LAFDS Sessions 1, 2 & 3 Homework

Full Name: _____

Group No.: ____

Lecturer Name: _____

Submission date: _/_/__ **Grade:**

Please write down all the steps not the final answer only

Questions:

1. (1 point) The angle between the vectors (1, 0, −1, 3) and (1, $\sqrt{3}$, 3, −3) in $\mathbb{R}^4$ is aπ, where a = ____.

2. (1 point) Which of the angles (if any) of triangle ABC, with A = (1, −2, 0), B = (2, 1, −2), and C = (6, −1, −3), is a right angle? Answer: the angle at vertex _____.

3. (7 points) Practice with numbers (if there is no answer, say so)

   a. $\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix} + \begin{bmatrix}10 & 20\\ 30 & 40\\ 50 & 60\end{bmatrix}$

   b. $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}3\\ 4\end{bmatrix}$

   c. $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}3 & 0\\ 4 & 1\end{bmatrix}$

   d. $\begin{bmatrix}1 & 2\\ 10 & 20\end{bmatrix}\begin{bmatrix}3 & 0\\ 4 & 1\end{bmatrix}$

   e. $\begin{bmatrix}1 & 2 & 7\end{bmatrix}\begin{bmatrix}3\\ 4\end{bmatrix}$

   f. $\begin{bmatrix}3\\ 4\end{bmatrix}\begin{bmatrix}1 & 2 & 7\end{bmatrix}$

   g. $\begin{bmatrix}0 & 1 & 2\\ 10 & -10 & 5\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}$

4. (4 points) Two containers contain a water of different temperatures. If we mix 240 g of water from the first container with 260 g of water from the second container, the resulting water temperature will be 52°C. If we mix 180 g of water from the first container with 120 g of water from the second container,

the resulting water temperature will be 46°C. What is the temperature of water in the containers? Write this in a matrix-vector form.

5. (3 point) Express the vector (9; 6) as a linear combination of the vectors (1; 2) and (1;-4).

6. (3 point) Determine whether the vector x1 = (2; 1; 3) lies in the span of the vectors x2 = (1; 2; 3) and x3 = (2; 3; 1).

7. (6 points) Express the vector "w" as a linear combination of the given vectors vi

- v1 = $\begin{bmatrix}-2\\3\end{bmatrix}$, w = $\begin{bmatrix}-8\\12\end{bmatrix}$
- v1 = $\begin{bmatrix}2\\0\\5\end{bmatrix}$, v2 = $\begin{bmatrix}0\\2\\0\end{bmatrix}$, w = $\begin{bmatrix}4\\-6\\10\end{bmatrix}$

8. (6 points) Match each set of vectors with their corresponding span

1. v1 = $\begin{bmatrix}3\\6\end{bmatrix}$	a) The span is the line y = 1/2 x
2. v1 = $\begin{bmatrix}1\\0\end{bmatrix}$, v2 = $\begin{bmatrix}0\\1\end{bmatrix}$	b) The span is the single point (0;0)
3. v1 = $\begin{bmatrix}0\\0\end{bmatrix}$	c) The span is the line y = 3x.
4. v1 = $\begin{bmatrix}2\\1\end{bmatrix}$, v2 = $\begin{bmatrix}-1\\5\end{bmatrix}$	d) The span is all of $\mathbb{R}^2$.
5. v1 = $\begin{bmatrix}2\\1\end{bmatrix}$, v2 = $\begin{bmatrix}0\\0\end{bmatrix}$	e) The span is the line y = 2x.
6. v1 = $\begin{bmatrix}1\\3\end{bmatrix}$, v2 = $\begin{bmatrix}2\\6\end{bmatrix}$	f) The span is all of $\mathbb{R}^2$

9. (2 points) Let S = {V1,V2,V3,V4,V5} where,

V1 = $\begin{bmatrix}1\\2\\2\\-1\end{bmatrix}$, V2 = $\begin{bmatrix}1\\3\\1\\1\end{bmatrix}$, V3 = $\begin{bmatrix}1\\5\\-1\\5\end{bmatrix}$, V4 = $\begin{bmatrix}1\\1\\4\\-1\end{bmatrix}$, V5 = $\begin{bmatrix}2\\7\\0\\2\end{bmatrix}$

Find a basis for the span Span(S).

