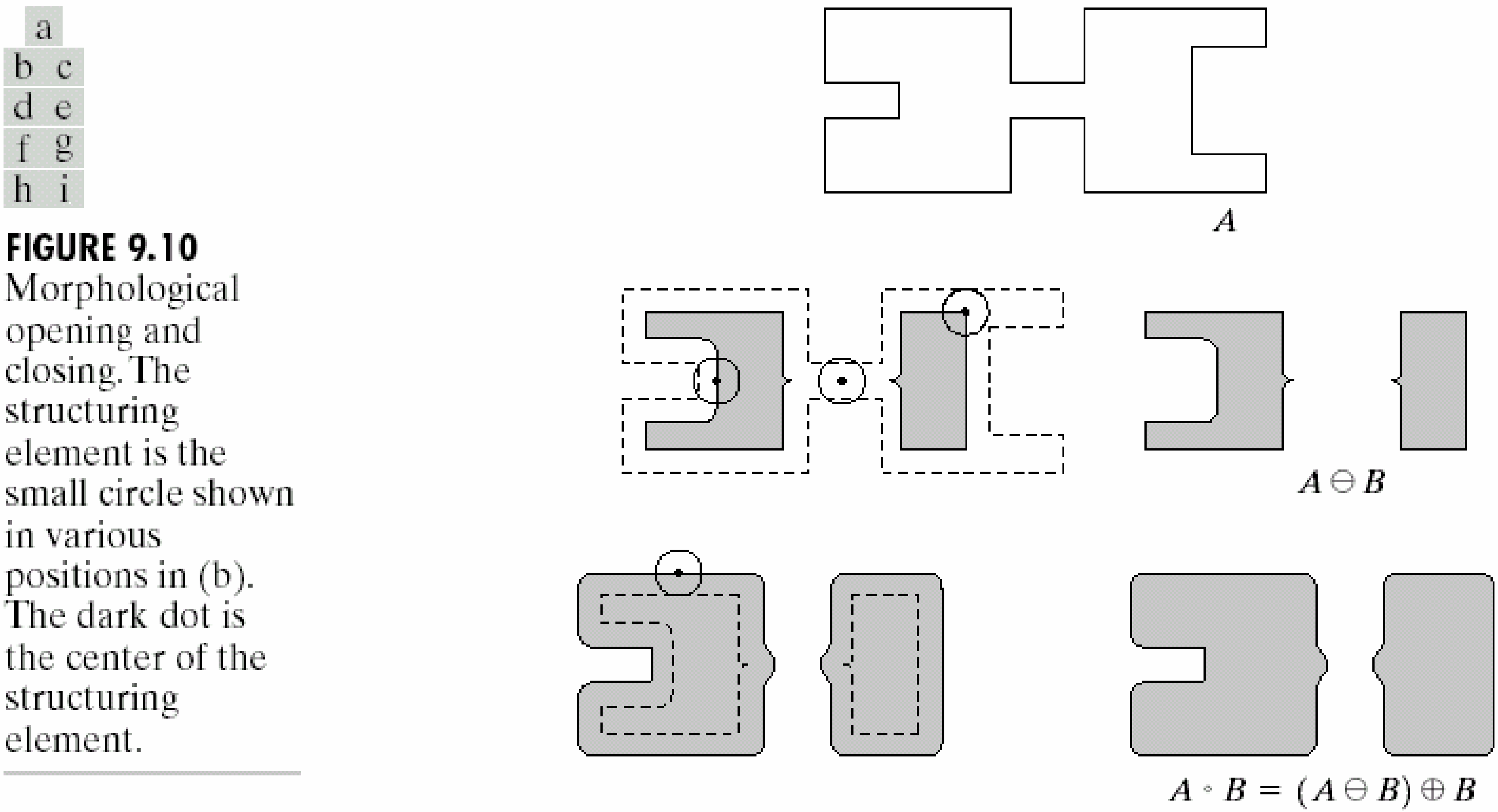
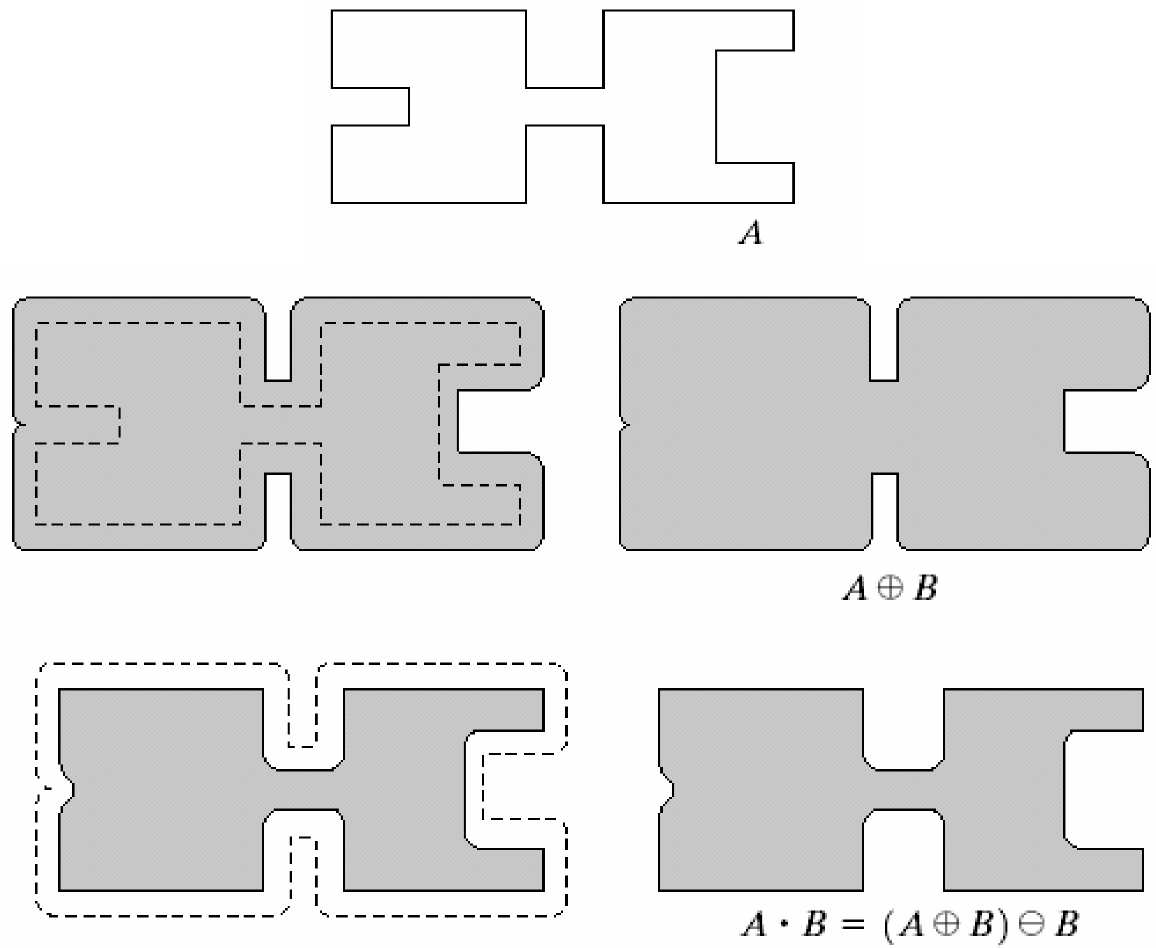



**FIGURE 9.10** Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

# Opening and Closing
# Closing Example

a
b c
d e
f g
h i

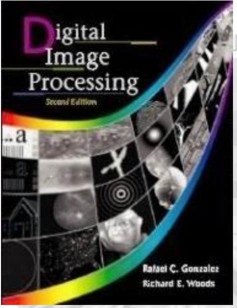
**FIGURE 9.10** Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



# Opening and Closing

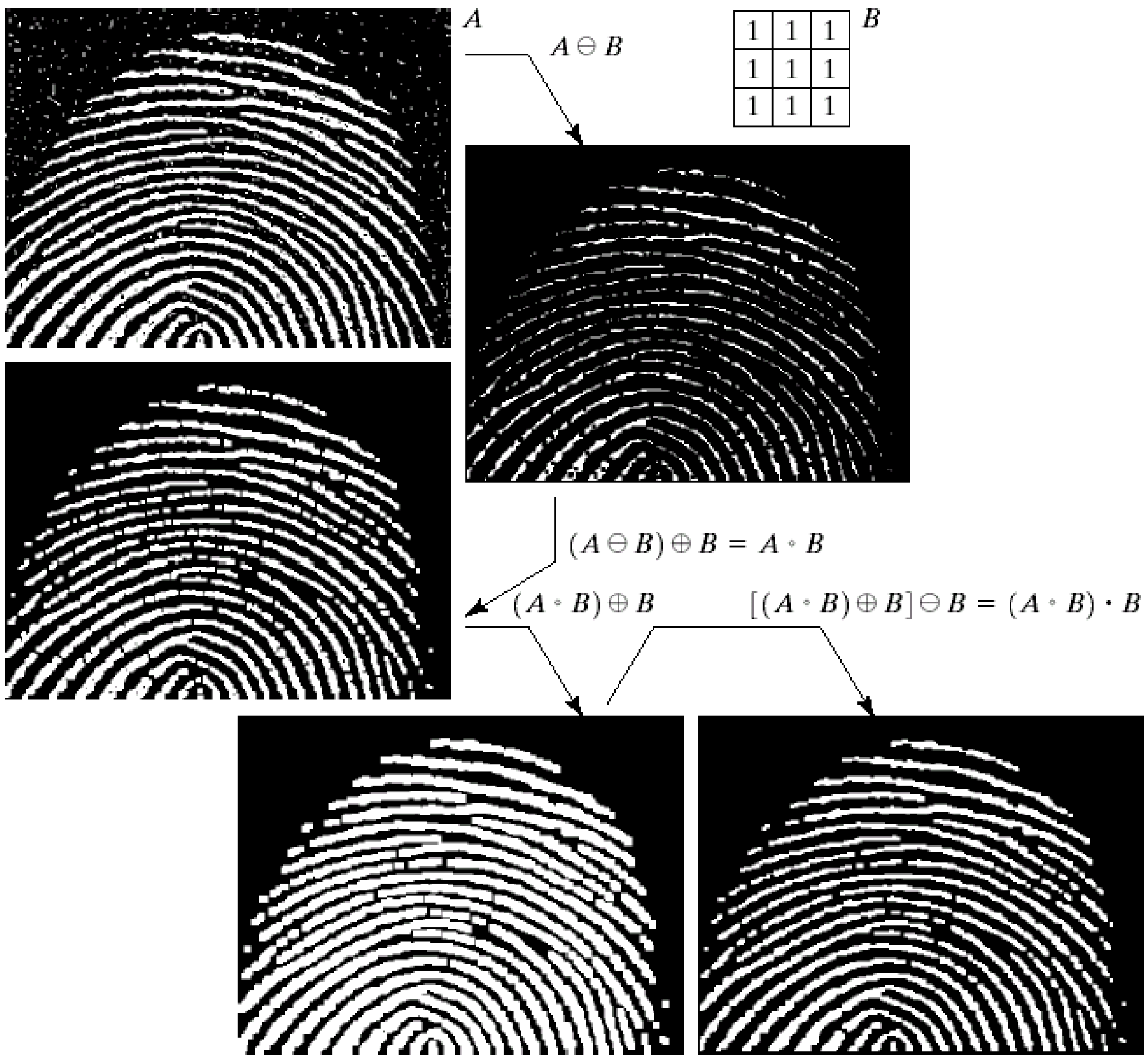
- The opening operation satisfies the following properties:
  - $A \circ B$ is a subset (subimage) of $A$.
  - If $C$ is a subset of $D$, then $C \circ B$ is a subset of $D \circ B$.
  - $(A \circ B) \circ B = A \circ B$

- The closing operation satisfies the following properties:
  - $A$ is a subset (subimage) of $A \bullet B$.
  - If $C$ is a subset of $D$, then $C \bullet B$ is a subset of $D \bullet B$.
  - $(A \bullet B) \bullet B = A \bullet B$






# Opening and Closing

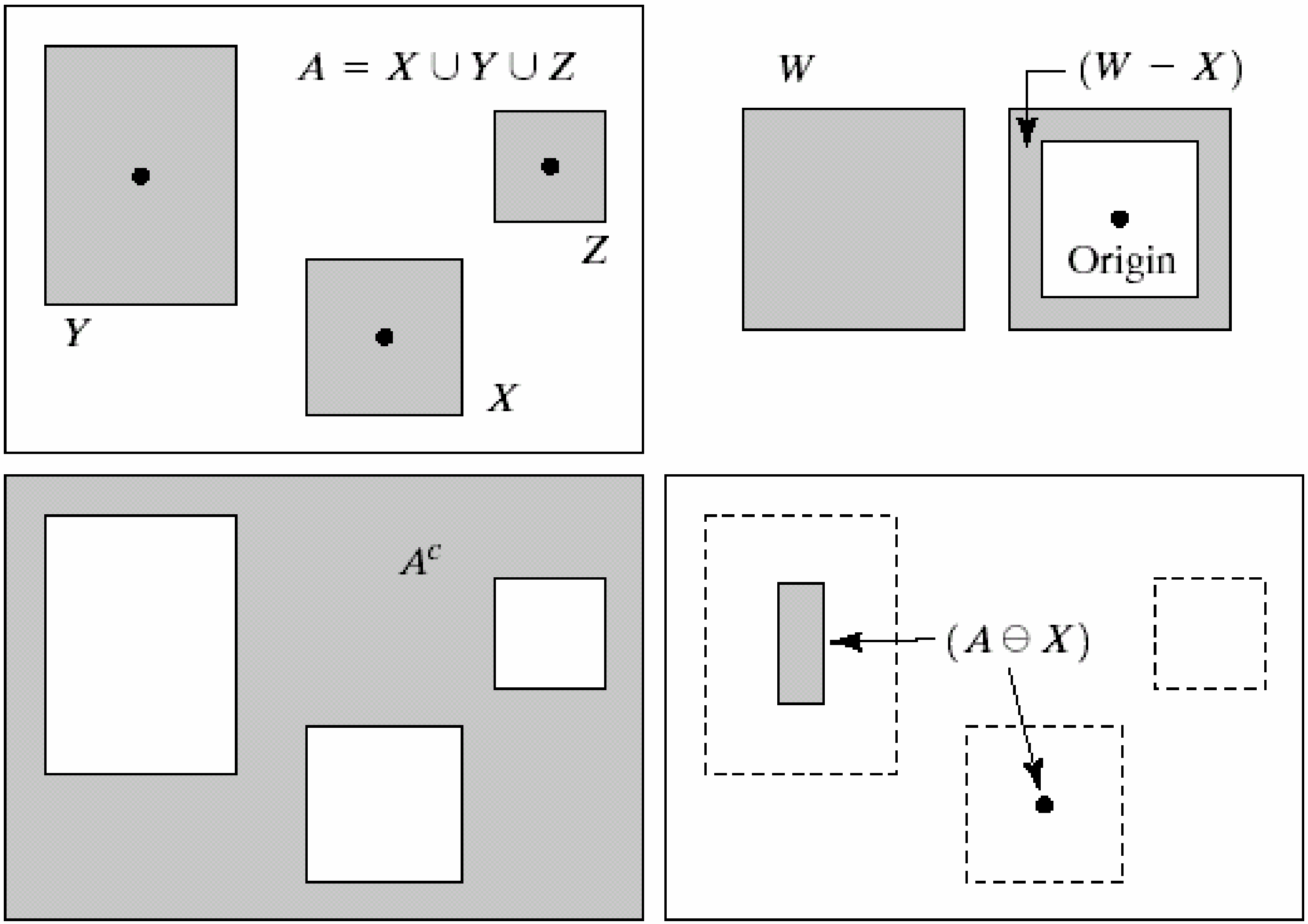



a b
d c
e f

**FIGURE 9.11** (a) Noisy image. (c) Eroded image. (d) Opening of $A$. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

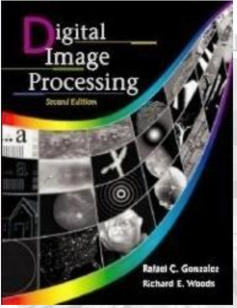
# The Hit-or-Miss Transformation

$$A \circledast B = (A \ominus X) \cap \left[A^c \ominus (W - X)\right] \qquad B = (X, W - X)$$




a b
c d
e
f

**FIGURE 9.12**
(a) Set $A$. (b) A window, $W$, and the local background of $X$ with respect to $W$, $(W - X)$. (c) Complement of $A$. (d) Erosion of $A$ by $X$. (e) Erosion of $A^c$ by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of $X$, as desired.





# The Hit-or-Miss Transformation

$$A \circledast B = (A \ominus X) \cap \left[A^c \ominus (W - X)\right] \qquad B = (X, W - X)$$

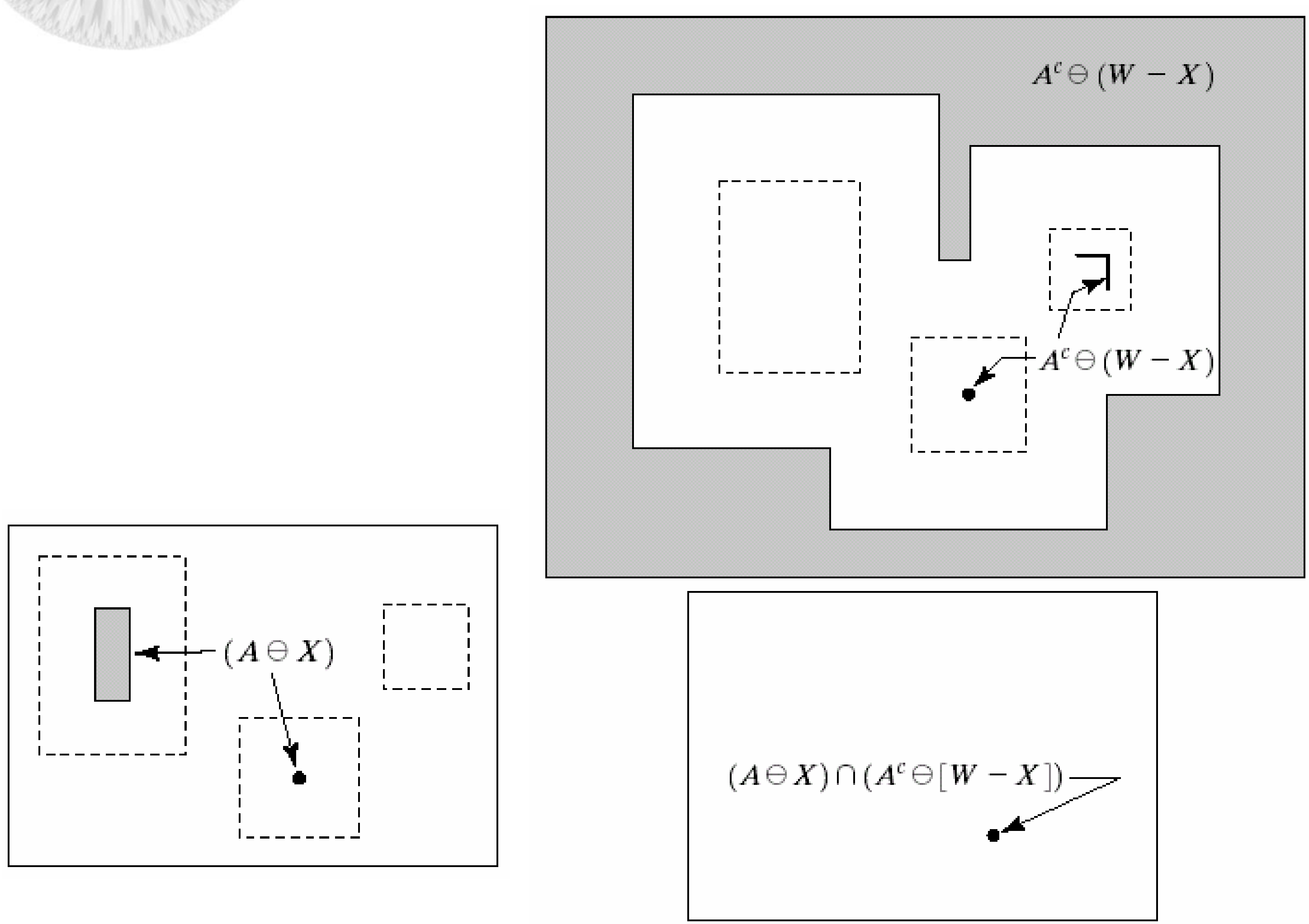



**FIGURE 9.12** (a) Set $A$. (b) A window, $W$, and the local background of $X$ with respect to $W$, $(W - X)$. (c) Complement of $A$. (d) Erosion of $A$ by $X$. (e) Erosion of $A^c$ by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of $X$, as desired.

