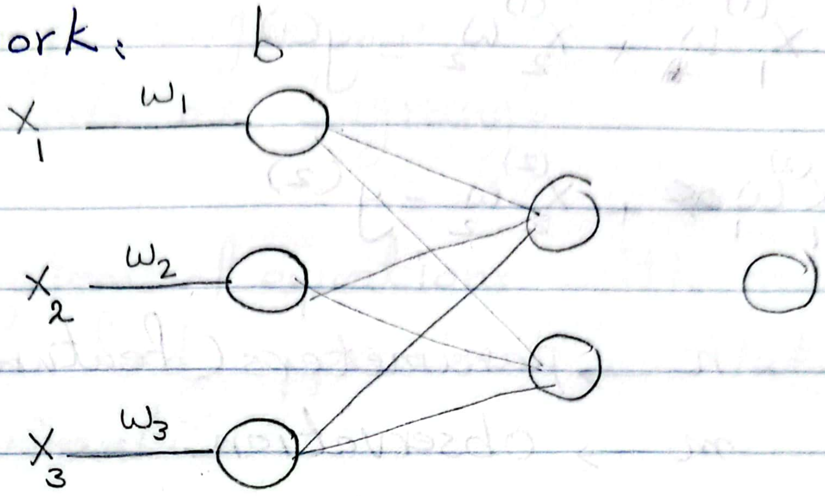
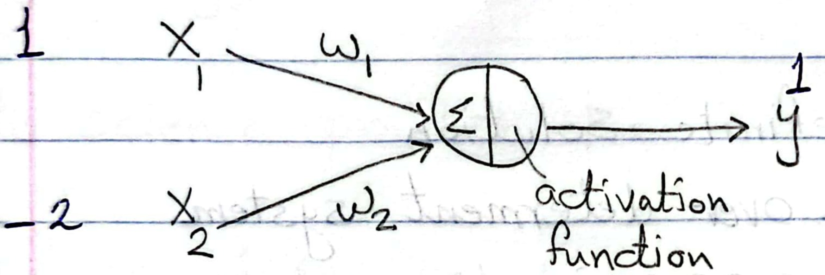
*Day_1 اللهم انفعني بهذا العلم وانفع الناس بي

why Math is important in Data Science?

Neural Network:





$$y = w_1 x_1 + w_2 x_2 \Rightarrow O/P$$

$x_1, x_2 - - - - \rightarrow x_n$

To find w_1, w_2

$w_1 - 2w_2 = 1$

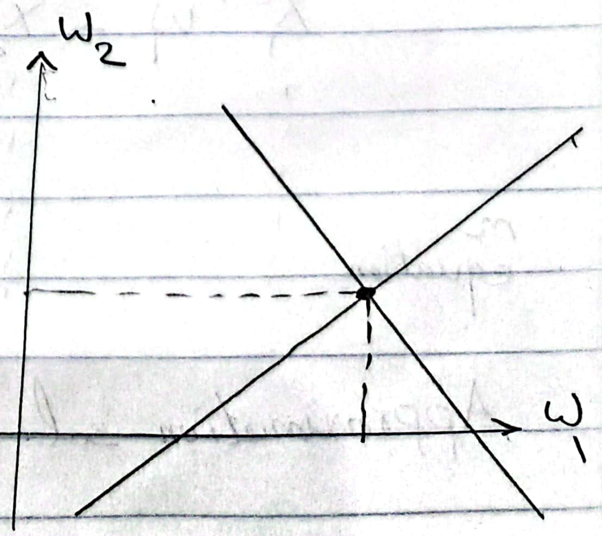
$3w_1 + 2w_2 = 11$

m

x_1	x_2	y
1	-2	1
3	2	11

$$w_1 = -\frac{2}{3}w_2 - \frac{11}{3}$$

w_1, w_2



Convert data to Linear equations

$$x_1^{(1)} w_1 + x_2^{(1)} w_2 = y^{(1)}$$

$$x_1^{(2)} w_1 + x_2^{(2)} w_2 = y^{(2)}$$

$n \rightarrow$ parameters (features)

$m \rightarrow$ observation

m equations , n unknowns

$m = n$ definite Solution

$m > n \rightarrow$ over determent system

approximation Solution

$m < n \Rightarrow$ infinite Solution.

$\longrightarrow n$ unknown

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m equation

Approximation sol. $\rightarrow$ قيمة w_1 ، w_2 ... أقرب ما يمكن للحل

$m > n \rightarrow$ least square

$m < n \rightarrow$ multi methods ex. Lagrange

$\rightarrow$ images "image recognition"

we represent system of equations with matrices of vector, and find approximation solution

$\hat{y} \rightarrow$ approximation

$$SE = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}-y)^2$$ $\leftarrow$ minimaxize error

Data science problem has mathematical meaning.

model: is relationship between Input and output.

* Image is stored as matrices.

* Gradient is a vector $\begin{bmatrix} \frac{\partial J}{\partial w_1} \\ \frac{\partial J}{\partial w_2} \end{bmatrix}$

* what is Linear Algebra:

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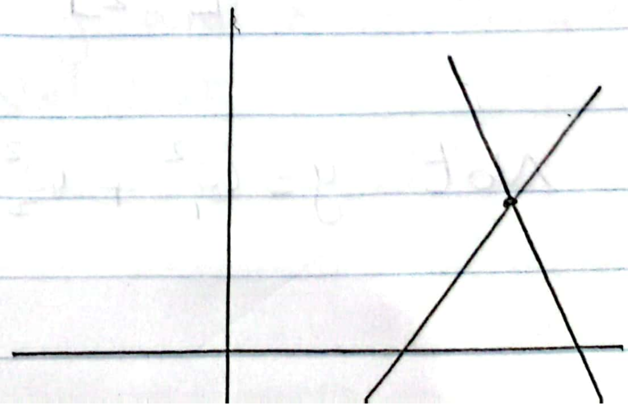
$$3w_1 + 2w_2 = 1$$



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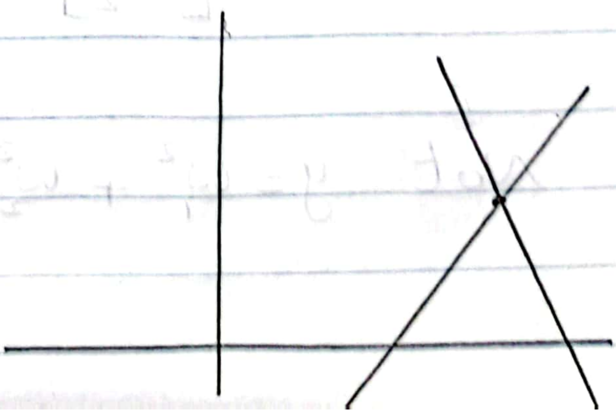
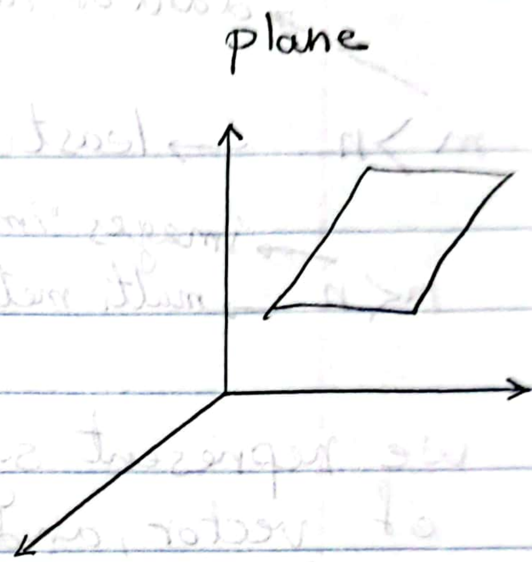
$$3\omega_1 + 2\omega_2 = 1$$





$$w_1 + w_2 + w_3 = \_\_$$
$$-w_1 + 2w_2 + w_3 = \_\_$$
$$w_1 + 4w_2 + 5w_3 = \_\_$$

the relationships are linear.
and can represent in matrices vector operation

$$\underbrace{\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}}_{\text{matrix}} \underbrace{\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}}_{\text{vector}} = \underbrace{\begin{bmatrix} 1 \\ 11 \end{bmatrix}}_{\text{vector}}$$

$$Aw = y \qquad \boxed{w = A^{-1}y}$$

- Solve using matrices vector operation

- the relationship between vectors are linear

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$c_1 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad c_2 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Not $y = w_1^2 + w_2^2$ or $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}^2$

If I have non-linear
Do approximation to → linear

Image as Matrix, Each pixel has intensity of (white or black Color)

As Data Scientist, you start with Data

n features → target

	x_1 Area	x_2 no. of bedroom	———— x_n	y
→				
m observation →				
→				

to predict houses price

Each row called observation.

Goal: General model

$$y = f(x_1, x_2, ------, x_n)$$

→ mathematical function

If relationship between y and x is linear
→ linear function

If relationship between y and x isnot linear
→ non-Linear function

If relationship between y and x is Complex nonlinear → Neural Network

model is just a mathematical function.

From mathematical point of view

$x_1, x_2, ---, x_n \rightarrow$ independent variable

$y \rightarrow$ dependent variable

problem (اختبار ال model يعتمد على شكل الداتا)

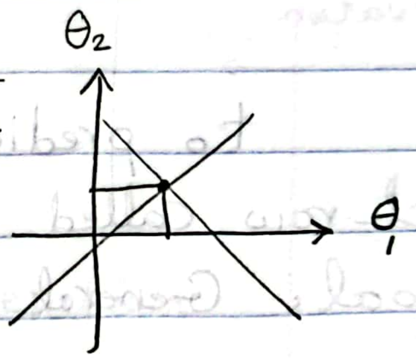
* Linear Models

After Analysis, we find that the relationship between Y, X Can represent by linear model.

Choose two variables x_1, x_2

x_1	x_2	y	$\hat{y}$ predicted
5	3	10	10
6	2	13	13

$$y = \theta_1 x_1 + \theta_2 x_2$$



$$5\theta_1 + 3\theta_2 = 10$$
$$6\theta_1 + 2\theta_2 = 13$$

In this case, After Solve equations, we have accurate model But in these only two observations

* To get general model, we find approximate solution

If $m > n$

no. of equations > no. of unknowns

↳ No definite solution, we find approximate Solution

x_1 x_2 y
5 6 11

$m < n$

↳ infinite number of solution.

x_1	x_2	y
5	3	4
6	2	8

$$\left.\begin{array}{l} 5\theta_1 + 3\theta_2 = 4 \\ 6\theta_1 + 2\theta_2 = 8 \end{array}\right] \rightarrow \text{System of linear equations.}$$

Can be presnted as

$$\underbrace{\overset{X}{\begin{bmatrix} 5 & 3 \\ 6 & 2 \end{bmatrix}}}_{\text{matrix}} \underbrace{\overset{\theta}{\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}}}_{\text{vector}} = \underbrace{\overset{y}{\begin{bmatrix} 4 \\ 8 \end{bmatrix}}}_{\text{vector}}$$

$$X\theta = y \Longrightarrow \boxed{\theta = X^{-1} y}$$

Matrix must be square → to have inverse

If $\begin{bmatrix} 5 & 3 \\ 2 & 4 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ $\quad m > n$

$$\theta = X^{-1} y \quad \times$$

because this matrix isn't square matrix, and hasn't inverse.

Case 1

$$\begin{bmatrix} 5 & 6 & 7 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$m < n$$

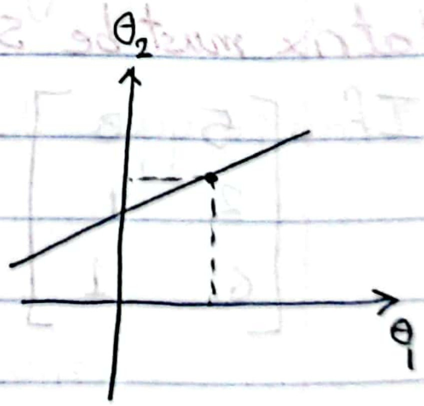
$$\cancel{\theta = X^{-1} y}$$

Case 2

$$\begin{bmatrix} 5 & 6 & 7 \\ 3 & 2 & 1 \\ 4 & 5 & 5 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$$

$$m = n \Rightarrow \theta = X^{-1} y \checkmark$$

If two Columns is redundant, it is singular matrix and hasn't inverse.
If two rows is redundant, it is singular matrix and hasn't inverse.



$$5\theta_1 + 6\theta_2 = 3$$

If I suppose θ_1 by any value,
I can get value of θ_2
Degree of freedom = 1
Infinite solution

$m < n$ ← no. of variables
↓
no. of

Case 3

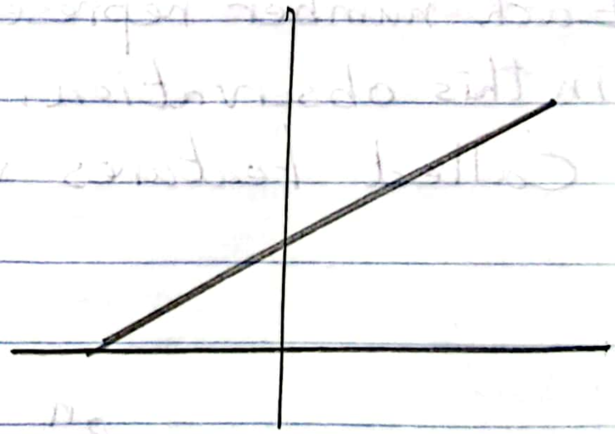
$$10x_1 + 5x_2 = 100$$
$$20x_1 + 10x_2 = 200$$

→ infinite solution
redundant observation
doesn't add any information

$$15x_1 + 16x_2 = 500$$

Can solve these system of equation

If redundant observation → when draw
two lines are identical منطبقين على بعض



Case 4

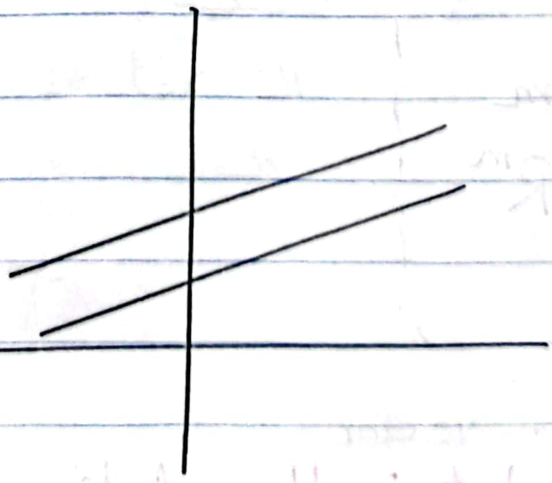
$$5x_1 + 10x_2 = 100$$
$$10x_1 + 20x_2 = 300$$

→ when draw, two parallel lines

no solution

$$\begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 300 \end{bmatrix}$$

redundant columns



* what is the vector?

Line has start point and end point and direction

Data In any dimension x, y, z

point نقطة

from CS, DS point of view

vector is list of Numbers

$$\overline{\quad R^n \quad}$$

$$\begin{array}{ccccc} x_1 & x_2 & \text{— — —} & x_n & \\ [\,5 & 6 & 7 & 6\,] & \leftarrow \text{row } \cancel{\text{of}} \text{ vector} \end{array}$$

Each number represent certain value of feature in this observation.

