# Practical Issues of Classification

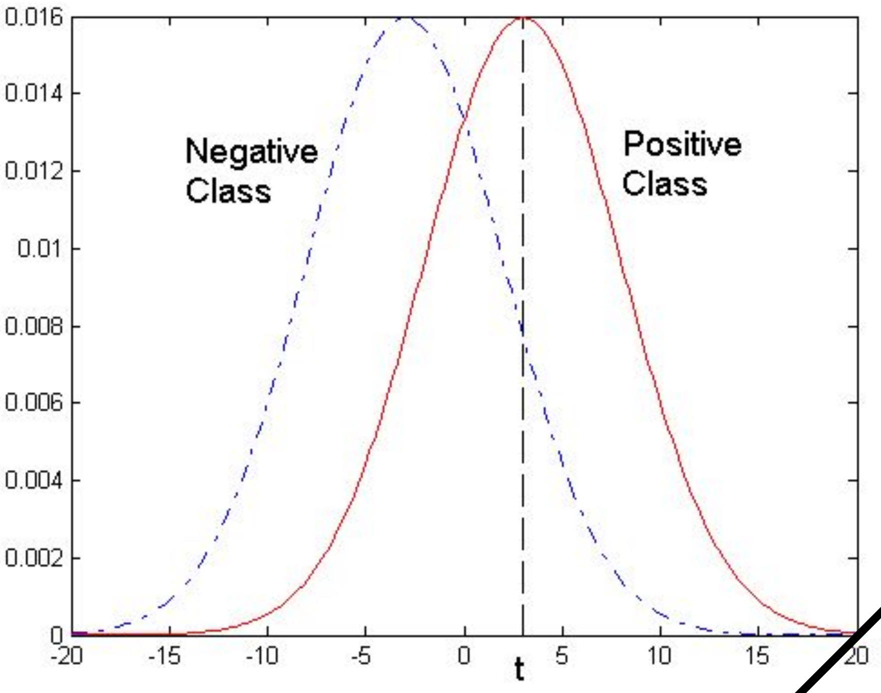
Dr. Faisal Kamiran

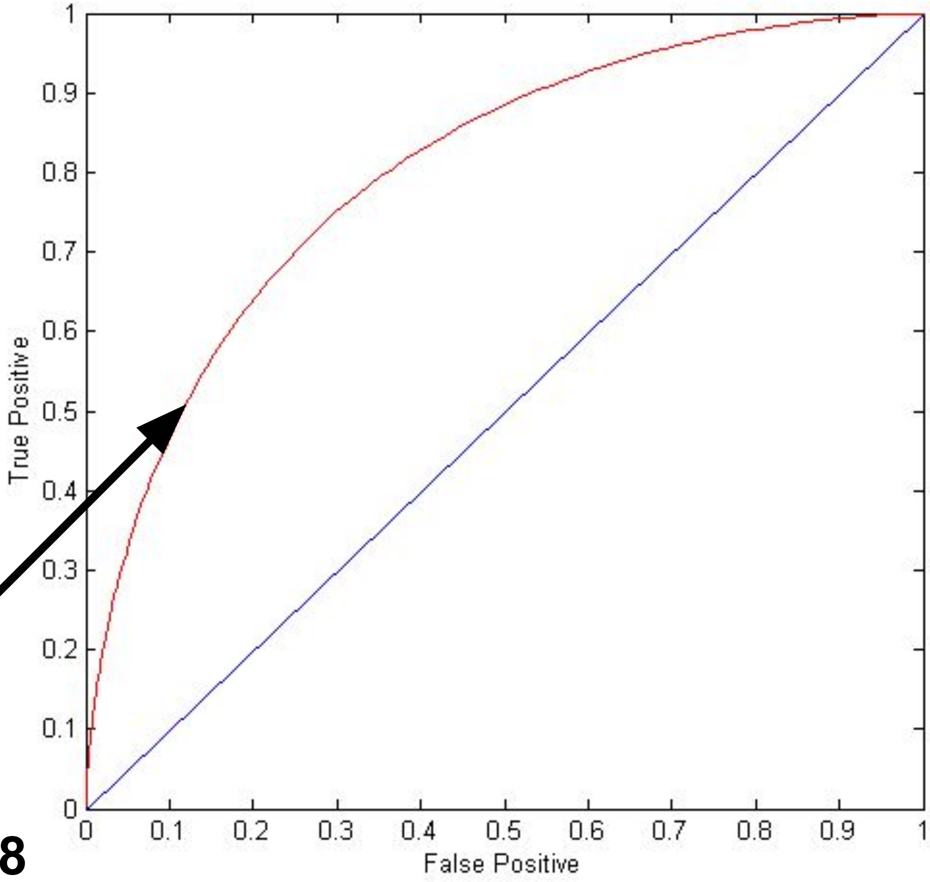
# ROC (Receiver Operating Characteristic)

- Developed in 1950s for signal detection theory to analyze noisy signals
  - Characterize the trade-off between positive hits and false alarms
- ROC curve plots TP (on the y-axis) against FP (on the x-axis)
- Performance of each classifier represented as a point on the ROC curve
  - changing the threshold of algorithm, sample distribution or cost matrix changes the location of the point

# ROC Curve

- 1-dimensional data set containing 2 classes (positive and negative)
- any points located at x > t is classified as positive





**At threshold t:**

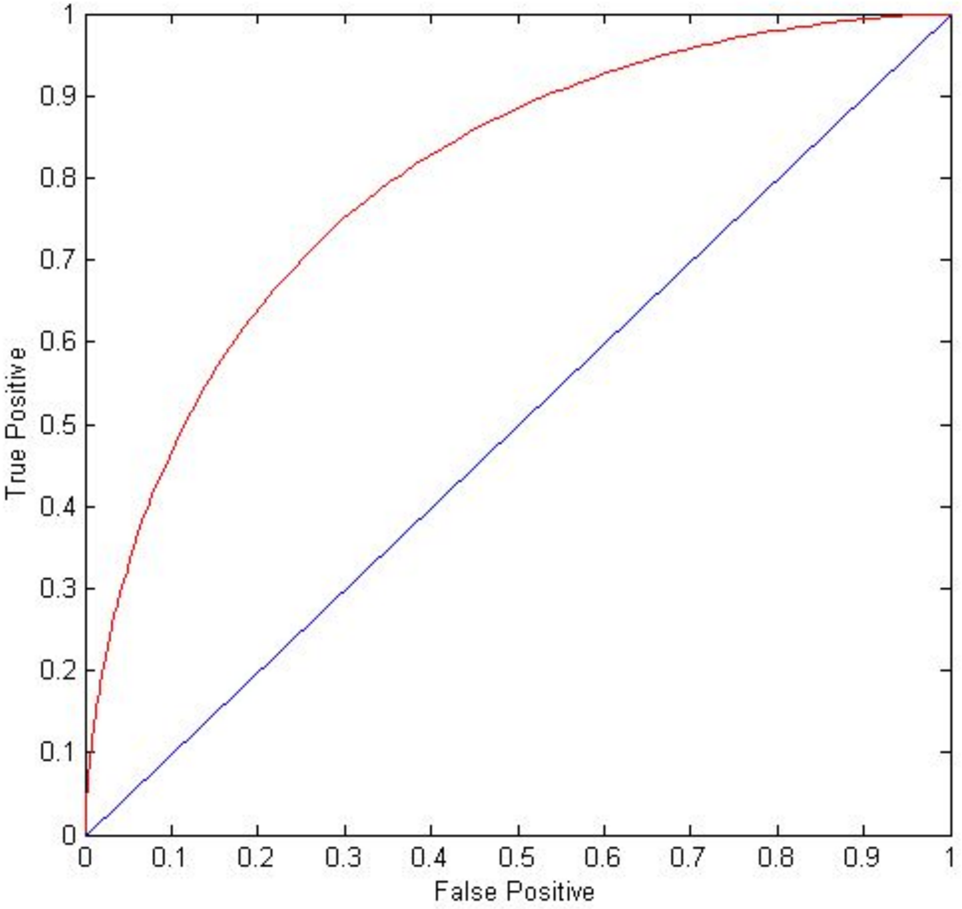
**TP=0.5, FN=0.5, FP=0.12, FN=0.88**

# ROC Curve

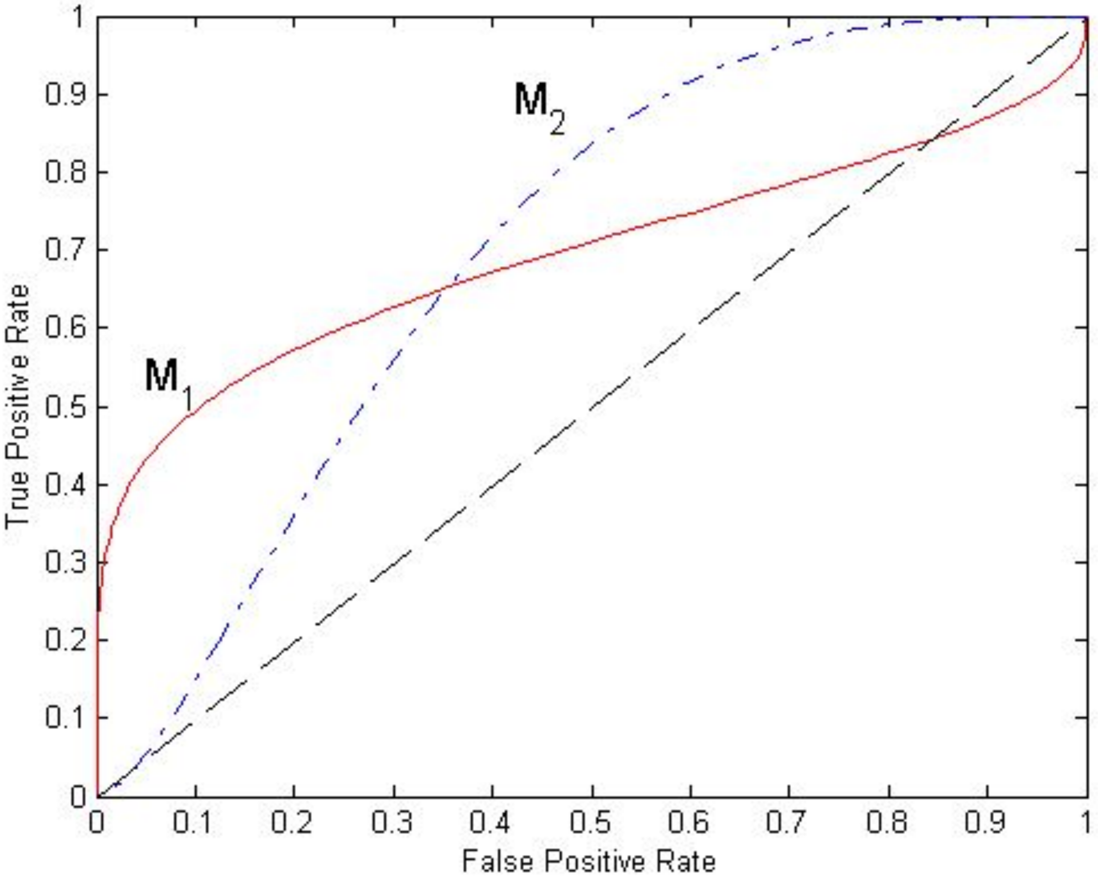
(TP,FP):

- (0,0): declare everything
  to be negative class
- (1,1): declare everything
  to be positive class
- (1,0): ideal

- Diagonal line:
  - Random guessing
  - Below diagonal line:
    - prediction is opposite of th
      true class



# Using ROC for Model Comparison



- No model consistently outperform the other
  - M$_1$ is better for small FPR
  - M$_2$ is better for large FPR
- Area Under the ROC curve
  - Ideal:
    - Area = 1
  - Random guess:
    - Area = 0.5

# How to Construct an ROC curve

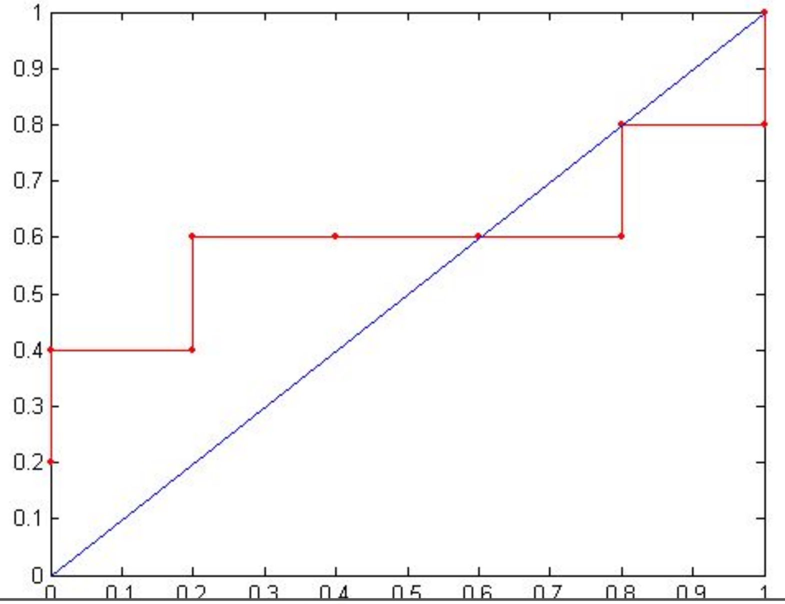
| Instance | P(+\|A) | True Class |
|---|---|---|
| 1 | 0.95 | + |
| 2 | 0.93 | + |
| 3 | 0.87 | - |
| 4 | 0.85 | - |
| 5 | 0.85 | - |
| 6 | 0.85 | + |
| 7 | 0.76 | - |
| 8 | 0.53 | + |
| 9 | 0.43 | - |
| 10 | 0.25 | + |

- Use classifier that produces posterior probability for each test instance P(+|A)
- Sort the instances according to P(+|A) in decreasing order
- Apply threshold at each unique value of P(+|A)
- Count the number of TP, FP, TN, FN at each threshold
- TP rate, TPR = TP/(TP+FN)
- FP rate, FPR = FP/(FP + TN)

# How to construct an ROC curve

| Class | + | - | + | - | - | - | + | - | + | + | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Threshold >= | 0.25 | 0.43 | 0.53 | 0.76 | 0.85 | 0.85 | 0.85 | 0.87 | 0.93 | 0.95 | 1.00 |
| TP | 5 | 4 | 4 | 3 | 3 | 3 | 3 | 2 | 2 | 1 | 0 |
| FP | 5 | 5 | 4 | 4 | 3 | 2 | 1 | 1 | 0 | 0 | 0 |
| TN | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 5 | 5 |
| FN | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 4 | 5 |
| TPR | 1 | 0.8 | 0.8 | 0.6 | 0.6 | 0.6 | 0.6 | 0.4 | 0.4 | 0.2 | 0 |
| FPR | 1 | 1 | 0.8 | 0.8 | 0.6 | 0.4 | 0.2 | 0.2 | 0 | 0 | 0 |

ROC Curve:



# Instance Based Classifiers

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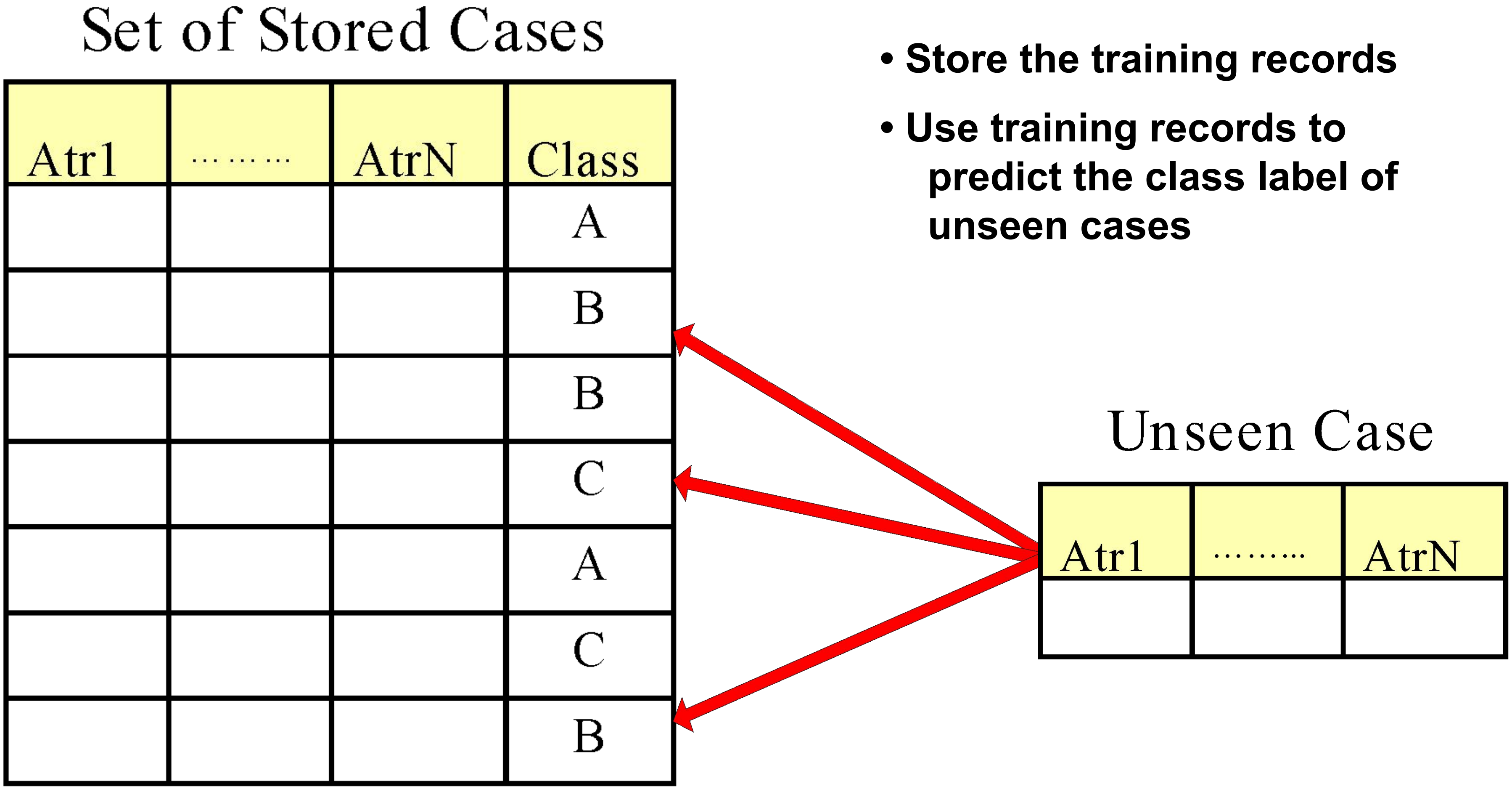
# Instance-Based Classifiers

- Store the training records
- Use training records to predict the class label of unseen cases



Set of Stored Cases

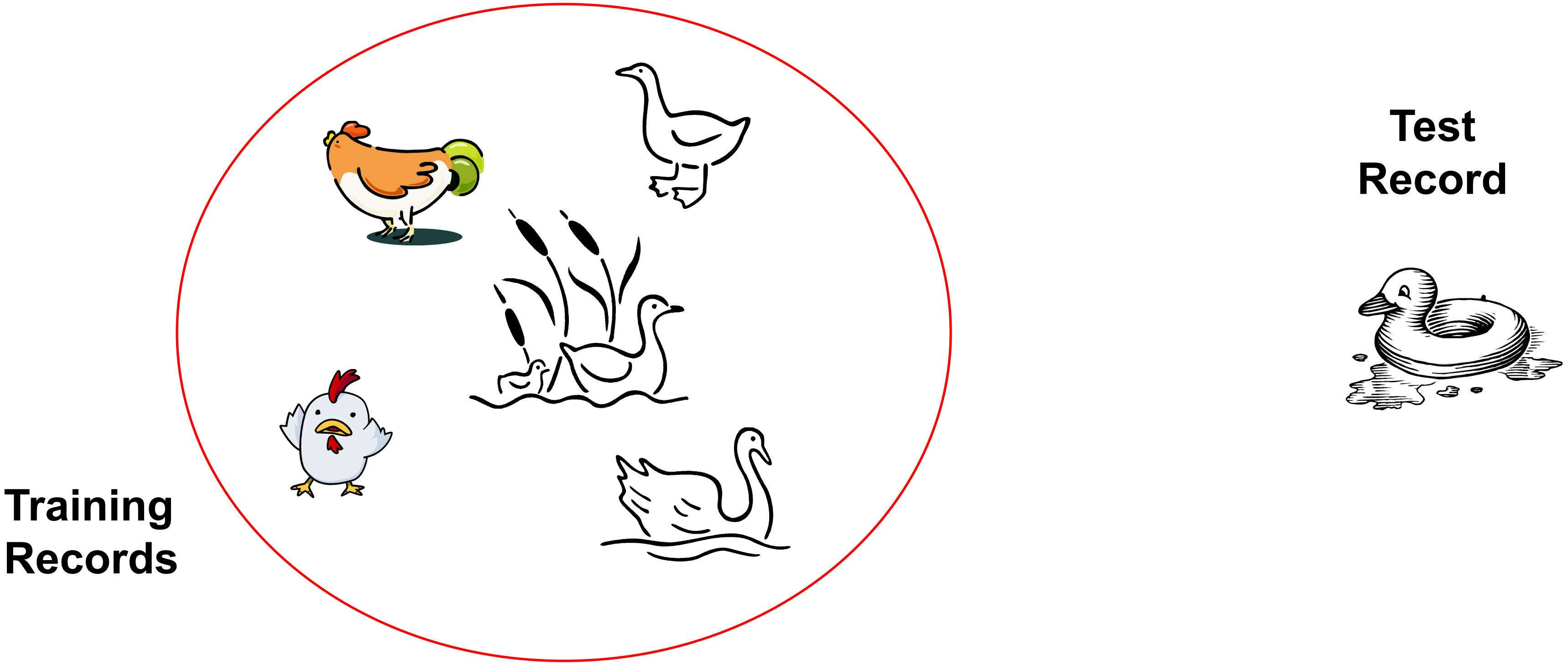
| Atr1 | ……… | AtrN | Class |
|---|---|---|---|
| | | | A |
| | | | B |
| | | | B |
| | | | C |
| | | | A |
| | | | C |
| | | | B |

Unseen Case

| Atr1 | ……… | AtrN |
|---|---|---|
| | | |

# Nearest Neighbor Classifiers

- Basic idea:
  - If it walks like a duck, quacks like a duck, then it's probably a duck



Training Records

Test Record

# Nearest Neighbor Classifiers

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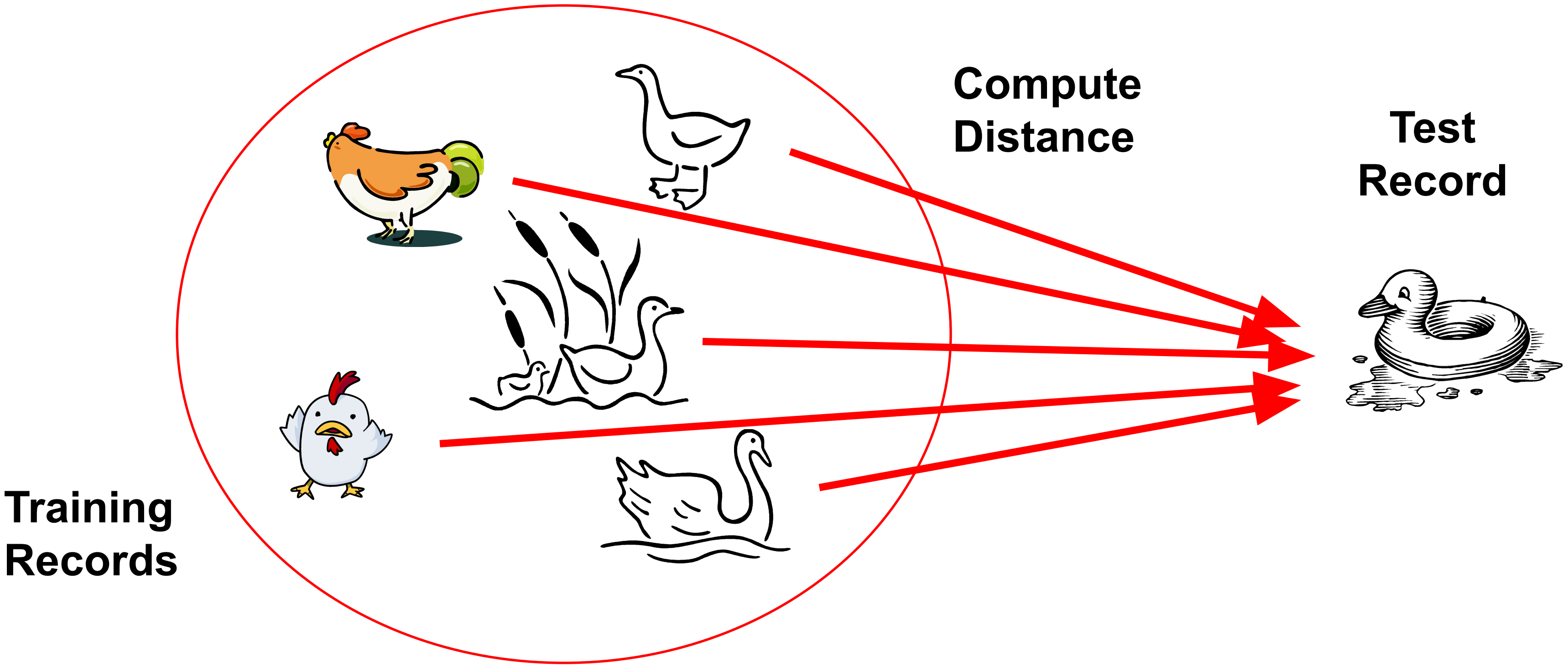

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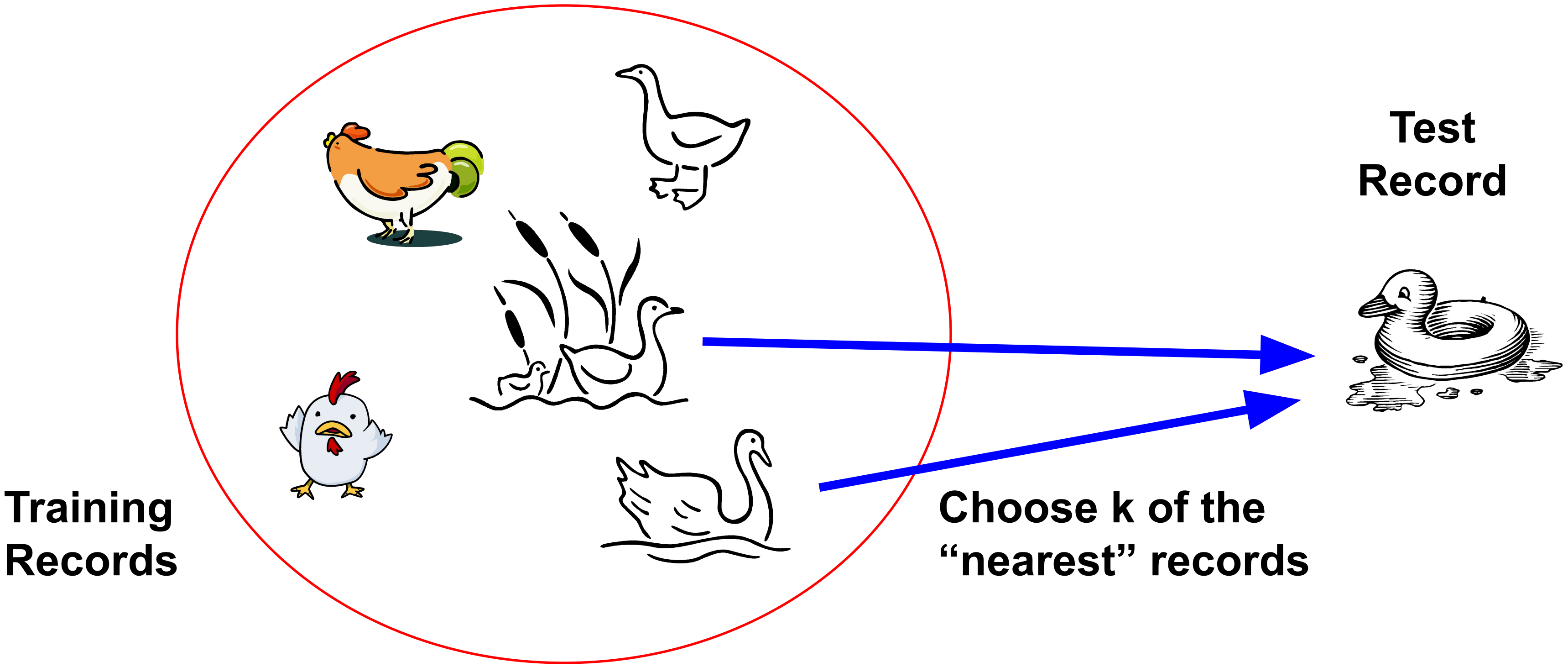
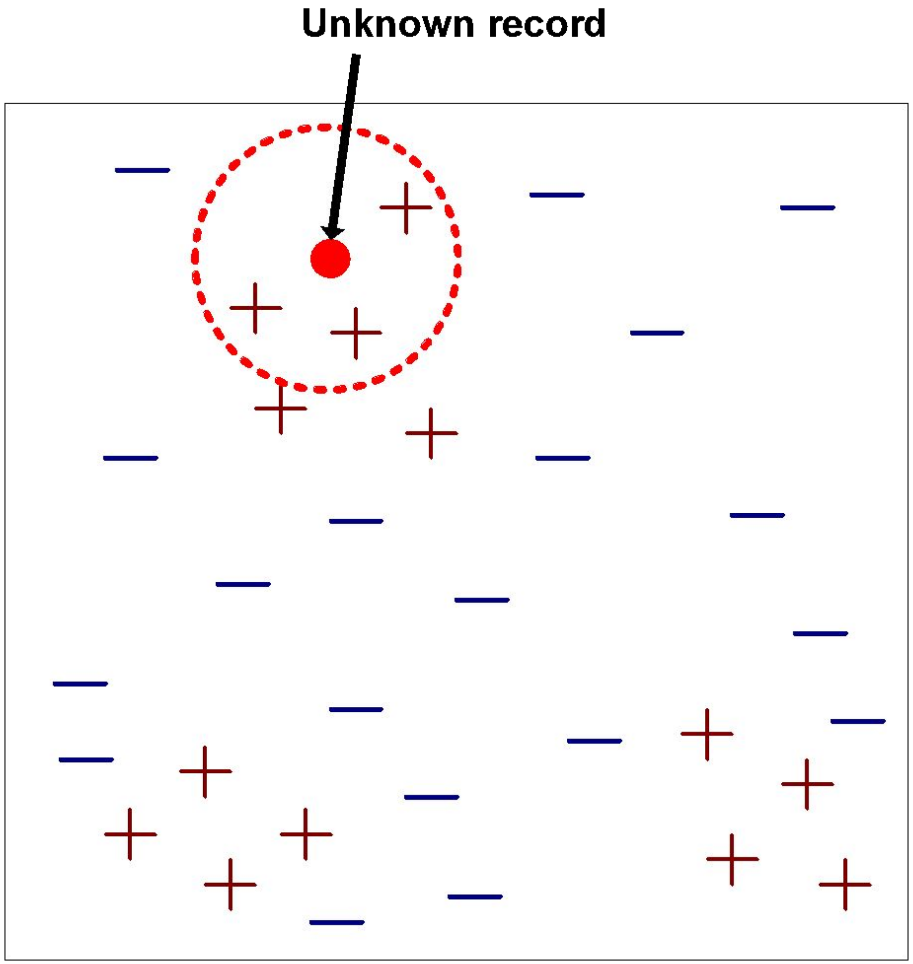


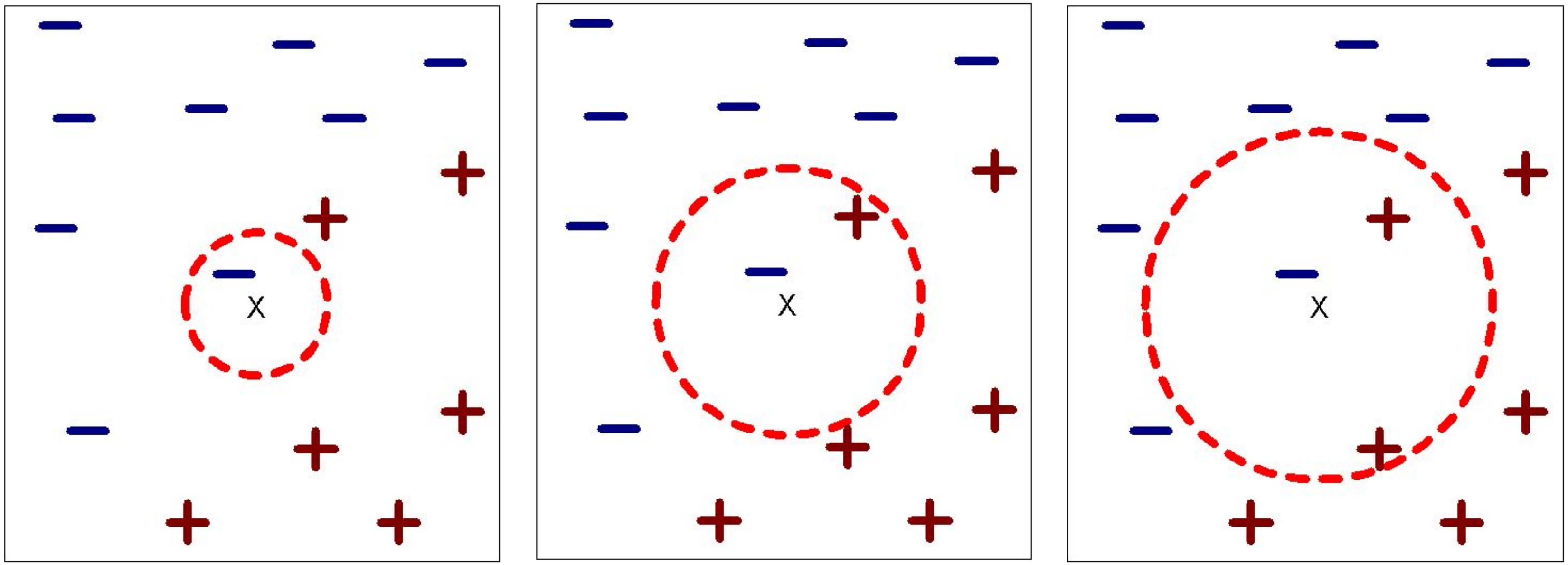
# Nearest-Neighbor Classifiers



- Requires three things
  - The set of stored records
  - Distance Metric to compute distance between records
  - The value of *k*, the number of nearest neighbors to retrieve

- To classify an unknown record:
  - Compute distance to other training records
  - Identify *k* nearest neighbors
  - Use class labels of nearest neighbors to determine the class label of unknown record (e.g., by taking majority vote)

# Definition of Nearest Neighbor



(a) 1-nearest neighbor (b) 2-nearest neighbor (c) 3-nearest neighbor

K-nearest neighbors of a record x are data points that have the k smallest distance to x

# Nearest Neighbor Classification

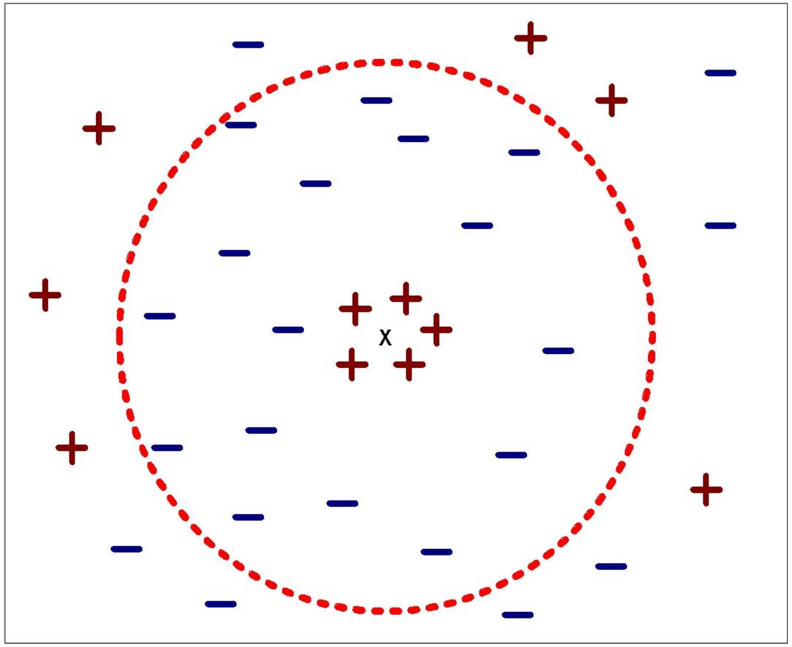
- Compute distance between two points:
  - Euclidean distance

$$d(p,q) = \sqrt{\sum_i (p_i - q_i)^2}$$

- Determine the class from nearest neighbor list
  - take the majority vote of class labels among the k-nearest neighbors
  - Weigh the vote according to distance
    - weight factor, w = $1/d^2$

# Nearest Neighbor Classification…

- Choosing the value of k:
  - If k is too small, sensitive to noise points
  - If k is too large, neighborhood may include points from other classes



# Nearest Neighbor Classification…

- Scaling issues
  - Attributes may have to be scaled to prevent distance measures from being dominated by one of the attributes
  - Example:
    - height of a person may vary from 1.5m to 1.8m
    - weight of a person may vary from 90lb to 300lb
    - income of a person may vary from $10K to $1M

# Nearest Neighbor Classification…

- k-NN classifiers are lazy learners
  - It does not build models explicitly
  - Unlike eager learners such as decision tree induction and rule-based systems
  - Classifying unknown records are relatively expensive

# Nearest Neighbor Classification…

- Problem with Euclidean measure:
  - High dimensional data
    - curse of dimensionality
  - Can produce counter-intuitive results

1 1 1 1 1 1 1 1 1 1 1 0

0 1 1 1 1 1 1 1 1 1 1 1

d = 1.4142

vs

1 0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0 1

d = 1.4142

# Distance Measures …

- Choosing the correct distance function is essential
  - Eucledian, Minkowski
  - Simple Matching Coefficient
  - Jaccard measure
  - Cosine Measure

- Example: distance measure for strings
  - Edit distance

# Edit Distance

- Distance between two strings: minimal number of operations to transform one into another
  - Insert a character
  - Delete a character
  - Replace a character with another
- Example:
  - Hello □ Jello distance = 1
  - Good □ Goodbye distance = 3

# Bayes Classifiers

# Bayes Classifier

- A probabilistic framework for solving classification problems
- Conditional Probability:

$$P(C \mid X) = \frac{P(X,C)}{P(X)}$$

$$P(X \mid C) = \frac{P(X,C)}{P(C)}$$

- Bayes theorem:

Posterior probability

Prior probability

$$P(C \mid X) = \frac{P(X \mid C)P(C)}{P(X)}$$

# Towards Naïve Bayesian Classifier

- Let D be a training set of tuples and their associated class labels, and each tuple is represented by an n attribute vector **X** = $(x_1, x_2, \ldots, x_n)$
- Suppose there are *m* classes $C_1, C_2, \ldots, C_m$.
- Classification is to derive the maximum posteriori, i.e., the maximal $P(C_i|\mathbf{X})$
- This can be derived from Bayes' theorem

$$P(C_i|\mathbf{X}) = \frac{P(\mathbf{X}|C_i)P(C_i)}{P(\mathbf{X})}$$

- Since P(X) is constant for all classes, only

$$P(C_i|\mathbf{X}) = P(\mathbf{X}|C_i)P(C_i)$$

needs to be maximized

# Derivation of Naïve Bayes Classifier

- A simplified assumption: attributes are conditionally independent (i.e., no dependence relation between attributes):

$$P(\mathbf{X} | C_i) = \prod_{k=1}^{n} P(x_k | C_i) = P(x_1 | C_i) \times P(x_2 | C_i) \times \ldots \times P(x_n | C_i)$$

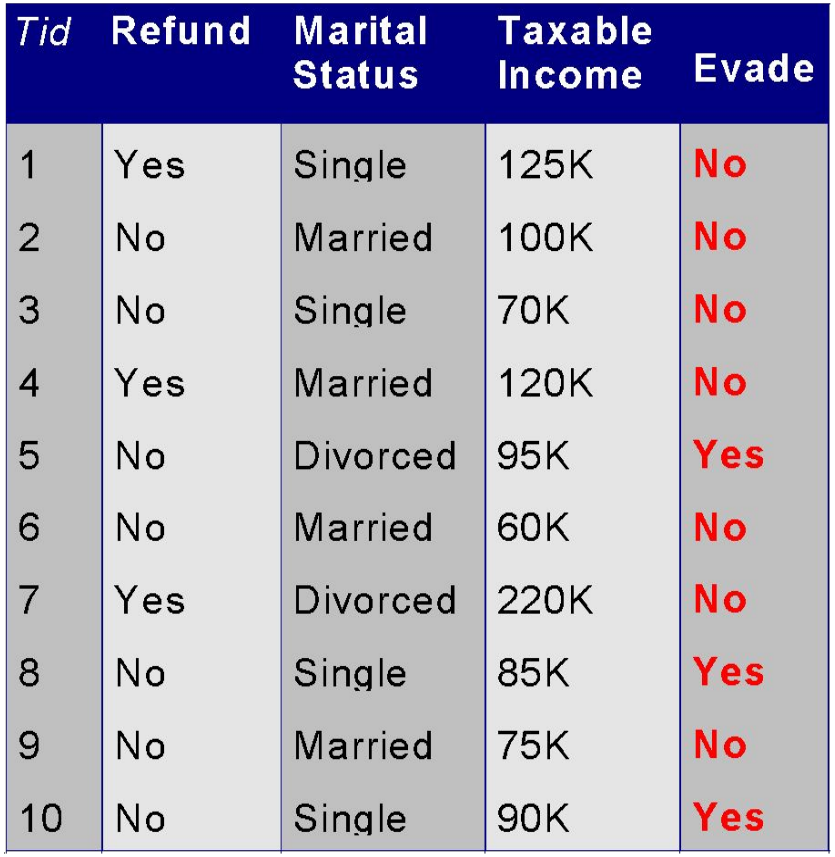
- This greatly reduces the computation cost: Only counts the class distribution
- If $A_k$ is categorical: $P(x_k|C_i)$ is the # of tuples in $C_i$ having value $x_k$ for $A_k$ divided by $|C_{i, D}|$ (# of tuples of $C_i$ in D)
- If $A_k$ is continous-valued: $P(x_k|C_i)$ is usually computed based on Gaussian distribution with a mean μ and standard deviation σ

$$g(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

and $P(x_k|C_i)$ is

$$P(\mathbf{X} | C_i) = g(x_k, \mu_{C_i}, \sigma_{C_i})$$

# How to Estimate Probabilities from Data?



| *Tid* | Refund | Marital Status | Taxable Income | Evade |
|---|---|---|---|---|
| 1 | Yes | Single | 125K | No |
| 2 | No | Married | 100K | No |
| 3 | No | Single | 70K | No |
| 4 | Yes | Married | 120K | No |
| 5 | No | Divorced | 95K | Yes |
| 6 | No | Married | 60K | No |
| 7 | Yes | Divorced | 220K | No |
| 8 | No | Single | 85K | Yes |
| 9 | No | Married | 75K | No |
| 10 | No | Single | 90K | Yes |

- Class: $P(C) = N_c/N$
  - e.g., $P(No) = 7/10$, $P(Yes) = 3/10$
- For discrete attributes:

  $$P(A_i \mid C_k) = |A_{ik}|/ N_{ck}$$

  - where $|A_{ik}|$ is number of instances having attribute $A_i$ and belongs to class $C_k$
  - Examples:

    P(Status=Married|No) = 4/7
    P(Refund=Yes|Yes)=0

# How to Estimate Probabilities from Data?

- For continuous attributes:
  - Discretization:
    - Replace the attribute with discrete intervals
  - Probability density estimation:
    - Assume attribute follows a normal distribution
    - Use data to estimate parameters of distribution (e.g., mean and standard deviation)
    - Once probability distribution is known, can use it to estimate the conditional probability $P(A_i|c)$

# How to Estimate Probabilities from Data?

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- Normal distribution:

$$P(A_i \mid c_j) = \frac{1}{\sqrt{2\pi\sigma_{ij}^2}} e^{-\frac{(A_i-\mu_{ij})^2}{2\sigma_{ij}^2}}$$

  - One for each $(A_i,c_i)$ pair

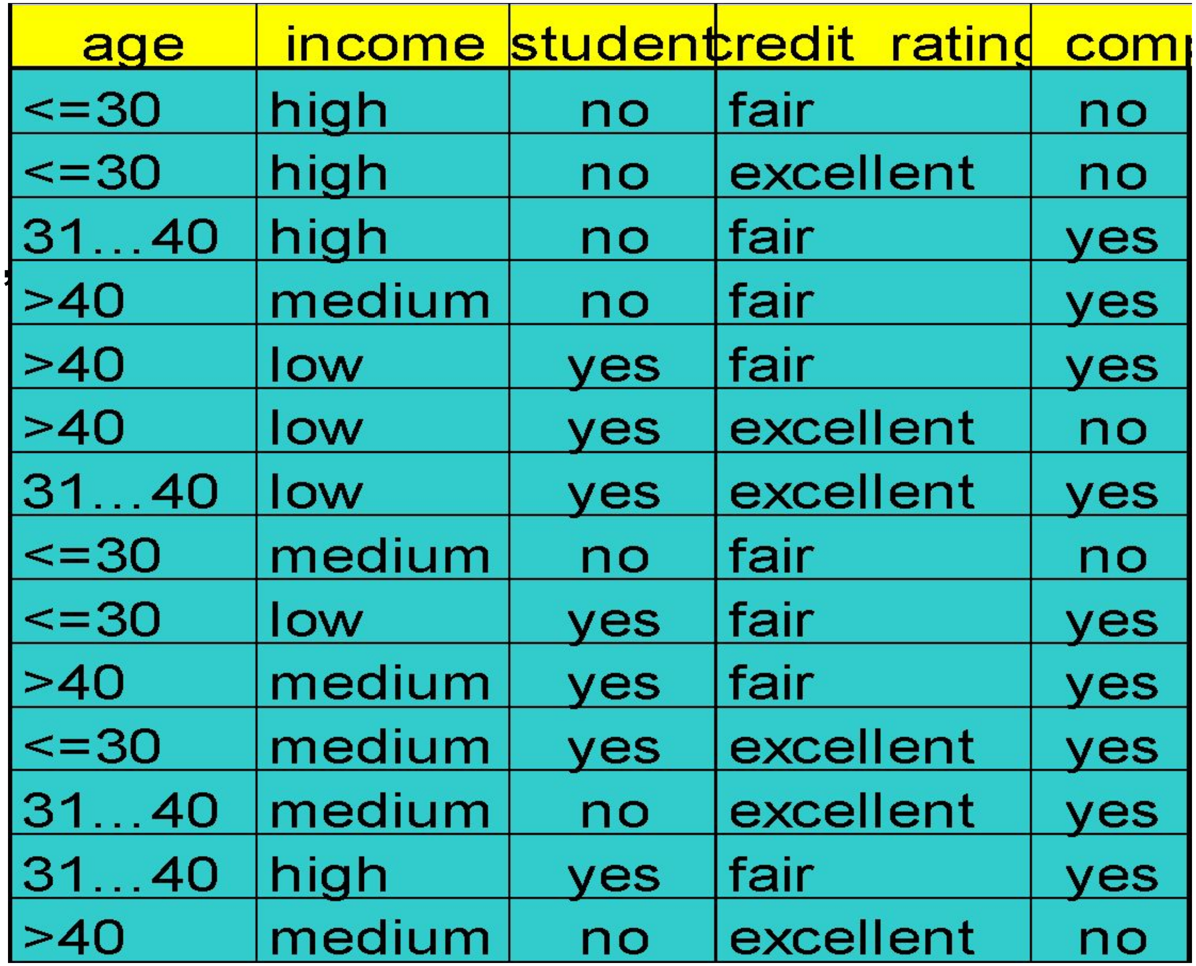
- For (Income, Class=No):
  - If Class=No
    - sample mean = 110
    - sample variance = 2975

$$P(Income = 120 \mid No) = \frac{1}{\sqrt{2\pi}(54.54)} e^{-\frac{(120-110)^2}{2(2975)}} = 0.0072$$

# Naïve Bayesian Classifier: Training Dataset

Class:
C1:buys_computer = ‘yes’
C2:buys_computer = ‘no’

Data sample
X = (age <=30,
Income = medium,
Student = yes
Credit_rating = Fair)



| age | income | student | credit rating | com |
|---|---|---|---|---|
| <=30 | high | no | fair | no |
| <=30 | high | no | excellent | no |
| 31…40 | high | no | fair | yes |
| >40 | medium | no | fair | yes |
| >40 | low | yes | fair | yes |
| >40 | low | yes | excellent | no |
| 31…40 | low | yes | excellent | yes |
| <=30 | medium | no | fair | no |
| <=30 | low | yes | fair | yes |
| >40 | medium | yes | fair | yes |
| <=30 | medium | yes | excellent | yes |
| 31…40 | medium | no | excellent | yes |
| 31…40 | high | yes | fair | yes |
| >40 | medium | no | excellent | no |

# Naïve Bayesian Classifier:  An Example

X = (age <=30, Income = medium, Student = yes,  Credit_rating = Fair)

- Naïve Bayes Classifier $$P(C_i|\mathbf{X}) = \frac{P(\mathbf{X}|C_i)P(C_i)}{P(\mathbf{X})}$$
- P(Buy=Yes|X)    = [P(X|Buy=yes) P(P(buy=yes)]/P(X)
- P(Buy=No|X)      = [P(X|Buy=No) P(P(buy=No)]/P(X)

| age | income | std | c rating | buy |
|---|---|---|---|---|
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| >40 | low | yes | excellent | no |
| 31…40 | low | yes | excellent | yes |
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| <=30 | low | yes | fair | yes |
| >40 | medium | yes | fair | yes |
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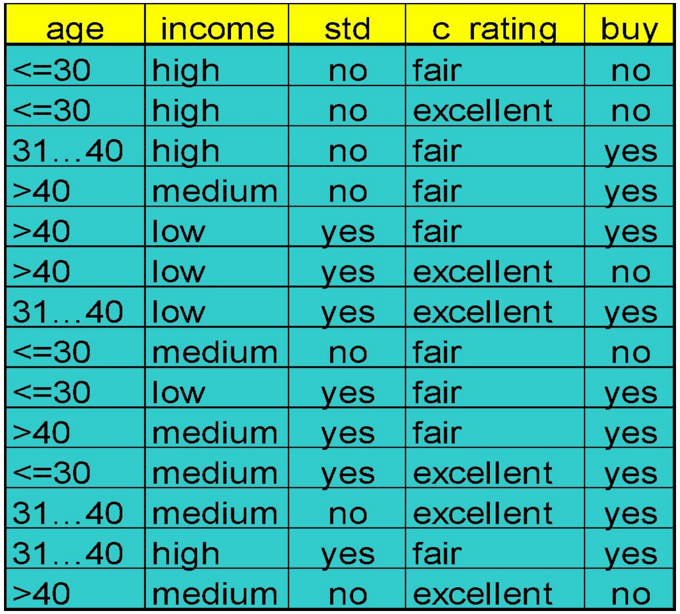
- P(Buy=Yes|X)    = [P(X|Buy=yes) P(P(buy=yes)]/P(X)
- P(Buy=No|X)      = [P(X|Buy=No) P(P(buy=No)]/P(X)

- P(X) would be same for both cases



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# Naïve Bayesian Classifier:  An Example

X = (age <=30, Income = medium, Student = yes,  Credit_rating = Fair)

- Naïve Bayes Classifier $$P(C_i|\mathbf{X}) = \frac{P(\mathbf{X}|C_i)P(C_i)}{P(\mathbf{X})}$$

- P(Buy=Yes|X)    = P(X|Buy=yes) P(buy=yes)
- P(Buy=No|X)      = P(X|Buy=No) P(buy=No)

- P($C_i$):    P(buy = "yes")  = 9/14 = 0.643
  P(buy = "no") = 5/14= 0.357

| age | income | std | c_rating | buy |
|---|---|---|---|---|
| <=30 | high | no | fair | no |
| <=30 | high | no | excellent | no |
| 31…40 | high | no | fair | yes |
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# Naïve Bayesian Classifier: An Example

X = (age <=30, Income = medium, Student = yes, Credit_rating = Fair)

- Naïve Bayes Classifier $$P(C_i|\mathbf{X}) = \frac{P(\mathbf{X}|C_i)P(C_i)}{P(\mathbf{X})}$$
- C = {Yes, No}

- P(Buy=Yes|X) = P(X|Buy=yes) P(buy=yes)
- P(Buy=No|X) = P(X|Buy=No) P(buy=No)

- P($C_i$): P(buy = yes) = 9/14 = 0.643
  P(buy = no) = 5/14= 0.357
- Compute P(X|Buy = yes) for each class
  - P(age = “<=30” | buys_computer = “yes”)
  - P(income = “medium” | buys_computer = “yes”)
  - P(student = “yes” | buys_computer = “yes)
  - P(credit_rating = “fair” | buys_computer = “yes”)

# Naïve Bayesian Classifier:  An Example

X = (age <=30, Income = medium, Student = yes,  Credit_rating = Fair)

- Naïve Bayes Classifier
- C = {Yes, No}

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  - P(age = "<=30" | buys_computer = "yes")
  - P(income = "medium" | buys_computer = "yes")
  - P(student = "yes" | buys_computer = "yes)
  - P(credit_rating = "fair" | buys_computer = "yes")
- Compute P(X|Buy = No) same as above

# Naïve Bayesian Classifier: An Example

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = 0.643
  P(buys_computer = "no") = 5/14= 0.357

- Compute $P(X|C_i)$ for each class

P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222
P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6

P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444
P(income = "medium" | buys_computer = "no") = 2/5 = 0.4

P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667
P(student = "yes" | buys_computer = "no") = 1/5 = 0.2

P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667
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**X = (age <=30, Income = medium, Student = yes, Credit_rating = Fair)**

# Naïve Bayesian Classifier:  An Example

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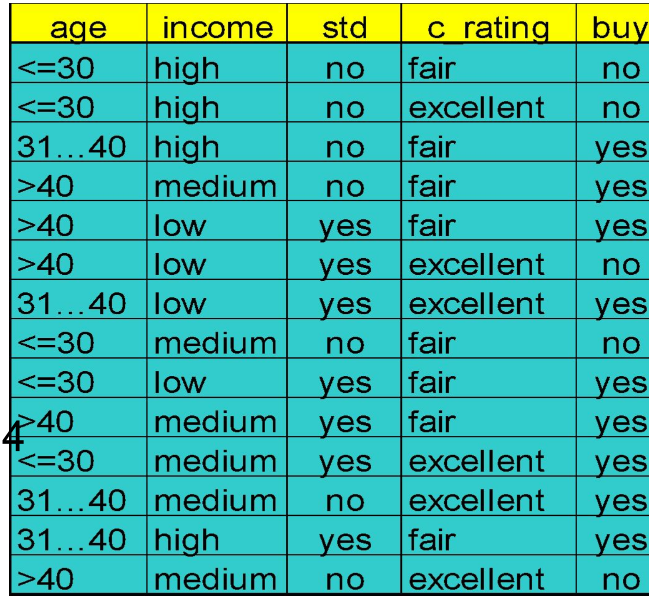

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| <=30 | medium | yes | excellent | yes |
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| >40 | medium | no | excellent | no |

- **X = (age <= 30 , income = medium, student = yes, credit_rating = fair)**

**$P(X|C_i)$ :** P(X|buys_computer = “yes”) = 0.222 x 0.444 x 0.667 x 0.667 = 0.044
P(X|buys_computer = “no”) = 0.6 x 0.4 x 0.2 x 0.4 = 0.019

**$P(X|C_i)*P(C_i)$ : P(X|buys_computer = “yes”) * P(buys_computer = “yes”) = 0.028**
P(X|buys_computer = “no”) * P(buys_computer = “no”) = 0.007

# Avoiding the 0-Probability Problem

- Naïve Bayesian prediction requires each conditional prob. be non-zero. Otherwise, the predicted prob. will be zero

$$P(X|C_i) = \prod_{k=1}^{n} P(x_k|C_i)$$

- Ex. Suppose a dataset with 1000 tuples, income=low (0), income= medium (990), and income = high (10),
- Use Laplacian correction (or Laplacian estimator)
  - Adding 1 to each case

    Prob(income = low) = 1/1003

    Prob(income = medium) = 991/1003

    Prob(income = high) = 11/1003
  - The "corrected" prob. estimates are close to their "uncorrected" counterparts

# Naïve Bayesian Classifier: Comments

- Advantages
  - Easy to implement
  - Good results obtained in most of the cases
- Disadvantages
  - Assumption: class conditional independence, therefore loss of accuracy
  - Practically, dependencies exist among variables
    - E.g., hospitals: patients: Profile: age, family history, etc.

      Symptoms: fever, cough etc., Disease: lung cancer, diabetes, etc.
    - Dependencies among these cannot be modeled by Naïve Bayesian Classifier
- How to deal with these dependencies?
  - Bayesian Belief Networks

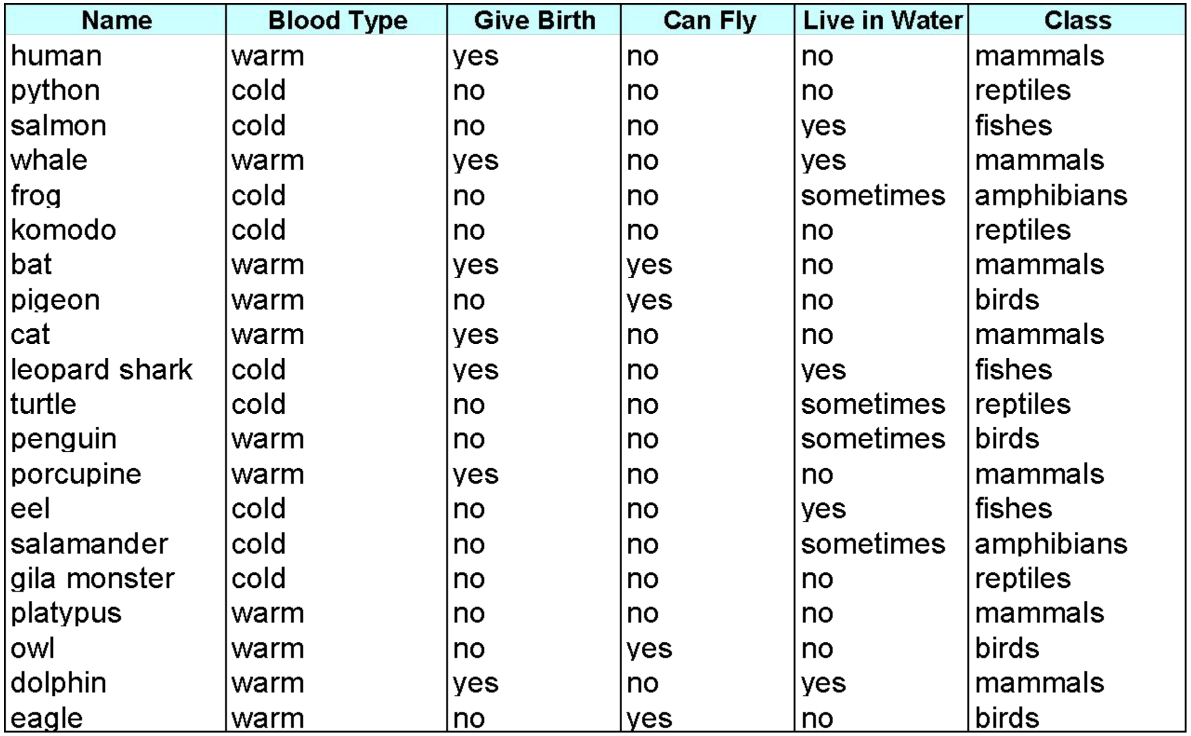
# Rule Based Classifiers

Dr. Faisal Kamiran

# Rule-Based Classifier

- Classify records by using a collection of "if…then…" rules
- Rule: (*Condition*) → *y*
  - where
    - *Condition* is a conjunctions of attributes
    - *y* is the class label
  - *LHS*: rule antecedent or condition
  - *RHS*: rule consequent
  - Examples of classification rules:
    - (Blood Type=Warm) ∧ (Lay Eggs=Yes) → Birds
    - (Taxable Income < 50K) ∧ (Refund=Yes) → Evade=No

# Rule-based Classifier (Example)



| Name | Blood Type | Give Birth | Can Fly | Live in Water | Class |
|---|---|---|---|---|---|
| human | warm | yes | no | no | mammals |
| python | cold | no | no | no | reptiles |
| salmon | cold | no | no | yes | fishes |
| whale | warm | yes | no | yes | mammals |
| frog | cold | no | no | sometimes | amphibians |
| komodo | cold | no | no | no | reptiles |
| bat | warm | yes | yes | no | mammals |
| pigeon | warm | no | yes | no | birds |
| cat | warm | yes | no | no | mammals |
| leopard shark | cold | yes | no | yes | fishes |
| turtle | cold | no | no | sometimes | reptiles |
| penguin | warm | no | no | sometimes | birds |
| porcupine | warm | yes | no | no | mammals |
| eel | cold | no | no | yes | fishes |
| salamander | cold | no | no | sometimes | amphibians |
| gila monster | cold | no | no | no | reptiles |
| platypus | warm | no | no | no | mammals |
| owl | warm | no | yes | no | birds |
| dolphin | warm | yes | no | yes | mammals |
| eagle | warm | no | yes | no | birds |

R1: (Give Birth = no) $\land$ (Can Fly = yes) → Birds

R2: (Give Birth = no) $\land$ (Live in Water = yes) → Fishes

R3: (Give Birth = yes) $\land$ (Blood Type = warm) → Mammals

R4: (Give Birth = no) $\land$ (Can Fly = no) → Reptiles

# Application of Rule-Based Classifier

- A rule *r* covers an instance **x** if the attributes of the instance satisfy the condition of the rule

R1: (Give Birth = no) ∧ (Can Fly = yes) → Birds

R2: (Give Birth = no) ∧ (Live in Water = yes) → Fishes

R3: (Give Birth = yes) ∧ (Blood Type = warm) → Mammals

R4: (Give Birth = no) ∧ (Can Fly = no) → Reptiles

R5: (Live in Water = sometimes) → Amphibians

| Name | Blood Type | Give Birth | Can Fly | Live in Water | Class |
|---|---|---|---|---|---|
| hawk | warm | no | yes | no | ? |
| grizzly bear | warm | yes | no | no | ? |

The rule R1 covers a hawk => Bird

The rule R3 covers the grizzly bear => Mammal

# Rule Coverage and Accuracy

- Coverage of a rule:
  - Fraction of records that satisfy the antecedent of a rule
- Accuracy of a rule:
  - Fraction of records that satisfy both the antecedent and consequent of a rule

| *Tid* | Refund | Marital Status | Taxable Income | Class |
|---|---|---|---|---|
| 1 | Yes | Single | 125K | No |
| 2 | No | Married | 100K | No |
| 3 | No | Single | 70K | No |
| 4 | Yes | Married | 120K | No |
| 5 | No | Divorced | 95K | Yes |
| 6 | No | Married | 60K | No |
| 7 | Yes | Divorced | 220K | No |
| 8 | No | Single | 85K | Yes |
| 9 | No | Married | 75K | No |
| 10 | No | Single | 90K | Yes |

**(Status=Single) → No**

**Coverage = 40%, Accuracy = 50%**

# How does Rule-based Classifier Work?

R1: (Give Birth = no) ∧ (Can Fly = yes) → Birds

R2: (Give Birth = no) ∧ (Live in Water = yes) → Fishes

R3: (Give Birth = yes) ∧ (Blood Type = warm) → Mammals

R4: (Give Birth = no) ∧ (Can Fly = no) → Reptiles

R5: (Live in Water = sometimes) → Amphibians

| Name | Blood Type | Give Birth | Can Fly | Live in Water | Class |
|---|---|---|---|---|---|
| lemur | warm | yes | no | no | ? |
| turtle | cold | no | no | sometimes | ? |
| dogfish shark | cold | yes | no | yes | ? |

A lemur triggers rule R3, so it is classified as a mammal

A turtle triggers both R4 and R5

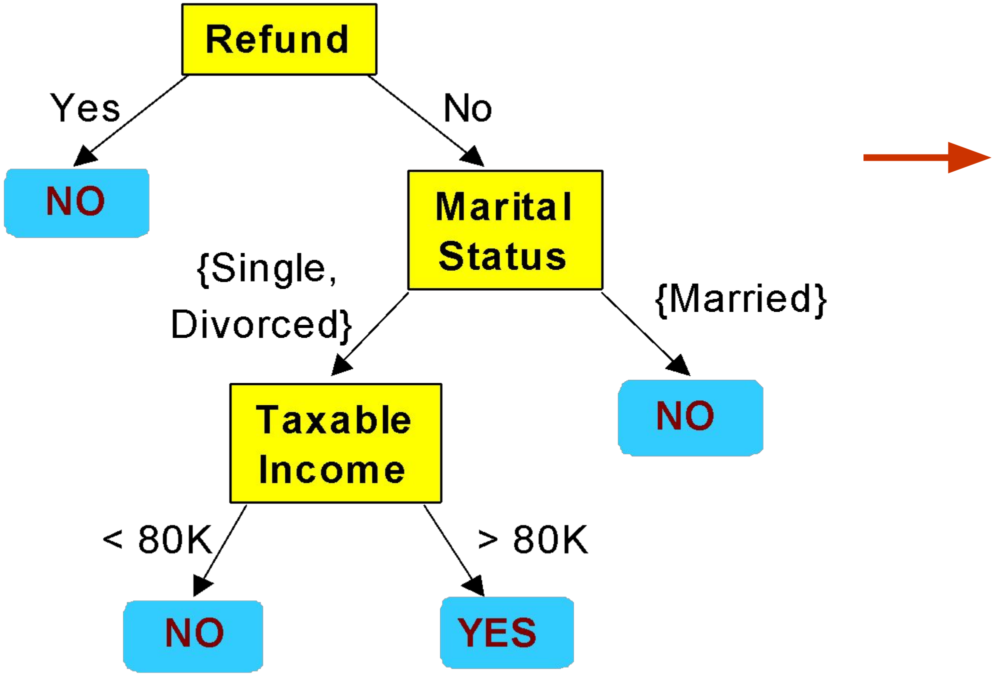
A dogfish shark triggers none of the rules

# Characteristics of Rule-Based Classifier

- Mutually exclusive rules
  - Classifier contains mutually exclusive rules if the rules are independent of each other
  - Every record is covered by at most one rule

- Exhaustive rules
  - Classifier has exhaustive coverage if it accounts for every possible combination of attribute values
  - Each record is covered by at least one rule

# From Decision Trees To Rules



## Classification Rules

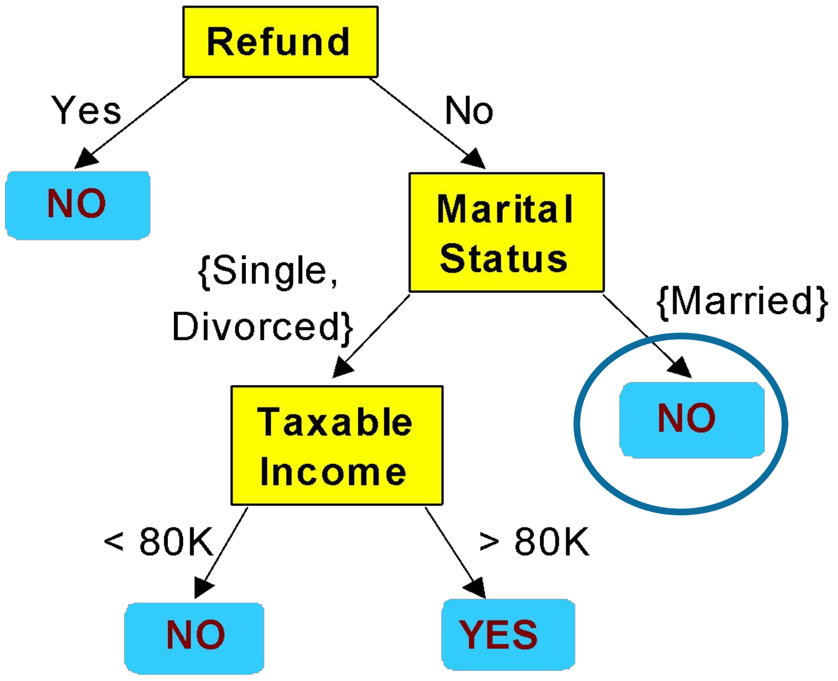
(Refund=Yes) ==> No

(Refund=No, Marital Status={Single,Divorced}, Taxable Income<80K) ==> No

(Refund=No, Marital Status={Single,Divorced}, Taxable Income>80K) ==> Yes

(Refund=No, Marital Status={Married}) ==> No

# Rules Can Be Simplified



| *Tid* | Refund | Marital Status | Taxable Income | Cheat |
|---|---|---|---|---|
| 1 | Yes | Single | 125K | No |
| 2 | No | Married | 100K | No |
| 3 | No | Single | 70K | No |
| 4 | Yes | Married | 120K | No |
| 5 | No | Divorced | 95K | Yes |
| 6 | No | Married | 60K | No |
| 7 | Yes | Divorced | 220K | No |
| 8 | No | Single | 85K | Yes |
| 9 | No | Married | 75K | No |
| 10 | No | Single | 90K | Yes |

**Initial Rule:** (Refund=No) $\wedge$ (Status=Married) → No

**Simplified Rule:** (Status=Married) → No

# Ordered Rule Set

- Rules are rank ordered according to their priority
  - An ordered rule set is known as a decision list
- When a test record is presented to the classifier
  - It is assigned to the class label of the highest ranked rule it has triggered
  - If none of the rules fired, it is assigned to the default class

