

MACHINE LEARNING SPRING 2022

Homework #1

Q: Hand code a neural network to compute &

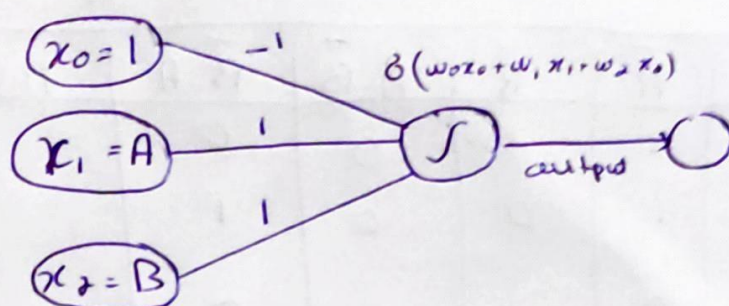
(A): AND

(B): XOR

(A) AND (using sigmoid)

The truth table of AND operator is as follows

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1



The output is considered as 1 if it is greater than 0.5.

Testing

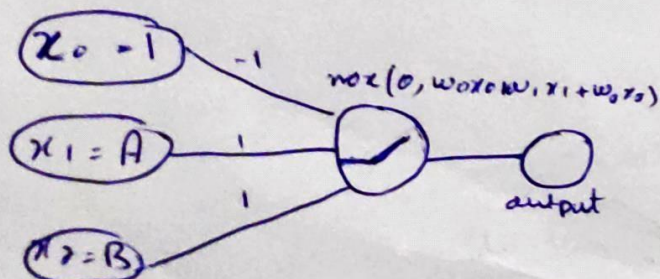
$$\sigma(-1 + 0 + 0) = 0.269 \leq 0.5 \Rightarrow \text{output} = 0 \quad (\text{For } A=0, B=0)$$

$$\sigma(-1 + 1 + 0) = 0 \leq 0.5 \Rightarrow \text{output} = 0 \quad (\text{For } A=1, B=0)$$

$$\sigma(-1 + 0 + 1) = 0 \leq 0.5 \Rightarrow \text{output} = 0 \quad (\text{For } A=0, B=1)$$

$$\sigma(-1 + 1 + 1) = 0.73 > 0.5 \Rightarrow \text{output} = 1 \quad (\text{For } A=1, B=1)$$

(A) AND (using relu)



The output is considered as 1 if it is greater than 0

Testing

$$① A=0, B=0$$

$$\max(0, -1 + 0 + 0) = 0 \Rightarrow \text{output} = 0$$

$$② A=1, B=0$$

$$\max(0, -1 + 1 + 0) = 0 \Rightarrow \text{output} = 0$$

$$③ A=0, B=1$$

$$\max(0, -1 + 0 + 1) = 0 \Rightarrow \text{output} = 0$$

$$④ A=1, B=1$$

$$\max(0, -1 + 1 + 1) = 1 \Rightarrow \text{output} = 1$$

(B) XOR

The truth table of XOR

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

$$\text{Now } A \oplus B = A \cdot \bar{B} + \bar{A} \cdot B \quad \text{--- (1)}$$