# Some Basic Morphological Algorithms
# Boundary Extraction

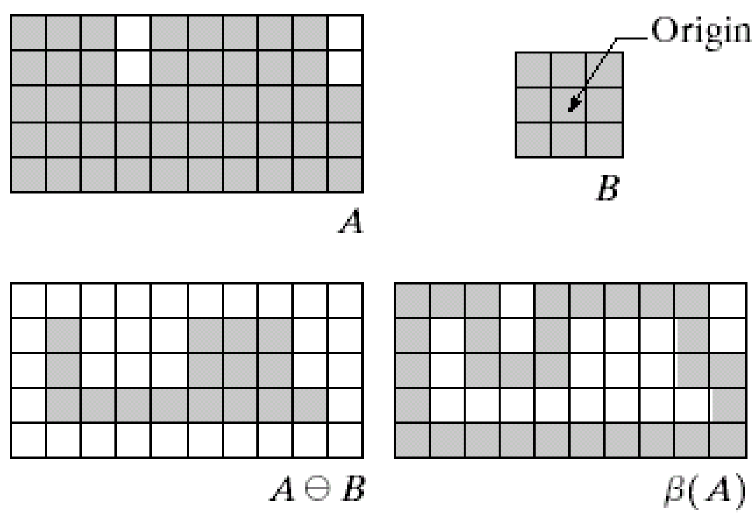
- The boundary of a set $A$,

$$\beta(A) = A - (A \ominus B)$$

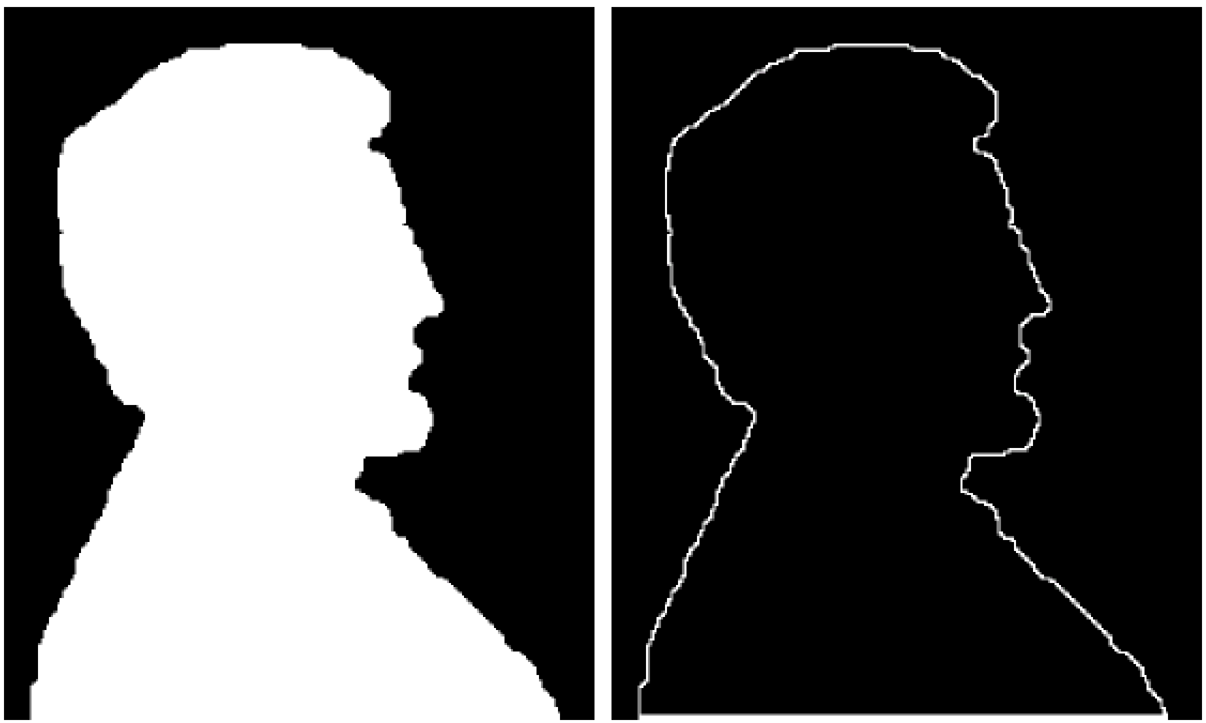
where $B$ is a suitable structuring element.

a b
c d

**FIGURE 9.13** (a) Set $A$. (b) Structuring element $B$. (c) $A$ eroded by $B$. (d) Boundary, given by the set difference between $A$ and its erosion.



# Some Basic Morphological Algorithms
# Boundary Extraction Example



a b

**FIGURE 9.14**
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

# Some Basic Morphological Algorithms
# Region Filling

- The following procedure can fill the region:

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

where $X_0 = p$, and $B$ is the symmetric structuring element shown in Fig. 9.15.

- Note that $p$ is the initial point we should assign.