R1: (Give Birth = no) ∧ (Can Fly = yes) → Birds

R2: (Give Birth = no) ∧ (Live in Water = yes) → Fishes

R3: (Give Birth = yes) ∧ (Blood Type = warm) → Mammals

R4: (Give Birth = no) ∧ (Can Fly = no) → Reptiles

R5: (Live in Water = sometimes) → Amphibians

| Name | Blood Type | Give Birth | Can Fly | Live in Water | Class |
|---|---|---|---|---|---|
| turtle | cold | no | no | sometimes | ? |

# Rule Ordering Schemes

- Size based ordering
  - High priority to the rules with toughest requirement
  - Toughness is measured by the size of antecedent
- Rule-based ordering
  - Individual rules are ranked based on their quality, i.e., accuracy, coverage, size and advice from the domain experts.
- Class-based ordering
  - Priority to the rules with most prevalent classes or most rare classes

# Building Classification Rules

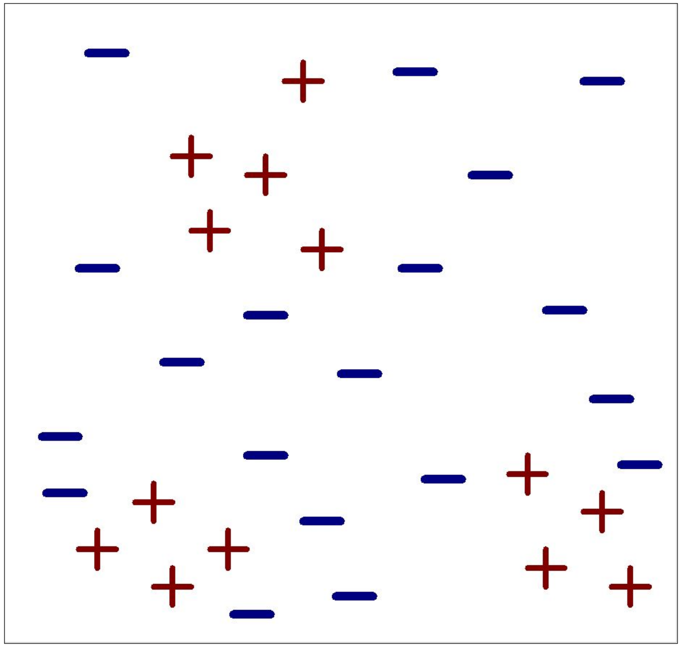
- Direct Method:
  - Extract rules directly from data
  - e.g.: RIPPER, CN2, Holte’s 1R

- Indirect Method:
  - Extract rules from other classification models (e.g. decision trees, neural networks, etc).
  - e.g: C4.5 rules

# Direct Method: Sequential Covering

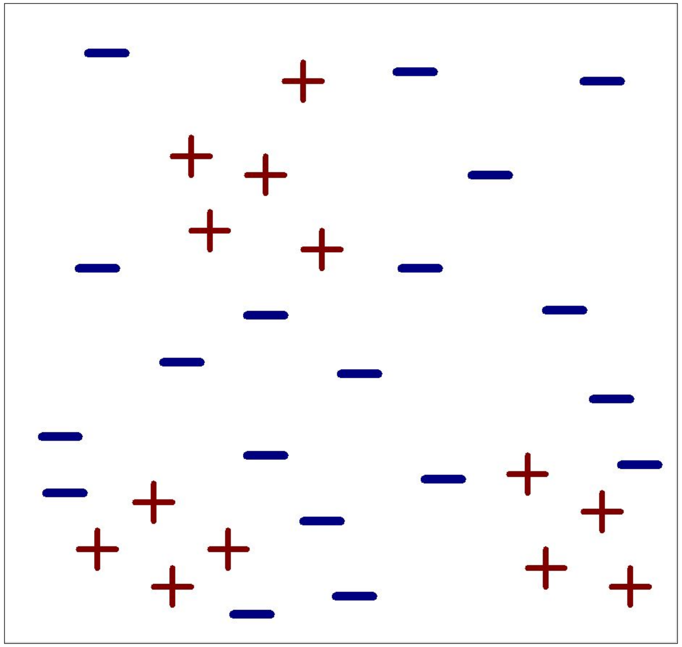
1. Start from an empty rule
2. Grow a rule using the Learn-One-Rule function
3. Remove training records covered by the rule
4. Repeat Step (2) and (3) until stopping criterion is met

# Example of Sequential Covering

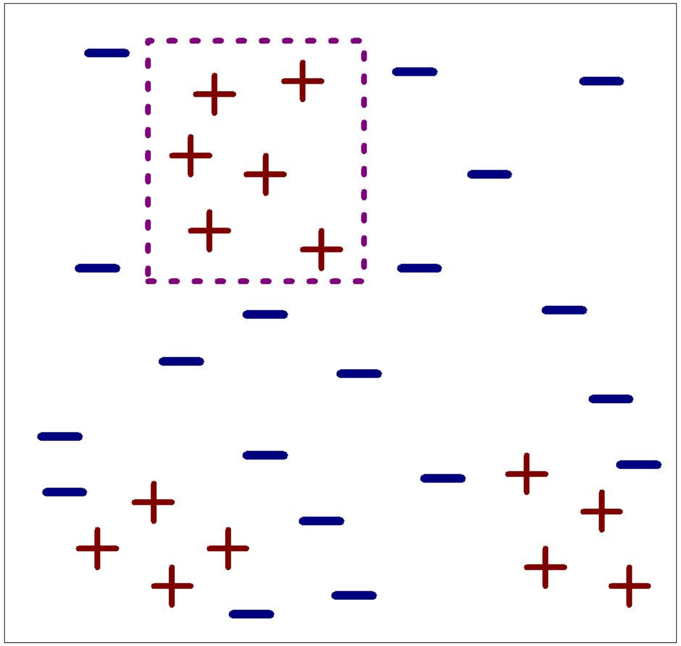


(i) Original Data

# Example of Sequential Covering

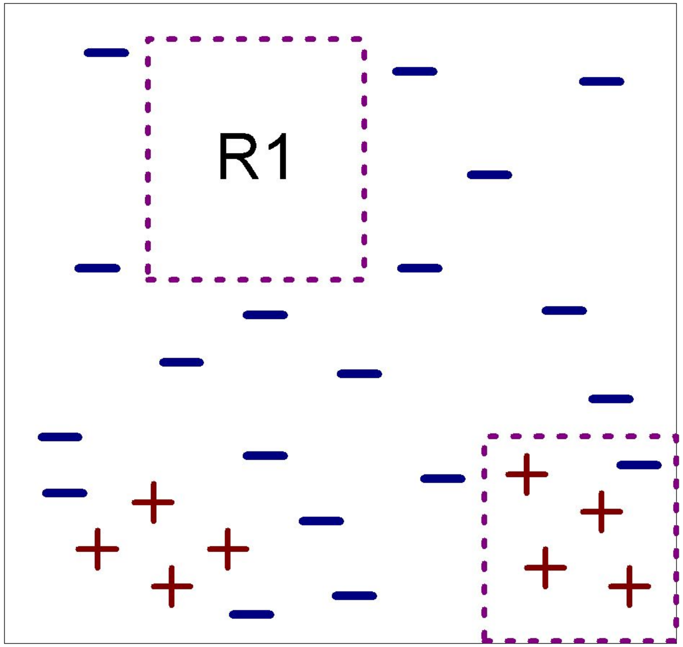


(i) Original Data



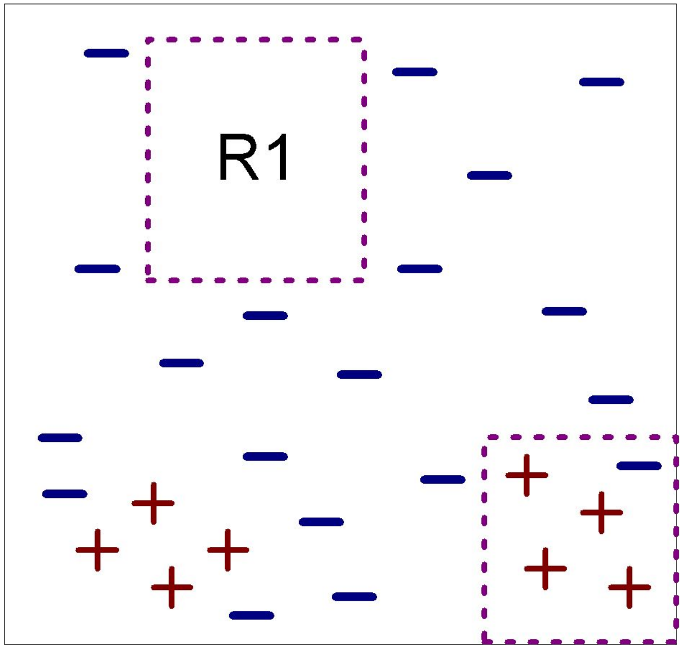
(ii) Step 1

# Example of Sequential Covering…

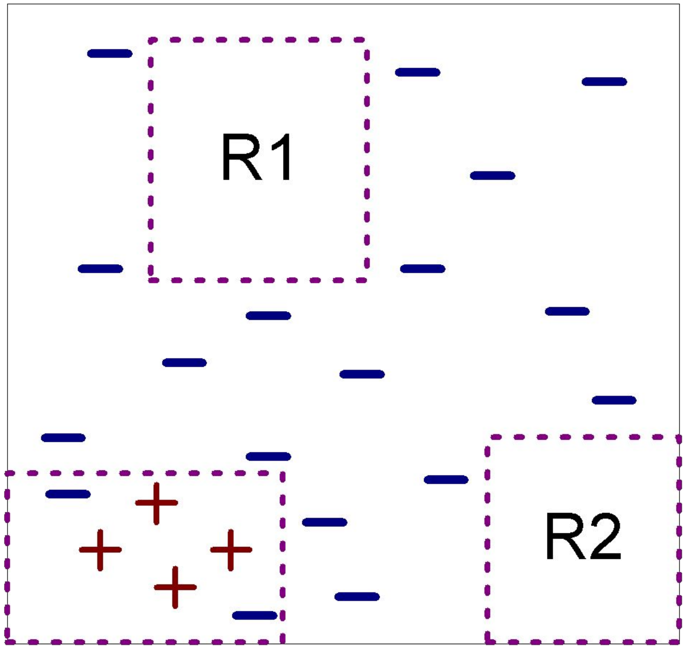


(iii) Step 2

# Example of Sequential Covering…



(iii) Step 2



(iv) Step 3