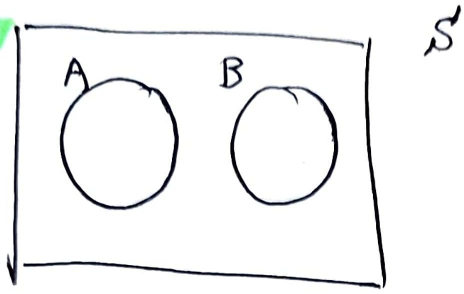
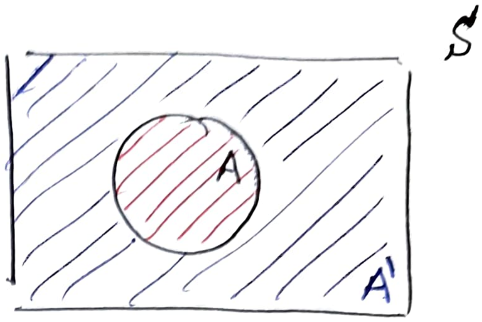
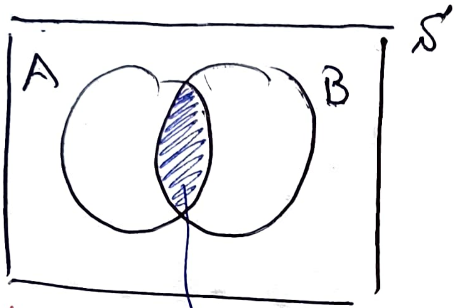
DASFML S1 Mansoura, 18/12/2024

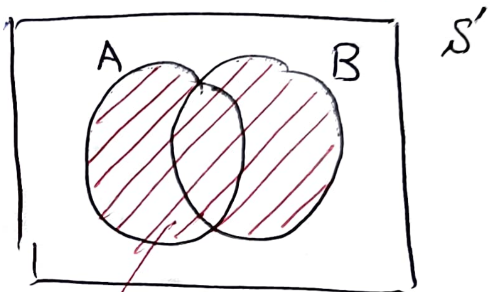
$A \cap B = \phi$
→ disjoint / → Mutually exclusive events







$A \cap B$

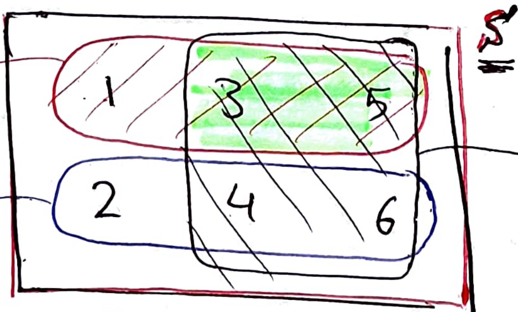


$A \cup B$

ex. Rolling one die once, oserving outcome: (random experiment)

$S = \{1, 2, 3, 4, 5, 6\}$

Outcomes: 1, 2, 3, 4, 5, 6



A ← event, B, C ← event, S ← sample space

A: odd #

B: even #

C: # > 2

→ A, B are disjoint (Mutually exclusive)

(A and B) (A & B) (AB) (A,B)

$A \cap B = \phi$ ⟷ (odd and even)

$A \cap C = \{3, 5\}$ ↔ (odd and greater than 2)

$B \cap C = \{4, 6\}$ ↔ (even and " " 2)

A, B are mutually exclusive ⟺ $A \cap B = \phi$

B, C are non-mutually exclusive ⟺ $A \cap C \neq \phi$

A, C " " " " "

→ Sample space $S = \{\text{all outcomes}\}$

→ event → $W \subset S$

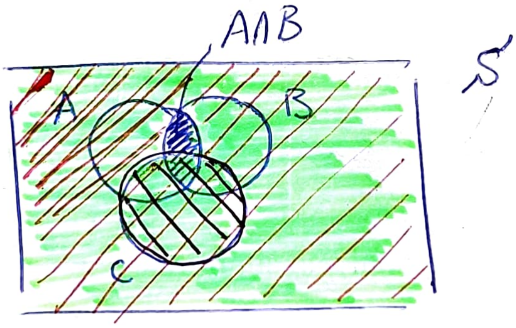
$S \subset S$

$A \subset B$

$B \subset S$

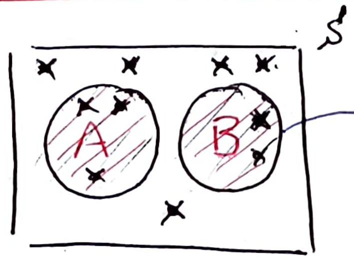
$\phi \subset S$

Review on set operations



$(A \cap B) \cup C$

Probability :



$\frac{N(A \cup B)}{N(S)} = \frac{N(A)+N(B)}{10}$

$[0, 1)$ $= \frac{N(A)}{N(S)} + \frac{N(B)}{N(S)}$

$= \frac{3}{10} + \frac{2}{10}$

P(B) $= P(A) + P(B)$

Classical def.