# Some Basic Morphological Algorithms
# Region Filling

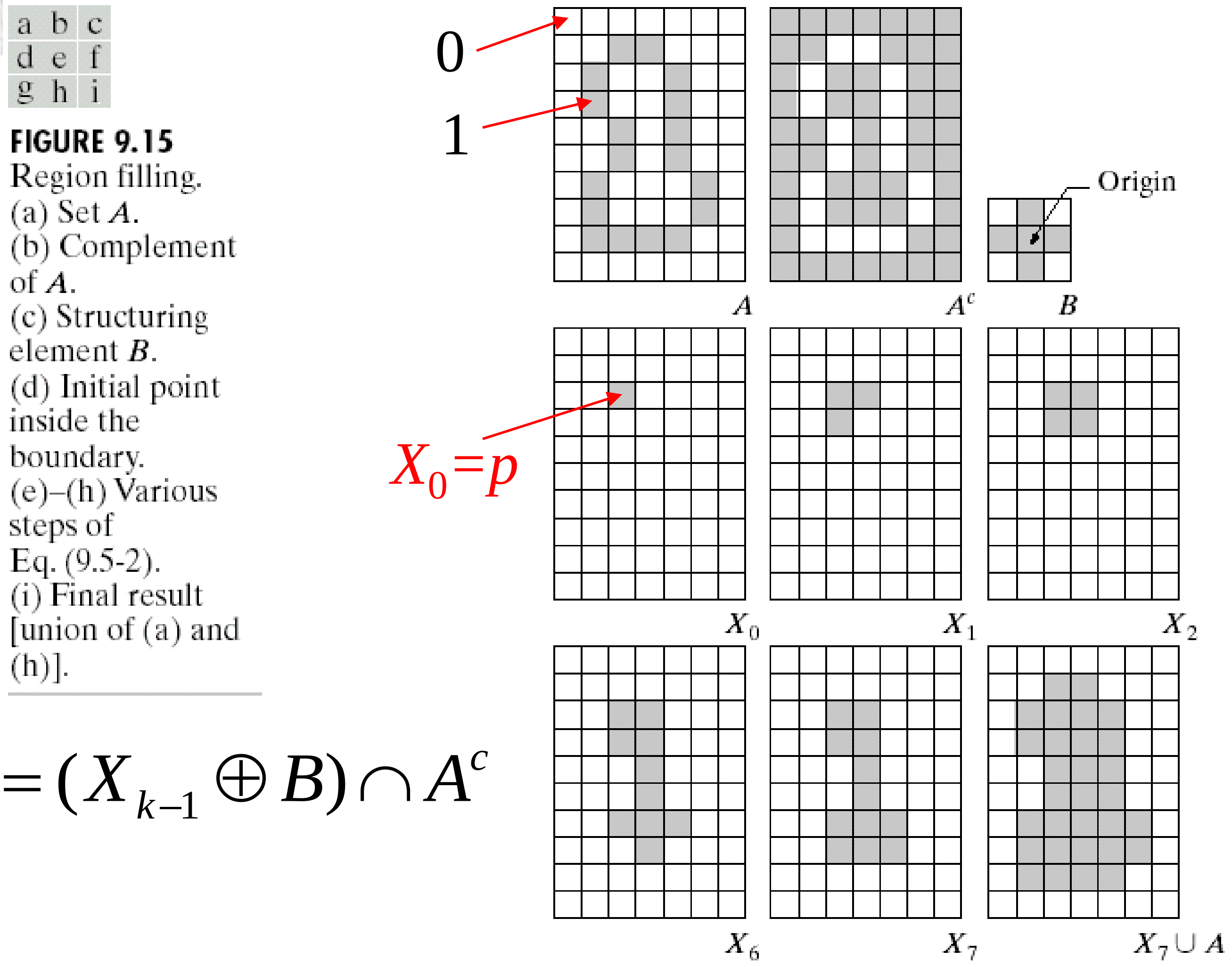
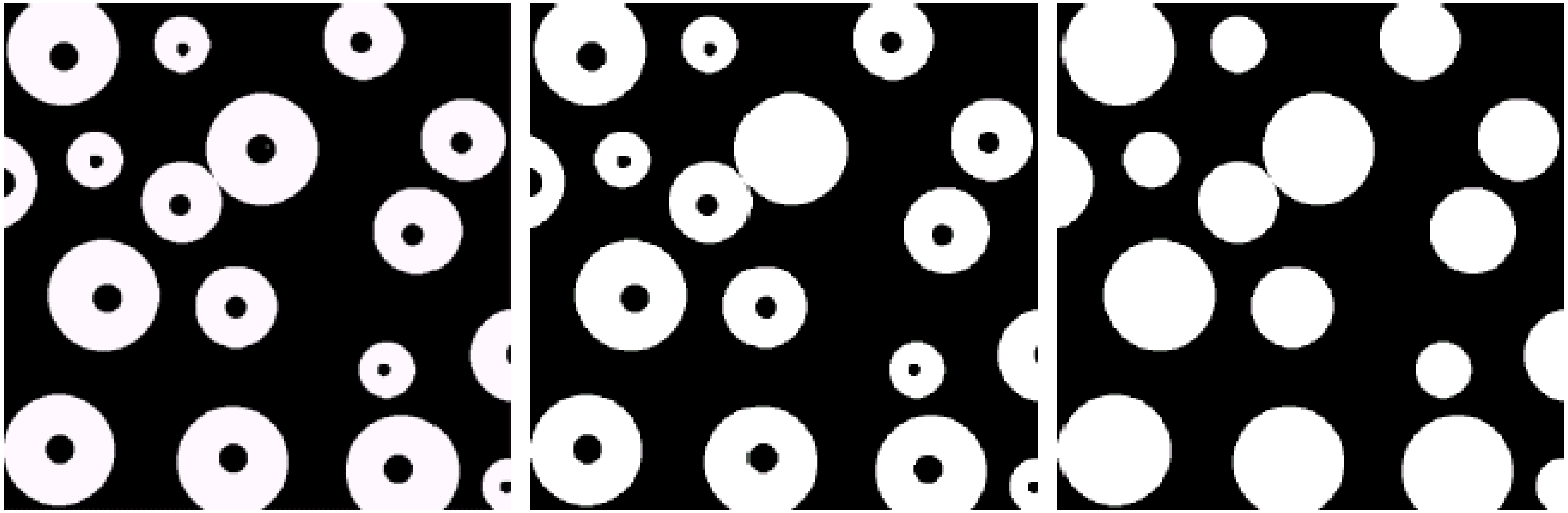



**FIGURE 9.15** Region filling. (a) Set $A$. (b) Complement of $A$. (c) Structuring element $B$. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

# Some Basic Morphological Algorithms
## Region Filling



a b c

**FIGURE 9.16** (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

# Some Basic Morphological Algorithms Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A$$

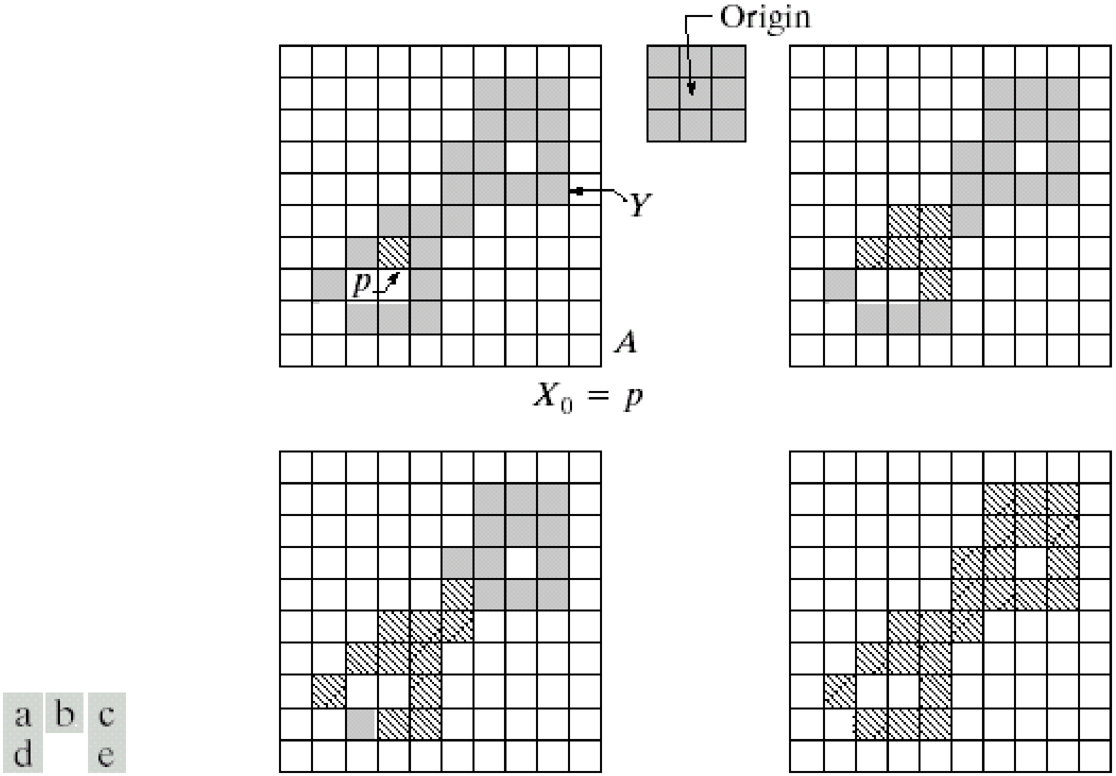
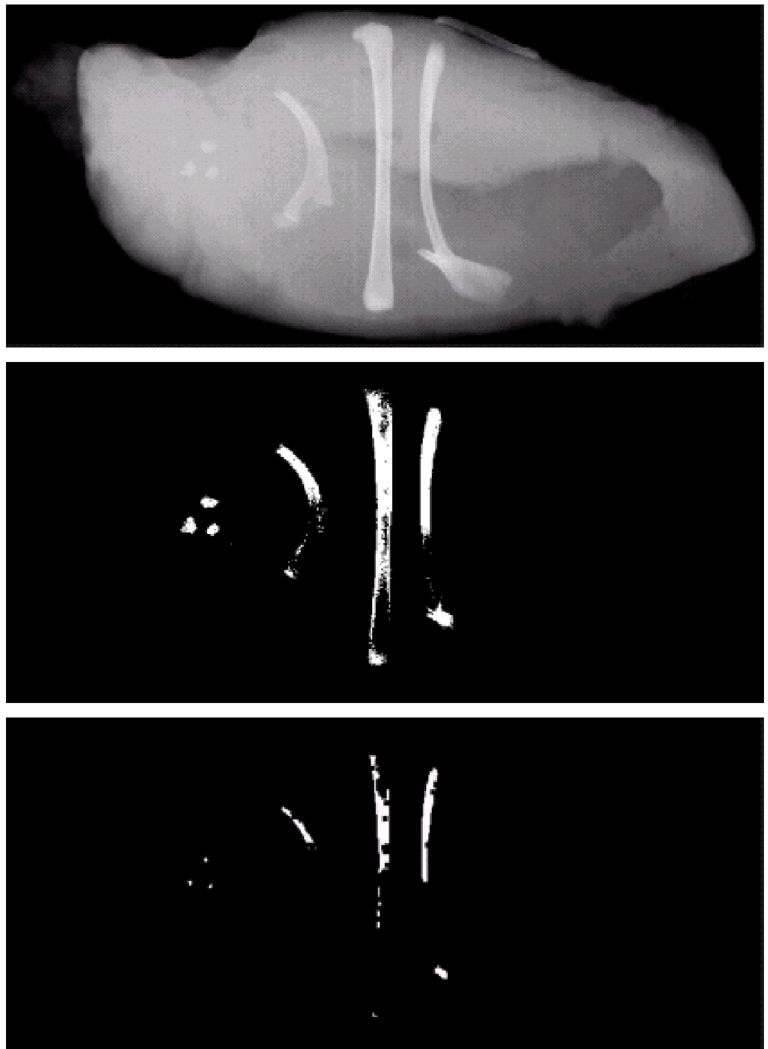