Called features vector

$$\begin{bmatrix} 5 \\ 6 \\ 7 \\ \vdots \\ 6 \end{bmatrix}$$

$n \times 1$

$$R^n$$

$$\begin{array}{cccccccc} & x_1 & x_2 & \text{— — —} & & x_n & & \\ \text{vector} \rightarrow & 5 & 6 & 7 & \ldots & 6 & y & \hat{y} \end{array}$$

vector

vector

m

R^m

Vector

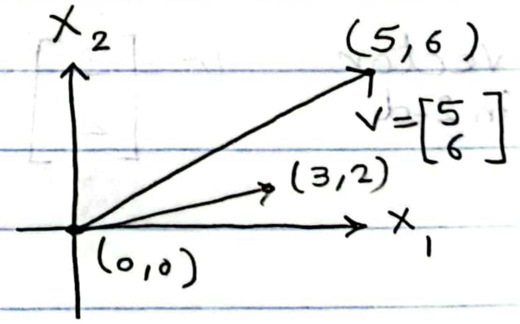
* what is the relationship between list of numbers with prespective of magnitude and direction?

magnitude and direction → physics prespective

In physics, they don't concern about vector origi

In math. (linear algebra)
Combine between CS, DS point of view and physics prespective, and this give us deep understand for vector

features	
x_1	x_2
5	6
3	2
4	1
1	0



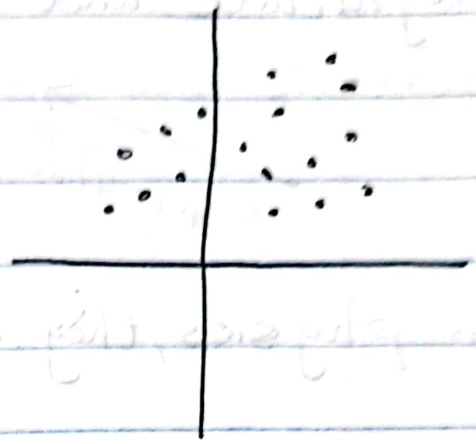
ملاحظة النقطة بال vector

In LA → ال vector لازم يكون طالع من ال origin

$v = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ feature vector in 2 dimension
↳ it represent point (5,6)

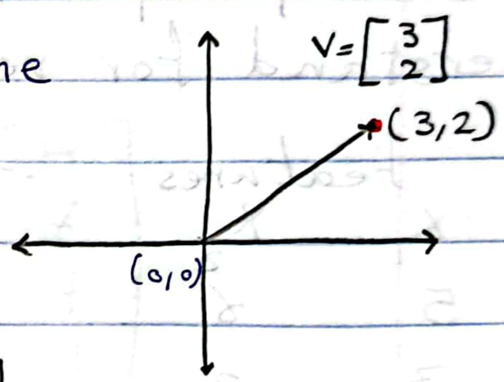
vector طالع من ال origin ورايح للنقطة (3,2)

these data points are vectors.

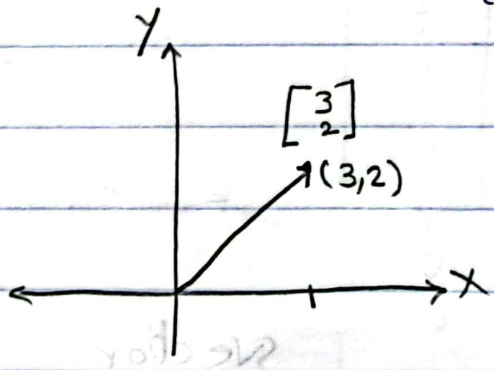


So, we deal with these points as vectors.
So, In dimensionality reduction related to vector operations.

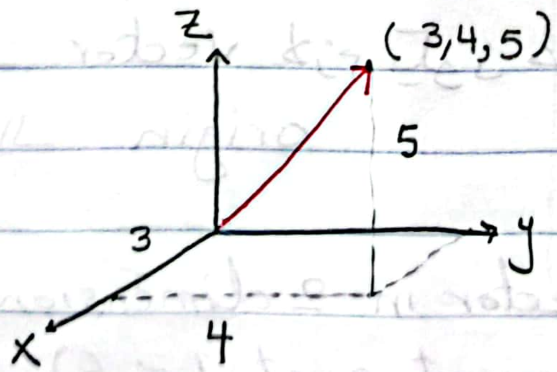
From Linear Algebra, ~~data~~ point of view
Data point, has Coordinates
So, we deal with vector in Cartesine Coordinate system



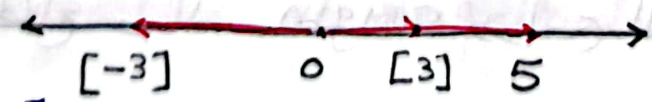
vector in 2-d $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$



$v = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ vector in 3d

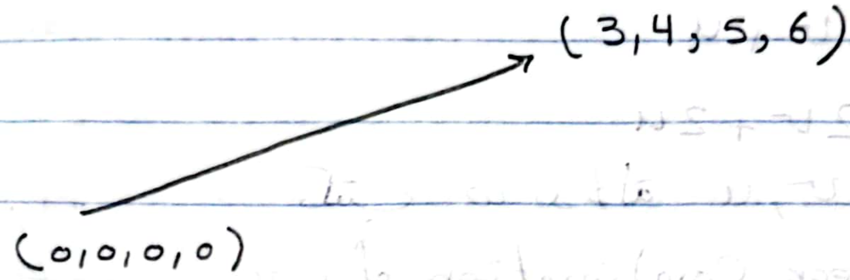


$v = [5]$ → vector in 1d "one-dimensional vector"



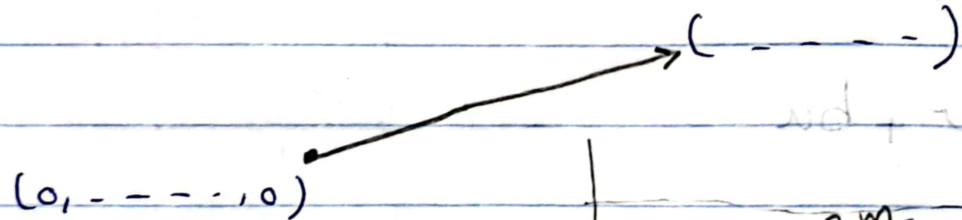
$v = [3]$

$v = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 2 \end{bmatrix}$ vector in 4 dimensional

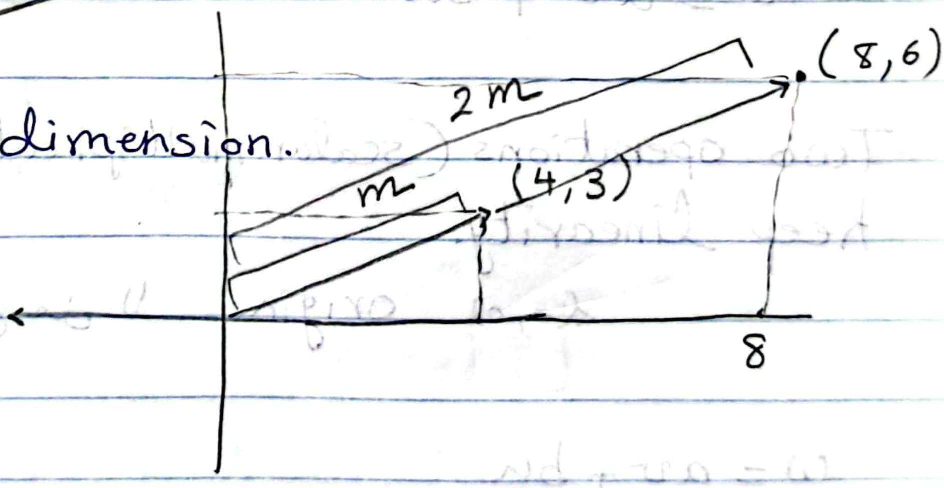


In n dimension → vector is arrow
طالع من نقطة ورايح لنقطة

A vector with 100 points



But I can't draw dimension.



$v = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$2v = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$ scaling

$2v = 2 \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

scalar

* vector addition.

$v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $v + u = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

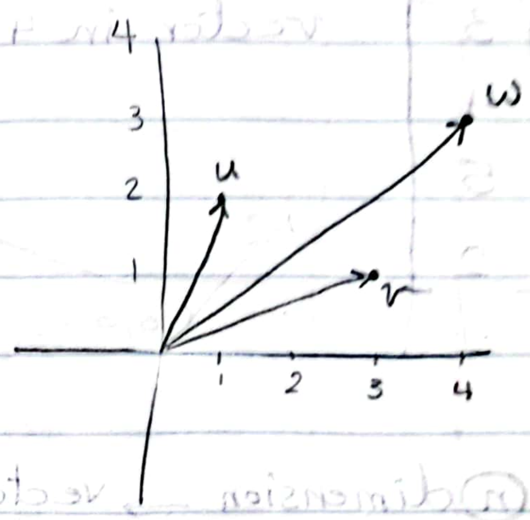
$w = u + v$

$w_2 = 2v + u$

$w_3 = 2v + 2u$

بكتب ω بدلالة v, u

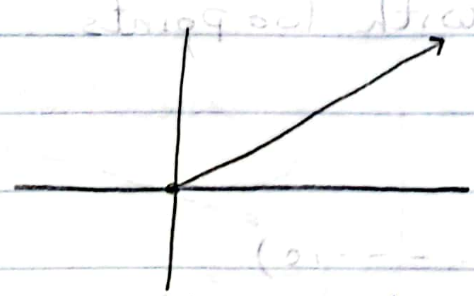
w is linear Combination of u, v



$w_3 = \underset{\uparrow}{2}v + 2u \leftarrow$ vector addition

scalar multiplication

$w = av + bu$



Two operations (scalar multiplication and vector addition) Keep linearity.

بيطلع ال vector طالع من ال origin (0,0)

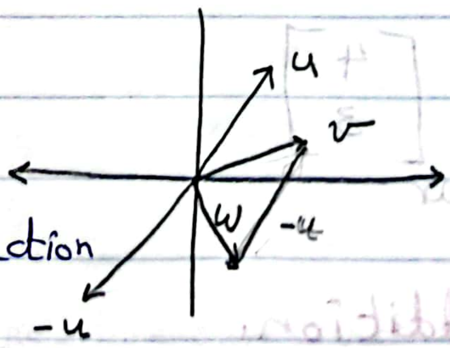
$w = av + bu$

a, b Can be positive or negative

$w = v - u$

subtracting is the same of addition but بتعكس direction

opposite



* what is linear Algebra?

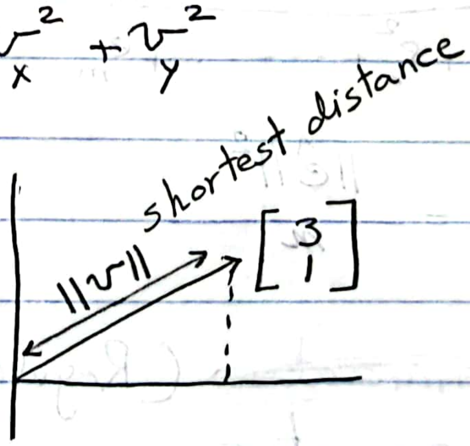
is the study of vectors and linear functions (Linear transformation)

- vectors are things you can add, and linear transformation are very special functions of vectors that respect vector addition.
- Linear Algebra allows only vector addition and scalar multiplication.

* vector norm:

$\|v\| \Leftarrow$ magnitude

$$v = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \Rightarrow \|v\| = \sqrt{v_x^2 + v_y^2}$$



$\|v\| = \sqrt{9+1} = \sqrt{10}$

↘ the length of vector

$\|v\|$ is Distance measure.

I want to find distance between origin (0,0) and (3,1)

L^2 norm $\rightarrow$ $\|v\| = (v_x^2 + v_y^2)^{1/2}$

there are different types of distance measure

L^1 norm	L^3 norm	L^5 norm
L^2 norm	L^4 norm	L^∞ norm

$$||v||_2 = \left(|v_x|^2 + |v_y|^2\right)^{1/2}$$

is called Euclidean dist. [in distance measure]

Mean square error (MSE)

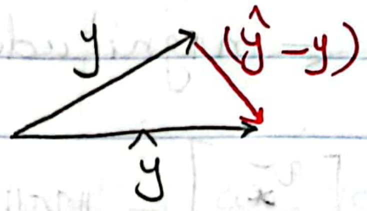
ك مرتبطة ارتباط وثيق بـ norm

In Machine learning "optimization" is called MSE

y	$\hat{y}$
5	6
2	4
3	1

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(\hat{y}_i - y_i)^2$$

$$\overset{\hat{e}}{\begin{bmatrix} \\ \\ \end{bmatrix}} = \overset{\hat{y}}{\begin{bmatrix} \\ \\ \end{bmatrix}} - \overset{y}{\begin{bmatrix} \\ \\ \end{bmatrix}}$$



$$||\hat{e}|| = \sqrt{e_1^2 + e_2^2 + \cdots + e_m^2}$$

$$MSE = \frac{||\hat{e}||^2}{m}$$

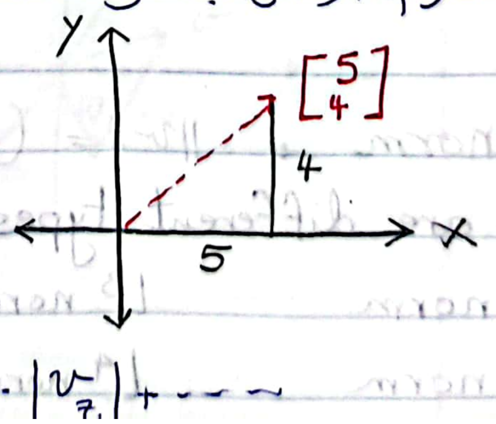
In ~~optimization~~ (Regularization) is called Ridge operator

what is L_1 norm? مش لازم أوصل مباشرة للنقطة من الـ origin

أمشي 5 على x ثم 4 على y

is called L1 norm

Taxi cab distance



$$L^1_{norm} = ||v||_1 = |v_x| + |v_y| + \cdots |v_z| + \cdots$$

In Distance measure called Manhattan dis.

In ML $\overset{\text{Regularization}}{\text{LASSO}}$

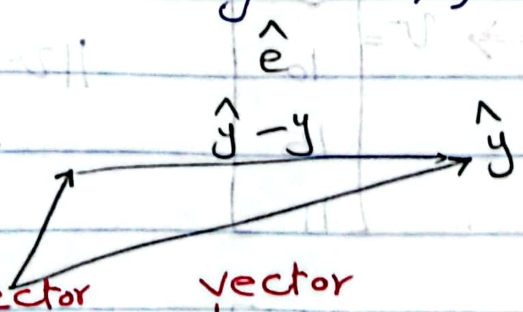
In optimization $MAE = \frac{1}{m} \sum_{i=1}^{m} \overbrace{|\hat{y} - y|}^{e}$

$$= \frac{L_1 \text{ norm}}{m}$$

	L_2 norm	L_1 norm
Distance measure	Euclidean distance	Manhattan dis.
Regularization	Ridge	LASSO
optimization	MSE	MAE
	بيروح للنقطة مباشرة	

* All vector in LA is come from origin (0,0)

* Any matrix has two pictures



row picture

column picture

row

$$\overset{\text{vector}\qquad\text{vector}}{\begin{bmatrix}5 & 10\\ 6 & 12\end{bmatrix}}\overset{\text{vector}}{\begin{bmatrix}x\\ y\end{bmatrix}} = \overset{\text{vector}}{\begin{bmatrix}5\\ 6\end{bmatrix}}$$

$5x + 10y = 5$

$6x + 12y = 6$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix}$$

$$L^1 = |v_1| + |v_2| + \cdots |v_n| \qquad \|v\|_1$$

$$L^2 = (|v_1|^2 + |v_2|^2 + \cdots |v_n|^2)^{1/2} \qquad \|v\|_2$$

$$L^3 = (|v_1|^3 + |v_2|^3 + \cdots + |v_n|^3)^{1/3} \qquad \|v\|_3$$

$$L^4 = (|v_1|^4 + |v_2|^4 + \cdots + |v_n|^4)^{1/4} \qquad \|v\|_4$$

$$L^P = (|v_1|^P + |v_2|^P + \cdots + |v_n|^P)^{1/P}$$

L_{max}

L_{∞} $\Rightarrow$ $v = \begin{bmatrix} 5 \\ 10 \\ 15 \\ 11 \end{bmatrix}$ $\qquad \|v\|_{max} = 15$

$v = \begin{bmatrix} 5 \\ 10 \\ -15 \\ 11 \end{bmatrix}$ $\qquad \|v\|_{max} = 15$

because we concern absolute value

Vector norm is important in ML
Cost function , Regularization, Distance measure

* Dot product:

$v_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ $u = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$u \cdot v_1 = 5 \times 2 + 1 \times 3 = 13$ Scalar

Each Component multiply by its equivalent element in other vector and sum

* Dot product return scalar

$v_1 \cdot u = u \cdot v_1 = 13$

$$\begin{bmatrix} 3 \\ 4 \\ 2 \\ 6 \\ \vdots \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 3 \\ 5 \\ \vdots \end{bmatrix} = 2 \times 3 + 4 \times 1 + 2 \times 3 + 6 \times 5 + \cdots = \boxed{} \text{ Scalar}$$

it tells us how much one vector is in the direction of another.

