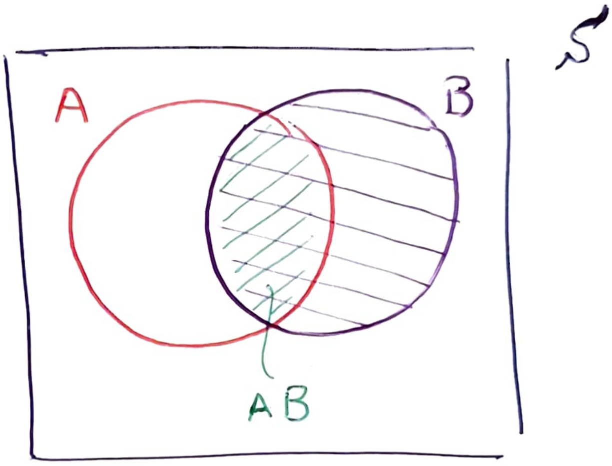
→ PASFML session 5, Mansoura, 1/1/2025 ①

- Conditional Probability, ...
- Bayes' Rule ⟷ Classification metrics
- Naïve Bayes' Classifier

$P(AB) \equiv P(A \cap B)$

$P(A|B) = \dfrac{P(AB)}{P(B)}$

$P(B|A) = \dfrac{P(AB)}{P(A)}$



→ if A, B are independent

$P(A|B) = P(A)$

$P(B/A) = P(B)$

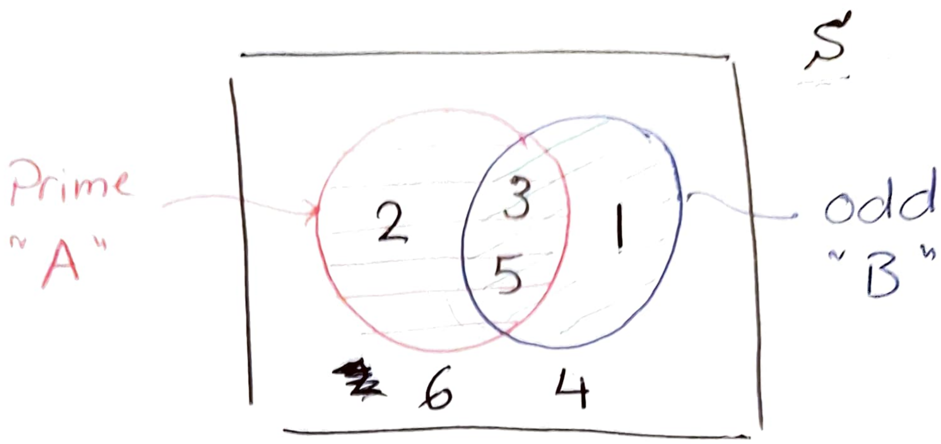
$P(AB) = P(A)\,P(B)$

$$P(AB) = P(B/A)\,P(A) = P(A/B)\,P(B)$$

$$\underbrace{P(A/B)}_{\text{Posterior}} = \frac{\overbrace{P(B/A)}^{\text{Likelihood}}\,\overbrace{P(A)}^{\text{Prior}}}{\underbrace{P(B)}_{\text{evidence}}}$$

Bayes' Rule *

ex.



$\rightarrow P(A,B) = \frac{2}{6}$ ← P(odd and Prime)

$\rightarrow P(A/B) = \frac{2}{3}$ P(Prime / odd)

$\rightarrow P(B/A) = \frac{2}{3}$

$$P(A/B) = \frac{P(AB)}{P(B)} = \frac{2/\cancel{6}}{3/\cancel{6}} = 2/3$$

$$P(B/A) = \frac{P(AB)}{P(A)} = \frac{2/\cancel{6}}{3/\cancel{6}} = 2/3$$

$P(A/B) = P(AB)/P(B)$ ← events A, B

$\underbrace{P_{X/Y}(x/y)}_{\text{Conditional Prob.}} = \underbrace{P_{X,Y}(x,y)}_{\text{Joint Prob.}} / \underbrace{P_Y(y)}_{\text{Marginal Prob.}}$ ← R.V.'s X, Y

