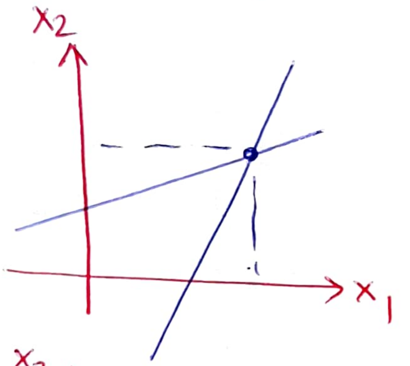
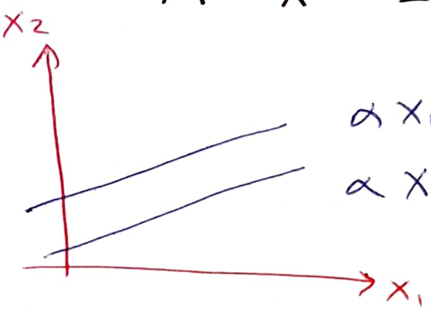
LA AI Mansoura session 2 30/11/2024 ①

$$\overset{0}{2x_1} + \overset{2}{3x_2} = 6$$
$$1\underset{0}{x_1} - 2\underset{2}{x_2} = -4$$

$$\Rightarrow \underbrace{\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 6 \\ -4 \end{bmatrix}}_{\vec{b}}$$

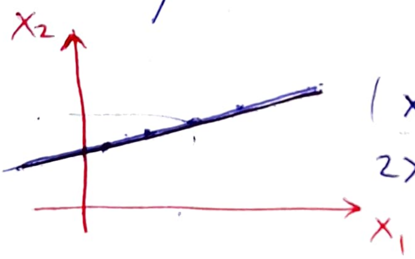
$$A \vec{x} = \vec{b}$$





$$\alpha x_1 + \beta x_2 = \gamma_1$$
$$\alpha x_1 + \beta x_2 = \gamma_2$$

$$x_1 + x_2 = 10$$
$$x_1 + x_2 = 20$$



$$(x_1 + 2x_2 = 5) \times 2$$
$$2x_1 + 4x_2 = 10$$

Identity matrix

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$IA = A$$
$$AI = A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$AA^{-1} = I$ — A is invertible

$A^{-1}A = I$ — A^{-1} is inverse of A

$$\begin{bmatrix} 2 & 3 \\ +1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

$$\cancel{A^{-1}A} \to I \; \vec{x} = A^{-1}\vec{b} \Rightarrow \boxed{\vec{x} = A^{-1}\vec{b}}$$

$$\vec{x} = ? \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

Gauss-Jordan Elimination

$$\begin{bmatrix} 1 & \cancel{0} \\ \cancel{0} & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix} \quad \text{row}_1 \div 2$$

$$\rightarrow \begin{bmatrix} 1 & 1.5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix} \quad \text{row}_2 - \text{row}_1$$

$$\Rightarrow \begin{bmatrix} 1 & 1.5 \\ 0 & -3.5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix} \quad r_2 \div -3.5$$

rref

$$\rightarrow \begin{bmatrix} 1 & 1.5 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \rightarrow \text{row}_2 \times -1.5 + \text{row}_1$$

$$0 \quad -1.5 \qquad -3$$

$$\rightarrow \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{I\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 3 & 6 \\ 1 & -2 & -4 \end{array}\right]$$

Augmented matrix

↓

Gauss elimination

$r_1 / 2$

$$\left[\begin{array}{cc|c} 1 & 1.5 & 3 \\ 1 & -2 & -4 \end{array}\right]$$

$r_1 \times -1 + r_2$

$$\left[\begin{array}{cc|c} 1 & 1.5 & 3 \\ 0 & -3.5 & -7 \end{array}\right]$$

$r_2 \div -3$

$$\left[\begin{array}{cc|c} 1 & 1.5 & 3 \\ 0 & 1 & +2 \end{array}\right]$$

$r_2 \times -1.5 + r_1$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 2 \end{array}\right]$$

⇐ rref (Augmented matrix)