**FIGURE 9.17** (a) Set $A$ showing initial point $p$ (all shaded points are valued 1, but are shown different from $p$ to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

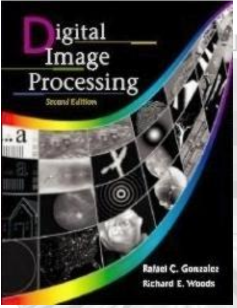
# Some Basic Morphological Algorithms
# Extraction of Connected Components



a
b
c d

**FIGURE 9.18** (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

| Connected component | No. of pixels in connected comp |
|---|---|
| 01 | 11 |
| 02 | 9 |
| 03 | 9 |
| 04 | 39 |
| 05 | 133 |
| 06 | 1 |
| 07 | 1 |
| 08 | 743 |
| 09 | 7 |
| 10 | 11 |
| 11 | 11 |
| 12 | 9 |
| 13 | 9 |
| 14 | 674 |
| 15 | 85 |






# Some Basic Morphological Algorithms
## Thinning

- The thinning of a set *A* by a structuring element *B* can be defined in terms of the hit-or-miss transform:

$$A \otimes B = A - (A \circledast B)$$

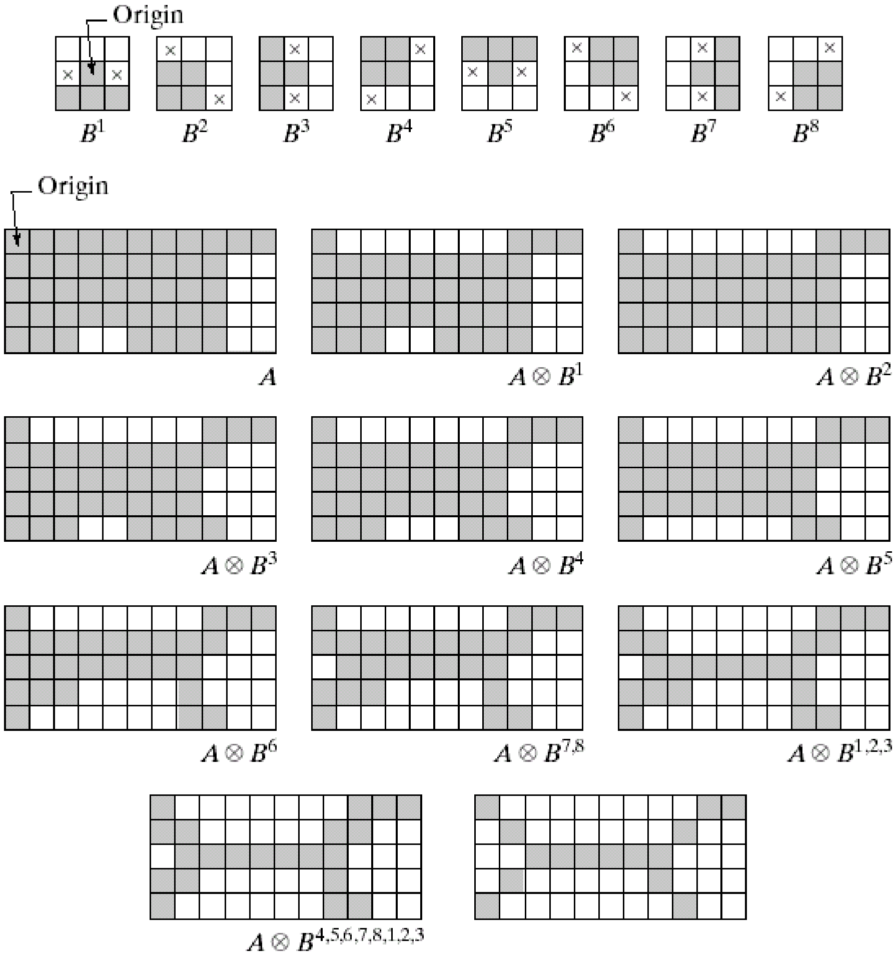
$$= A \cap (A \circledast B)^c$$

- A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

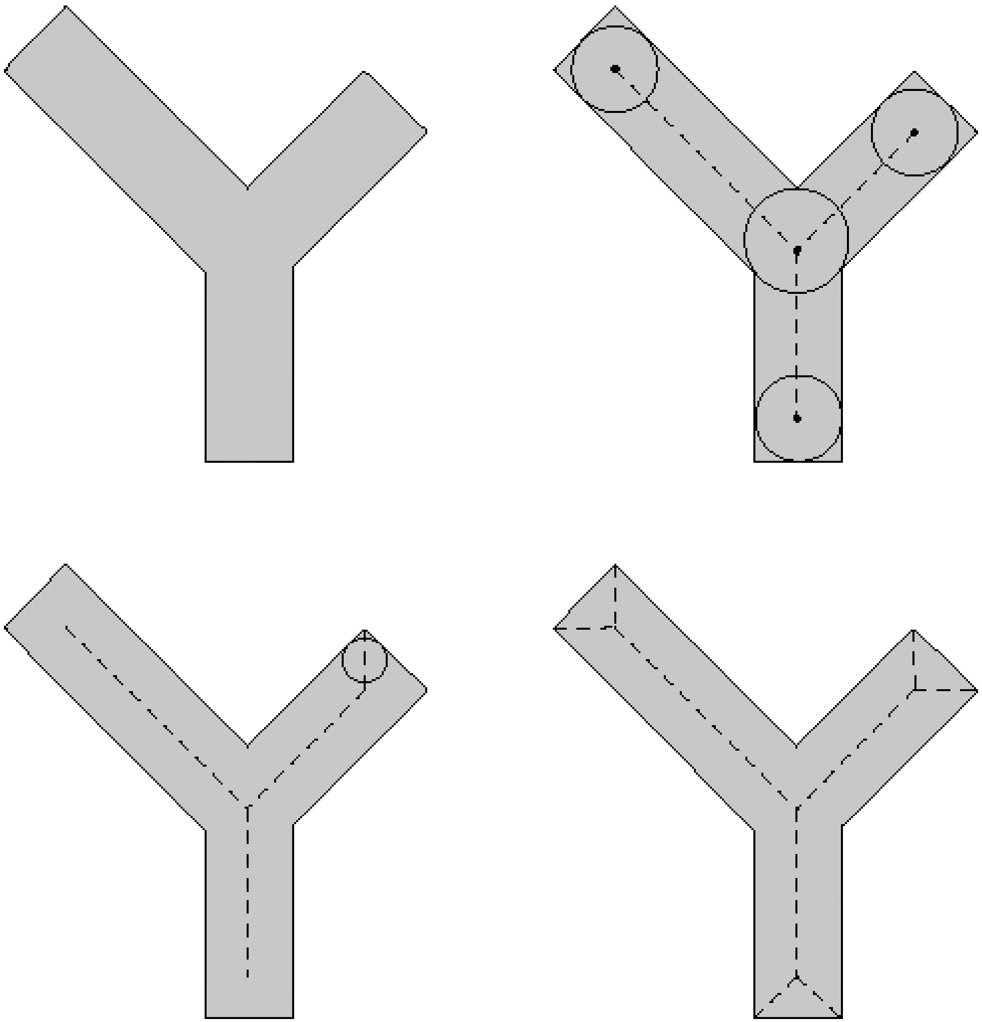
# Some Basic Morphological Algorithms
## Thinning




a
b c d
e f g
h i j
k l

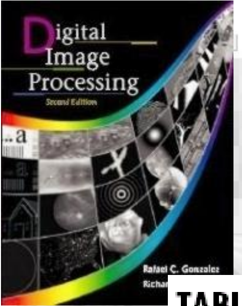
**FIGURE 9.21** (a) Sequence of rotated structuring elements used for thinning. (b) Set $A$. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to $m$-connectivity.

# Some Basic Morphological Algorithms
## Skeletons



a b
c d

**FIGURE 9.23**
(a) Set $A$.
(b) Various positions of maximum disks with centers on the skeleton of $A$.
(c) Another maximum disk on a different segment of the skeleton of $A$.
(d) Complete skeleton.





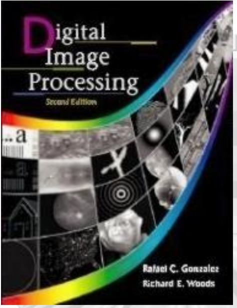

# Summary of Morphological Operations and Their Properties

**TABLE 9.2** Summary of morphological operations and their properties.

| **Operation** | **Equation** | **Comments** (The Roman numerals refer to the structuring elements shown in Fig. 9.26). |
|---|---|---|
| Translation | $(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$ | Translates the origin of $A$ to point $z$. |
| Reflection | $\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$ | Reflects all elements of $B$ about the origin of this set. |
| Complement | $A^c = \{w \mid w \notin A\}$ | Set of points not in $A$. |