→ $0 \leq \frac{N(B)}{N(S)} = P(B) \leq 1$

$P(S) = 1$

$\frac{0}{N(S)} = \frac{N(\phi)}{N(S)} = P(\phi) = 0$

OR

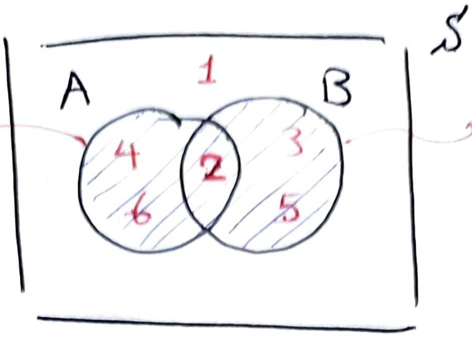
$P(A \cup B) = \frac{N(A \cup B)}{N(S)}$

$= N(A$

— if A, B are disjoint

$P(A \cap B) = P(\phi) = 0$

$P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}$ → zero



even

$A = \{2, 4, 6\}$

prime

$B = \{2, 3, 5\}$

→ A, B are non-mutually exclusive

$A \cap B \neq \phi$ ~~A∩~~ ex $A \cap B = \{2\}$

and $P(A \cap B) = \dfrac{N(A \cap B)}{N(S)}$

$P(A \cap B) = \dfrac{1}{6}$

or $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

$A \cup B = \{2, 3, 4, 5, 6\}$

$P(A \cup B) = \dfrac{N(A \cup B)}{N(S)}$

$= 5/6$

1 -

H	T

S' $P(H) = \frac{1}{2}$

2 - flipping a coin two times ≡ flipping two coins

HH	HT
TH	TT

S'

$P(2\text{ heads}) = 1/4$

$P(1H \& 1T) = 2/4$

$P(H^1, T^2) = P(H^1T^2) = 1/4$

or $P(T^1, \cancel{H}^2) = P(T^1H^2) = 1/4$

3/4

$P(1\text{ head} \& \text{one tail}) \equiv P((1^{st} H \text{ and } 2^{nd} T) \text{ or } (1^{st} T \& 2^{nd} H)$

$= P(1^{st} H \& 2^{nd} T) + P(1^{st} T \& 2^{nd} H)$

$= 1/4 + 1/4 = 1/2$

flipping 3 fair coins, observing outcomes.

3 -

HHH	HHT
HTH	HTT
THH	THT
TTH	TTT

S

$P(2H \& 1T) = \boxed{3/8}$

2 heads, 1 tail

$= P(H^1H^2T^3 \text{ or } H^1T^2H^3 \text{ or } T^1H^2H^3)$

$P(\_\_\_) + P(\_\_\_) + P(\_\_\_)$

$1/8 + 1/8 + 1/8$

Prob. (2 heads and one tail)

$= 3/8$

- two dice are rolled

S

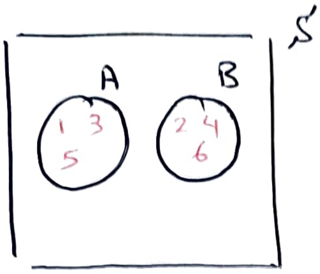
(1,1)	(1,2)	...	(1,6)
(2,1)	(2,2)	—	(2,6)
(3,1)			(3,6)
(4,1)			(4,6)
(5,1)			(5,6)
(6,1)		......	(6,6)

a) - Prob. of two even numbers? $= \frac{9}{36}$

b) - Prob. of two numbers whose sum = 7? $\frac{6}{36}$

Intersection of two events

1



$P(A) = 1/2$

$P(B) = 1/2$

$P(A,B) = 0 \neq P(A) \cdot P(B)$

Mutually exclusive

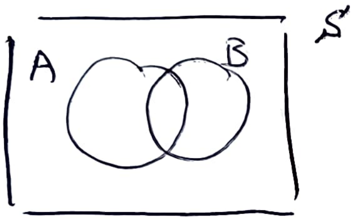
if A, B are disjoint

$P(AB) \equiv P(A,B) \equiv P(A \cap B)$

$\equiv P(A,B) = 0$

$P(A) \cdot P(B) \neq 0$

2



A, B ~~are~~ in general

A, B are → independant

$P(A|B) = P(A)$

$P(B/A) = P(B)$

$$\boxed{P(AB) = P(A) \cdot P(B)}$$

A, B are not indep.