I

solution

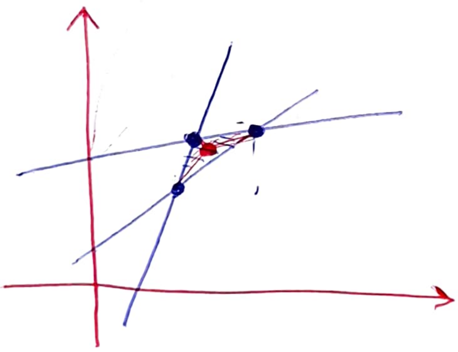
$$x + y = 5$$
$$2x + 2y = 10$$

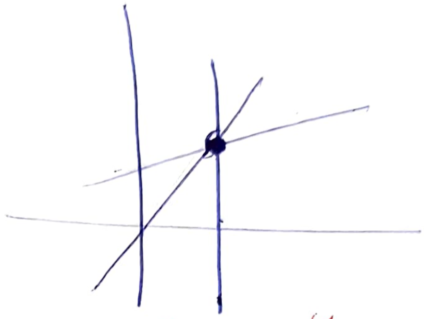
$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 2 & 2 & 10 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right]$$

Overdetermined system of equations

$$\begin{cases} x_1 + x_2 = 2 & \rightarrow ① \\ x_1 + 2x_2 = 5 & \rightarrow ② \\ 2x_1 + 3x_2 = \cancel{7}\ 9 & ③ \end{cases}$$





$x_1 + x_2 = 2 \longrightarrow ①$ (×2: $2x_1 + 2x_2 = 4$)

$x_1 + 2x_2 = 5 \longrightarrow ②$ (×3: $3x_1 + 6x_2 = 15$)

$\boxed{5x_1 + 8x_2 = 17} \longrightarrow ③$

eq. ③ $= 2① + 3②$

eq. 3 : Linear combination of ①, ②

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ex

$$x_1 + x_2 = b_1$$
$$x_1 + 2x_2 = b_2$$
$$\boxed{2x_1 + 3x_2 = b_3} \rightarrow \text{if} \rightarrow \text{linear combination.}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

augmented matrix $\longrightarrow$

$$\left[\begin{array}{cc|c} 1 & 1 & b_1 \\ 1 & 2 & b_2 \\ 2 & 3 & b_3 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & b_1 \\ 0 & 1 & b_2 - b_1 \\ 0 & 1 & b_3 - 2b_1 \end{array}\right]$$

$0-0 \quad 1-1 \quad b_3 - 2b_1 - (b_2 - b_1)$

$b_3 - b_2 - b_1$

$$\xrightarrow[r_3 - r_2]{r_1 - r_2} \left[\begin{array}{cc|c} 1 & 0 & \cancel{b_1 - (b_2 - b_1)}\ 2b_1 - b_2 \\ 0 & 1 & b_2 - b_1 \\ 0 & 0 & b_3 - b_2 - b_1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2b_1 - b_2 \\ b_2 - b_1 \\ b_3 - b_2 - b_1 \end{bmatrix}$$

free variable

$$x_1 = 2b_1 - b_2$$

$$x_2 = b_2 - b_1$$

$$\boxed{0 = b_3 - b_2 - b_1} \rightarrow \text{condition}$$

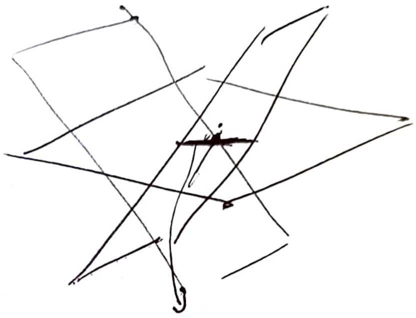
$$b_3 = b_2 + b_1$$

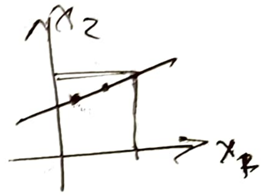
$$x_1 + 3x_2 + 2x_3 = 1 \quad b_1$$
$$2x_1 + x_2 + x_3 = 2 \quad b_2$$

0
1
$x_1 + 2x_2 = 3$

many solution
"inf. # of sol."

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$





$x_2 = 0 \Rightarrow x_1 = 3$
$x_2 = 1 \Rightarrow x_1 = 1$
$x_2 = 2 \Rightarrow x_1 =$ ✓

free variable

augmented matrix

$$\begin{array}{c} \times -2 \\ +r_2 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 2 & 1 & 1 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -5 \to 1 & +3/5 & 0 \end{array}\right] \div -5$$

$$\begin{array}{c} r_2 \times -3 \\ +r_1 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & 3/5 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1/5 & 1 \\ 0 & 1 & 3/5 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 1/5 \\ 0 & 1 & 3/5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

free variable

$$x_1 + \tfrac{1}{5}x_3 = 1 \Rightarrow x_1 = 1 - \tfrac{1}{5}x_3$$
$$x_2 + \tfrac{3}{5}x_3 = 0 \Rightarrow x_2 = -\tfrac{3}{5}x_3$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - - - -$$