Iran university of science and technology

Discrete mathematics
Problem Set #2

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1 Connected graph

QUESTION HERE.

Theorem 1.1. A graph is connected if and only if for every partition of its vertices into two nonempty sets, there is an edge with endpoints in both sets.

Answer. ANSWER HERE.

2 n-vertex graph

QUESTION HERE.

Theorem 2.1. Every n-vertex graph with at least n edges contains a cycle.

Answer. ANSWER HERE.

3 connected simple n-vertex graph

QUESTION HERE.

Theorem 3.1. If l, m, n are nonnegative integers with l + m = n âĽě 1, then there exists a connected simple nvertex graph with l vertices of even degree and m vertices of odd degree if and only if m is even, except for (l, m, n) = (2, 0, 2).

Answer. ANSWER HERE.

4 k_cube graph

what is k_cube(Qk) graph? Count number of its Vertices and Edges and prove that it is bipartite.

Answer. *Proof.* PROOF HERE. □

5 Diameter of graph

diameter of G is length of the longest path between two vertices in it. Show that if GâĂŹs diameter is greater than 3 its complementâĂŹs diameter would be less than 3.

Answer. *Proof.* PROOF HERE. □

6 title

Necessary and sufficient conditions for a list d to be graphic when d consists of k copies of a and n âĹŠ k copies of b, with a âĽě b âĽě 0. Since the degree sum must be even, the quantity k a + (n âĹŠ k)b must be even. In addition, the inequality k a âĽď k(k âĹŠ 1) + (n âĹŠ k) mink, b must hold, since eac h vertex with degree b has at most mink, b incident edges whose other endpoint has degree a. We construct graphs with the desired degree sequence when these conditions hold. Note that the inequality implies a âĽď n âĹŠ 1.

Answer. *Proof.* PROOF HERE. □

7 Graph orientation

QUESTION HERE.

Theorem 7.1. Every graph G has an orientation such that |d +(v) âĹŠ dâĹŠ(v)| âĽď 1 for all v.

Answer. ANSWER HERE.

8 TITLE HERE

A strong orientation of a graph that has an odd cycle also has an odd (directed) cycle. Suppose that D is a strong orientation of a graph G that has an odd cycle v1, . . . , v2k+1. Since D is strongly connected, for eac h i there is a vi, vi+1-path in D. If for some i every suc h path has even length, then the edge between vi and vi+1 points from vi+1 to vi, since the other orientation would be a vi, vi+1-path of length 1 (odd). In this case, we have an odd cycle through vi and vi+1. Otherwise, we have a path of odd length from eac h vi to vi+1. Combining these gives a closed trail of odd length. In a digraph as well as in a graph (by the same proof), a closed odd trail contains the edges of an odd cycle

Answer. ANSWER HERE.

9 TITLE HERE

Tournaments with all players kings. a) If n is odd, then there is an tournament with n vertices suc h that every player is a king.

Answer. ANSWER HERE.

10 TITLE HERE

For a tree T with vertex degrees in 1, k, the possible values of n(T) are the positive integers that are 2 more than a multiple of k âĹŠ 1.

Answer. ANSWER HERE.

11 TITLE HERE

QUESTION HERE.

Theorem 11.1. A tree has exactly one center or has two adjacent centers.

Answer. ANSWER HERE.

Theorem 11.2. A tree has exactly one center if and only if its diameter is twice its radius.

Answer. ANSWER HERE.

12 TITLE HERE

QUESTION HERE.

Theorem 12.1. e = xy is a cutting edge if and only if a) G-e does not have ha xâĘŠy path. b) it does not belong to any cycle.

Answer. ANSWER HERE.

13 TITLE HERE

how many 5-cycles does peterson graph have?

Answer. ANSWER HERE.

14 TITLE HERE

A certain bridge club has a special rule to the effect thal four members may play together only if no two of them have previously partnered one another. At one meeting fourteen members, each of whom has previously partnered five others, turn up. Three games are played, and then proceedings come to a halt because of the club rule. Just as the members are preparing to leave, a new member, unknown to any of them, arrives. Show that at least on'e more g'ame can now be played..

Answer. ANSWER HERE.