
Association Rule Mining -

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Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

$\{\text{Diaper}\} \rightarrow \{\text{Beer}\},$
 $\{\text{Milk, Bread}\} \rightarrow \{\text{Eggs, Coke}\},$
 $\{\text{Beer, Bread}\} \rightarrow \{\text{Milk}\},$

Implication means co-occurrence,
not causality!

Definition: Frequent Itemset

- **Itemset**
 - A collection of one or more items
 - ◆ Example: {Milk, Bread, Diaper}
 - k-itemset
 - ◆ An itemset that contains k items
- **Support count (σ)**
 - Frequency of occurrence of an itemset
 - E.g. $\sigma(\{\text{Milk, Bread, Diaper}\}) = 2$
- **Support**
 - Fraction of transactions that contain an itemset
 - E.g. $s(\{\text{Milk, Bread, Diaper}\}) = 2/5$
- **Frequent Itemset**
 - An itemset whose support is greater than or equal to a *minsup* threshold

<i>TID</i>	<i>Items</i>
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4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

- **Association Rule**

- An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
- Example:
 $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

<i>TID</i>	<i>Items</i>
1	Bread, Milk
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- **Rule Evaluation Metrics**

- Support (s)
 - ◆ Fraction of transactions that contain both X and Y
- Confidence (c)
 - ◆ Measures how often items in Y appear in transactions that contain X

Example:

$\{\text{Milk, Diaper}\} \Rightarrow \text{Beer}$

$$s = \frac{\sigma(\text{Milk, Diaper, Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk, Diaper, Beer})}{\sigma(\text{Milk, Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

- Given a set of transactions T , the goal of association rule mining is to find all rules having
 - support $\geq \textit{minsup}$ threshold
 - confidence $\geq \textit{minconf}$ threshold
- Brute-force approach:
 - List all possible association rules
 - Compute the support and confidence for each rule
 - Prune rules that fail the *minsup* and *minconf* thresholds

⇒ **Computationally prohibitive!**

Mining Association Rules

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

$\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$ ($s=0.4$, $c=0.67$)
 $\{\text{Milk, Beer}\} \rightarrow \{\text{Diaper}\}$ ($s=0.4$, $c=1.0$)
 $\{\text{Diaper, Beer}\} \rightarrow \{\text{Milk}\}$ ($s=0.4$, $c=0.67$)
 $\{\text{Beer}\} \rightarrow \{\text{Milk, Diaper}\}$ ($s=0.4$, $c=0.67$)
 $\{\text{Diaper}\} \rightarrow \{\text{Milk, Beer}\}$ ($s=0.4$, $c=0.5$)
 $\{\text{Milk}\} \rightarrow \{\text{Diaper, Beer}\}$ ($s=0.4$, $c=0.5$)

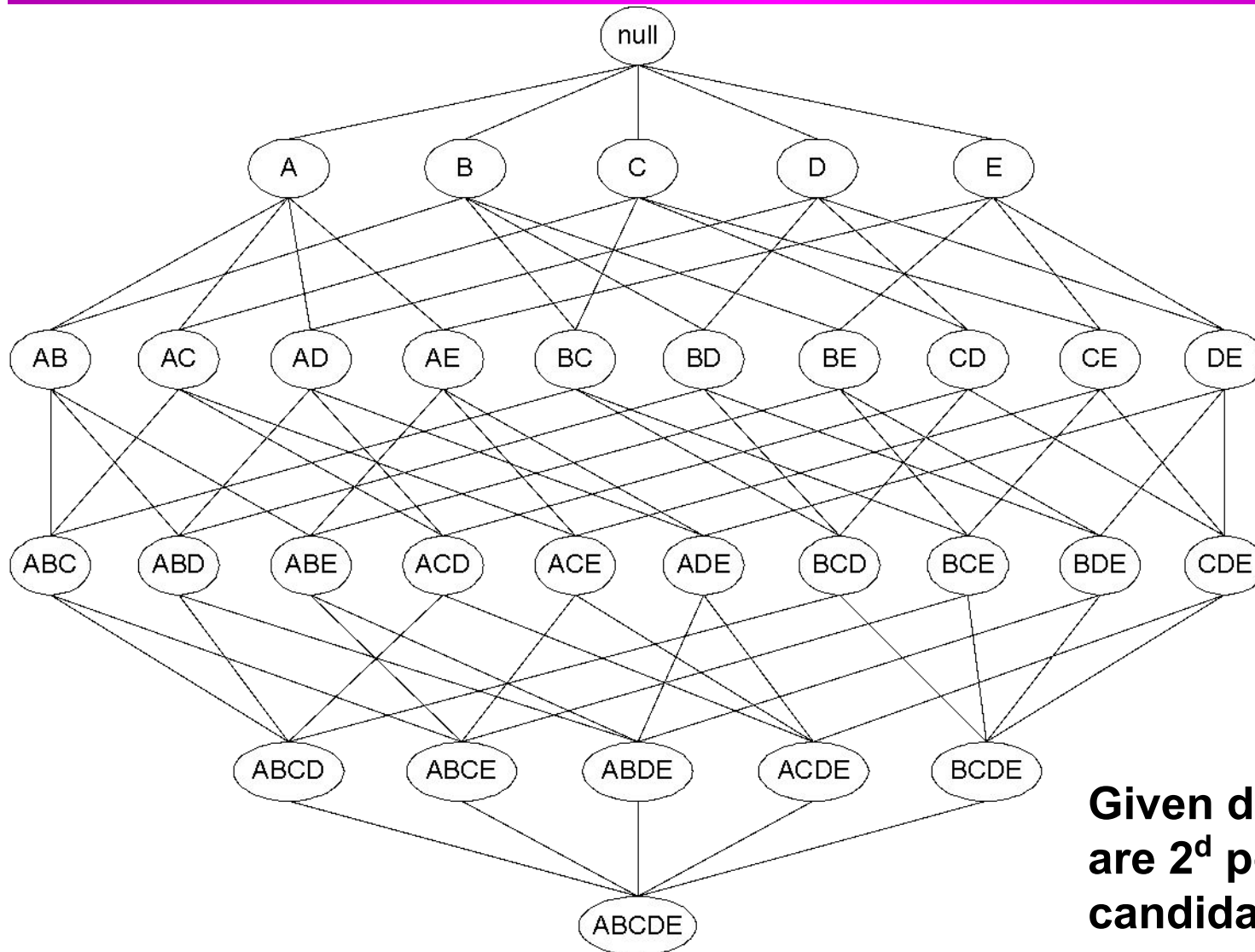
Observations:

- All the above rules are binary partitions of the same itemset:
 $\{\text{Milk, Diaper, Beer}\}$
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- Two-step approach:
 1. Frequent Itemset Generation
 - Generate all itemsets whose support \geq minsup
 2. Rule Generation
 - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

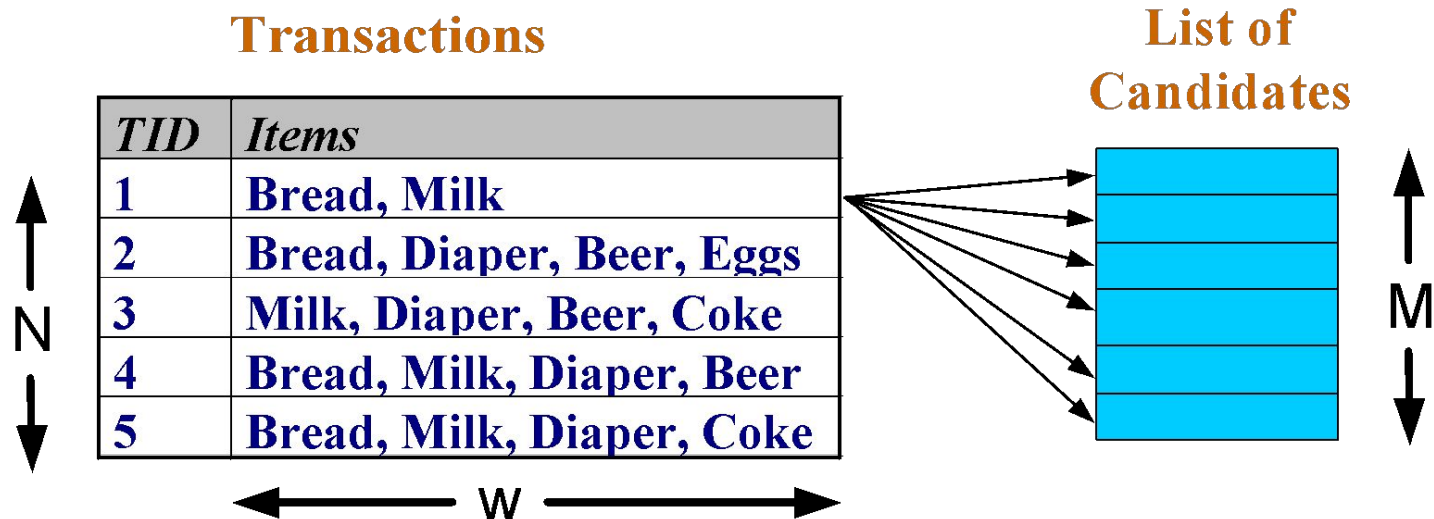
Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

