# Statistical and Mathematical Methods for Data Analysis

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# Textbooks

- **Probability & Statistics for Engineers & Scientists**, Ninth Edition, Ronald E. Walpole, Raymond H. Myer

- **Elementary Statistics: Picturing the World,** 6th Edition, Ron Larson and Betsy Farber

- **Elementary Statistics**, 13th Edition, Mario F. Triola

# Reference books

- **Probability and Statistical Inference, Ninth Edition,** Robert V. Hogg, Elliot A. Tanis, Dale L. Zimmerman

- **Probability Demystified**, Allan G. Bluman

- **Practical Statistics for Data Scientists: 50 Essential Concepts,** Peter Bruce and Andrew Bruce

- **Schaum's Outline of Probability,** Second Edition, Seymour Lipschutz, Marc Lipson

- **Python for Probability, Statistics, and Machine Learning,** José Unpingco

# References


- Probability & Statistics for Engineers & Scientists, Ninth edition, Ronald E. Walpole, Raymond H. Myer
- Elementary Statistics, Tenth Edition, Mario F. Triola
- https://libguides.library.kent.edu/spss/chisquare


These notes contain material from the above resources.

# Chi-Square Test of Independence

- The **Chi-Square Test of Independence determines** whether there is an association between **categorical variables** (i.e., whether the variables are independent or related). It is a nonparametric test.

- This test **utilizes a contingency table** to analyze the data. A contingency table (also known as a *cross-tabulation*, *crosstab*, or *two-way table*) is an arrangement in which data is classified according to **two categorical variables**. The categories for one variable appear in the rows, and the categories for the other variable appear in columns. Each variable must **have two or more categories**. Each cell reflects the total count of cases for a specific pair of categories.

# Commons Uses

The Chi-Square Test of Independence is commonly used to test the following:

- Statistical independence or association between **two or more categorical variables**.

- The Chi-Square Test of Independence can only compare **categorical variables**. It cannot make comparisons between continuous variables or between categorical and continuous variables. Additionally, the Chi-Square Test of Independence only assesses *associations* between categorical variables, and can not provide any inferences about causation

# Data Requirements

Data must meet the following requirements:

- ❑ Two categorical variables.
- ❑ Two or more categories (groups) for each variable.
- ❑ Independence of observations.
  - o There is no relationship between the subjects in each group.
  - o The categorical variables are not "paired" in any way (e.g. pre-test/post-test observations).
- ❑ Relatively large sample size.
  - o Expected frequencies for each cell are at least 1.
  - o Expected frequencies should be at least 5 for the majority (80%) of the cells.

# Contingency Tables for Testing Independence of Attributes

- Suppose a medical researcher wants to determine whether there is a **relationship** between **caffeine consumption** and **heart attack risk**. Are these variables independent or are they dependent?
- We use the **chi-square test for independence** to answer such a question.
- To perform a **chi-square test for independence**, we will use sample data that are organized in a **contingency table**.

# Contingency table

- An **$r \times c$ contingency table** shows the observed frequencies for two variables.
- The observed frequencies are arranged in **r rows** and **c columns**.
- The intersection of a row and a column is called a **cell**.

**Example:** The contingency table shows the results of a random sample of 2200 adults classified by their favorite way to eat ice cream and gender. At a = 0.01, can you conclude that the variables favorite way to **eat ice cream** and **gender are related**?

| | Favorite way to eat ice cream | | | | |
|---|---|---|---|---|---|
| **Gender** | **Cup** | **Cone** | **Sundae** | **Sandwich** | **Other** |
| **Male** | 592 | 300 | 204 | 24 | 80 |
| **Female** | 410 | 335 | 180 | 20 | 55 |

1. **We state our hypothesis as**:

$H_0$: The variables favorite way to eat ice cream and gender are independent.

$H_1$: The variables favorite way to eat ice cream and gender are dependent. (Claim)

2. **The level of significance is set** $\alpha = 0.01$.

3. **Test statistic to be used is**

$$\chi^2_{cal} = \sum \frac{(O_f - E_f)^2}{E_f}$$

# 4. Calculations

| | Favorite way to eat ice cream | | | | | Total |
|---|---|---|---|---|---|---|
| Gender | Cup | Cone | Sundae | Sandwich | Other | |
| Male | 592 | 300 | 204 | 24 | 80 | 1200 |
| Female | 410 | 335 | 180 | 20 | 55 | 1000 |
| Total | 1002 | 635 | 384 | 44 | 135 | **2200** |

