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assessment 1

Dot Product For 2 vectors (scalar) - Inner Product
Cross " " " " (vector)
$a \times b = |a|\,|b| \sin(\theta)\, k$ → unit vector it's magnitude is 1
$\|v\|\,\|w\| \cos\theta$, $\cos\theta = \frac{v \cdot w}{\|v\| \cdot \|w\|}$

$v \cdot w = w \cdot v$
The length of vector is square root of $v \cdot v$
Length $= \|v\| = \sqrt{v \cdot v} = (v_1^2 + v_2^2 + \dots v_n^2)^{\frac{1}{2}}$

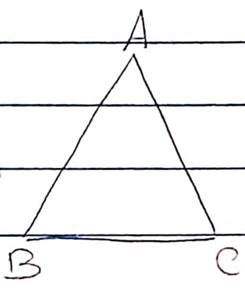
Questions

1- The angle between 2 vectors:

$$\cos(\theta) = \frac{v \cdot w}{\|v\| \cdot \|w\|} = \frac{(1 \times 1) + (0 \times \sqrt{3}) + (-1 \times 3) + (3 \times -3)}{\sqrt[2]{1^2 + 0^2 + (-1)^2 + (3)^2} \times \sqrt{(1)^2 + (\sqrt{3})^2 + (3)^2 + (-3)^2}}$$
$= -1.91$

$\cos(\theta) = -1.91$ $\boxed{=135}$ ← θ

2- angle at vertex



We have a tringle ABC,
The question wants to know if any 2 vectors making a right angle
(90) ← right angel means it is $\boxed{90^\circ}$

and we will be able to know that if we get the dot product of
any 2 vectors and was $= \boxed{0}$ ← this means they are perpendicular
متعامدين يعني الزاوية 90.

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$A=(1,-2,0)$, $B=(2,1,-2)$, $C=(6,-1,-3)$

We need first to calculate AB , AC, BC

$AB = B - A = \sqrt{(2-1)^2+(1-(-2))^2+(-2-0)^2} = \sqrt{14}$

Which is $(1,3,-2)$

$AC = C - A = \sqrt{(6-1)^2+(-1-(-2))^2+(-3-0)^2} = \sqrt{35}$

Which is $(5,1,-3)$

$BC = C - B = \sqrt{(6-2)^2+(-1-1)^2+(-3-(-2))^2} = \sqrt{21}$

Which is $(4,-2,-1)$

Now, let's calculate dot product

$\boxed{AB \cdot AC} = (1,3,-2)\cdot(5,1,-3) = (1\times5)+(3\times1)+(-2\times-3) =$

$= 5+3+6 = 14$

their dot product not $\boxed{0}$, so they not perpendicular

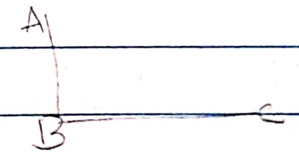
So there is no right angle between them.

$AB \cdot BC = (1,3,-2)\cdot(4,-2,-1) = (1\times4)+(3\times-2)+(-2\times-1)$

$4 - 6 + 2 = 0$

their dot product is $= \boxed{0}$, so they are perpendicular

So there is a right angle between them (which is $\boxed{B}$)



Since we are in triangle, if B is right angle (90°)

it's certain that A, C won't be a right angle

Just for sure

$AC \cdot BC = (5,1,-3)\cdot(4,-2,-1) = 20-2-3 = 15$

So the angle will be at vertex $\boxed{B}$

3- a. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 20 \\ 30 & 40 \\ 50 & 60 \end{bmatrix} = \begin{bmatrix} 11 & 22 \\ 33 & 44 \\ 55 & 66 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = (1\times3)+(2\times4) = 11$

c. $\underset{1\times2}{\begin{bmatrix} 1 & 2 \end{bmatrix}} \underset{2\times2}{\begin{bmatrix} 3 & 0 \\ 4 & 1 \end{bmatrix}} = \begin{bmatrix} (1\times3)+(2\times4) & (1\times0)+(2\times1) \end{bmatrix}$

