Iran university of science and technology

Discrete mathematics
Problem Set #4

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May 4, 2018

1 Prove the sequence

Consider the sequence defined recursively by:

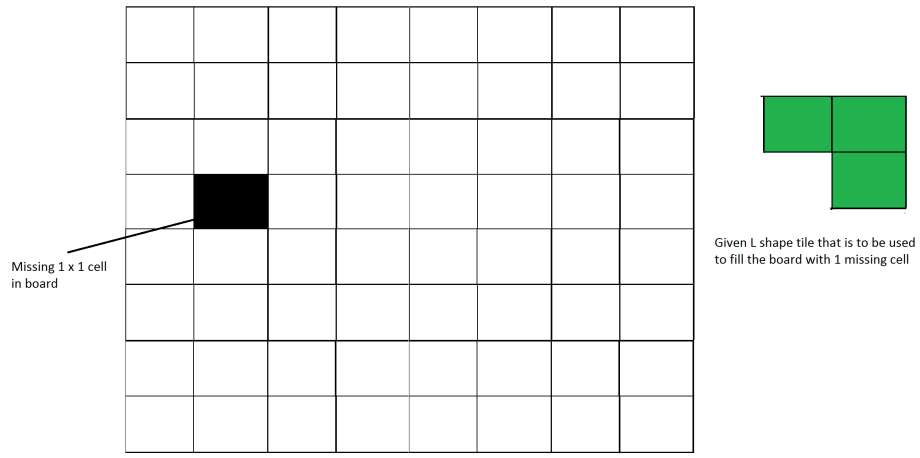
$$a_0 = 1$$
$$a_n = a_{n-1} + a_{n-2} + \cdots + a_0 + 1$$

Prove that $a_n = 2^n$ by strong induction.

Answer. *Proof.* PROOF HERE. □

2 Fill the board using L shaped tiles

Given a n by n board where n is of form $2k$ where $k \geq 1$ (Basically n is a power of 2 with minimum value as 2). The board has one missing cell (of size 1×1). Prove that the board can be filled using L shaped tiles.
A L shaped tile is 2×2 square with one cell of size 1×1 missing.



Answer. *Proof.* PROOF HERE. □

3 Prime number and positive number

prove that every positive integer n, $n \geq 2$, can be expressed as the product of one or more prime numbers.

Answer. *Proof.* PROOF HERE. □

4 The game

In a game , there are two players and two piles of matches. At each turn, a player removes some (non-zero) number of matches from one of the piles. The player who removes the last match wins.
Prove that if the two piles contain the same number of matches at the start of the game, then the second player can always win.

Answer. *Proof.* PROOF HERE. □

5 ATM Machine

Suppose an ATM machine has only \$2 and \$5 bills. Claim: The ATM can generate any output amount $n \geq 4$. Prove the claim.

Answer. *Proof.* PROOF HERE. □

6 Colored computers

We have n computers ($n \geq 4$), each pair of them are connected by a wire, each wire is colored in blue, red, yellow or green. At least there is a wire colored in each color.
Prove that we can choose a number of computers so that the wires connecting computers, include exactly 3 different colors.

Answer. *Proof.* PROOF HERE. □