$\underbrace{f_{X/Y}(x/y)}_{\text{Conditional PDF}} = \underbrace{f_{X,Y}(x,y)}_{\text{Joint PDF}} / \underbrace{f_Y(y)}_{\text{Marginal PDF}}$

also can be written in terms of CDF!

ex. two jointly Gaussian R.V.'s X, Y

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho_{XY}^2}}\, e^{-\left(\frac{1}{2(1-\rho_{XY}^2)}\left[\frac{(x-\mu_x)^2}{\sigma_X^2} + \frac{(y-\mu_y)^2}{\sigma_Y^2} - 2\rho_{XY}\left(\frac{x-\mu_x}{\sigma_X}\right)\left(\frac{y-\mu_y}{\sigma_Y}\right)\right]\right)}$$

Joint PDF

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}}\, e^{-\left(\frac{(x-\mu_x)^2}{2\sigma_X^2}\right)}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_Y^2}}\, e^{-\frac{(y-\mu_y)^2}{2\sigma_Y^2}}$$

if x, y are uncorrelated

$\Rightarrow \rho_{XY} = 0$

$$\Rightarrow f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y}\, e^{-\left[\frac{(x-\mu_x)^2}{2\sigma_X^2} + \frac{(y-\mu_y)^2}{2\sigma_Y^2}\right]}$$

$$= \frac{1}{\sqrt{2\pi\sigma_X^2}} \times \frac{1}{\sqrt{2\pi\sigma_Y^2}} \times e^{-\frac{(x-\mu_x)^2}{2\sigma_X^2}} \times e^{-\frac{(y-\mu_y)^2}{2\sigma_Y^2}}$$

$$f_X(x) \times f_Y(y)$$

given $f_{X,Y}(x,y) \Rightarrow$ we can find $f_X(x)$ & $f_Y(y)$

then we can find $f_{X/Y}(x/y) = \dfrac{f_{X,Y}(x,y)}{f_Y(y)}$

$$f_{Y/X}(y/x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

Multiplication Rule . "Chain Rule"

$$P(ABCD) = P(A)P(B/A)P(C/AB)P(D/ABC)$$

~~P(A/X) = X P(A) P(X/A)~~

→ Drawing cards without Replacement

$$P(A^1A^2A^3A^4) = P(A^1)P(A^2/A^1)P(A^3/A^1A^2)P(A^4/A^1A^2A^3)$$

$$= \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49}$$

$$= 0.000\ldots$$

Birthday Paradox

n persons   P(two persons at least share the same birthday).

= 1 − P(no two persons share the same —),

= 1 − P(each person was born on a different day)

$= 1 - \left(\frac{365}{365}\right)\left(\quad \right)$

1 2 3 4

monty hall problem

Bayes' Rule (and total Probability theorem)

ex.

→ 1 % of Population suffer certain allergy.

→ test → 98% accurate in detecting Allergy.

recall

$\frac{TN}{N}$ → 97% accurate in detecting no Allergy.

→ if a person took the test, and tested +ve,

what is the probability that this person actually has allergy.

A : has allergy   A' : does not have allergy.

$\boxed{P(A) = 0.01}$ ✓ $P(A') = 0.99$

B : test +ve   B' : test -ve

belief updating

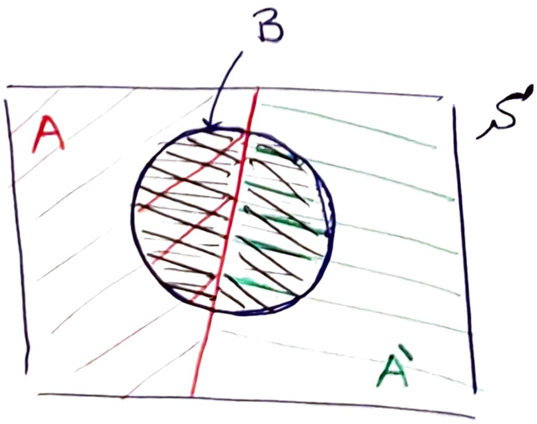
$$P(A/B) = \frac{\overset{0.01}{P(A)}\ \overset{0.98}{P(B/A)}}{\boxed{P(B)}} = 0.248$$

?   0.0395   ?