$= \begin{bmatrix} 11 & 2 \end{bmatrix}$

d. $\underset{2\times2}{\begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}} \underset{2\times2}{\begin{bmatrix} 3 & 0 \\ 4 & 1 \end{bmatrix}} = \begin{bmatrix} (1\times3+2\times4) & (1\times0+2\times1) \\ (10\times3+20\times4) & (10\times0+20\times1) \end{bmatrix}$

$= \begin{bmatrix} 11 & 2 \\ 110 & 20 \end{bmatrix}$

e. $\begin{bmatrix} 1 & 2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ Invalid Operation

$1\times3 \neq 2\times1$

f. $\underset{2\times1}{\begin{bmatrix} 3 \\ 4 \end{bmatrix}} \underset{1\times3}{\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}} \begin{bmatrix} 3\times1 & 3\times2 & 3\times7 \\ 4\times1 & 4\times2 & 4\times7 \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 & 21 \\ 4 & 8 & 28 \end{bmatrix}$

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9. $\begin{bmatrix} 0 & 1 & 2 \\ 10 & -10 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$2\times3 \quad 3\times3$

$$\begin{bmatrix} (0\times1 + 1\times4 + 2\times7) & (0\times2 + 1\times5 + 2\times8) & (0\times3 + 1\times6 + 2\times9) \\ (10\times1 - 10\times4 + 5\times7) & (10\times2 - 10\times5 + 5\times8) & (10\times3 - 10\times6 + 5\times9) \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 21 & 24 \\ 5 & 10 & 15 \end{bmatrix}$$

4. Now! We have 2 Containers Contains Some Water With different temperature

When We mixed 240 From First Container and 260 From the Second Container We got (52°)

and 180 From First Container, 120 From the Second We got (46)°

our goal is to Find the initial temprature and then Put the equation in a matrix-vector Form

$\boxed{1}$ $240(T_1 - 52) + 260(T_2 - 52) = 0$ First equation

$\boxed{2}$ $180(T_1 - 46) + 120(T_2 - 46) = 0$ Second "

$\boxed{1}$ $240T_1 - 12480 + 260T_2 - 13520 = 0$ Solve

$240T_1 + 260T_2 = 26000$

$\boxed{2}$ $180T_1 - 8280 + 120T_2 - 5520 = 0$

$180T_1 + 120T_2 = 13800$

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The result after solving the equation

$$240T_1 + 260T_2 = 26000$$

$$180T_1 + 120T_2 = 13800$$

in matrix-vector form

$$\begin{bmatrix} 240 & 260 \\ 180 & 120 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 26000 \\ 13800 \end{bmatrix}$$

5. $v_1(9,6) \xrightarrow[\text{Combination}]{\text{linear}} (1,2), (1,-4)$

To express the vector (v_1) is a linear Combination of $(1,2)$, $(1,-4)$, we need to solve them as an equation and find (x, y)

$$(1,2)x + (1,-4)y = (9,6)$$

① $1x + y = 9$ (by multiplying $(x4)$)

② $2x - 4y = 6$

$4x + 4y = 36$ | $6x = 42$, $x = \boxed{7}$

$2x - 4y = 6$ | $(4\times7) + 4y = 36$, $y = \boxed{2}$

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So, v_1 $\boxed{(9,6)}$ as a linear Combination of (1,2), (1,-4)

Will be

$x(1,2) + y(1,-4)$

$7(1,2) + 2(1,-4)$

$(7,14) + (2,-8) = \boxed{(9,6)}$

6- To Know Wether the vector x_1 lies in the span between (x_2, x_3)
We should Convert them as an equation (With rref)
If We Can find an exact Solution for them, so x_1 is lies in the span between (x_2, x_3)

ده ① لازم
كل اللي بعده
يكونوا أصفار

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 3 \end{array}\right]$$

augmented matrix

$r_2 = 2r_1 - r_2$

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -1 & -3 \\ 3 & 1 & 3 \end{array}\right]$$

$r_3 = 3r_1 - r_3$

عاوزينه ده
يصفر

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 5 & 3 \end{array}\right]$$

عاوزينه
ده
يبقى واحد

$r_2 = r_2 \div -1$

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عاوزينه دي تبقى صفر

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 3 \\ 0 & 5 & 3 \end{array}\right] \qquad r_1 = 2r_2 - r_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 5 & 3 \end{array}\right] \qquad r_3 = 5r_2 - r_3$$