$$A \oplus B = (A+B) \cdot (\bar{A} + \bar{B}) \quad \text{--- (2)}$$

Verifying these two equations using truth table

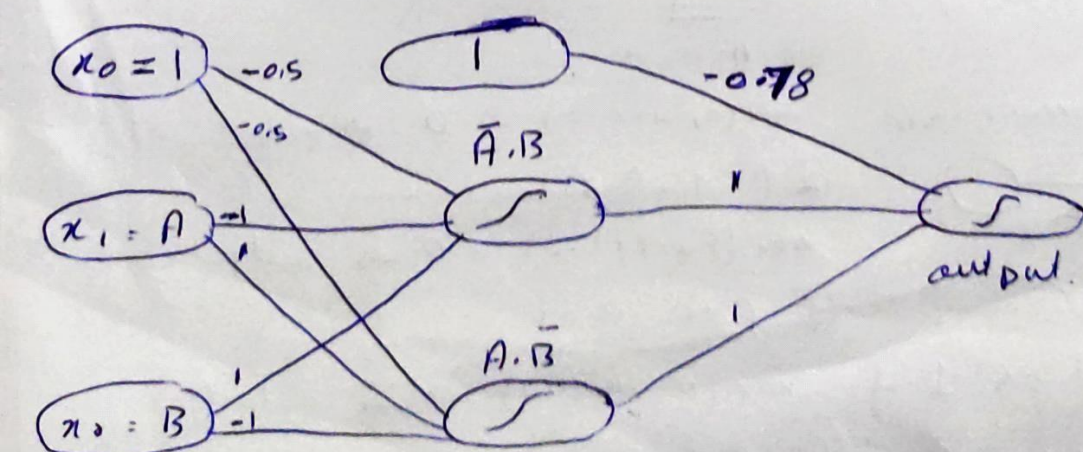
A	B	\bar{A}	\bar{B}	$A \cdot \bar{B}$	$\bar{A} \cdot B$	$A \cdot \bar{B} + \bar{A} \cdot B$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

verified

A	B	\bar{A}	\bar{B}	$A+B$	$\bar{A} + \bar{B}$	$(A+B) \cdot (\bar{A} + \bar{B})$
0	0	1	1	0	1	0
0	1	1	0	1	1	1
1	0	0	1	1	1	1
1	1	0	0	1	0	0

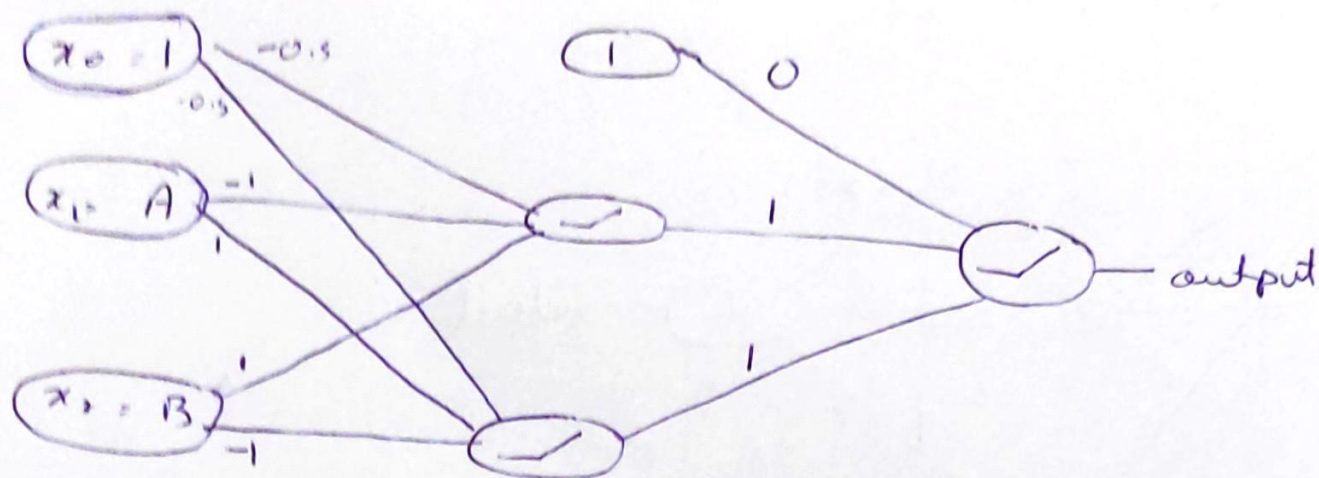
verified.