$P(A/B) \neq P(A)$

$P(B/A) \neq P(B)$

$P(AB) \neq P(A) \cdot P(B)$

Conditional Prob!

$P(AB) = P(A|B)\, P(B)$

$= P(AB) = P(B|A)\, P(A)$

to be discussed in detail later. ISA.

"fair coins"

→ Tossing 3 coins, observing outcomes. ⑦

Prob. of getting 2 heads and one tail.

$= P(H^1H^2T^3 \text{ or } H^1T^2H^3 \text{ or } T^1H^2H^3)$

$= P(H^1 \cap H^2 \cap T^3) + P(H^1T^2H^3) + P(T^1H^2H^3)$

$= P(H^1)\cdot P(H^2)P(T^3) + P(H^1)P(T^2)P(H^3) + P(T^1)P(H^2)P(H^3)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= 3 \times \frac{1}{8} = \frac{3}{8}$

$3 = \binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1} \times 1}$

Binomial distribution

→ sessions 2, 3, 4

→ $\boxed{P(\text{"}K\text{" heads in "}n\text{" trials})}$

→ $P(2 \text{ heads in } 3 \text{ coin tosses}) = 3/8$

$= \binom{n}{K} \times P(H)^K \times P(H')^{n-K} = \binom{3}{2} 0.5^2 \times (1-0.5)^{3-2}$

$0.7 \qquad 0.3$

⇒ $\boxed{{}^nC_K = \binom{n}{K} = \frac{n!}{K!(n-K)!}}$ ← عدد الطرق الممكنة

- Combinations
- التوافيق

$\frac{3!}{2!(3-2)!} \times 0.5^2 \times 0.5^1$

$= 3 \times 0.5^3 = \frac{3}{8}$

tossing 10 coins

$\vdots$

$P(8H, 2T) = ?$

$P(7H, 3T) = ?$

$\vdots$

10 → H

9H, 1T

← 8H, 2T

$\vdots$

10T

$$P(8H \text{ in } 10 \text{ tosses}) = \binom{10}{8} P(H)^8 \, P(H)^2$$

$$= \binom{10}{8} (0.5)^8 \times (0.5)^2$$

$$\binom{10}{8} \, 0.5^{10} = ?$$

---

10 tosses ; K head → 10 − K tails

8 H 2 tail

Prob (K successes in n trials)

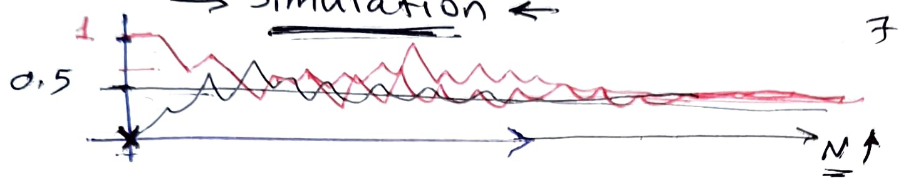
experimental Prob.

$P(H) = \frac{1}{2}$?

$0 \Rightarrow H \quad , \quad 1 \Rightarrow T$

H | 0 0 1 0 1 1 1 0 1 0 1 1 0 | $\frac{N(H)}{N}$

→ simulation ←



#	
1	H
2	H
3	T
4	H
5	T
6	T
7	T
⋮	⋮
1000	
10000	

$N \to \infty$

$N \to$

#	

---

tossing 3 coins

$3 \times N$

~~10,000~~

8000

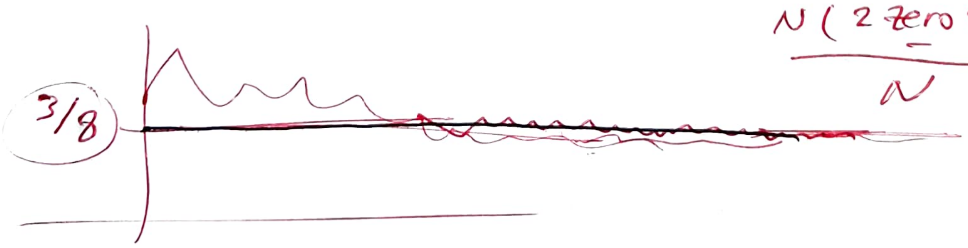
C_1	0	0	1
C_2	0	0	1
C_3	1	0	0

2H, T

3H

2T, ?

$\frac{N(2H, T)}{N} \quad \frac{N(2\text{ zeros}, 1)}{N}$



A, B	mutually exclusive	non-mutually exclusive
indep.	✗	✓ ?
not indep.	✓ ?	✓ ?

True