عاوزينه دي تبقى صفر

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 12 \end{array}\right] \leftarrow \text{IndePendent Variable}$$

So $x = 4$, $y = 3$

Since we Found an exact solution, So x_1 lies in the span of x_2, x_3

7. $V_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$, $W = \boxed{\begin{bmatrix} -8 \\ 12 \end{bmatrix}}$

linear Combination of V_1

لو ضربنا $V_1 \times 4$ هنطلع W | مش محتاجين معادلة

$$\begin{bmatrix} -2 \\ 3 \end{bmatrix} \cdot (4) = \boxed{\begin{bmatrix} -8 \\ 12 \end{bmatrix}}$$

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$$V_1 = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \quad W = \begin{bmatrix} 4 \\ -6 \\ 10 \end{bmatrix}$$

هنحلها ٢٢ بحيث لو طلعوا exact solution يبقى W is linear combination of V_1, V_2

عاوزينها ١ دي

$$\left[\begin{array}{cc|c} 2 & 0 & 4 \\ 0 & 2 & -6 \\ 5 & 0 & 10 \end{array}\right] \qquad r_1 = r_1 \div 2$$

عاوزينها ١ دي

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 2 & -6 \\ 5 & 0 & 10 \end{array}\right] \qquad r_2 = r_2 \div 2$$

عاوزينها صفر

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 5 & 0 & 10 \end{array}\right] \qquad r_3 = 5r_1 - r_3$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right]$$

- So we can express W as a linear combination of V_1 and V_2

$a = 2$, $b = -3$

$$\boxed{\text{So, } W = 2V_1 - 3V_2}$$

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8.

Span القاعدة

لو انا عندى vector فى dimension (R_1, R_2, R_3)

الـ span هيكون كل الـ vectors اللى بيمشى على الـ line (بتاع الـ vector ده)

لو انا عندى 2 vectors ← 3 R_2

هيبقى عندى حالتين

① لو هما independent | ② لو هما dependent

فالـ linear combination منهم يقدر يـ span الفضاء كله R_2 | فالـ span هيكون كل الـ vectors اللى بيمشى على (line اللى هو الـ linear combination بتاعهم.

ده فيكتور ① فى R_2 يبقى الـ span هيكون line ← $1-V_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

هنجيبه ازاى ؟ ← line

slope $= \frac{6}{3}$

so $y = 2x$

$y = 2x$

الإجابة هتكون $\boxed{\text{e) the span is line } y = 2x}$

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2- $V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

they are independent, so the span will be

d) The span is all of R^2

3- $V_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) The span is the single point (0, 0)

4. $V_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

they are independent, so

f) The span is all of R^2

5- $V_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

The are dependent, so the span will be a line, slope $= \frac{1}{2}$

So

a) The span is the line $y = 1/2 X$

6- $V_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, V_2 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ The are dependent, so the span will be a line, slope $= \frac{3}{1}$ or $\frac{6}{2} = 3$, $y = 3X$

So c) the span is the line $y = 3X$

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9. To Find the Span We Will need to Solve them as a linear equation, So We Can Solve it by rref

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & 3 & 5 & 1 & 7 \\ 2 & 1 & -1 & 4 & 0 \\ -1 & 1 & 5 & -1 & 2 \end{array}\right]$$

عاوزين دي صفر

$R_2 = R_2 - 2R_1$

(1)

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 2 & 1 & -1 & 4 & 0 \\ -1 & 1 & 5 & -1 & 2 \end{array}\right]$$

عاوزين دي صفر

$R_3 = R_3 - 2R_1$

(2)

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & -1 & -3 & 2 & -4 \\ -1 & 1 & 5 & -1 & 2 \end{array}\right]$$

عاوزين دي صفر

$R_4 = R_4 + R_1$

(3)

عاوزين دي صفر

دي تمام

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & -1 & -3 & 2 & -4 \\ 0 & 2 & 6 & 0 & 4 \end{array}\right]$$

$R_1 = R_1 - R_2$

(4)

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & -1 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & -1 & -3 & 2 & -4 \\ 0 & 2 & 6 & 0 & 4 \end{array}\right]$$

عاوزين دي صفر

(5) $R_3 = R_3 + R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & -1 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 2 & 6 & 0 & 4 \end{array}\right]$$

عاوزين دي صفر

$R_4 = R_4 - 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & -1 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 2 & -2 \end{array}\right]$$

عاوزين دي واحد

ليه مش هينفع

(لو جربنا كل حاجة مش هينفع)

- الحل في الموضوع ده إننا نروح للـ Row اللي بعده (R_4) ونبدلهم هنا shift للـ column

column 3 = 0 ... عشان

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & -1 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

عاوزين ده واحد

$R_3 = R_3 / 2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & -1 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

عاوزين دي صفر

Date: / / object:

$R_4 = R_4 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & -1 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

عاوزاسيب دي صفر

$R_1 = R_1 - 2R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

عاوزاسيب دي صفر

$R_2 = R_3 + R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Final rref

Since he wants the basis vectors (they are the independents ones) which is $\rightarrow$ $\boxed{v_1 , v_2 , v_4}$

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10.

$$A = \begin{bmatrix} 17 & -11 \\ 6 & -3 \end{bmatrix}$$

$$\det(A) = (17 \times -3) - (-11 \times 6)$$
$$= \boxed{15}$$

$$A = \begin{bmatrix} \overset{+}{1} & \overset{-}{1} & \overset{+}{2} \\ 2 & 3 & 1 \\ 3 & 4 & -5 \end{bmatrix}$$

$$\det(A) = 1\Big(\underset{}{\overset{-15}{(3 \times -5)}} - \overset{4}{(1 \times 4)}\Big) - \Big(1\big(\overset{-10}{(2 \times -5)} - \overset{3}{(1 \times 3)}\big)\Big)$$
$$+2\Big(\overset{8}{(2 \times 4)} - \overset{9}{(3 \times 3)}\Big)$$

$$-19 + 13 - 2 = \boxed{-8}$$

11-

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix}$$

$$\det(A) = (-3 \times 3) - (-2 \times 3)$$
$$= -9 + 6$$
$$= \boxed{-3}$$

$$\mathrm{Inv}(A) = \frac{1}{\det(A)} \times \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix}$$

$$\mathrm{Inv}(A) = \frac{1}{-3} \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & \frac{2}{-3} \\ 1 & 1 \end{bmatrix}$$

$$A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Let's get the det(A) first

$$\det(A) = 1(1\times1 - 1\times1) - 0(0\times1 - 1\times1) + 1(0\times1 - 1\times1)$$

$$0 \quad - 0 \quad - 1$$

$$= \boxed{-1}$$

then (calculate minors)

det

$$a_{11} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = \boxed{0}$$

$$a_{12} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = \boxed{-1}$$

$$a_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = \boxed{-1}$$

$$a_{21} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = \boxed{-1}$$

$$a_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = \boxed{0}$$

$$a_{23} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = \boxed{1}$$

$$a_{31} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = \boxed{-1}$$

$$a_{32} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = \boxed{0}$$

$$a_{33} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = \boxed{1}$$

Cofactor matrix $= \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

applying it to the minors

$$\text{adj}(A) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Inv}(A) = \frac{1}{\det(A)} \times \text{Adj}(A)$$

$$= \frac{1}{-1} \times \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Inv}(A) = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

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12-

$$M = \begin{bmatrix} 3 & 1 & 0 & -1 \\ 2 & 4 & 3 & 2 \end{bmatrix}$$

Rank - number of Independent row ($\underline{2}$)

$$M = \begin{bmatrix} 5 & 2 & 3 \\ 7 & 2 & 2 \\ 9 & -1 & 1 \end{bmatrix}$$

Rank - $\boxed{3}$

all the rows are Independent.