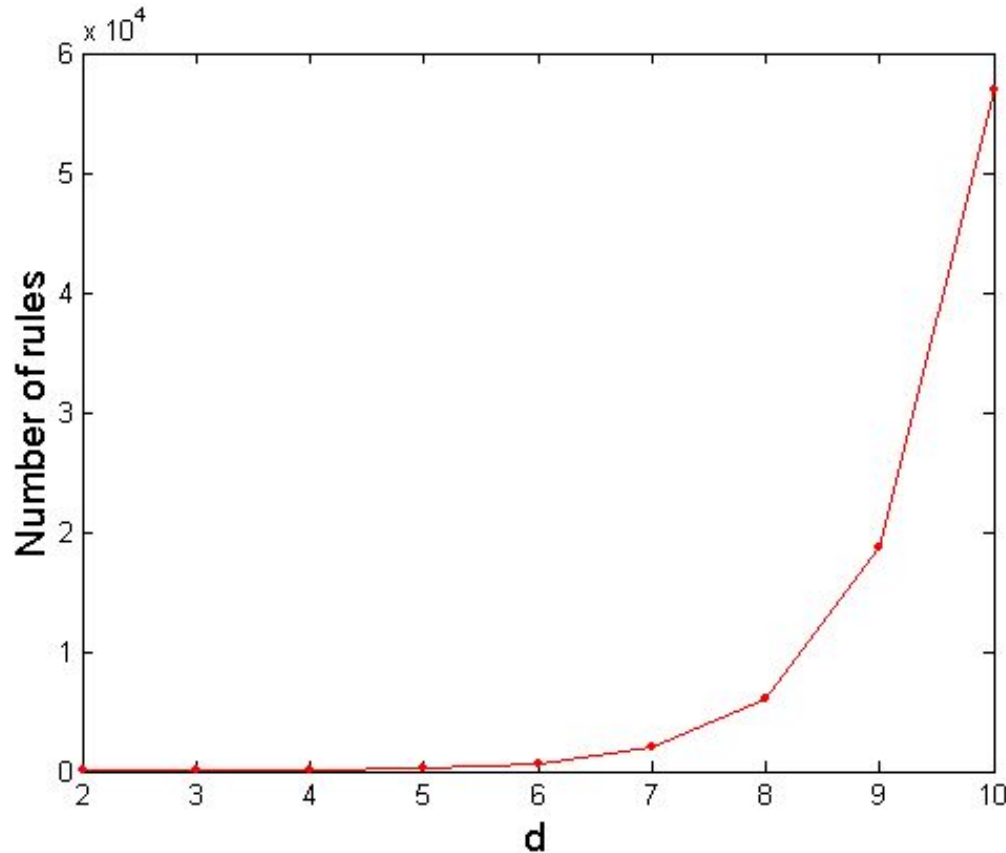
- Brute-force approach:
 - Each itemset in the lattice is a **candidate** frequent itemset
 - Count the support of each candidate by scanning the database



- Match each transaction against every candidate
- Complexity $\sim O(NMw) \Rightarrow$ **Expensive since $M = 2^d$!!!**

Computational Complexity

- Given d unique items:
 - Total number of itemsets = 2^d
 - Total number of possible association rules:



$$= 3^d - 2^{d+1} + 1$$

If $d=6$, $R = 602$ rules

Frequent Itemset Generation Strategies

- Reduce the **number of candidates** (M)
 - Complete search: $M=2^d$
 - Use pruning techniques to reduce M
- Reduce the **number of transactions** (N)
 - Reduce size of N as the size of itemset increases
 - Used by DHP and vertical-based mining algorithms
- Reduce the **number of comparisons** (NM)
 - Use efficient data structures to store the candidates or transactions
 - No need to match every candidate against every transaction

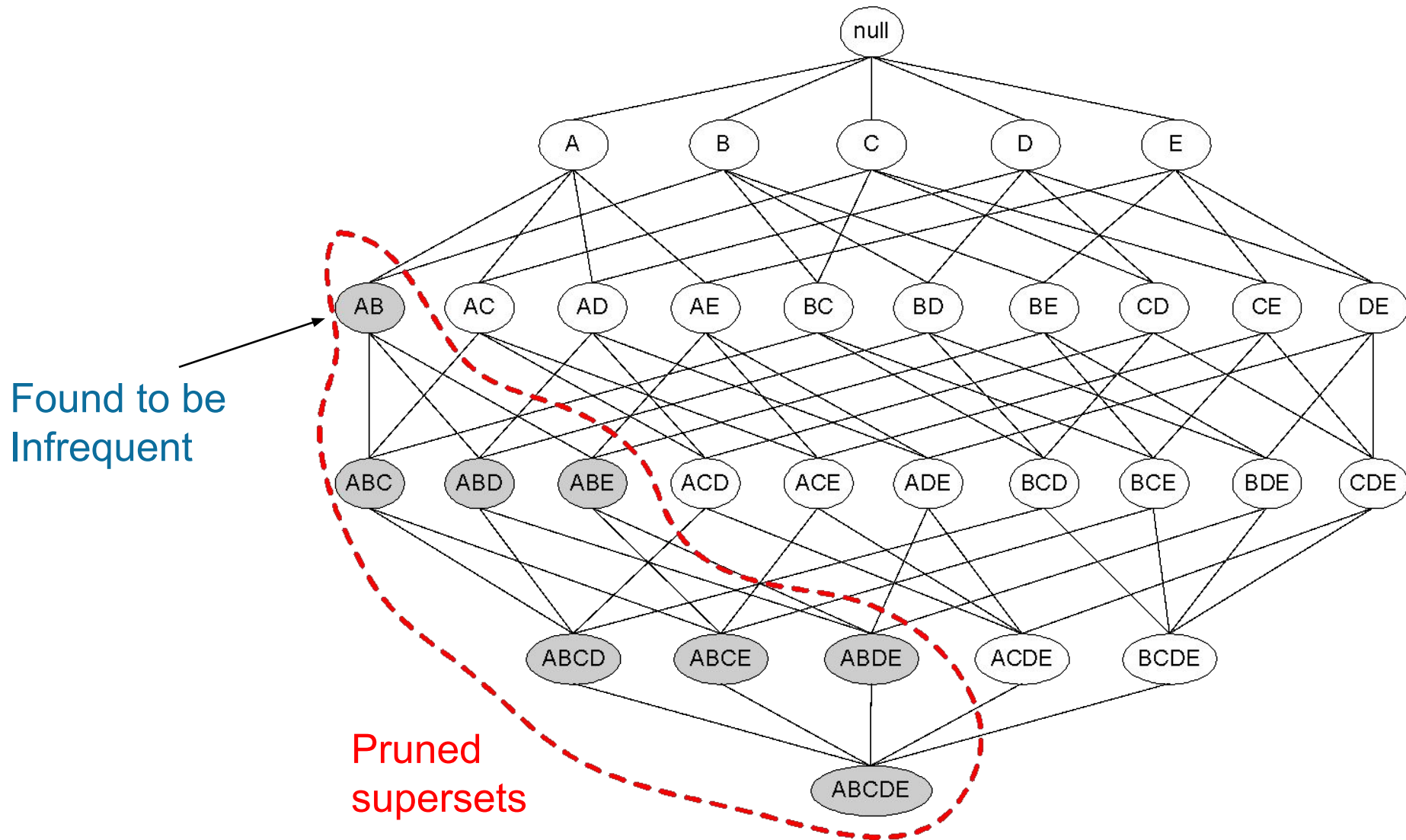
Reducing Number of Candidates

- **Apriori principle:**
 - If an itemset is frequent, then all of its subsets must also be frequent
- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the **anti-monotone** property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)



Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,
 ${}^6C_1 + {}^6C_2 + {}^6C_3 = 41$
 With support-based pruning,
 $6 + 6 + 1 = 13$



Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3



What about {**Beer**, Diaper, **Milk**} ?
 And, {Bread, **Milk**, **Beer**}?
 And, {**Bread**, Diaper, **Beer**}?

Apriori Algorithm

- Method:
 - Let $k=1$
 - Generate frequent itemsets of length 1
 - Repeat until no new frequent itemsets are identified
 - ◆ Generate length $(k+1)$ candidate itemsets from length k frequent itemsets
 - ◆ Count the support of each candidate by scanning the DB
 - ◆ Eliminate candidates that are infrequent, leaving only those that are frequent

The Apriori Algorithm—An Example

$\text{Sup}_{\min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

The Apriori Algorithm—An Example

$\text{Sup}_{\min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

C_1

1st scan

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

The Apriori Algorithm—An Example

$\text{Sup}_{\min} = 2$

Database TDB

Tid	Items
10	A, C, D
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30	A, B, C, E
40	B, E

C_1

1st scan

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

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40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

The Apriori Algorithm—An Example

$\text{Sup}_{\min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

2nd scan

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

The Apriori Algorithm—An Example

$\text{Sup}_{\min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

2nd scan

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

The Apriori Algorithm—An Example

$\text{Sup}_{\min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

2nd scan

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

The Apriori Algorithm—An Example

$\text{Sup}_{\min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

2nd scan

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

3rd scan

L_3

Itemset	sup
{B, C, E}	2

C_3

Itemset
{B, C, E}