| Difference | $A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$ | Set of points that belong to $A$ but not to $B$. |
| Dilation | $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$ | "Expands" the boundary of $A$. (I) |
| Erosion | $A \ominus B = \{z \mid (B)_z \subseteq A\}$ | "Contracts" the boundary of $A$. (I) |
| Opening | $A \circ B = (A \ominus B) \oplus B$ | Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I) |
| Closing | $A \bullet B = (A \oplus B) \ominus B$ | Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I) |

# Summary of Morphological Operations and Their Properties

| | | |
|---|---|---|
| Hit-or-miss transform | $A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$<br>$= (A \ominus B_1) - (A \oplus \hat{B}_2)$ | The set of points (coordinates) at which, simultaneously, $B_1$ found a match ("hit") in $A$ and $B_2$ found a match in $A^c$. |
| Boundary extraction | $\beta(A) = A - (A \ominus B)$ | Set of points on the boundary of set $A$. (I) |
| Region filling | $X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$ | Fills a region in $A$, given a point $p$ in the region. (II) |
| Connected components | $X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$ | Finds a connected component $Y$ in $A$, given a point $p$ in $Y$. (I) |
| Convex hull | $X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$. | Finds the convex hull $C(A)$ of set $A$, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III) |






# Summary of Morphological Operations and Their Properties

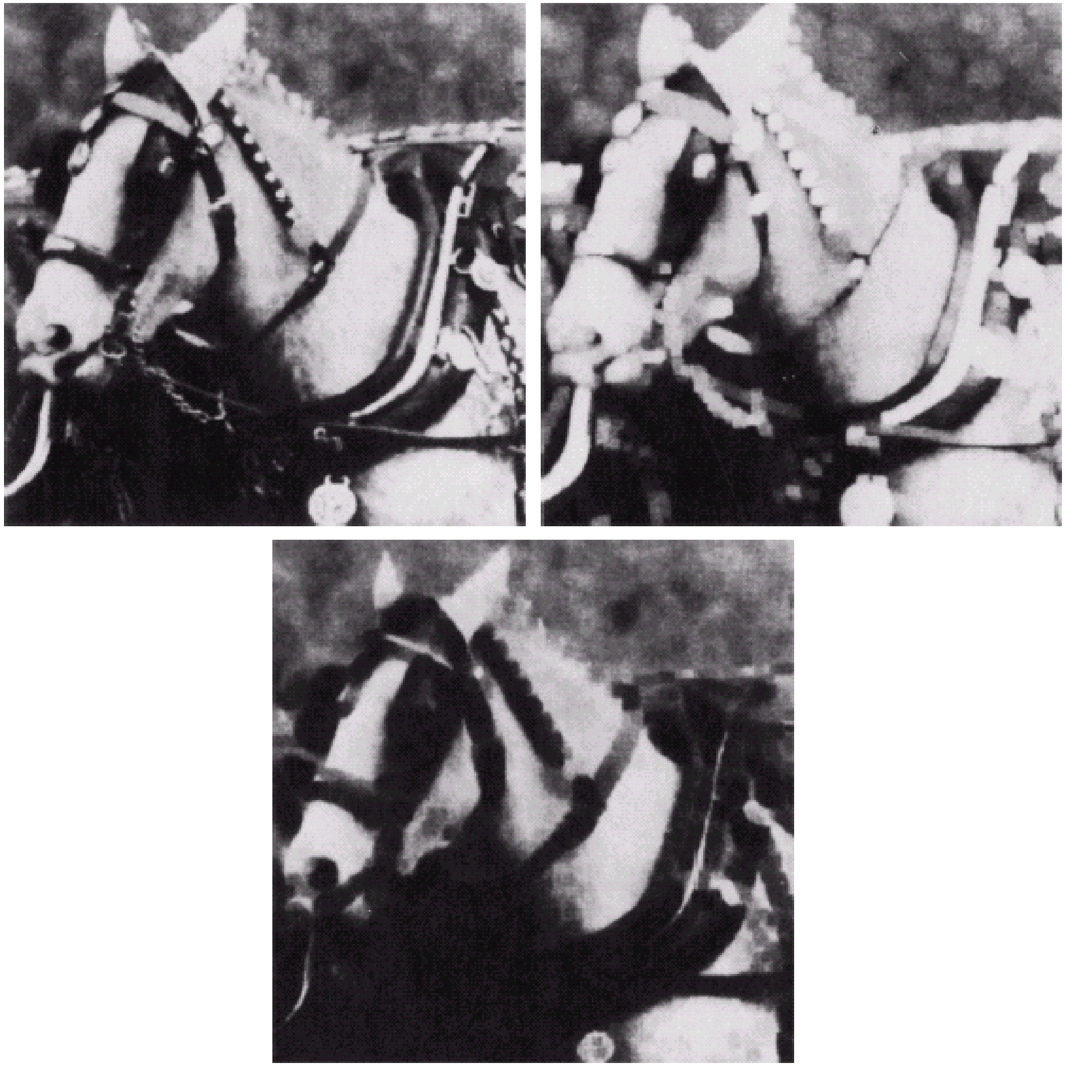
| Operation | Equation | Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26). |
|---|---|---|
| Thinning | $A \otimes B = A - (A \circledast B)$ <br> $= A \cap (A \circledast B)^c$ <br><br> $A \otimes \{B\} =$ <br> $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ <br> $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$ | Thins set $A$. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV) |
| Thickening | $A \odot B = A \cup (A \circledast B)$ <br><br> $A \odot \{B\} =$ <br> $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$ | Thickens set $A$. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed. |