13-

a) $X_1 + 4X_2 + 3X_3 - X_4 = 5$

$X_1 - X_2 + X_3 + 2X_4 = 6$

$4X_1 + X_2 + 6X_3 + 5X_4 = 9$

rref

$$\left[\begin{array}{cccc|c} 1 & 4 & 3 & -1 & 5 \\ 1 & -1 & 1 & 2 & 6 \\ 4 & 1 & 6 & 5 & 9 \end{array}\right]$$

$R_2 = R_2 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 4 & 3 & -1 & 5 \\ 0 & -5 & -2 & 3 & 1 \\ 4 & 1 & 6 & 5 & 9 \end{array}\right]$$

$R_3 = R_3 - 4R_1$

$$\left[\begin{array}{cccc|c} 1 & 4 & 3 & -1 & 5 \\ 0 & -5 & -2 & 3 & 1 \\ 0 & -15 & -6 & 9 & -11 \end{array}\right]$$

$R_2 = \frac{R_2}{-5}$

$$\left[\begin{array}{cccc|c} 1 & 4 & 3 & -1 & 5 \\ 0 & 1 & 2/5 & -3/5 & -1/5 \\ 0 & -15 & -6 & 9 & -11 \end{array}\right]$$

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$R_1 = R_1 - 4R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 7/5 & 7/5 & 29/5 \\ 0 & 1 & 2/5 & -3/5 & -1/5 \\ 0 & 15 & 6 & 9 & -11 \end{array}\right]$$

$R_3 = R_3 + 15R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 7/5 & 7/5 & 29/5 \\ 0 & 1 & 2/5 & -3/5 & -1/5 \\ 0 & 0 & 0 & 0 & -14 \end{array}\right]$$

(no Solution)

لان مستحيل حاجه عليه اصفار
فـ ٥ تطلع ١٤-

2-

$$x_1 - 2x_2 + x_3 - x_4 = 3$$
$$2x_1 - 4x_2 + x_3 + x_4 = 2$$
$$x_1 - 2x_2 - 2x_3 + 3x_4 = 1$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 2 & -4 & 1 & 1 & 2 \\ 1 & -2 & -2 & 3 & 1 \end{array}\right]$$

$R_2 = R_2 - 2R_1$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 0 & 0 & -1 & 3 & -4 \\ 1 & -2 & -2 & 3 & 1 \end{array}\right]$$

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$R_3 = R_3 - R_1$

عاوزين دي بواحد

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 0 & 0 & -1 & 3 & -4 \\ 0 & 0 & -3 & 4 & -2 \end{array}\right]$$

$R_2 = -R_2$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 0 & 0 & 1 & -3 & 4 \\ 0 & 0 & -3 & 4 & -2 \end{array}\right]$$

$R_1 = R_1 - R_2$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 2 & -1 \\ 0 & 0 & 1 & -3 & 4 \\ 0 & 0 & -3 & 4 & -2 \end{array}\right]$$

$R_3 = R_3 + 3R_2$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 2 & -1 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & -5 & 10 \end{array}\right]$$

$R_3 = \frac{R_3}{-5}$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 2 & -1 \\ 0 & 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$R_1 = R_1 - 2R_3$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 0 & 3 \\ 0 & 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$R_2 = R_2 + 3R_3$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$$\Downarrow$$

$x_1 - 2x_2 = 3$

$x_3 = -2$

$x_4 = -2$

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3- $x_1 + 2x_2 + 3x_3 = 1$
$2x_1 - x_2 + x_3 = 2$
$3x_1 + x_2 + x_3 = 4$
$5x_2 + 2x_3 = 1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & -1 & 1 & 2 \\ 3 & 1 & 1 & 4 \\ 0 & 5 & 2 & 1 \end{array}\right]$$

$R_2 = R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -5 & -5 & 0 \\ 3 & 1 & 1 & 4 \\ 0 & 5 & 2 & 1 \end{array}\right]$$

$R_3 = R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -5 & -5 & 0 \\ 0 & -5 & -8 & 1 \\ 0 & 5 & 2 & 1 \end{array}\right]$$

$R_2 = \frac{R_2}{-5}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -5 & -8 & 1 \\ 0 & 5 & 2 & 1 \end{array}\right]$$

$R_3 = R_3 + R_4$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -6 & 2 \\ 0 & 5 & 2 & 1 \end{array}\right]$$

$R_4 = R_4 - 5R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

$R_1 = R_1 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

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$R_3 = \frac{R_3}{-6}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1/3 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

$R_1 = R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1/3 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

$R_2 = R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & -1/3 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

$X_1 = 4/3$

$X_2 = 1/3$

$X_3 = -1/3$

$R_4 = R_4 + 3R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & -1/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$