10. (5 points) Find the determinant of the matrix M

$$M = \begin{pmatrix} 17 & -11 \\ 6 & -3 \end{pmatrix} \qquad M = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 4 & -5 \end{pmatrix}$$

11. (5 points) Find the inverse of the matrix A

$$A = \begin{pmatrix} -3 & -2 \\ 3 & 3 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

12. (5 points) Find the rank of the matrix M

$$M = \begin{pmatrix} 3 & 1 & 0 & -1 \\ 2 & 4 & 3 & 2 \end{pmatrix} \qquad M = \begin{pmatrix} 5 & 2 & 3 \\ 7 & 2 & 2 \\ 9 & -1 & 1 \end{pmatrix}$$

13. (15 points) Find the solution set of the following systems of linear equations.

a)
$$\begin{aligned} x_1 + 4x_2 + 3x_3 - x_4 &= 5 \\ x_1 - x_2 + x_3 + 2x_4 &= 6 \\ 4x_1 + x_2 + 6x_3 + 5x_4 &= 9 \end{aligned}$$

b)
$$\begin{aligned} x_1 - 2x_2 + x_3 - x_4 &= 3 \\ 2x_1 - 4x_2 + x_3 + x_4 &= 2 \\ x_1 - 2x_2 - 2x_3 + 3x_4 &= 1 \end{aligned}$$

c)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_1 - x_2 + x_3 &= 2 \\ 3x_1 + x_2 + x_3 &= 4 \\ 5x_2 + 2x_3 &= 1 \end{aligned}$$

Practice with Code (Optional):

1. Write a NumPy code line(s) to get and print your numpy library version

2. Write a NumPy code line(s) to get help on the “add” function.

3. Write a NumPy code line(s) to test whether any of the elements of an input array is non-zero
4. Write a NumPy code line(s) to compute the x and y coordinates for points on a sine curve and plot the points using matplotlib.
5. Write a NumPy code line(s) to add elements in a matrix. If an element in the matrix is 0, we will not add the element below this element (in red)

$$M = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 3 & 0 & 3 \\ 1 & 0 & 4 & 4 \end{bmatrix}$$

6. Write a NumPy code line(s) to extract all numbers which are less and greater than a specified integer in an input array
7. Write a NumPy code line(s) to find the missing (hint: undefined) data in an input array

8. Calculating the inverse of a 2x2 matrix without using numpy

   - Code a function to calculate the determinant of 2x2 matrix
   - Code a function that interchange the diagonal elements of a 2x2 matrix and inverse the sign of the off diagonal elements
   - Code a function to compute the inverse of 2x2 matrix based on the two previous functions if it exists

9. Coding a Python code to inverse a 3x3 matrix in order to solve a linear system (no numpy.linalg.inv allowed) with 3 constraints and 3 variables:
   - Coding a function that checks if a 3x3 matrix is invertible
   - Coding a function that generates the transpose of a 3x3 matrix
   - Coding a function that generates the matrix of minors of a 3x3 matrix
   - Coding a function that generates the matrix of cofactors of a 3x3 matrix
   - Coding a function that generates the inverse of a 3x3 matrix if it exists

Reading homework:

- Numpy vs. Scipy: https://bit.ly/3vURVkl
- Numpy documentation: https://numpy.org/doc/stable/user/quickstart.html
- "Python for Data Analysis by Wes McKinney" Chapter 4: https://www.oreilly.com/library/view/python-for-data/9781449323592/ch04.html
- Scalars and vectors: https://www.mathsisfun.com/algebra/scalar-vector-matrix.html
- Vectors and matrices: https://www.statlect.com/matrix-algebra/vectors-and-matrices
- Dot product: https://www.mathsisfun.com/algebra/vectors-dot-product.html
- Operations on matrices: https://medium.com/linear-algebra/part-2-operations-on-matrices-3caab542aebd

- Linear Transformation and matrices: http://amsi.org.au/ESA_Senior_Years/SeniorTopic8/8a/8a_2content_3.html
- Linear combination, span, linear independence: https://medium.com/linear-algebra-basics/vector-span-f90b989d712d
- **Determinants:** https://www.youtube.com/watch?v=Ip3X9LOh2dk (video 3blue1brown)
  - https://medium.com/sho-jp/linear-algebra-101-part-5-determinants-b54f990782cc
  - https://www.mathsisfun.com/algebra/matrix-determinant.html
  - https://medium.com/linear-algebra/part-20-determinants-e4b2fbcce883
  - https://medium.com/linear-algebra/part-21-properties-of-determinants-1af8a231fd2b?source=---------0----------
- **Inverse of a matrix**
  - https://www.mathsisfun.com/algebra/matrix-inverse.html
  - https://www.mathsisfun.com/algebra/matrix-inverse-minors-cofactors-adjugate.html