# Expected Frequency ($E_f$)

$$E_{r,c} = \frac{(\text{Sum of row } r)(\text{Sum of column } c)}{\text{Sample size}}$$

$$E_{1,1} = \frac{(1200)(1002)}{2200} = 546.55$$

| | Favorite way to eat ice cream | | | | | Total |
|---|---|---|---|---|---|---|
| **Gender** | **Cup** | **Cone** | **Sundae** | **Sandwich** | **Other** | |
| **Male** | 546.55 | 346.36 | 209.45 | 24 | 73.64 | 1200 |
| **Female** | 455.45 | 288.64 | 174.55 | 20 | 61.36 | 1000 |
| **Total** | 1002 | 635 | 384 | 44 | 135 | **2200** |

| $O_f$ | $E_f$ | $(O_f - E_f)$ | $(O_f - E_f)^2$ | $\frac{(O_f - E_f)^2}{E_f}$ |
|---|---|---|---|---|
| 592 | 546.55 | 45.45 | 2065.7025 | 3.7795 |
| 300 | 346.36 | - 46.36 | 2149.2496 | 6.2052 |
| 204 | 209.45 | - 5.45 | 29.7025 | 0.1418 |
| 24 | 24 | 0 | 0 | 0 |
| 80 | 73.64 | 6.36 | 40.4496 | 0.5493 |
| 410 | 455.45 | - 45.45 | 2065.7025 | 4.5355 |
| 335 | 288.64 | 46.36 | 2149.2496 | 7.4461 |
| 180 | 174.55 | 5.45 | 29.7025 | 0.1702 |
| 20 | 20 | 0 | 0 | 0 |
| 55 | 61.36 | - 6.36 | 40.4496 | 0.6592 |
| | | | | $\chi^2_{cal}$ **= 23.487** |

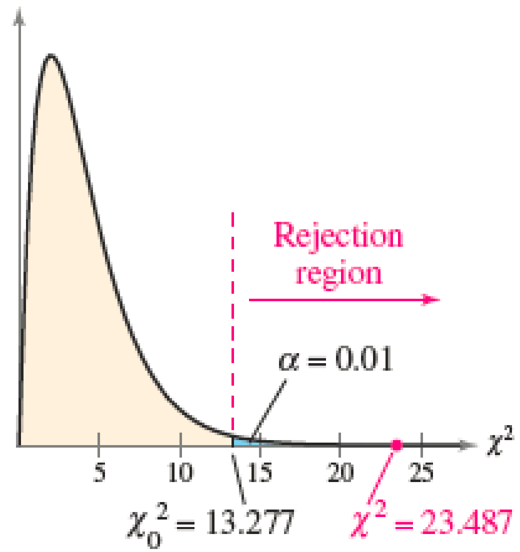
5. **Critical region:**

$\chi^2_{cal} > \chi^2_{tab}$

Where $\chi^2_{tab} = \chi^2_{\alpha,(r-1)(c-1)} = \chi^2_{0.01,(2-1)(5-1)}$

= 13.277

23.487 > 13.277 (true)



## 6. Conclusion

There is enough evidence at the 1% level of significance to conclude that the variables *favorite way to eat ice cream* and *gender* are dependent.