**TABLE 9.2**
Summary of morphological results and their properties. *(continued)*

# Summary of Morphological Operations and Their Properties

| | | |
|---|---|---|
| Skeletons | $S(A) = \bigcup_{k=0} S_k(A)$ <br> $S_k(A) = \bigcup_{k=0}^{K}\{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ <br> Reconstruction of $A$: <br> $A = \bigcup_{k=0}^{K}(S_k(A) \oplus kB)$ | Finds the skeleton $S(A)$ of set $A$. The last equation indicates that $A$ can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, $K$ is the value of the iterative step after which the set $A$ erodes to the empty set. The notation $(A \ominus kB)$ denotes the $k$th iteration of successive erosion of $A$ by $B$. (I) |
| Pruning | $X_1 = A \otimes \{B\}$ <br> $X_2 = \bigcup_{k=1}^{8}(X_1 \circledast B^k)$ <br> $X_3 = (X_2 \oplus H) \cap A$ <br> $X_4 = X_1 \cup X_3$ | $X_4$ is the result of pruning set $A$. The number of times that the first equation is applied to obtain $X_1$ must be specified. Structuring elements V are used for the first two equations. In the third equation $H$ denotes structuring element I. |

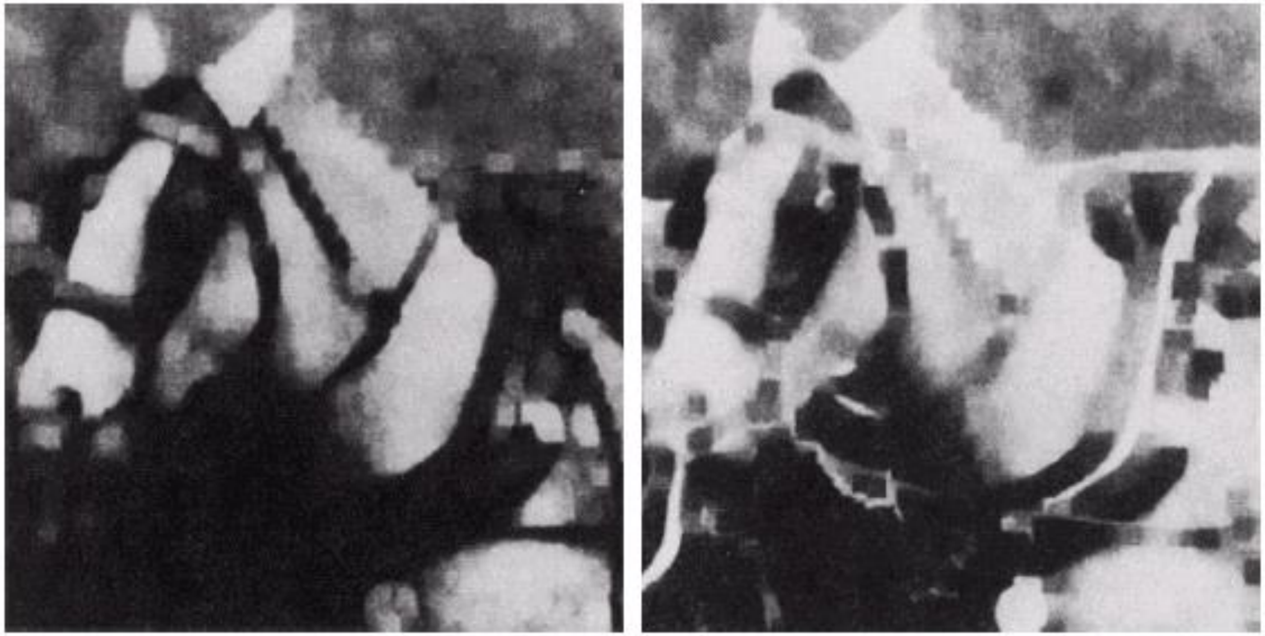
# Extensions to Gray-Scale Images



a b
c

**FIGURE 9.29** (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

# Extensions to Gray-Scale Images



a b

**FIGURE 9.31** (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

# Extensions to Gray-Scale Images

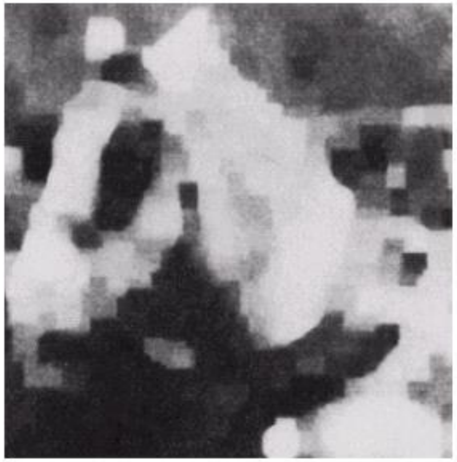


**FIGURE 9.32** Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

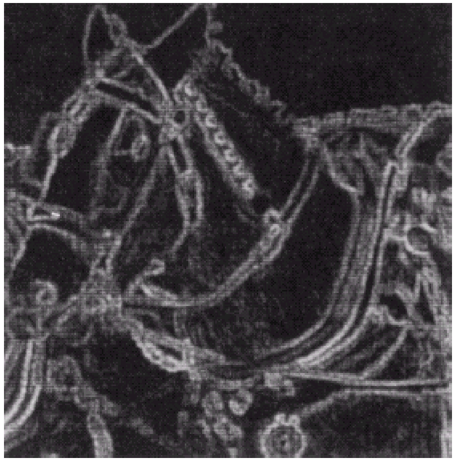


**FIGURE 9.33** Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)