



IRAN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Discrete mathematics
Problem Set #5

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1 Connected graph

Prove the theorem below.

Theorem 1.1. A graph is connected if and only if for every partition of its vertices into two nonempty sets, there is an edge with endpoints in both sets.

Answer. ANSWER HERE.

2 n -vertex graph

Prove the theorem below.

Theorem 2.1. Every n -vertex graph with at least n edges contains a cycle.

Answer. ANSWER HERE.

3 Connected simple n -vertex graph

Prove the theorem below.

Theorem 3.1. If l, m, n are nonnegative integers with $l + m = n \geq 1$, then there exists a connected simple n -vertex graph with l vertices of even degree and m vertices of odd degree if and only if m is even, except for $(l, m, n) = (2, 0, 2)$.

Answer. ANSWER HERE.

4 k_cube graph

what is $k_cube(Q_k)$ graph?

Count number of it's Vertices and Edges and prove that it is bipartite.

Answer. *Proof.* PROOF HERE. □

5 Diameter of graph

diameter of G is length of the longest path between two vertices in it. Show that if G 's diameter is greater than 3 its complement's diameter would be less than 3.

Answer. *Proof.* PROOF HERE. □

6 About Graph

Necessary and sufficient conditions for a list d to be graphic when d consists of k copies of a and nk copies of b , with $a \geq b \geq 0$. Since the degree sum must be even, the quantity $k a + (nk)b$ must be even. In addition, the inequality $k a \leq k(k+1) + (nk) \min\{k, b\}$ must hold, since each vertex with degree b has at most $\min\{k, b\}$ incident edges whose other endpoint has degree a . We construct graphs with the desired degree sequence when these conditions hold. Note that the inequality implies $a \leq n+1$.

Answer. *Proof.* PROOF HERE. □

7 Graph orientation

Prove the theorem below.

Theorem 7.1. Every graph G has an orientation such that $|d^+(v) - d^-(v)| \leq 1$ for all v .

Answer. ANSWER HERE.

8 Strong orientation of a graph

A strong orientation of a graph that has an odd cycle also has an odd (*directed*) cycle. Suppose that D is a strong orientation of a graph G that has an odd cycle v_1, \dots, v_{2k+1} . Since D is strongly connected, for each i there is a v_i, v_{i+1} -path in D . If for some i every such path has even length, then the edge

between v_i and $v_i + 1$ points from $v_i + 1$ to v_i , since the other orientation would be a $v_i, v_i + 1$ - path of length 1 (*odd*). In this case, we have an odd cycle through v_i and $v_i + 1$. Otherwise, we have a path of odd length from each v_i to $v_i + 1$. Combining these gives a closed trail of odd length. In a digraph as well as in a graph (by the same proof), a closed odd trail contains the edges of an odd cycle.

Answer. ANSWER HERE.

9 The Tournaments

Tournaments with all players kings. a) If n is odd, then there is a tournament with n vertices such that every player is a king.

Answer. ANSWER HERE.

10 About Tree

For a tree T with vertex degrees in $1, k$, the possible values of $n(T)$ are the positive integers that are 2 more than a multiple of $k-1$.

Answer. ANSWER HERE.

11 More about Tree

Prove the theorems below.

Theorem 11.1. A tree has exactly one center or has two adjacent centers.

Answer. ANSWER HERE.

Theorem 11.2. A tree has exactly one center if and only if its diameter is twice its radius.

Answer. ANSWER HERE.

12 Cutting edge

Prove the theorem below.

Theorem 12.1. $e = xy$ is a cutting edge if and only if a) $G - e$ does not have a $x \rightarrow y$ path. b) it does not belong to any cycle.

Answer. ANSWER HERE.

13 $5 - cycles$ in peterson graph

how many $5 - cycles$ does **peterson** graph have?

Answer. ANSWER HERE.

14 Certain bridge club

A certain bridge club has a special rule to the effect that four members may play together only if no two of them have previously partnered one another. At one meeting fourteen members, each of whom has previously partnered five others, turn up. Three games are played, and then proceedings come to a halt because of the club rule. Just as the members are preparing to leave, a new member, unknown to any of them, arrives. Show that at least one more game can now be played.

Answer. ANSWER HERE.