# Chi-Squared Table



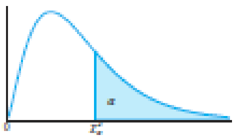


Table A.5 Critical Values of the Chi-Squared Distribution

| $v$ | 0.995 | 0.99 | 0.98 | 0.975 | 0.95 | 0.90 | 0.80 | 0.75 | 0.70 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | $0.0^4393$ | $0.0^3157$ | $0.0^3628$ | $0.0^3982$ | 0.00393 | 0.0158 | 0.0642 | 0.102 | 0.148 | 0.455 |
| **2** | 0.0100 | 0.0201 | 0.0404 | 0.0506 | 0.103 | 0.211 | 0.446 | 0.575 | 0.713 | 1.386 |
| **3** | 0.0717 | 0.115 | 0.185 | 0.216 | 0.352 | 0.584 | 1.005 | 1.213 | 1.424 | 2.366 |
| **4** | 0.207 | 0.297 | 0.429 | 0.484 | 0.711 | 1.064 | 1.649 | 1.923 | 2.195 | 3.357 |
| **5** | 0.412 | 0.554 | 0.752 | 0.831 | 1.145 | 1.610 | 2.343 | 2.675 | 3.000 | 4.351 |
| **6** | 0.676 | 0.872 | 1.134 | 1.237 | 1.635 | 2.204 | 3.070 | 3.455 | 3.828 | 5.348 |
| **7** | 0.989 | 1.239 | 1.564 | 1.690 | 2.167 | 2.833 | 3.822 | 4.255 | 4.671 | 6.346 |
| **8** | 1.344 | 1.647 | 2.032 | 2.180 | 2.733 | 3.490 | 4.594 | 5.071 | 5.527 | 7.344 |
| **9** | 1.735 | 2.088 | 2.532 | 2.700 | 3.325 | 4.168 | 5.380 | 5.899 | 6.393 | 8.343 |
| **10** | 2.156 | 2.558 | 3.059 | 3.247 | 3.940 | 4.865 | 6.179 | 6.737 | 7.267 | 9.342 |
| **11** | 2.603 | 3.053 | 3.609 | 3.816 | 4.575 | 5.578 | 6.989 | 7.584 | 8.148 | 10.341 |
| **12** | 3.074 | 3.571 | 4.178 | 4.404 | 5.226 | 6.304 | 7.807 | 8.438 | 9.034 | 11.340 |
| **13** | 3.565 | 4.107 | 4.765 | 5.009 | 5.892 | 7.041 | 8.634 | 9.299 | 9.926 | 12.340 |
| **14** | 4.075 | 4.660 | 5.368 | 5.629 | 6.571 | 7.790 | 9.467 | 10.165 | 10.821 | 13.339 |
| **15** | 4.601 | 5.229 | 5.985 | 6.262 | 7.261 | 8.547 | 10.307 | 11.037 | 11.721 | 14.339 |
| **16** | 5.142 | 5.812 | 6.614 | 6.908 | 7.962 | 9.312 | 11.152 | 11.912 | 12.624 | 15.338 |
| **17** | 5.697 | 6.408 | 7.255 | 7.564 | 8.672 | 10.085 | 12.002 | 12.792 | 13.531 | 16.338 |
| **18** | 6.265 | 7.015 | 7.906 | 8.231 | 9.390 | 10.865 | 12.857 | 13.675 | 14.440 | 17.338 |
| **19** | 6.844 | 7.633 | 8.567 | 8.907 | 10.117 | 11.651 | 13.716 | 14.562 | 15.352 | 18.338 |
| **20** | 7.434 | 8.260 | 9.237 | 9.591 | 10.851 | 12.443 | 14.578 | 15.452 | 16.266 | 19.337 |
| **21** | 8.034 | 8.897 | 9.915 | 10.283 | 11.591 | 13.240 | 15.445 | 16.344 | 17.182 | 20.337 |
| **22** | 8.643 | 9.542 | 10.600 | 10.982 | 12.338 | 14.041 | 16.314 | 17.240 | 18.101 | 21.337 |
| **23** | 9.260 | 10.196 | 11.293 | 11.689 | 13.091 | 14.848 | 17.187 | 18.137 | 19.021 | 22.337 |
| **24** | 9.886 | 10.856 | 11.992 | 12.401 | 13.848 | 15.659 | 18.062 | 19.037 | 19.943 | 23.337 |
| **25** | 10.520 | 11.524 | 12.697 | 13.120 | 14.611 | 16.473 | 18.940 | 19.939 | 20.867 | 24.337 |
| **26** | 11.160 | 12.198 | 13.409 | 13.844 | 15.379 | 17.292 | 19.820 | 20.843 | 21.792 | 25.336 |
| **27** | 11.808 | 12.878 | 14.125 | 14.573 | 16.151 | 18.114 | 20.703 | 21.749 | 22.719 | 26.336 |
| **28** | 12.461 | 13.565 | 14.847 | 15.308 | 16.928 | 18.939 | 21.588 | 22.657 | 23.647 | 27.336 |
| **29** | 13.121 | 14.256 | 15.574 | 16.047 | 17.708 | 19.768 | 22.475 | 23.567 | 24.577 | 28.336 |
| **30** | 13.787 | 14.953 | 16.306 | 16.791 | 18.493 | 20.599 | 23.364 | 24.478 | 25.508 | 29.336 |
| **40** | 20.707 | 22.164 | 23.838 | 24.433 | 26.509 | 29.051 | 32.345 | 33.66 | 34.872 | 39.335 |
| **50** | 27.991 | 29.707 | 31.664 | 32.357 | 34.764 | 37.689 | 41.449 | 42.942 | 44.313 | 49.335 |
| **60** | 35.534 | 37.485 | 39.699 | 40.482 | 43.188 | 46.459 | 50.641 | 52.294 | 53.809 | 59.335 |