$$P(B) = P(A \cap B) + P(A' \cap B)$$

$P(A \cap B) = \underset{0.98}{P(B/A)}\,\underset{0.01}{P(A)}$

$P(A' \cap B) = \underset{0.03}{P(B/A')}\,\underset{0.99}{P(A')}$



$$P(B) = 0.98 \times 0.01 + 0.03 \times 0.99$$
$$= 0.0395$$

total Probability

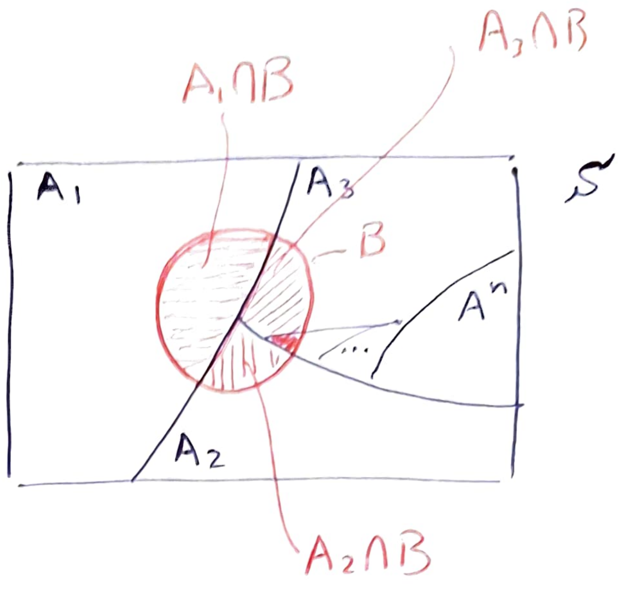
if

$S = A_1 \cup A_2 \cdots \cup A_n$

A_i are disjoint events

" $i = 1 \rightarrow n$ "

$\bigcup_{i=1}^{n} A_i = S$



$B = (A_1 \cap B) \cup (A_2 \cap B) \cup \cdots \cup (A_n \cap B)$

$$P(B) = \sum_{i=1}^{n} P(A_i B)$$

Confusion Matrix

(Binary Classifiers)

Predictions

		P	N
ground truth	✓	TP ✓	FN
	X	FP	TN ✓

nesbet el homa 3ndhom w elmodel tl3 eno 3ndhom 3la kol ely el model tl3 enhom 3ndhom

$$\text{Precision} = \frac{TP}{\text{all Pos.}} = \frac{TP}{TP+FP}$$

nesbet el homa 3ndhom w elmodel tl3 eno 3ndhom 3la kol ely bgd 3ndhom

$$\text{Recall} = \frac{TP}{\text{real} \checkmark} = \frac{TP}{TP+FN}$$

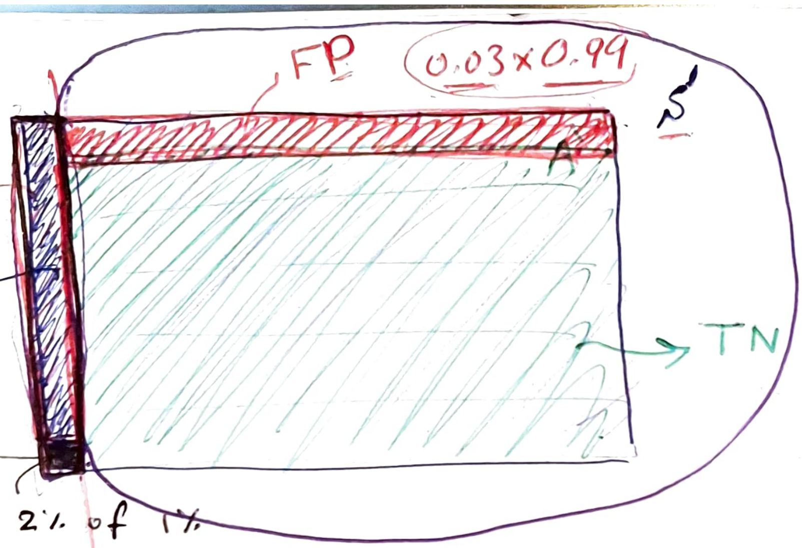
$$\frac{\text{Prec.} \times \text{Recall}}{\text{Prec.} + \text{Recall}} \times 2$$

$$\text{F1-score} = \text{harmonic mean of Pr. \& Recall} = \frac{2}{1/\text{Prec.} + 1/\text{recall}}$$

$$\text{Accuracy} = \frac{\text{Correct classifications}}{\text{all classifications}} = \frac{TP+TN}{TP+TN+FP+FN}$$

$P(A) = \underline{0.01}$

FP 0.03×0.99



A

TP

$.98 \times 0.01$

TN

FN

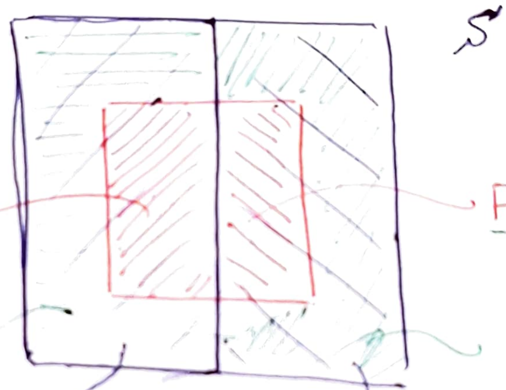
2% of 1%

0.01 0.99

0.98 *

$$\frac{\overbrace{0.98 \times 0.01}^{\text{True Pos.}}}{\underbrace{0.03 \times 0.99 + 0.98 \times 0.01}_{\text{tot. Pos.}}}$$

Positive test = اجمالی



0.98×0.01 = TP

FP = 0.03×0.99

FN

TN

A

A'

Naive Bayes Classifier

$$P(A/B) = \frac{P(A)\,P(B/A)}{P(B)}$$

Binary classification

$X \rightarrow$ male or female

$Y \rightarrow$ cat or not cat

~~PXX~~ $P(\text{male} / \text{features}) = ?$ a $a > b \Rightarrow$ male

$P(\text{female} / \text{features}) = ?$ b $b > a \Rightarrow$ female

$P(\text{cat} / \text{features}) = 0.35$

$P(\text{not cat} / \text{features}) = 0.65$ $\Rightarrow$ not cat

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training $\longrightarrow$

$$f(h/m) = \frac{1}{\sqrt{2\pi\sigma_{h_m}}}\, e^{-\frac{(h - \mu_{h_m})^2}{2\sigma_{h_m}^2}}$$

$f(w/m) = \checkmark$

$f(ss/m) = \checkmark$

$f(h/f) = \checkmark$

$f(w/f) = \checkmark$

$f(ss/f) = \checkmark$

~~male~~	height	weight	shoe size
male	170	x	–
male	175	x	–
male	165	x	–
male	173	x	
	$\mu = \checkmark$ $\sigma = \checkmark$	✓ ✓	✓ ✓
female	160	x	–
"	175	x	–
"	155	x	–
"	165	x	–
	✓ ✓	✓ ✓	✓ ✓

$$P(\text{male}/\overrightarrow{\text{features}}) = \frac{P(\text{male})\,P(\overrightarrow{\text{features}}/\text{male})}{P(\overrightarrow{\text{features}})} \quad P(B)$$

$$P(\text{female}/\overrightarrow{\text{features}}) = \frac{P(\text{female})\,P(\overrightarrow{\text{features}}/\text{female})}{P(\overrightarrow{\text{features}})} \quad P(B)$$

Naïve assumption $\longrightarrow$ assume independence!

$$P(\overrightarrow{\text{features}}/\overset{\text{class}}{\cancel{\text{male}}})$$

$$= P(\text{weight, height, shoesize}/\overset{\text{class}}{\cancel{\text{male}}})$$

joint probability

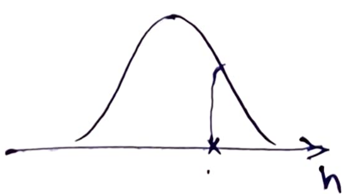
features are dependent (not indep.)

assuming independence

$$p(\overrightarrow{\text{features}}/\text{male}) = p(\text{height}/\text{male}) \cdot p(w/\text{male}) \cdot p(\text{s.s.}/m)$$

$$p(\overrightarrow{f\text{——}}/\text{female}) =$$

$$\frac{1}{\sqrt{2\pi}\,\sigma_{h/m}}\, e^{-\frac{(\overset{=6}{h} - \mu_{h/m})^2}{2\sigma_{h/m}^2}}$$



h Random variable

h = 6