$u \cdot v_1 = v_1 \cdot u \longrightarrow$ بيوضح مقدار تعامد v_1 على u

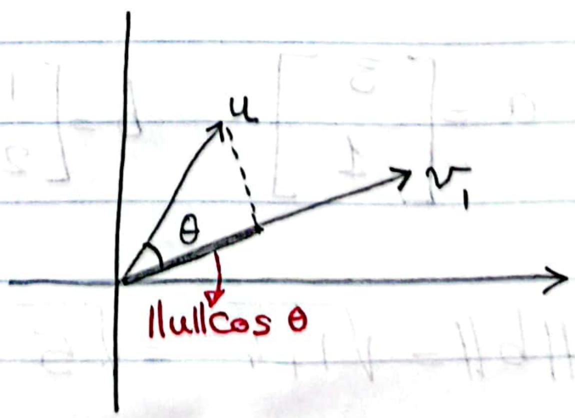
magnitude

$u \cdot v_1 = v_1 \cdot u = \|v_1\| \, \|u\| \cos\theta$

$v_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$u = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$u \cdot v_1 = 13 = \|v_1\| \, \|u\| \cos\theta$



Dot product:

It tells us something about how much two vectors point in the same direction.

- measure of how closely two vectors align, in terms of the directions they point.

other.

$u \cdot v_1 = 13 = \underbrace{\|v_1\|}_{\text{magnitude } v_1} \boxed{\|u\| \cos\theta}$ → projection of u on v_1

If v_1 unit vector

$\|v_1\| = 1$

$v_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$\|v_1\| = \sqrt{25+1} = \sqrt{26}$

unit vector

$\frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{26}}$ normalization of v_1

$\begin{bmatrix} \frac{5}{\sqrt{26}} \\ \frac{1}{\sqrt{26}} \end{bmatrix} \rightarrow \text{norm} = \sqrt{\frac{25}{26} + \frac{1}{26}} = \sqrt{\frac{26}{26}} = \sqrt{1} = 1$

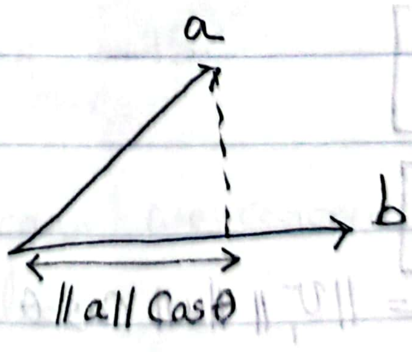
If norm vector $v_1 = 1$

$u \cdot v_1 = \|u\| \cos\theta$

$\boxed{} = \|u\| \cos\theta$

Dot Product = "equivalent" projection

$a = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$



$\|b\| = \sqrt{1+4} = \sqrt{5}$

$$b' = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$$

projection $a \rightarrow b'$ and you don't know angle θ

$$a \cdot b' = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

نجيب Dot product, because in this case dot product is equivalent to projection

كأني عملت projection لـ a على b'

If I have vector a, b such b is unit vector

$$\boxed{a \cdot b = \text{projection of } a \rightarrow b}$$

$$a = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \quad b = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

Convert b to unit vector

$$\|b\| = \sqrt{49 + 64} = \sqrt{113}$$

$$b' = \frac{b}{\|b\|} = \begin{bmatrix} 7/\sqrt{113} \\ 8/\sqrt{113} \end{bmatrix}$$

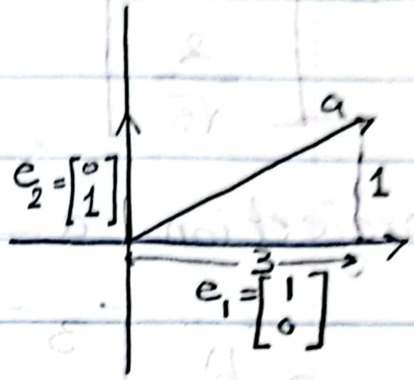
$$\therefore \|b'\| = 1$$

$$a \cdot b' = \|a\| \cos\theta$$

$$a = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

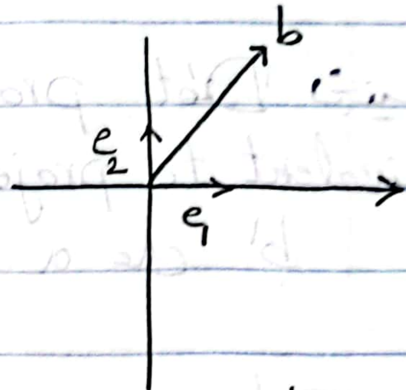
$$a = 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$a = 3\hat{e}_1 + 1\hat{e}_2$$

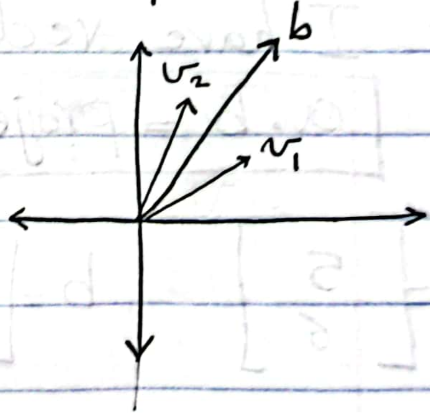


$$b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$b = 2\hat{e}_1 + 3\hat{e}_2$$



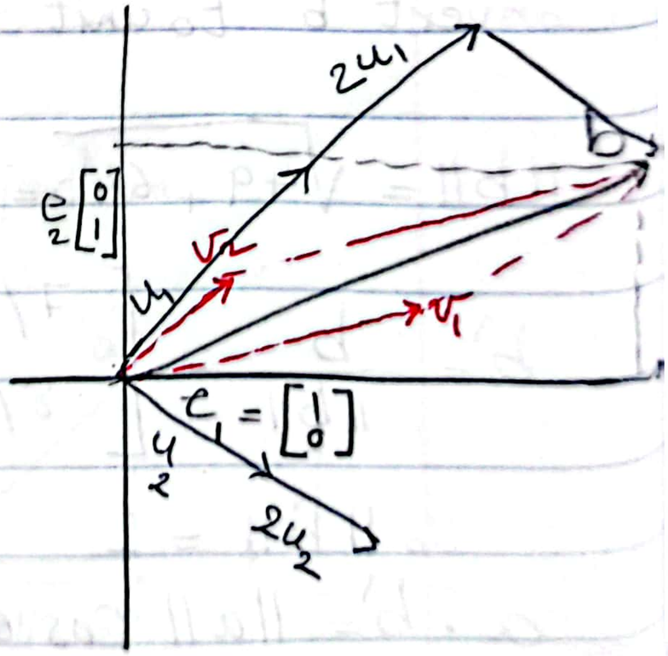
$$b = v_1 + v_2$$



$$b = \begin{bmatrix} 5 \\ 2 \end{bmatrix} = 5\hat{e}_1 + 2\hat{e}_2$$

$$b = v_1 + v_2$$

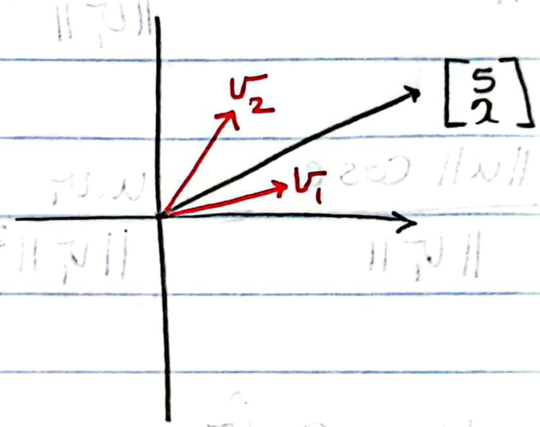
$$b = 2u_1 + 2u_2$$



vector b don't change , كتبته مرة بدلالة $\hat{e}_2 , \hat{e}_1$
ومرة بدلالة u_2 , u_1
ومرة بدلالة v_2 , v_1

If I have data point (vector)

x_1	x_2
5	2

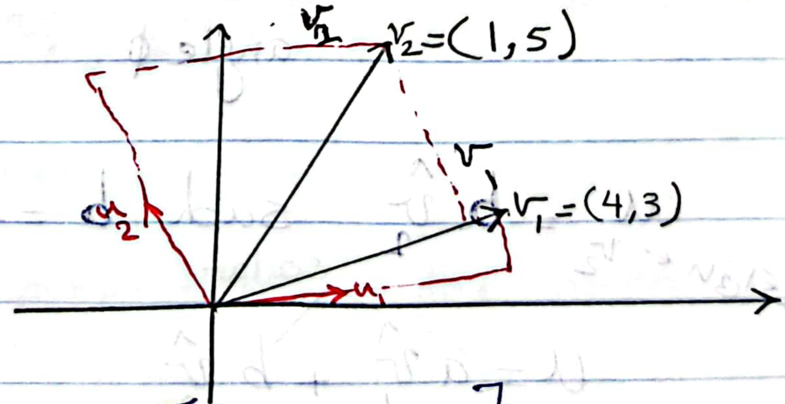


z_1	z_2
2	0.5

Changing basis
Changing Coordinates
principle Component Analysis

بسيب الداتا زي ماهي ولكن نغير الـ Reference اللي بكتب بالنسبة له الداتا

$v_1 = \begin{bmatrix}4\\3\end{bmatrix} \qquad v_2 = \begin{bmatrix}1\\5\end{bmatrix}$



$v_1 = 4u_1 + 3u_2$

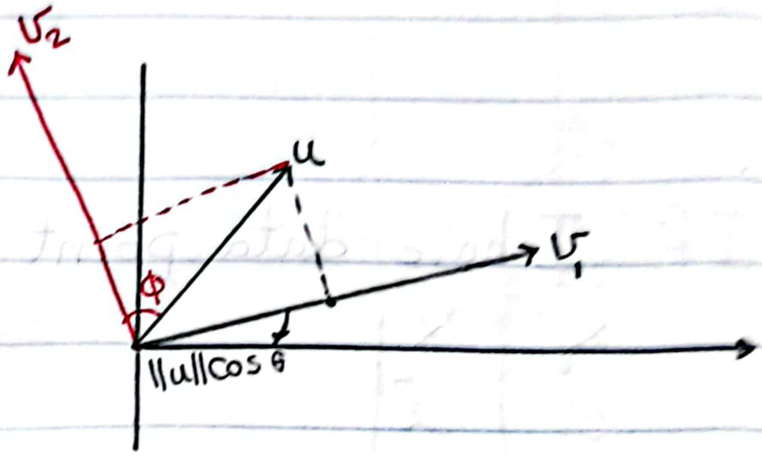
$v_2 = 1\hat{e}_1 + 5\hat{e}_2$

$v_1 = \begin{bmatrix}2\\0.5\end{bmatrix}$

$v_1 = 2u_1 + 0.5u_2$

$v_2 = 1.25u_1 + 1.5u_2 \qquad v_2 = \begin{bmatrix}1.25\\1.5\end{bmatrix}$

*what is the relation of perivous and dot product?



$u \cdot v_1 = \|v_1\| \, \|u\| \cos\theta$

$$\|u\| \cos\theta = \frac{u \cdot v_1}{\|v_1\|}$$

$$\frac{\|u\| \cos\theta}{\|v_1\|} = \frac{u \cdot v_1}{\|v_1\|^2} \rightarrow a$$

عايز اعرف $\|u\|\cos\theta$ بالنسبة لـ v_1
لـ اقسم على طوله

$$u_{v_1} = a \hat{v}_1 \quad , \text{such} \quad a = \frac{u \cdot v_1}{\|v_1\|^2}$$

عرفت بال dot product والقسمة على norm²

عرفت Component u في اتجاه v_1

If I have v_2 prependeclar عمودي on v_1

اعمل projection u on v_2

angle ϕ

$$u_{v_2} = b \hat{v}_2 \quad \text{such} \quad b = \frac{u \cdot v_2}{\|v_2\|^2}$$

u_{v_2} ← فى اتجاه v_2

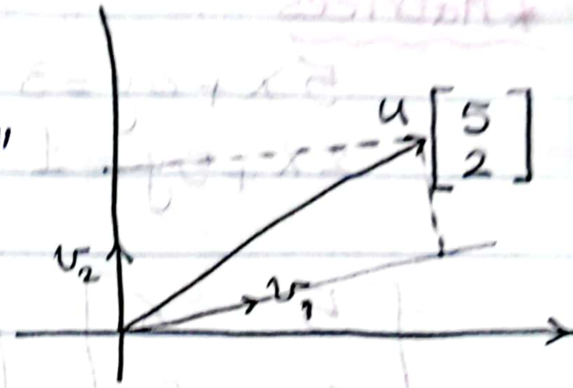
scalar

$$u = a \hat{v}_1 + b \hat{v}_2$$

$$u = \frac{u \cdot v_1}{\|v_1\|^2} \hat{v}_1 + \frac{u \cdot v_2}{\|v_2\|^2} \hat{v}_2$$

كتبت ال u بدلالة v_1 ، v_2 وعرفت ال Components

$u = 5\hat{e}_1 + 2\hat{e}_2$



v_1, v_2 prependicular "orthogonal"
must be prependicular to can make projection in two direction

$$u = \frac{u \cdot \hat{v}_1}{\|v_1\|^2}\hat{v}_1 + \frac{u \cdot \hat{v}_2}{\|v_2\|^2}\hat{v}_2$$

Component in v_1 direction

Component in v_2 direction

$$\|v_1\|^2 = v_1 \cdot v_1$$

$$\|v_2\|^2 = v_2 \cdot v_2$$

$$u = \frac{u \cdot \hat{v}_1}{v_1 \cdot v_1}\hat{v}_1 + \frac{u \cdot v_2}{v_2 \cdot v_2}\hat{v}_2$$

Not scalar multiplication, it is dot product
so, can't eliminate nemoritor with demenortar

لايمكن ان نختصر البسط مع المقام !!

