Resolution Example and Exercises

Solutions to Selected Problems

Example:

Consider the following axioms:

1. All hounds howl at night.
2. Anyone who has any cats will not have any mice.
3. Light sleepers do not have anything which howls at night.
4. John has either a cat or a hound.
5. (Conclusion) If John is a light sleeper, then John does not have any mice.

The conclusion can be proved using Resolution as shown below. The first step is to write each axiom as a well-formed formula in first-order predicate calculus. The clauses written for the above axioms are shown below, using LS(x) for `light sleeper'.

1. $\forall x\ (HOUND(x) \rightarrow HOWL(x))$
2. $\forall x\ \forall y\ (HAVE\ (x,y) \wedge CAT\ (y) \rightarrow \neg \exists z\ (HAVE(x,z) \wedge MOUSE\ (z)))$
3. $\forall x\ (LS(x) \rightarrow \neg \exists y\ (HAVE\ (x,y) \wedge HOWL(y)))$
4. $\exists x\ (HAVE\ (John,x) \wedge (CAT(x) \vee HOUND(x)))$
5. $LS(John) \rightarrow \neg \exists z\ (HAVE(John,z) \wedge MOUSE(z))$

The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

1. $\forall x\ (HOUND(x) \rightarrow HOWL(x))$

   $\neg\ HOUND(x) \vee HOWL(x)$
2. $\forall x\ \forall y\ (HAVE\ (x,y) \wedge CAT\ (y)$ &rarr $\neg \exists z\ (HAVE(x,z) \wedge MOUSE\ (z)))$

   $\forall x\ \forall y\ (HAVE\ (x,y) \wedge CAT\ (y)$ &rarr $\forall z\ \neg\ (HAVE(x,z) \wedge MOUSE\ (z)))$

   $\forall x\ \forall y\ \forall z\ (\neg\ (HAVE\ (x,y) \wedge CAT\ (y)) \vee \neg\ (HAVE(x,z) \wedge MOUSE\ (z)))$

   $\neg\ HAVE(x,y) \vee \neg\ CAT(y) \vee \neg\ HAVE(x,z) \vee \neg\ MOUSE(z)$
3. $\forall x\ (LS(x)$ &rarr $\neg \exists y\ (HAVE\ (x,y) \wedge HOWL(y)))$

   $\forall x\ (LS(x) \rightarrow \forall y\ \neg\ (HAVE\ (x,y) \wedge HOWL(y)))$

   $\forall x\ \forall y\ (LS(x) \rightarrow \neg\ HAVE(x,y) \vee \neg\ HOWL(y))$

   $\forall x\ \forall y\ (\neg\ LS(x) \vee \neg\ HAVE(x,y) \vee \neg\ HOWL(y))$

   $\neg\ LS(x) \vee \neg\ HAVE(x,y) \vee \neg\ HOWL(y)$
4. $\exists x\ (HAVE\ (John,x) \wedge (CAT(x) \vee HOUND(x)))$

   $HAVE(John,a) \wedge (CAT(a) \vee HOUND(a))$
5. $\neg\ [LS(John) \rightarrow \neg \exists z\ (HAVE(John,z) \wedge MOUSE(z))]$ *(negated conclusion)*

   $\neg\ [\neg\ LS\ (John) \vee \neg \exists z\ (HAVE\ (John,\ z) \wedge MOUSE(z))]$

   $LS(John) \wedge \exists z\ (HAVE(John,\ z) \wedge MOUSE(z))$

   $LS(John) \wedge HAVE(John,b) \wedge MOUSE(b)$

The set of CNF clauses for this problem is thus as follows:

1. $\neg\ HOUND(x) \vee HOWL(x)$
2. $\neg\ HAVE(x,y) \vee \neg\ CAT(y) \vee \neg\ HAVE(x,z) \vee \neg\ MOUSE(z)$
3. $\neg\ LS(x) \vee \neg\ HAVE(x,y) \vee \neg\ HOWL(y)$
4.
   1. $HAVE(John,a)$
   2. $CAT(a) \vee HOUND(a)$
5.
   1. $LS(John)$
   2. $HAVE(John,b)$
   3. $MOUSE(b)$

Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

[1.,4.(b):]	6.	$CAT(a) \vee HOWL(a)$
[2,5.(c):]	7.	$\neg\ HAVE(x,y) \vee \neg\ CAT(y) \vee \neg\ HAVE(x,b)$
[7,5.(b):]	8.	$\neg\ HAVE(John,y) \vee \neg\ CAT(y)$
[6,8:]	9.	$\neg\ HAVE(John,a) \vee HOWL(a)$
[4.(a),9:]	10.	$HOWL(a)$
[3,10:]	11.	$\neg\ LS(x) \vee \neg\ HAVE(x,a)$
[4.(a),11:]	12.	$\neg\ LS(John)$
[5.(a),12:]	13.	$\square$

Exercises:

1. Unify (if possible) the following pairs of predicates and give the resulting substitutions. *b* is a constant.

a. $P(x, f(x), z)$
   $\neg\ P(g(y),f(g(b)),y)$

b. $P(x, f(x))$
   $\neg\ P(f(y), y)$

c. $P(x, f(z))$
   $\neg\ P(f(y), y)$

2. Consider the following axioms: