

IRAN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Discrete mathematics Problem Set #3

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1 Give the predicates

In each of the following, write a statement in first-order logic that expresses the indicated sentence. Your statement may use any first-order construct (equality, connectives, quantifiers, etc.), but you must only use the predicates, functions, and constants provided. You do not need to provide the simplest formula possible, though we'd appreciate it if you made an effort to do so.

- i. Given the predicates

$Natural(x)$, which states that x is a natural number

and the functions

$x + y$, which represents the sum of x and y , and

$x.y$, which represents the product of x and y

write a statement in first-order logic that says "for any $n \in \mathbb{N}$, n is even if and only if n^2 is even."

Answer. ANSWER HERE.

- ii. Given the predicate

$Person(p)$, which states that p is a person;

$Kitten(k)$, which states that k is a kitten; and

$HasPet(o, p)$, which states that o has p as a pet,

write an FOL statement that says "someone has exactly two pet kittens and no other pets."

Answer. ANSWER HERE.

- iii. The **axiom of pairing** is the following statement: given any two distinct objects x and y , there's a set containing x and y and nothing else. Given the predicates

$x \in y$, which states that x is an element of y , and

$Set(S)$, which states that S is a set,

write a statement in first-order logic that expresses the axiom of pairing.

Answer. ANSWER HERE.

iv. Given the predicates

$x \in y$, which states that x is an element of y , and

$Set(S)$, which states that S is a set,

write a statement in first-order logic that says "every set has a power set."

Answer. ANSWER HERE.

v. Given the predicates

$Lady(x)$, which states that x is a lady;

$Glitters(x)$, which states that x glitters;

$SureIsGold(x, v)$, which states that x is sure that y is gold;

$Buying(x, v)$, which states that x buys y ; and

$StuirwayToHeaven(A)$, which states that x is a Stairway to Heaven;

write a statement in first-order logic that says "there's a lady who's sure all that glitters is gold, and she's buying a Stairway to Heaven."

Answer. ANSWER HERE.

2 Consistent vocabulary

Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

a. Some students took French in spring 2001.

Answer.

$$\exists x, \exists y : Student(x) \wedge French(y) \wedge TakeInSpring2001(x, y)$$

b. Every student who takes French passes it.

Answer.

$$\forall x, \forall y : (Student(x) \wedge French(y) \wedge Take(x, y)) \Rightarrow Pass(x, y)$$

c. Only one student took Greek in spring 2001.

Answer.

$\exists x, \exists y, \forall z :$

$$Student(x) \wedge Greek(y) \wedge TakeInSpring2001 \wedge \underbrace{(Student(z) \wedge TakeInSpring2001(z, y))}_{x=z}$$

- d. The best score in Greek is always higher than the best score in French.

Answer. ANSWER HERE.

- e. Every person who buys a policy is smart.

Answer. ANSWER HERE.

- f. No person buys an expensive policy.

Answer. ANSWER HERE.

- g. There is an agent who sells policies only to people who are not insured.

Answer. ANSWER HERE.

- h. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

Answer. ANSWER HERE.

- i. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

Answer. ANSWER HERE.

- j. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

Answer. ANSWER HERE.

3 Germans

Represent the sentence "All Germans speak the same languages" in predicate calculus. Use $Speaks(x, l)$, meaning that person x speaks language l , and $German(y)$, meaning that y is a *German* person.

Answer.

$$\forall x, y, l \quad (German(x) \wedge German(y) \wedge Speaks(x, l) \Rightarrow Speaks(y, l))$$

or

$$\forall x, y \quad (German(x) \wedge German(y) \Rightarrow \forall l \quad (Speaks(x, l) \Leftrightarrow Speaks(y, l)))$$

4 Jim & Laura

What axiom is needed to infer the fact $Female(Laura)$ given the facts $Male(Jim)$ and $Spouse(Jim, Laura)$?

Answer. ANSWER HERE.

5 Describing the predicates

Write axioms describing the predicates: $GrandChild$, $GreatGrandparent$, $Brother$, $Sister$, $Daughter$, Son , $Aunt$, $Uncle$, $BrotherInLaw$, $SisterInLaw$ and $FirstCousin$. Find out the proper definition of math cousin n times removed, and write the definition in first-order logic.

Answer.

- $GrandChild(a, b) \Rightarrow parent(b, x) \wedge parent(x, a)$
- $GreatGrandparent(a, b) \Rightarrow parent(a, x) \wedge GrandChild(b, x)$
- The Sibling relationship is added to make the expression of some future relationships simpler. In this situation, Sibling encompasses full, half and step siblings.
 - $Sibling(a, b) \Rightarrow parent(x, a) \wedge parent(x, b) \wedge not_equal(a, b)$
 - $Sibling(a, b) \Rightarrow Sibling(b, a)$
- $Brother(a, b) \Rightarrow Sibling(a, b) \wedge gender(a, 'male')$
- $Sister(a, b) \Rightarrow Sibling(a, b) \wedge gender(a, 'female')$
- $Son(a, b) \Rightarrow parent(b, a) \wedge gender(a, 'male')$
- $Daughter(a, b) \Rightarrow parent(b, a) \wedge gender(a, 'female')$
- $Uncle(a, b) \Rightarrow parent(x, b) \wedge Sibling(x, a) \wedge gender(a, 'male')$
- $Aunt(a, b) \Rightarrow parent(x, b) \wedge Sibling(x, a) \wedge gender(a, 'female')$
- married is a primitive relation, meaning it is not defined in terms of any other relations. It is necessary however to note that it is reflexive.
 - $married(a, b) \Rightarrow married(b, a)$
- $BrotherInLaw(a, b) \Rightarrow married(b, x) \wedge Sibling(a, x) \wedge gender(a, 'male')$
- $SisterInLaw(a, b) \Rightarrow married(b, x) \wedge Sibling(a, x) \wedge gender(a, 'female')$
- $FirstCousin(a, b) \Rightarrow parent(x, a) \wedge parent(y, b) \wedge Sibling(x, y)$

6 Humanoid wolf

QUESTION HERE.

i. ITEM

Answer. ANSWER HERE.

ii. ITEM

Answer. ANSWER HERE.

iii. ITEM

Answer. ANSWER HERE.

iv. ITEM

Answer. ANSWER HERE.

v. ITEM

Answer. ANSWER HERE.

vi. ITEM

Answer. ANSWER HERE.