* Matrices

$$\left.\begin{aligned} 5x+6y&=3 \\ 2x+5y&=1 \end{aligned}\right\} \rightarrow \text{linear equation}$$

can be represented as:

$$\begin{bmatrix} 5 & 6 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Linear function applied to the vector

I have vector x, y and when multiply this matrix to vector x, y $\xrightarrow{\text{يعني}}$ linear function affected on it and convert it to another vector $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 5 & 6 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5\times1+6\times3 \\ 2\times1+5\times3 \end{bmatrix} \cancel{+2\times1+5\times3}$$

$$= \begin{bmatrix} 23 \\ 17 \end{bmatrix}$$

* Any linear function can be represented in matrix

$$\begin{bmatrix} 5 & 2 & 1 \\ 6 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

linear fun affected on vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and transformed it to another vector $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$

*operations on Matrices:

1_ Addition
2_ subtraction
3_ Matrix Multiplication $\quad AB \neq BA$
4_ Matrix Transpose

$$(AB)^T = B^T A^T$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

*Different Types Matrices:

1_ Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2_ Diagonal Matrix:

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

اضربها في نفسها 100 مره ← A^{100} ~~DDD~~ $= \begin{bmatrix} 2^{100} & 0 & 0 \\ 0 & 8^{100} & 0 \\ 0 & 0 & 1^{100} \end{bmatrix}$

3_ Scalar Matrix:

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= 5 \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

4_ Lower Triangular Matrix:

$$\begin{bmatrix} 5 & 0 & 0 \\ 2 & 1 & 0 \\ 7 & 6 & -3 \end{bmatrix}$$

5_ upper triangular Matrix:

$$\begin{bmatrix} 2 & -2 & 7 \\ 0 & 4 & 11 \\ 0 & 0 & 5 \end{bmatrix}$$

useful for back substitution

If I have a system of linear equations:

$$5x + 2y + 3z = 10$$
$$6x + 1y + 5z = 12$$
$$1x + 2y + 13z = 100$$

$$\begin{bmatrix} 5 & 2 & 3 \\ 6 & 1 & 5 \\ 1 & 2 & 13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 100 \end{bmatrix}$$

By using Gaussian elimination, I want to convert matrix to upper triangular matrix

$\Downarrow$

$$\begin{matrix} 3 & 2 & 1 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{matrix} \quad \begin{matrix} x \\ y \\ z \end{matrix} \quad \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$$3x + 2y + z = 1$$
$$2y + 6z = 2$$
$$5z = 3$$
$$z = \frac{3}{5}$$

back substitution

6_ symmetric Matrix:

$$\begin{bmatrix} 3 & -2 & 11 & 5 \\ -2 & 4 & -1 & 6 \\ 11 & -1 & 6 & 7 \\ 5 & 6 & 7 & 9 \end{bmatrix}$$

① $A = A^{-1}$

② Save memory in Computation, you need to store only half of it.

③ Symmetric matrix can be decomposed into matrix of orthonormal eigenvectors and diagonal matrix of eigenvalues. So it is very important

7_ Row Echelon form (REF)

$$\begin{bmatrix} 5 & 6 & 7 \\ 3 & 2 & 1 \\ 4 & 6 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} \boxed{3} & 2 & 1 \\ 0 & \boxed{1} & 2 \\ 0 & 0 & \boxed{3} \end{bmatrix}$$

pivot 1

pivot in row 2

pivot in row

the first non-zero element in first row is called pivot

similar to upper triangular matrix if it's square matrix

أول عنصر ليس صفر فى الصف الأول يسمى pivot1 ويكون القيم اللى تحته أصفار "فى العمود"

وكذلك أول عنصر فى الصف الثانى ليس صفر يسمى pivot2 ويكون القيمة اللى تحته صفر "فى العمود"

$$\begin{bmatrix} 5 & 6 & 7 \\ 3 & 2 & 1 \\ 4 & 8 & 11 \\ 1 & 6 & 7 \end{bmatrix}_{4\times3} \xrightarrow{\text{Convert REF}} \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \\ \boxed{0 \quad 0 \quad 0} \end{bmatrix}$$

Non-square matrix

$$\begin{bmatrix} 5 & 6 & 3 & 2 & 1 \\ 7 & 8 & 9 & 10 & 11 \\ 2 & 3 & 1 & 8 & 6 \end{bmatrix} \xrightarrow[\text{By BEF}]{\text{Reduction}} \begin{bmatrix} \boxed{3} & 2 & 1 & 3 & 5 \\ 0 & \boxed{1} & \boxed{2} & 6 & 7 \\ 0 & 0 & 0 & \boxed{1} & 1 \end{bmatrix}$$

* Echelon Form of Matrix:

Row echelon form (REF)

A matrix is in echelon form if it has the shape resulting from a Gaussian elimination and has the following properties:

- All rows consisting of only zeros are at the bottom.
- The leading coefficient (also called the pivot) of a nonzero row is always strictly to the right of the leading coefficient of the row above it.

. These two conditions imply that all entries into a column below a leading coefficient are zeros.

$$\begin{bmatrix} 1 & a_0 & a_1 & a_2 & a_3 \\ 0 & 0 & 2 & a_4 & a_5 \\ 0 & 0 & 0 & 1 & a_6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & -2 & 0 & 6 \\ 0 & 1 & 7 & -5 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

what's the benefit?

To solve system of linear equations, and then use back substitution.

8_ Reduced Row Echelon Form (RREF)

$$\underset{\text{REF}}{\begin{bmatrix} 5 & 3 & 2 \\ 0 & 6 & 1 \\ 0 & 0 & 8 \end{bmatrix}} \longrightarrow \underset{\text{RREF}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \rightarrow I$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$x = 3$, $y = 2$, $z = 1$

Can find solution

Non-square

$$\begin{bmatrix} 1 & 0 & 0 & 5 & 2 & 1 \\ 0 & 1 & 0 & 8 & 1 & 4 \\ 0 & 0 & 1 & 3 & 5 & 3 \end{bmatrix}$$

General formula of RREF

* Echelon Form of Matrix

Reduced row echelon form (RREF):

- A matrix is in reduced row echelon form (also called row canonical form) if it satisfies the following conditions:
- It is in row echelon form.
- The leading entry in each nonzero row is a 1 (called a leading 1)
- Each column containing a leading 1 has zeros in all its other entries.

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 6 \\ 0 & 1 & 7 & 0 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

Non-square matrix

Tensors in DL "Images"

1D Tensor vector

8
7
3
2
-4
3

2D Tensor/Matrix

-9	4	2	5	7
3	0	12	8	61
1	23	-6	45	2

3D Tensor/cube

-9	4	2	5
3	8	12	8
1	23	-6	45
22	3	-1	72

4D tensor
vector of cubes

5D Tensor
Matrix of cubes

* System of Linear Equations.

$$5x + 6y = 100$$
$$3x + 5y = 15$$

$$\begin{bmatrix} 5 & 6 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 100 \\ 15 \end{bmatrix}$$

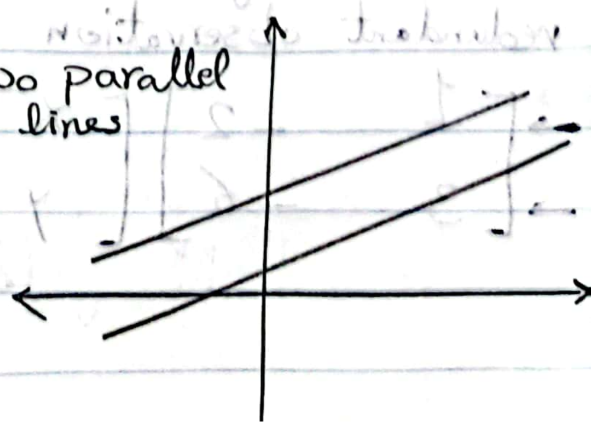
x_1	x_2	y
3	4	2
5	6	3

$\Rightarrow$

$$3\theta_1 + 4\theta_2 = 2$$
$$5\theta_1 + 6\theta_2 = 3$$

$$x - 2y = 1$$
$$3x - 6y = 11$$

no_solution



Redundant column

$$\begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

$$\boxed{A\, x = b}$$

No solution

$$C_1 \times -2 = C_2$$

If there is no solution → so I can't get inverse for matrix A

what is the meaning of No solution?

there isn't vector x, can give to matrix A and convert it to vector b

If I take matrix A, and transform it to Row Echelon form

$$\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$$

$$x - 2y = 1$$

$$\boxed{0y = 8} \leftarrow$$

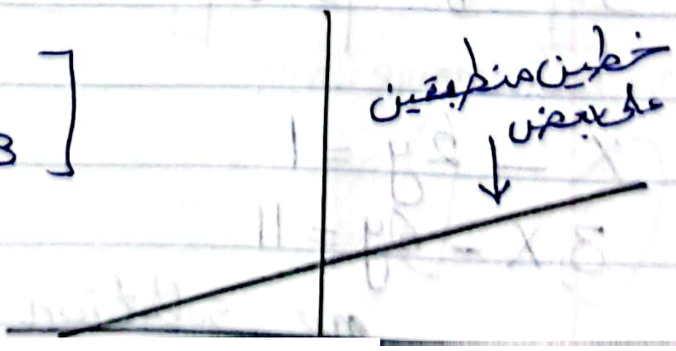
there is no value of y can satisfy this eq.

$$x - 2y = 1 \longleftarrow$$ لم يتم إضافة معلومة

$$3x - 6y = 3$$ لأنها نفس المعادلة الأولى مضروبة في 3

redundant observation

$$\begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$



Infinite no. of solutions

$$\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x - 2y = 1$$
$$0y = 0$$

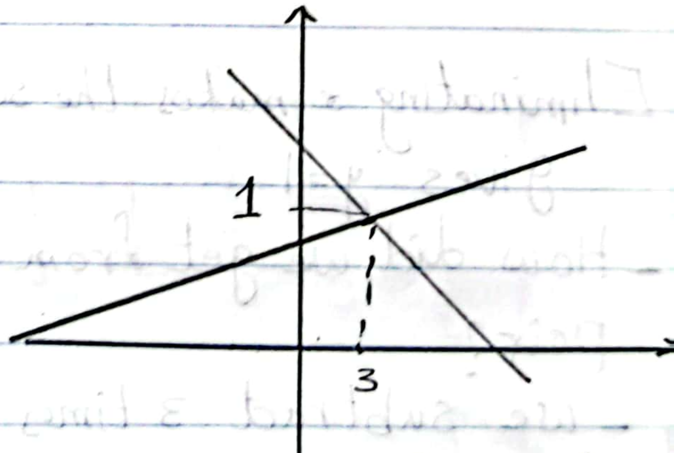
infinite no. of solutions.

Any value of y → can achieve this eq. $0y = 0$

There are alot of vectors x, y with different values, can transform by using matrix to find this vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Definite solution:

$$x - 2y = 1$$
$$3x + 2y = 11$$



خطين متقاطعين
نقطة واحدة (3,1)

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

System Reduction

$$\begin{bmatrix} 1 & -2 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$$

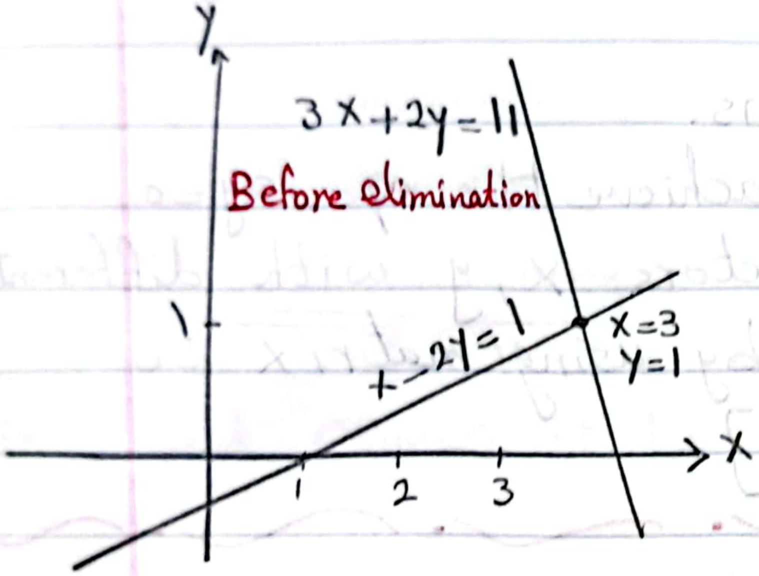
$$x - 2y = 1$$
$$8y = 8 \rightarrow \boxed{y = 1}$$

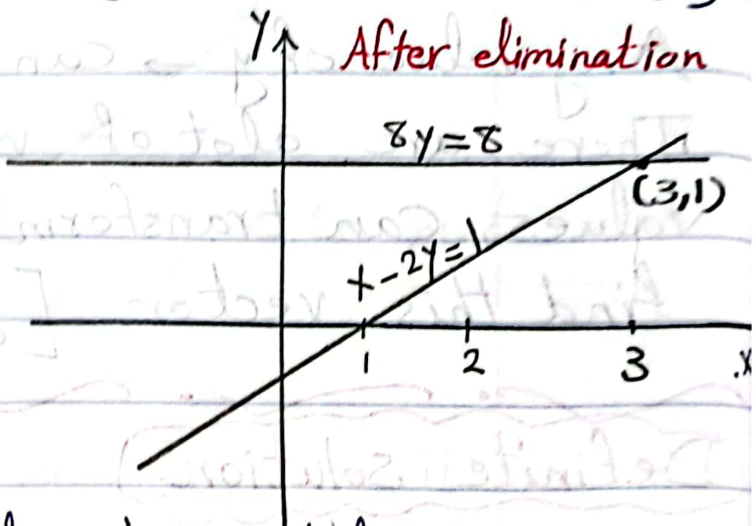
$$x - 2 = 1 \rightarrow \boxed{x = 3}$$

* Gaussian Elimination.

Before $x - 2y = 1$
$3x + 2y = 11$

After $x - 2y = 1$
$8y = 8$
(multiply equation 1 by 3)
(subtract to eliminate $3x$)





Eliminating x makes the second line horizontal. Then $8y = 8$ gives $y = 1$.

- How did we get from the first pair of lines to the second pair?
- we subtract 3 times the first equation from the second equation.

To eliminate x: subtract a multiple of equation I from equation 2.