Table A.5 (continued) Critical Values of the Chi-Squared Distribution

| | $\alpha$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $v$ | 0.30 | 0.25 | 0.20 | 0.10 | 0.05 | 0.025 | 0.02 | 0.01 | 0.005 | 0.001 |
| 1 | 1.074 | 1.323 | 1.642 | 2.706 | 3.841 | 5.024 | 5.412 | 6.635 | 7.879 | 10.827 |
| 2 | 2.408 | 2.773 | 3.219 | 4.605 | 5.991 | 7.378 | 7.824 | 9.210 | 10.597 | 13.815 |
| 3 | 3.665 | 4.108 | 4.642 | 6.251 | 7.815 | 9.348 | 9.837 | 11.345 | 12.838 | 16.266 |
| 4 | 4.878 | 5.385 | 5.989 | 7.779 | 9.488 | 11.143 | 11.668 | 13.277 | 14.860 | 18.466 |
| 5 | 6.064 | 6.626 | 7.289 | 9.236 | 11.070 | 12.832 | 13.388 | 15.086 | 16.750 | 20.515 |
| 6 | 7.231 | 7.841 | 8.558 | 10.645 | 12.592 | 14.449 | 15.033 | 16.812 | 18.548 | 22.457 |
| 7 | 8.383 | 9.037 | 9.803 | 12.017 | 14.067 | 16.013 | 16.622 | 18.475 | 20.278 | 24.321 |
| 8 | 9.524 | 10.219 | 11.030 | 13.362 | 15.507 | 17.535 | 18.168 | 20.090 | 21.955 | 26.124 |
| 9 | 10.656 | 11.389 | 12.242 | 14.684 | 16.919 | 19.023 | 19.679 | 21.666 | 23.589 | 27.877 |
| 10 | 11.781 | 12.549 | 13.442 | 15.987 | 18.307 | 20.483 | 21.161 | 23.209 | 25.188 | 29.588 |
| 11 | 12.899 | 13.701 | 14.631 | 17.275 | 19.675 | 21.920 | 22.618 | 24.725 | 26.757 | 31.264 |
| 12 | 14.011 | 14.845 | 15.812 | 18.549 | 21.026 | 23.337 | 24.054 | 26.217 | 28.300 | 32.909 |
| 13 | 15.119 | 15.984 | 16.985 | 19.812 | 22.362 | 24.736 | 25.471 | 27.688 | 29.819 | 34.527 |
| 14 | 16.222 | 17.117 | 18.151 | 21.064 | 23.685 | 26.119 | 26.873 | 29.141 | 31.319 | 36.124 |
| 15 | 17.322 | 18.245 | 19.311 | 22.307 | 24.996 | 27.488 | 28.259 | 30.578 | 32.801 | 37.698 |
| 16 | 18.418 | 19.369 | 20.465 | 23.542 | 26.296 | 28.845 | 29.633 | 32.000 | 34.267 | 39.252 |
| 17 | 19.511 | 20.489 | 21.615 | 24.769 | 27.587 | 30.191 | 30.995 | 33.409 | 35.718 | 40.791 |
| 18 | 20.601 | 21.605 | 22.760 | 25.989 | 28.869 | 31.526 | 32.346 | 34.805 | 37.156 | 42.312 |
| 19 | 21.689 | 22.718 | 23.900 | 27.204 | 30.144 | 32.852 | 33.687 | 36.191 | 38.582 | 43.819 |
| 20 | 22.775 | 23.828 | 25.038 | 28.412 | 31.410 | 34.170 | 35.020 | 37.566 | 39.997 | 45.314 |
| 21 | 23.858 | 24.935 | 26.171 | 29.615 | 32.671 | 35.479 | 36.343 | 38.932 | 41.401 | 46.796 |
| 22 | 24.939 | 26.039 | 27.301 | 30.813 | 33.924 | 36.781 | 37.659 | 40.289 | 42.796 | 48.268 |
| 23 | 26.018 | 27.141 | 28.429 | 32.007 | 35.172 | 38.076 | 38.968 | 41.638 | 44.181 | 49.728 |
| 24 | 27.096 | 28.241 | 29.553 | 33.196 | 36.415 | 39.364 | 40.270 | 42.980 | 45.558 | 51.179 |
| 25 | 28.172 | 29.339 | 30.675 | 34.382 | 37.652 | 40.646 | 41.566 | 44.314 | 46.928 | 52.619 |
| 26 | 29.246 | 30.435 | 31.795 | 35.563 | 38.885 | 41.923 | 42.856 | 45.642 | 48.290 | 54.051 |
| 27 | 30.319 | 31.528 | 32.912 | 36.741 | 40.113 | 43.195 | 44.140 | 46.963 | 49.645 | 55.475 |
| 28 | 31.391 | 32.620 | 34.027 | 37.916 | 41.337 | 44.461 | 45.419 | 48.278 | 50.994 | 56.892 |
| 29 | 32.461 | 33.711 | 35.139 | 39.087 | 42.557 | 45.722 | 46.693 | 49.588 | 52.335 | 58.301 |
| 30 | 33.530 | 34.800 | 36.250 | 40.256 | 43.773 | 46.979 | 47.962 | 50.892 | 53.672 | 59.702 |
| 40 | 44.165 | 45.616 | 47.269 | 51.805 | 55.758 | 59.342 | 60.436 | 63.691 | 66.766 | 73.403 |
| 50 | 54.723 | 56.334 | 58.164 | 63.167 | 67.505 | 71.420 | 72.613 | 76.154 | 79.490 | 86.660 |
| 60 | 65.226 | 66.981 | 68.972 | 74.397 | 79.082 | 83.298 | 84.58 | 88.379 | 91.952 | 99.608 |