(B) XOR (using eq 1 and sigmoid)



output is considered as 1 if it is greater than 0.5

(B) XOR (using Relu and eq. 1)



output is consider as 1 if it is greater than 0

Testing (for Sigmoid and Relu Both)

① $A=0, B=0$ (Sigmoid)

$$\delta(-0.78 + \delta(-0.5+0+0) + \delta(-0.5+0+0)) = \delta(-0.78 + 0.38 + 0.38) = 0.5 \leq 0.5 \Rightarrow \text{output} = 0$$

$A=0, B=0$ (Relu)

$$\text{Relu}(0 + \text{Relu}(-0.5+0+0) + \text{Relu}(-0.5+0+0)) = \text{Relu}(0 + 0 + 0) = 0 \leq 0 \Rightarrow \text{output} = 0$$

② $A=1, B=0$ (Sigmoid)

$$\delta(-0.78 + \delta(-0.5+1+0) + \delta(-0.5-1+0)) = \delta(-0.78 + 0.68 + 0.18) = 0.54 > 0.5 \Rightarrow \text{output} = 1$$

$A=1, B=0$ (Relu)

$$\text{Relu}(0 + \text{Relu}(0.5+1+0) + \text{Relu}(-0.5-1+0)) = \text{Relu}(0 + 0.5 + 0) = 0.5 > 0 \Rightarrow \text{output} = 1$$

③ $A=0, B=1$ (Sigmoid)

Same as ②

$A=0, B=1$ (Relu)

Same as ②

④ $A=1, B=1$ (Sigmoid)

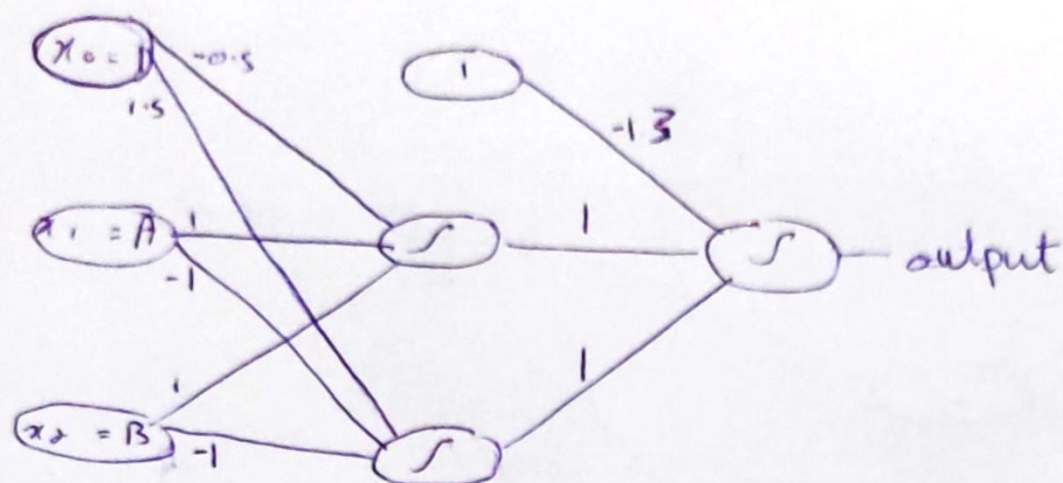
$$\delta(-0.78 + \delta(-0.5+1-1) + \delta(-0.5-1+1)) = \delta(-0.78 + 0.38 + 0.38) = 0.5 \leq 0.5 \Rightarrow \text{output} = 0$$

$A=1, B=1$ (Relu)

$$\text{Relu}(0 + \text{Relu}(-0.5+1-1) + \text{Relu}(-0.5-1+1)) = \text{Relu}(0 + 0 + 0) = 0 \leq 0 \Rightarrow \text{output} = 0$$

EXTRA WORK

(B) using sigmoid and eq (2)



Testing

① $A=0$, $B=0$

$$6(-1.5 + 6(-0.5 + 0 + 0) + 6(1.5 + 0 + 0)) = 6(-1.3 + 0.38 + 0.82) = 0.43 < 0.5 \\ \Rightarrow \text{output} = 0$$

② $A=1$, $B=1$

same as ①

③ $A=0$, $B=1$

$$6(-1.5 + 6(-0.5 + 0 + 1) + 6(1.5 + 0 - 1)) = 6(-1.3 + 0.68 + 0.68) = 0.517 > 0.5 \\ \Rightarrow \text{output} = 1$$

④ $A=1$, $B=0$

same as ③