*System of Linear Equations:

$$x + 2y + 3z = 6$$
$$2x + 5y + 2z = 4$$
$$6x - 3y + z = 2$$

Coefficient matrix

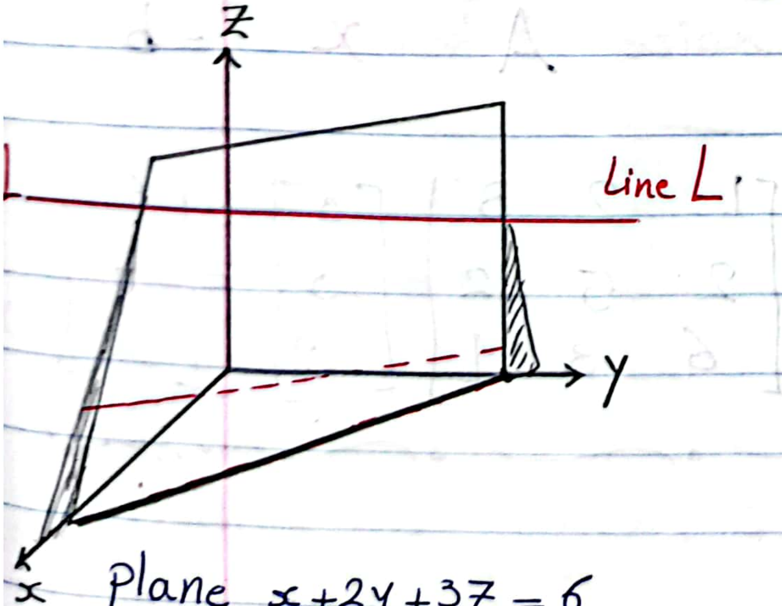
$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 6 & -3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}}_{b}$$

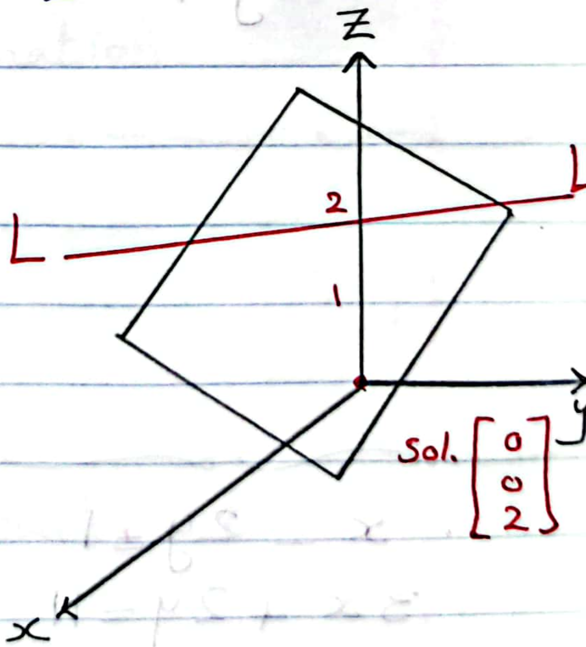
$$x\begin{bmatrix}1\\2\\6\end{bmatrix} + y\begin{bmatrix}2\\5\\-3\end{bmatrix} + z\begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{bmatrix}6\\4\\2\end{bmatrix}$$

$$0\begin{bmatrix}1\\2\\6\end{bmatrix} + 0\begin{bmatrix}2\\5\\-3\end{bmatrix} + 2\begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{bmatrix}6\\4\\2\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}6\\4\\2\end{bmatrix} = \begin{bmatrix}6\\4\\2\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 3\\2 & 5 & 2\\6 & -3 & 1\end{bmatrix}\begin{bmatrix}0\\0\\2\end{bmatrix} = \begin{bmatrix}6\\4\\2\end{bmatrix}$$





Plane $x+2y+3z=6$

plane $2x+5y+2z=4$

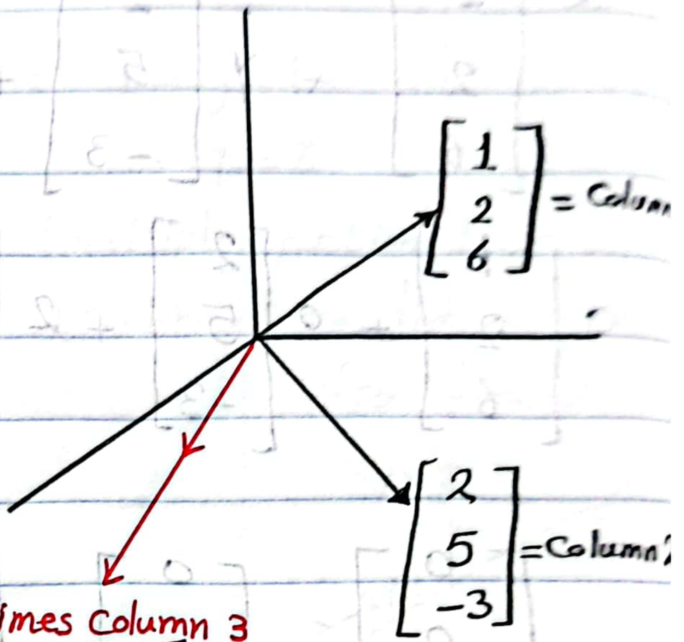
3rd plane $6x-3y+z=2$

$(0,0,2)$ is on all three planes

Row picture: Two planes meet at a line L. Three planes meet at a point

Column picture:

Combine the columns with weights $(x, y, z) = (0, 0, 2)$



2 times Column 3 is $b = \begin{bmatrix}6\\4\\2\end{bmatrix}$

$x + 2y + 3z = 6$

$2x + 5y + 2z = 4$

$6x - 3y + z = 2$

$$\underset{A}{\begin{bmatrix}1 & 2 & 3\\2 & 5 & 2\\6 & 3 & -1\end{bmatrix}}\underset{x}{\begin{bmatrix}x\\y\\z\end{bmatrix}} = \underset{b}{\begin{bmatrix}6\\4\\2\end{bmatrix}}$$

$$\begin{bmatrix}1 & 2 & 3\\2 & 5 & 2\\6 & 3 & 1\end{bmatrix}\begin{bmatrix}0\\0\\2\end{bmatrix} = \begin{bmatrix}6\\4\\2\end{bmatrix}$$

$x - 2y = 1$

$3x + 2y = 11$

$$\begin{bmatrix}1 & -2\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\11\end{bmatrix}$$

Augmented matrix

$$\times 3 \begin{bmatrix} \boxed{1} & -2 & \vdots & 1 \\ 3 & 2 & \vdots & 11 \end{bmatrix}$$

multiply first row by 3 and subtract

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -8 & -8 \end{bmatrix} \rightarrow \text{matrix in Row Echlon form}$$

$$\begin{bmatrix} 1 & -2 \\ 0 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$$

$x - 2y = 1$ ← back subtitution

$-8y = -8 \rightarrow y = +1$

x [redacted]2 = [redacted] → $\boxed{x = 3}$

Solution: $x = 3$, $y = 1$

This is called Gaussian Elimination

$x - 2y = 1$

$3x - 6y = 11$

$$\times 3 \begin{bmatrix} 1 & -2 & \vdots & 1 \\ 3 & -6 & \vdots & 11 \end{bmatrix}$$

multiply first row by 3 and subtract

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & -8 \end{array}\right]$$

$x - 2y = 1$

$0y = -8$ (No solution)

$x - 2y = 1$
$3x - 6y = 3$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & -6 & 3 \end{array}\right]$$

multiply first row by 3 and subtract

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$x - 2y = 1$
$0y = 0$

Infinite solution

System of linear equations

$2x + 4y - 2z = 2$
$4x + 9y - 3z = 8$
$-2x - 3y + 7z = 10$

$$\begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 2 \\ 8 \\ 10 \end{matrix}$$

Augmented Matrix

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 4 & 9 & -3 & 8 \\ -2 & -3 & 7 & 10 \end{array}\right]$$

multiply first row by 2 and subtract

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 1 & 5 & 12 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} \boxed{2} & 4 & -2 & 2 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & \boxed{4} & 8 \end{array}\right]$$

Matrix is in Row Echlon form

$$2x + 4y - 2z = 2$$

$$y + z = 4$$

$$4z = 8$$

$$\boxed{z = 2}$$

$$\boxed{y = 2}$$

$$2x + 8 - 4 = 2$$

$$2x = -2$$

$$\boxed{x = -1}$$

To get Reduced Row Echelon form

Divide third row by 4

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & 1 & \boxed{1} & 4 \\ 0 & 0 & 1 & 2 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{array}\right]$$

$$R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 2 & 4 & \boxed{-2} & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$2R_3 + R_1$

$$\left[\begin{array}{ccc|c} 2 & \boxed{4} & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$4R_2 - R_1$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Divide R_1 by 2

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

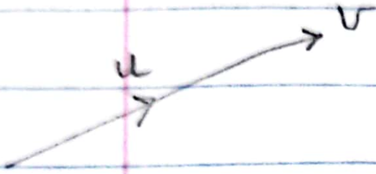
$$\boxed{\begin{array}{c} x = -1 \\ y = 2 \\ z = 2 \end{array}}$$

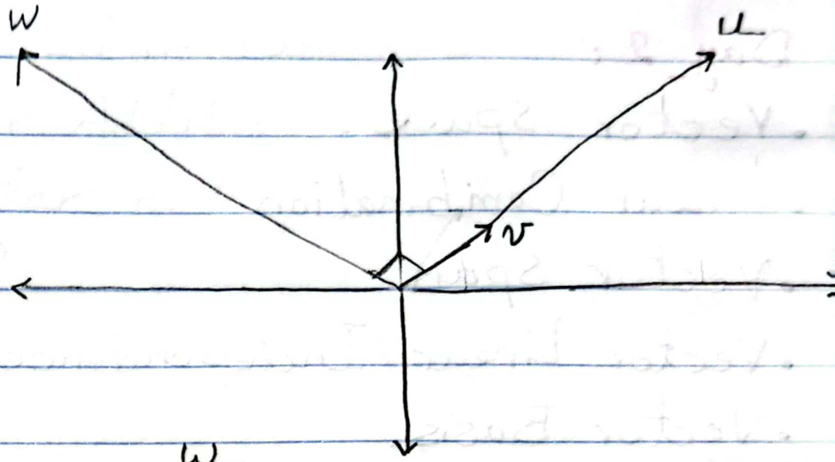
Gauissan Gurdan method: to convert matrix to identity matrix

* Augmented matrix in python (Numpy)

$$\begin{array}{cc} & \xrightarrow{\text{axis } 1} \\ \text{axis } 0 \downarrow & \begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix} \end{array}$$

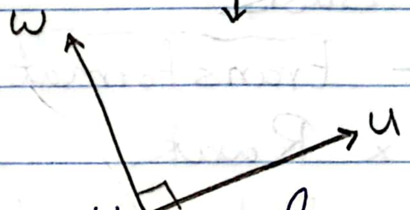
we must define axis = 1, to work horizontally.





$\cos(0) = 1$

$u \cdot v = \|u\| \, \|v\|$



$u, w = 0$

$u \cdot w = \|u\| \, \|w\| \boxed{\cos\theta} = 0$

To know if two vectors are orthogonal or Not
Calculate Dot product
If Dot product $= 0 \rightarrow$ orthogonal

* We must take care if the array is row vector or Column vector
If you do matrix vector multiplication, define your Shape to avoid any errors.

Day_2 :

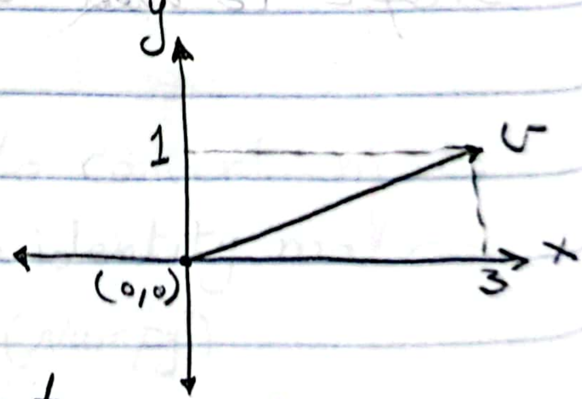
- Vector space.
- Linear Combination
- Vector span.
- Vector Linear Independence.
- Vector Basis.

- Linear transformation.
- Matrix Rank.
- Matrix Determinant.
- Matrix inverse.

- Changing Basis.
- Orthonormal and Non_orthonormal space.
- Transformation in Non_orthonormal space.
- Gram_schmidt Process (orthogonalization).

• Vector space.

$$v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



Any vector in LA must be start from origin (0,0)

v is Consisted of two Components

$v \in R^2$ space

R^2 space is contain all vectors in this dimension.

In R^2 space → Any 2 vector can
1- vector Addition
2- Scalar multiplication

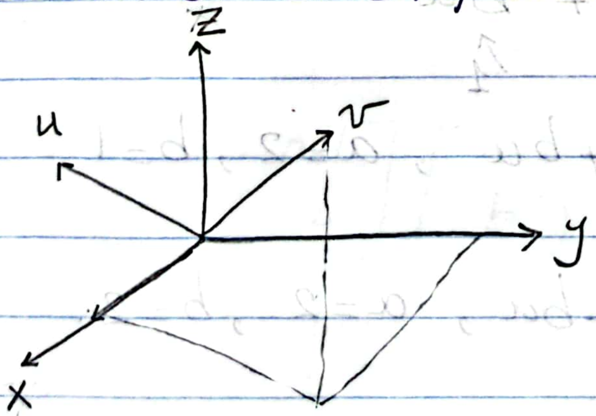
$R^1, R^2, \dots R^n$? هل الكلام ده يختلف في
لا يختلف

vector → Arrow عبارة عن

$v = [5] \in R^1$

$v - u = [5] - [-2] = [7]$

R^3 → Consist of 3 Components : x, y, z



$v = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

$v, u \in R^3$

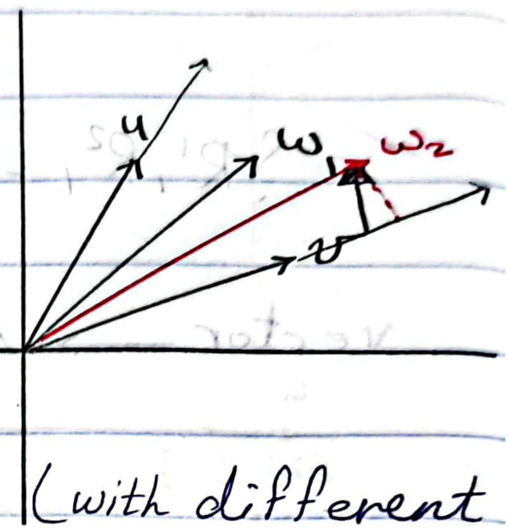
vector space → المكان اللي فيه كل ال vector دي في
ال dimension د.

* Linear Combination,

$w_1 = v + u$

$w_2 = 2v + u$

$w_3 = 2v + 2u$



w_1 is a linear Combination of v, u

w_2 is a linear Combination of v, u (with different scaling)

w_3 is a linear Combination of v, u

$$w_1 = \underset{1}{a}v + \underset{1}{b}u$$

$w_2 = av + bu$, $a = 2$, $b = 1$

$w_3 = av + bu$, $a = 2$, $b = 2$

If you have u, v and check if w is a linear Combination from u, v?

$w_5 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ Is it a linear Combination from

$v = \begin{bmatrix} 1.5 \\ 0.3 \end{bmatrix}$, $u = \begin{bmatrix} -0.1 \\ -4 \end{bmatrix}$?

check can I write

$$\begin{bmatrix} 3 \\ 5 \end{bmatrix} = a \begin{bmatrix} 1.5 \\ 0.3 \end{bmatrix} + b \begin{bmatrix} -0.1 \\ -4 \end{bmatrix}$$

If I can find a value for a, b that satisfy this system of linear equations $\rightarrow \therefore w_5$ linear Combination from v, u

If I can't find a solution $\rightarrow \therefore w_5$ isn't a linear Combination from v, u

If Inifinite solution $\rightarrow \therefore w_5$ is a linear Combination from v, u

$$\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1.5 & -0.1 \\ 0.3 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

بيعرفني Inverse If it there a definite Solution or not

$$\begin{bmatrix} 1.5 & -0.1 \\ 0.3 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Augmented matrix

$$\left[\begin{array}{cc|c} 1.5 & -0.1 & 3 \\ 0.3 & -4 & 5 \end{array}\right]$$

$$R_1 / \tfrac{3}{2} \longrightarrow R_1$$

$$\left[\begin{array}{cc|c} 1 & -1/15 & 2 \\ 0.3 & -4 & 5 \end{array}\right]$$

$$R_2 - \frac{3}{10} \times R_1 \longrightarrow R_2$$

$$\left[\begin{array}{cc|c} 1 & -1/15 & 2 \\ 0 & -199/50 & 22/5 \end{array}\right]$$

$$R_2 / \frac{-199}{50} \longrightarrow R_2$$

$$\left[\begin{array}{cc|c} 1 & -1/15 & 2 \\ 0 & 1 & -220/199 \end{array}\right]$$

$$R_1 - 1/15\, R_2 \longrightarrow R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 1150/597 \\ 0 & 1 & -220/199 \end{array}\right] \quad \text{RREF}$$

$x_1 = 1150/597$, $x_2 = -220/199$

$a = 1150/597$, $b = -220/199$

$\therefore$ vector $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ is a linear combination from u, v

---

If $w = \begin{bmatrix} 11 \\ 16 \end{bmatrix}$ is alinear Combination from

$v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

* If matrix hasn't an inverse → haven't solution
OR ~~but~~ can be infinite solution

Haven't ~~Solution~~ inverse → no solution
→ infinite solution

Have inverse → definite solution

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11 \\ 16 \end{bmatrix}$$

Augmented matrix

$$\left[\begin{array}{cc|c} 1 & 3 & 11 \\ 2 & 4 & 16 \end{array}\right]$$

$$R_2 - 2R_1 \longrightarrow R_2$$

$$\left[\begin{array}{cc|c} 1 & 3 & 11 \\ 0 & -2 & -6 \end{array}\right]$$

$$R_2 / -2 \longrightarrow R_2$$

$$\left[\begin{array}{cc|c} 1 & 3 & 11 \\ 0 & 1 & 3 \end{array}\right]$$

$$R_1 - 3R_2 \longrightarrow R_1$$

"RREF" → $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$

$x_1 = 2$

$x_2 = 3$

$a = 2$, $b = 3$

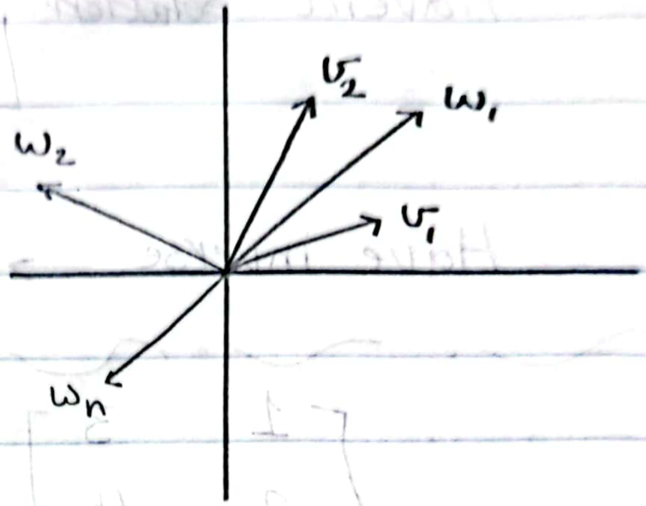
$\therefore w = \begin{bmatrix} 11 \\ 16 \end{bmatrix}$ is a linear Combination from v_1 , v_2

If I have 2 vector $\in R^2$

vector in R^2 space أى

Can be written بدلالة v_1, v_2

✓



If I have 1 vector $\in R^2$

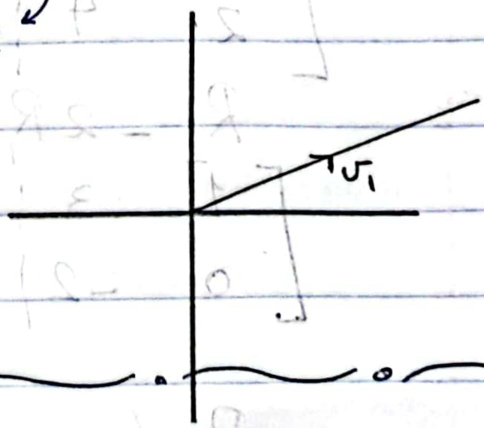
هل أقدر اكتب أى vector in space بدلالته ؟

<u>No</u>

أقدر اكتب الـ vectors اللى على الخط بس

يحصل له scaling

سواء negative or positive



$w = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$

$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

w is a linear Combination from v_1, v_2, v_3 ?

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

Augmented Matrix

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 2 \\ 0 & 2 & 1 & 5 \\ 1 & 1 & 4 & 3 \end{array}\right]$$

$$R_3 - R_1 \longrightarrow R_3$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 2 \\ 0 & 2 & 1 & 5 \\ 0 & -1 & 1 & 1 \end{array}\right]$$

$$R_2/2 \longrightarrow R_2$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1/2 & 5/2 \\ 0 & -1 & 1 & 1 \end{array}\right]$$

$$R_3 + R_2 \longrightarrow R_3$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1/2 & 5/2 \\ 0 & 0 & 3/2 & 7/2 \end{array}\right]$$

$$R_3 / \tfrac{3}{2} \longrightarrow R_3$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1/2 & 5/2 \\ 0 & 0 & 1 & 7/3 \end{array}\right]$$

$$R_2 - \tfrac{1}{2} R_3 \longrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 0 & 4/3 \\ 0 & 0 & 1 & 7/3 \end{array}\right]$$

$$R_1 - 3R_3 \longrightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -5 \\ 0 & 1 & 0 & 4/3 \\ 0 & 0 & 1 & 7/3 \end{array}\right]$$

$$R_1 - 2R_2 \longrightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -23/3 \\ 0 & 1 & 0 & 4/3 \\ 0 & 0 & 1 & 7/3 \end{array}\right] \leftarrow \text{RREF}$$

$$x = -23/3$$

$$y = 4/3$$

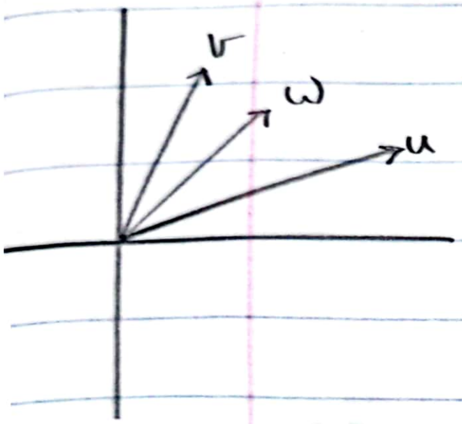
$$z = 7/3$$

$\therefore$ w is a linear combination from v_1, v_2, v_3

$$w = av_1 + bv_2 + cv_3$$

$$w = -\frac{23}{3}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \frac{4}{3}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + \frac{7}{3}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

$w = av_1 + bv_2$ هل أقدر أكتب w بدلالة v_1
لوحدها أو بدلالة v_2 لوحدها؟
لا، لانها ليست على نفس الـ line



If in R^3 $\rightarrow w = av_1 + bv_2 + cv_3$

هل أقدر أكتب w بدلالة two vectors فقط؟

Is w a linear combination from v_1, v_2 only?

$$w = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad \cancel{v_3 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}}$$

$$\begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{array}\right]$$ $\rightarrow$ unsquare matrix since the third column is output

$R_3 - R_1 \rightarrow R_3$

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 2 & 5 \\ 0 & -1 & 1 \end{array}\right]$$

$R_2/2 \rightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 5/2 \\ 0 & -1 & 1 \end{array}\right]$$

$$R_3 + R_2 \longrightarrow R_3$$

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 5/2 \\ 0 & 0 & 7/2 \end{array}\right]$$

$$a_1 + 2b_2 = 2$$

$$b = 5/2$$

$$0 = 7/2$$

There are no solutions.

By using smpython package

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{0 \quad 0} & & 1 \end{array}\right]$$

$a=0$, $b=0$ trivial solution

Can't write w as a linear Combination from v_1, v_2

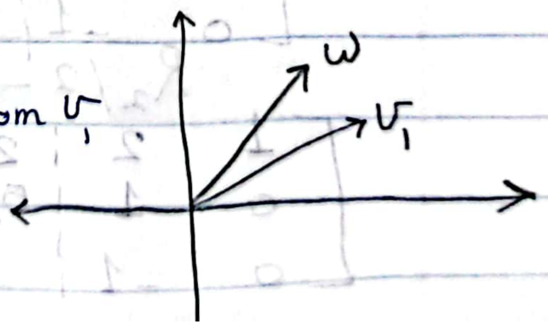
$\therefore$ w Not a linear Combination v_1, v_2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \cancel{0} & \cancel{0} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$a=0$, $b=0$ Trivial solution

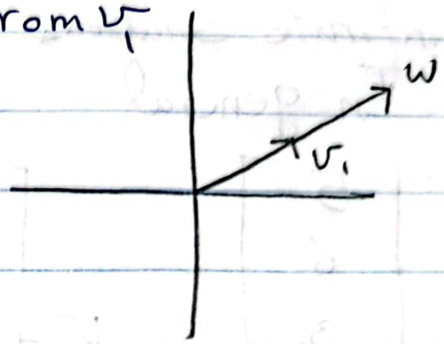
* w Not a linear Combination from v_1

$w = a v_1$ ✗



w is a linear Combination from v_1

$w = a v_1$ ✓



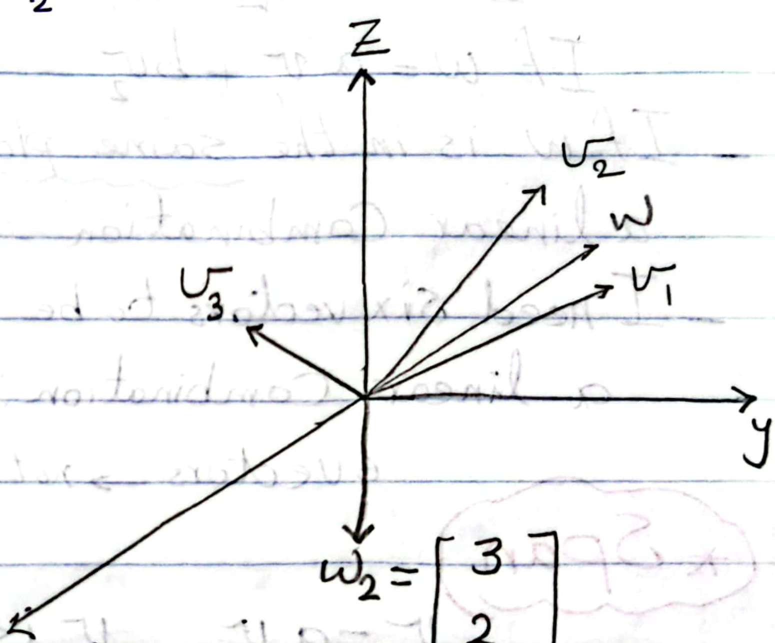
What's intitution?

أي two vector أكيد موجودين في ال plane

The meaning of w isn't a linear Combination from $v_1, v_2 \longrightarrow w$ isn't existing in the same plane with v_1, v_2

$$v_1 = \begin{array}{c} \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \end{array} \begin{array}{l} x \text{ Component} \\ y \\ z \end{array} \qquad v_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$w = \begin{bmatrix} 0 \\ 1.5 \\ 1 \end{bmatrix}$$



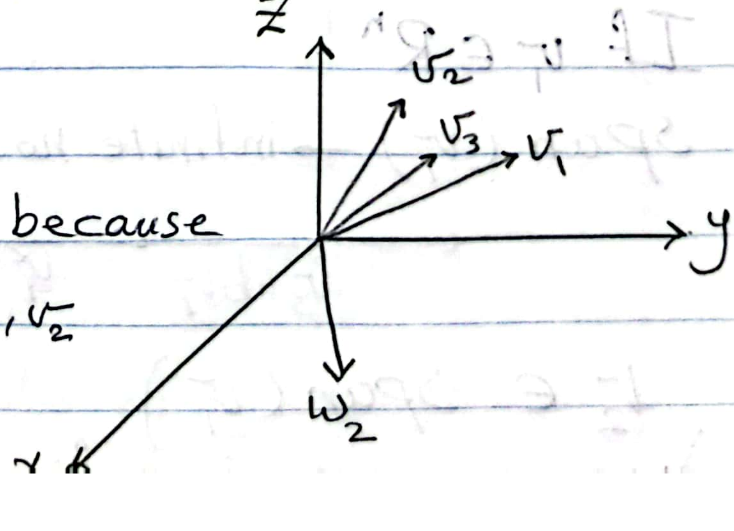
$$w_2 = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

w, v_1, v_2 in yz plane.

* هل ممكن أقدر أكتب w_2 بدلالة: v_1 ، v_2 ؟

لا ممكن ، لأنه خارج ال Plane

Any vector in R^3 plane, can be written بدلالة v_1, v_2, v_3



v_3 don't add information, because it's redundant of v_1, v_2

$v_3 = a v_1 + b v_2$

لو w تنطبق على v_1 or v_2 — infinite solutions

In general

$$v_1 = \begin{bmatrix} 5 \\ 6 \\ 3 \\ 4 \\ 1 \\ 2 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \\ 6 \\ 0 \end{bmatrix} \qquad R^6$$

If $w = av_1 + bv_2$

- If w is in the same plane with $v_1, v_2 \rightarrow \therefore w$ is a linear Combination
- I need six vectors to be sure that any vector is a linear Combination. $v_1, v_2, \ldots, v_6$

6 vectors → not combination يكونوا مختلفين

* Span

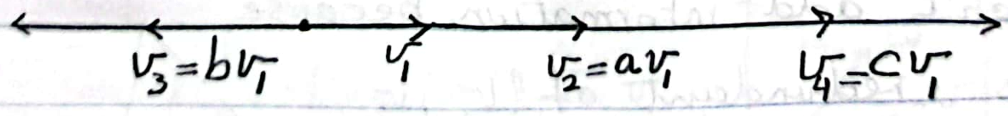
$v_1 = av_2$ v_1 is a linear Combination from v_2

$v_1 = av_2 + bv_3$

↳ v_1 is a linear Combination from v_2, v_3

If $v_1 \in R^n$

span (v_1) → infinite no. of vectors on the line.



$v_2 \in$ Span (v_1)

↳ meaning v_2 is a linear Combination from v_1

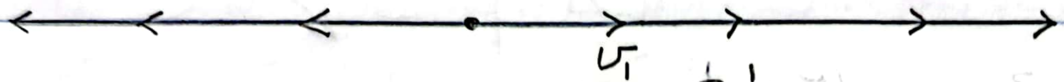
One vector can span line.

$v_1 \in R^1$

span (v_1) All vectors in $R^{(1)}$

$v_1 \rightarrow$ أقدر اكتب بدلالته infinite no. of vector على نفس ال line

span of $v_1 \longrightarrow$ All vectors in $R^{(1)}$

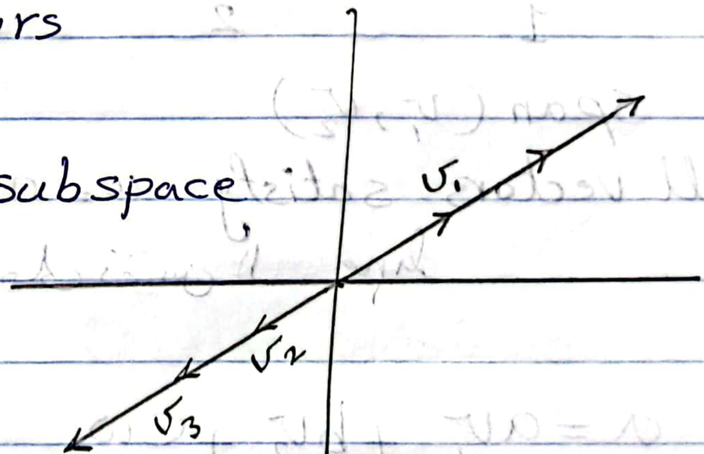


$[5] \in R^1$

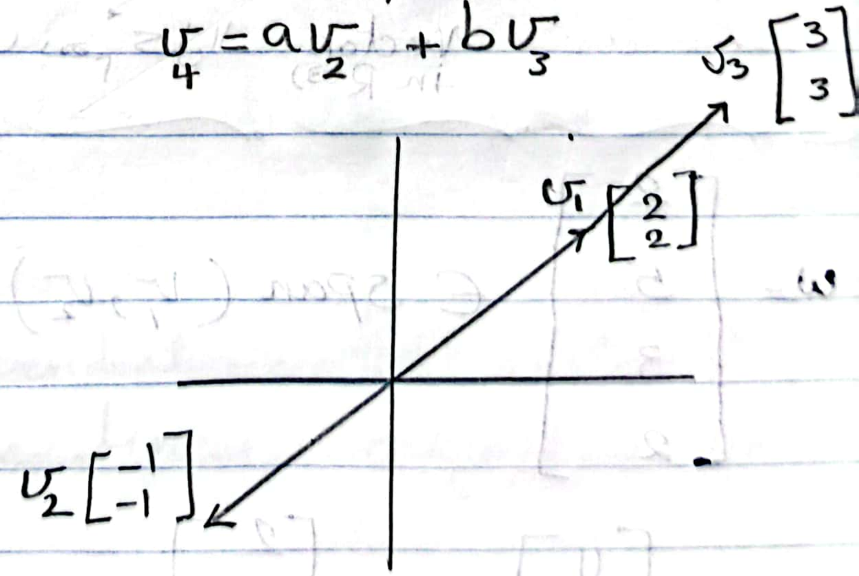
$R^2 \rightarrow$ its span is all vectors that on the same line.

v_1 span one dimensional subspace exist in R^2

لازم كل ال vectors تكون على نفس line



$v_4 = a v_2 + b v_3$



$v_2 = -0.5 v_1$

$v_3 = v_1 - v_2$

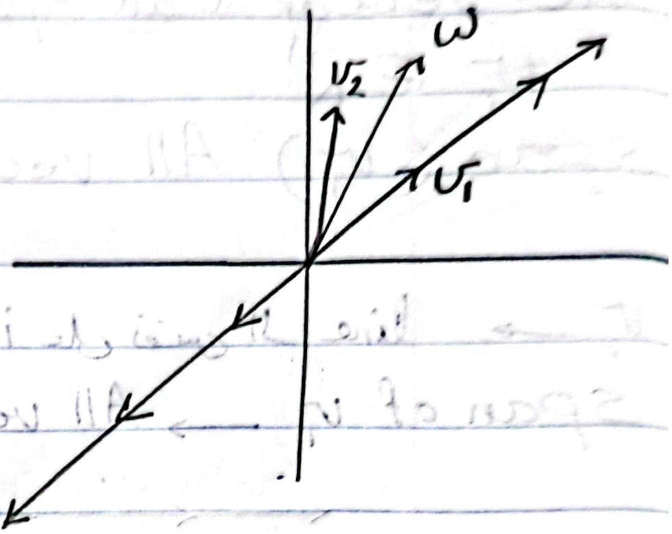
$v_3 = v_1 + 0.5 v_1$

$R^1 \rightarrow$ one dimensional vector space

$R^2 \rightarrow$ two dimensional vector space.

$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

span (v_1)



$w \in$ span (v_1) No
must be on the same line.

$v_1 = \begin{matrix} 3 \\ 1 \end{matrix}$ $v_2 = \begin{matrix} 1 \\ 2 \end{matrix}$

span (v_1, v_2)
all vectors satisfy $w = av_1 + bv_2$
بشرط v_1, v_2 مش على نفس الـ line

$u = av_1 + bv_2 + cw$
أقدر اجيب بهم كل الـ vectors in $R^{(3)}$

$w = \begin{bmatrix} 2 \\ 5 \\ 3 \\ 2 \end{bmatrix} \in$ span (v_1, v_2)

$v_1 = \begin{bmatrix} 11 \\ 5 \\ -7 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 13 \\ 0 \\ -7 \end{bmatrix}$

Is w is alinear Combination of v_1, v_2

$$w = a v_1 + b v_2$$

$$\begin{bmatrix} 2 \\ 5 \\ 3 \\ 2 \end{bmatrix} = a \begin{bmatrix} 11 \\ 5 \\ -7 \\ 0 \end{bmatrix} + b \begin{bmatrix} 2 \\ 13 \\ 0 \\ -7 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 11 & 2 & 2 \\ 5 & 13 & 5 \\ -7 & 0 & 3 \\ 0 & -7 & 2 \end{array}\right]$$ using sympython

RREF $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $a = 0$, $b = 0$.

trivial solution

$\therefore$ w is not a linear Combination from v_1, v_2

Since I haven't a solution satisfy $w = a v_1 + b v_2$

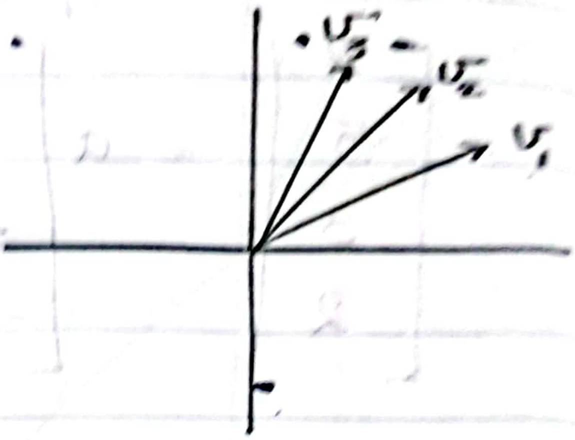
* Vector Linear Independence.

- A vector is linearly dependent on other vectors if it Can be expressed as the linear Combination of other vectors.

$$v_1 = 5 v_2 + 7 v_3$$

v_1, v_2, v_3 are linearly dependent.

$v_3 = a_1 v_1 + b_1 v_2$



$v_1 = a_2 v_2 + b_2 v_3$

$v_2 = a_3 v_1 + b_3 v_3$

v_1, v_2, v_3 linearly dependent vector

$$a v_1 + b v_2 + c v_3 = 0$$

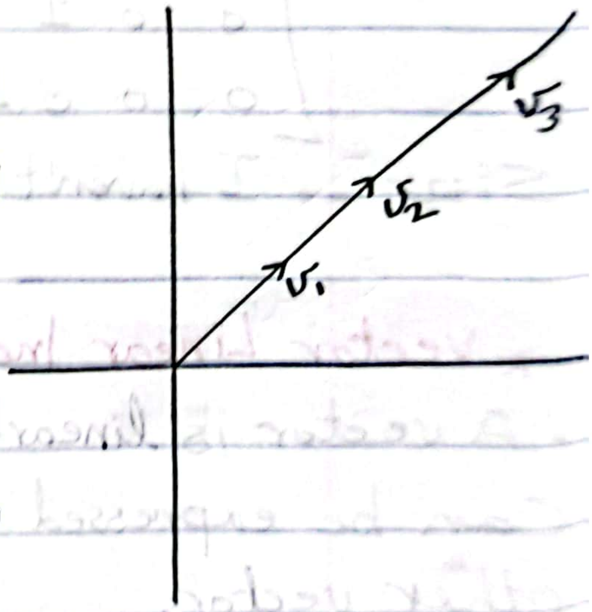
I have 3 vectors in R^2, Can أقدر اكتب اي واحد فيهم بدلالة الاثنين الثانيين

If I remove $v_2 \longrightarrow v_1, v_3$ linearly independent

$$a v_1 + b v_2 + c v_3 = 0$$

still dependent ← vector لو شيلت

remain vector independent ← 2 vector لو شيلت



Any n vectors in R^n

$$a_1 v_1 + a_2 v_2 + \dots + a_n v_n = 0$$

If I have values for $a_1 \dots a_n$

$\therefore v_1, v_2, \dots, v_n$ Dependent

$v_1 = \begin{bmatrix} 5 \\ 2 \\ 6 \end{bmatrix}$ $v_2 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ $v_3 = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$

Dependent → 1- In the same plane
2- on the same line (same span)

$$\begin{bmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 6 & 4 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If a, b, c have value → Dependent
If $a, b, c = 0$ → Independent

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$a = 0, b = 0, c = 0$ trivial solution
$\therefore v_1, v_2, v_3$ independent.

If I have 1 vector in R^1 ⇒ Span(v_1) ⇒ R^1
〃 〃 〃 2 vectors in R^2 → span(v_1, v_2) ⇒ R^2 يكون
↳ if v_1, v_2 independent

3 vectors R^3 → span(v_1, v_2, v_3) → R^3
↳ independent

* 2 independent vectors in R^3 → give me Plane in R^3

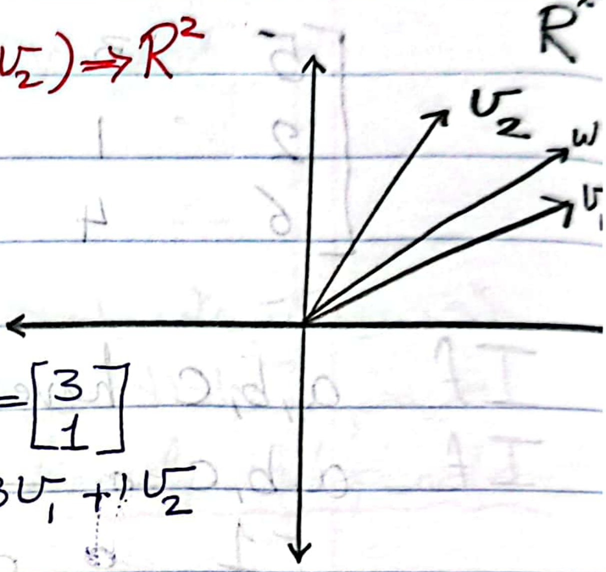
I need third vector (independent) to span R^3

If third vector (dependent) → can't span R^3

(v_1, v_2) independent → span$(v_1, v_2) \Rightarrow R^2$

∴ v_1, v_2 Basis of R^2

v_1, v_2 are reference vectors



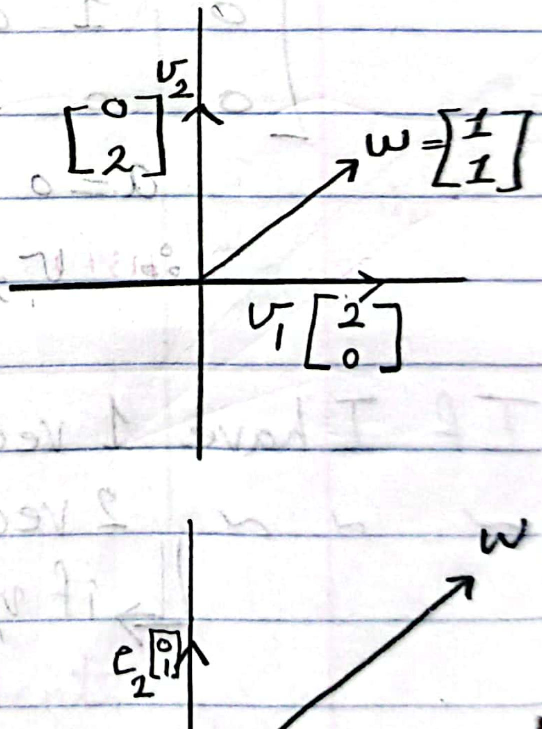
$w = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$w = 3v_1 + 1 v_2$

$w = 1v_1 + 1v_2$

v_1, v_2 special case of basis

v_1, v_2 are orthogonal basis

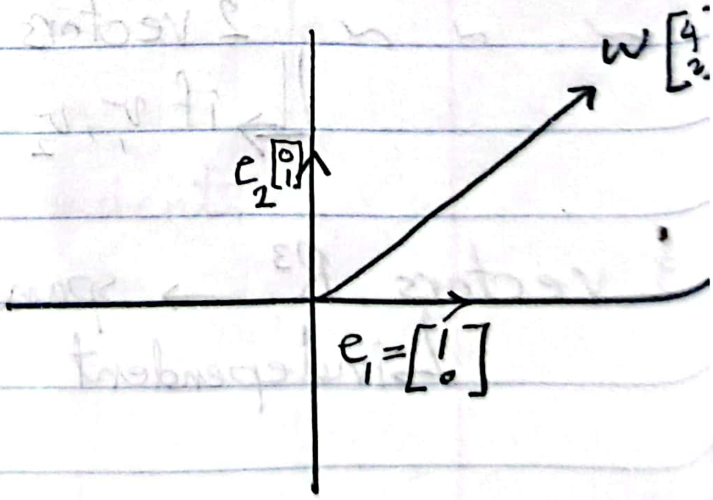


$w = 4\hat{e}_1 + 2\hat{e}_2$

e_1, e_2 → orthonormal Basis

$\|\hat{e}_1\| = 1$

$\|\hat{e}_2\| = 1$

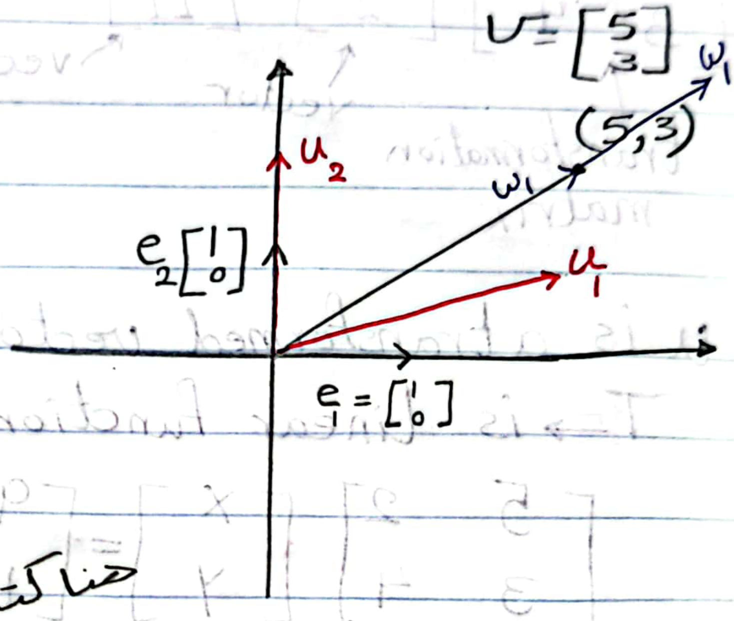


Basis $\longrightarrow$ بكتب بدلالته أي vector in space

$v = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$v = 5\hat{e}_1 + 3\hat{e}_2$

$= 5\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\longleftarrow$ هنا كتبنا v بدلالة e_1, e_2



$v = u_1 + 0.5u_2$

$v_{(u_1, u_2)} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$ $\longleftarrow$ هنا كتابة v بدلالة u_1, u_2

$w = 2\hat{e}_1 + 0.5e_2$

$w_{(e_1, e_2)} = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$

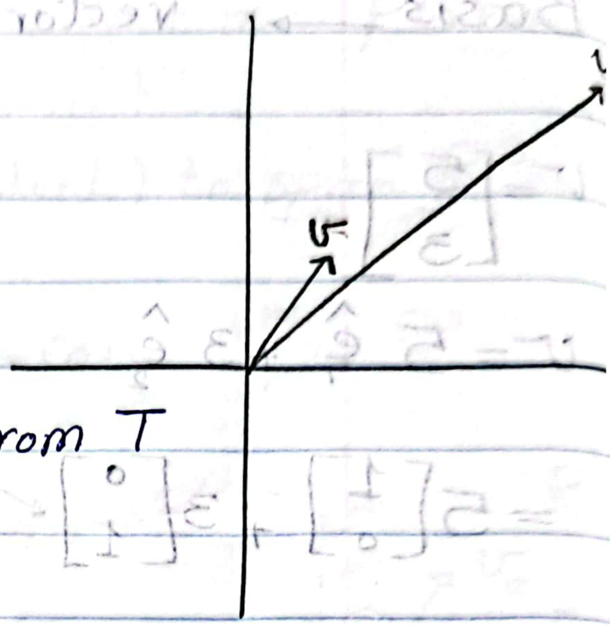
---

* Linear transformation preserves the vector space structure i.e, preserving linear Combinations.

- Rotation 45° counterclockwise
- shrink (scaling down)
- Scaling up
- scaling up in x, scaling down in y
- share

$$\overset{T}{\begin{bmatrix}5&2\\3&4\end{bmatrix}}\overset{v}{\begin{bmatrix}1\\2\end{bmatrix}}=\overset{u}{\begin{bmatrix}9\\11\end{bmatrix}}$$

transformation matrix   vector   vector



u is a transformed vector v from T

T → is linear function

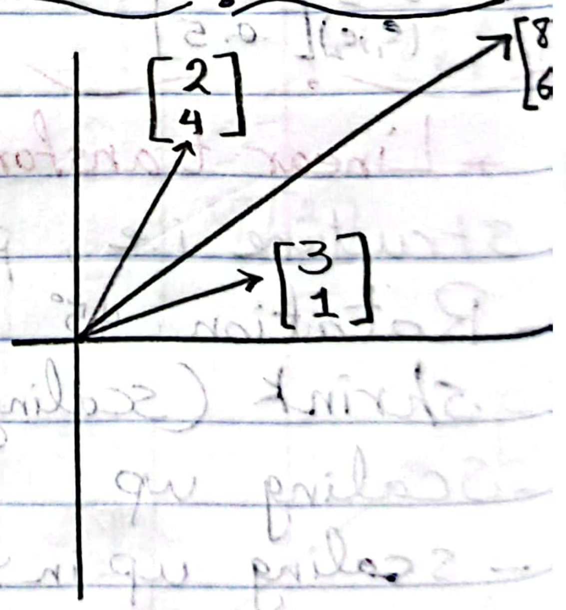
$$\begin{bmatrix}5&2\\3&4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}9\\11\end{bmatrix}$$

$$\begin{bmatrix}5x+2y\\3x+4y\end{bmatrix}=\begin{bmatrix}9\\11\end{bmatrix}$$

Resulted vector u → it is from origin (0,0)

→ it is still vector

$$\begin{bmatrix}2&3\\4&1\end{bmatrix}\overset{v}{\begin{bmatrix}1\\2\end{bmatrix}}=\overset{u}{\begin{bmatrix}8\\6\end{bmatrix}}$$



$$\begin{bmatrix}8\\6\end{bmatrix}=1\begin{bmatrix}2\\4\end{bmatrix}+2\begin{bmatrix}3\\1\end{bmatrix}$$

$$\begin{bmatrix}8\\6\end{bmatrix}=1\begin{bmatrix}2\\4\end{bmatrix}+2\begin{bmatrix}3\\1\end{bmatrix}$$

$$\begin{bmatrix}8\\6\end{bmatrix}=8\hat{e}_1+6\hat{e}_2$$

$$v = 1\hat{e}_1 + 2\hat{e}_2 = \underline{\underline{1}}\begin{bmatrix}1\\0\end{bmatrix} + \underline{\underline{2}}\begin{bmatrix}0\\1\end{bmatrix}$$

$$\underline{\underline{1}}\begin{bmatrix}2\\4\end{bmatrix} + \underline{\underline{2}}\begin{bmatrix}3\\1\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}\overset{\hat{e}_1}{\begin{bmatrix}1\\0\end{bmatrix}} = \begin{bmatrix}2\\4\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}\overset{\hat{e}_2}{\begin{bmatrix}0\\1\end{bmatrix}} = \begin{bmatrix}3\\1\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}\left(1\begin{bmatrix}1\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\end{bmatrix}\right)$$

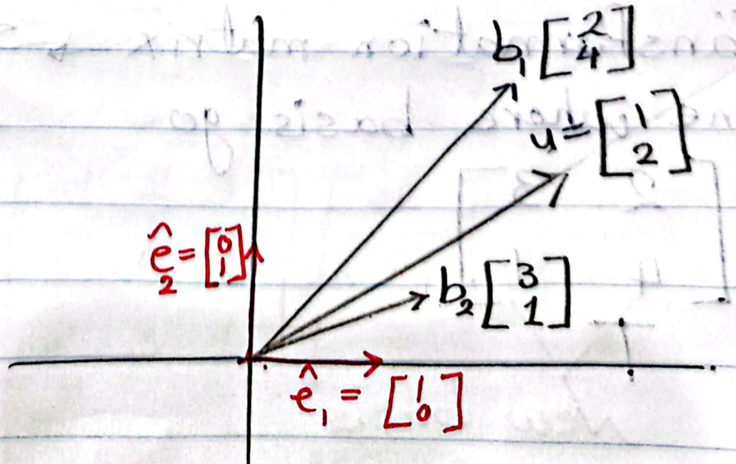
$$= 1\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} + 2\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$$

$$= 1\begin{bmatrix}2\\4\end{bmatrix} + 2\begin{bmatrix}3\\1\end{bmatrix}$$

Matrix make effect on basis vector of space

$$u_{b_1, b_2} = \begin{bmatrix}1\\2\end{bmatrix}$$

$$u_{\hat{e}_1, \hat{e}_2} = \begin{bmatrix}8\\6\end{bmatrix}$$

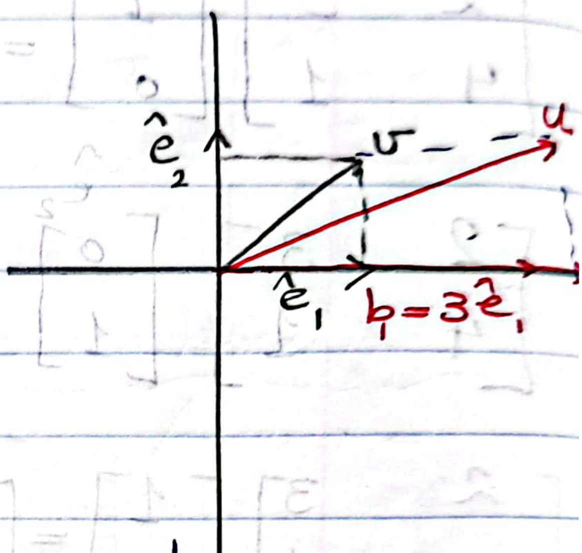


transformation Basis.

- A transformation is $\boxed{\text{linear}}$ if:
  - All lines remains line.
  - The origin doesn't move.

$v = 2\hat{e}_1 + \hat{e}_2$

$v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$



$v = 2\hat{e}_1$

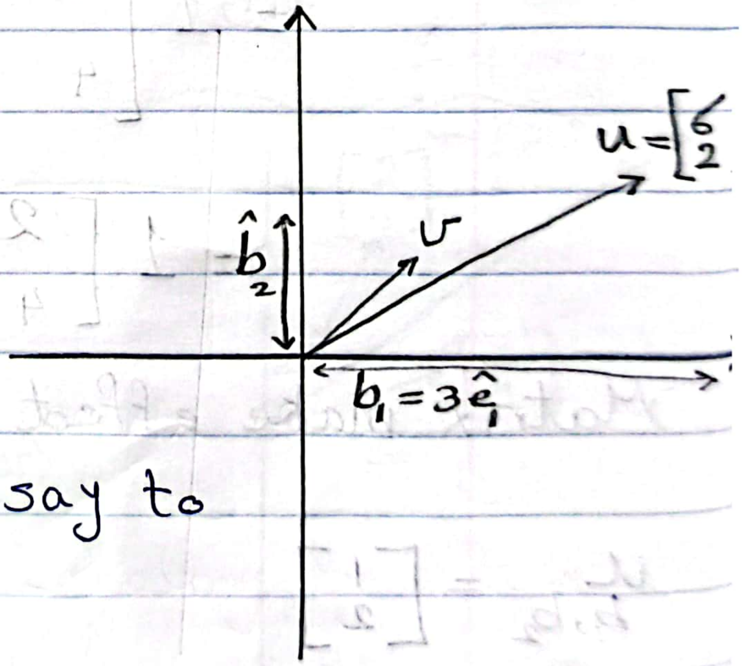
$u = 2(3\hat{e}_1) = 6\hat{e}_1$

v Convert from v to u → because change its basis but Combination is as it كماهو

$u = 2(\underbrace{3\hat{e}_1}_{\hat{b}_1}) + \hat{b}_2$

$u = 2(3\hat{e}_1) + 1(2\hat{e}_2)$

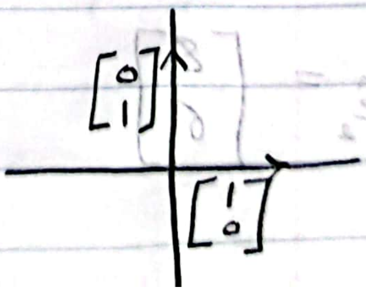
$u = \underline{\underline{6}}\hat{e}_1 + \underline{\underline{2}}\hat{e}_2$



Transformation matrix → say to me where basis go

$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

New basis.



$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 18 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 10 \\ 15 \end{bmatrix} = \begin{bmatrix} 18 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{\begin{matrix} 2 \\ 5 \end{matrix}} & \begin{matrix} 3 \\ 1 \end{matrix} \end{bmatrix}^T$$

$$\hat{e}_2 \longrightarrow \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\hat{e}_1 \longrightarrow \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$v = a\,\hat{e}_1 + b\,\hat{e}_2$$

$$v_{transformed} = u = a\left(\begin{bmatrix} 5 \\ 2 \end{bmatrix}\right) + b\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right)$$

---

* System of Linear Equations Revisited

$$x + 2y + 3z = 6$$
$$2x + 5y + 2z = 4$$
$$6x - 3y + z = 2$$

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 6 & 3 & 1 \end{bmatrix}}_{\text{coefficient}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{vector}} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

Thinking of solving system of linear equation as finding the vector x that when we apply the transformation matrix A becomes vector b.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 6 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

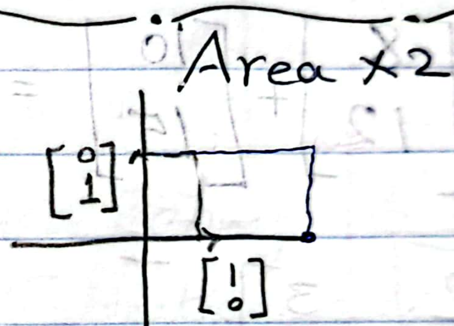
$$Ax = b \longrightarrow x = A^{-1}b$$

∴ inverse → Reverse transformation.

* **Matrix Determinant:**

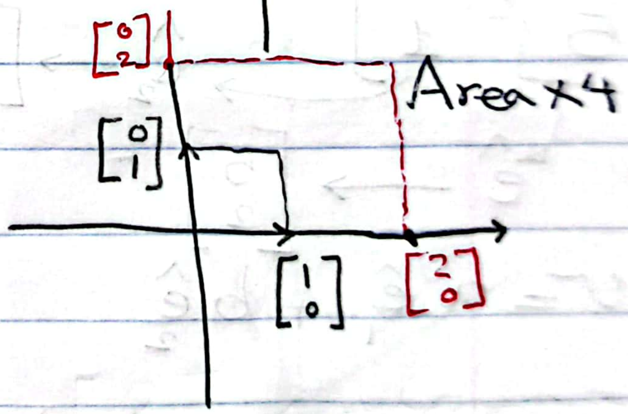
$$\begin{bmatrix} \boxed{2} & 0 \\ 0 & 1 \end{bmatrix}$$ Scaling → 2 ,ضعف

$\det(2-0) = 2$



$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$ Scaling by 4

$\det(4-0) = 4$



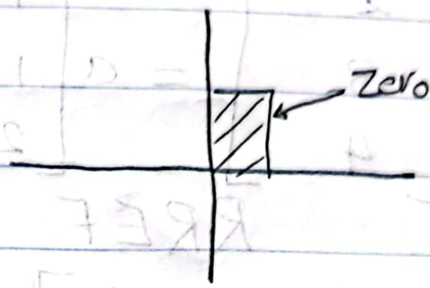
Determinant of transformation matrix represent C. Area of space

Area (its vector space) → scaling by det. value.

* If det. is negative, it means that the transformation flips the space, i.e. the basis vector switches its positions

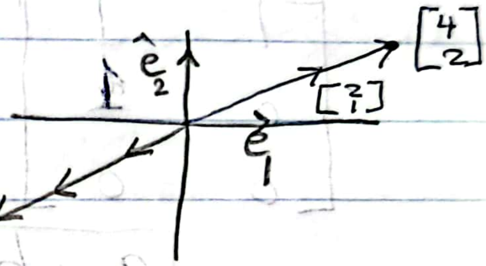
علاقة ال Axes ببعضها اتعكست

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$



$$\det \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} = 4 - 4 = 0$$

Area $= 0$



matrix takes all vectors in R^2 and Collapse them on one line. R^1

- Collapsing the space into a line.
- Transformation couldn't be restored (can't get inverse)
- singular matrix $\longrightarrow$ can't get inversed $\begin{cases} \rightarrow \text{have no solution} \\ \rightarrow \text{infinite sol.} \end{cases}$

  since Columns of this matrix are dependent vectors.

$$\left.\begin{aligned} 4x + 2y &= 0 \\ 2x + y &= 0 \end{aligned}\right\} \text{Dependent vector}$$

numpy.linalg.inv($\overset{\text{matrix}}{\downarrow}$)

numpy.linalg.det( )

* Connection between row and Column pictures of the matrix:

Ex.1 $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$, $w = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$

Is w a linear Combination from v_1, v_2?

$w = a v_1 + b v_2$

$$\begin{bmatrix}2\\3\\4\end{bmatrix} = a\begin{bmatrix}1\\1\\2\end{bmatrix} + b\begin{bmatrix}1\\3\\1\end{bmatrix} \Rightarrow \begin{matrix}a+b=2\\a+3b=4\\2a+b=3\end{matrix}$$

RREF

$$\begin{bmatrix}1 & 1\\0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\1\\0\end{bmatrix} \qquad \begin{matrix}a+b=2\\b=1\end{matrix}$$

$\boxed{a=1,\ b=1}$

$\boxed{\begin{matrix}a+b=2\\a+3b=4\\2a+b=3\end{matrix}}$ → Row picture

If there is a solution (3 eq. in 2 variables)

∴ there is one eq. is redundant of two eq.

a, b have values,

and w is a linear Combination from v_1, v_2

If there is no solution → "3 different eq. in 2 variables"

and w isnot a linear Combination from v_1, v_2

$$\begin{bmatrix}1 & 1\\1 & 3\\2 & 1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\4\\3\end{bmatrix}$$ ← Column picture

- Row picture, Column picture are Consistent

$$\begin{bmatrix}1 & 1\\0 & 1\\\boxed{0 \quad 0}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\1\\\boxed{0}\end{bmatrix}$$ ← Redundant eq.

two independent eq. in 2 variables.

From Row picture → I solved system of linear eq.
From Col. picture → I understand why these system of linear eq. has solution.

$w = a v_1 + b v_2$

* System of linear eq. has solution because output of these system of linear eq. is a vector, and this vector can be obtained as a linear combination from two vectors. $w = \hat{v}_1 + \hat{v}_2$

∴ System has definite sol.

Ex-2

$v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \quad w = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$

$$w = a \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + b \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + b \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$a + b = 5$

$a + 3b = 3$

$2a + b = 4$

RREF

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -7 \end{bmatrix}$$

$$\boxed{a = 6}$$

$$a + b = 5$$

$$2b = -2 \longrightarrow \boxed{b = -1}$$

$$0a + 0b = -7$$

a, b Solve first two eq. only.

But third eq. doesn't intersect with (first two eq
because it isn't redundant eq.

$\therefore$ System has no solution.

$\therefore$ w isn't a linear combination from v_1, v_2