Digital Logic Design :

Lecture 5

The Standard POS form :

A Standard POS expression is one in which all the variables in the domain appear in each sum term in the expression.

example : $(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+D)$

Convert the following Boolean expression into standard POS form.

$(A+\bar{B})(B+\bar{C}+D)$

now,

$$A+\bar{B} = A+\bar{B}+C.\bar{C}$$

$$= (A+\bar{B}+C)(A+B+\bar{C}) \quad [\text{Rule 12: } A+BC=(A+B)(A+C)]$$

$$= (A+\bar{B}+C+D.\bar{D})(A+\bar{B}+\bar{C}+D.\bar{D})$$

$$= (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})$$

$$B+\bar{C}+D = B+\bar{C}+D+A.\bar{A}$$

$$= (A+B+\bar{C}+D)(\bar{A}+B+\bar{C}+D)$$

$$\therefore (A+\bar{B})(B+\bar{C}+D) = (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$$
$$(A+\bar{B}+\bar{C}+\bar{D})(A+B+\bar{C}+D)(\bar{A}+B+\bar{C}+D)$$

Developing truth table for standard SOP expression :

let $X = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + ABC$

Inputs			Output	Output	Output
A	B	C	X	Y	Z
0	0	0	0	0	1
0	0	1	1	1	0
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	0	0	0
1	1	0	0	0	1
1	1	1	1	1	1

Converting POS Expression to Truth Table Format :

let $Y = (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

Determining Standard Expression from a Truth Table

standard SOP expression :

$Z = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$

Standard POS expression :

$Z = (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

3 variable karnaugh map :

AB \ C	0	1
00	0	1
01	0	0
11	0	1
10	1	0

Map the following SOP expression in the karnaugh map.

$\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

Map the following SOP expression on a K map :

$\bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$

001 010 110 111

AB \ C	0	1
00	0	1
01	1	0
11	1	1
10	0	0

Map the following SOP expression on a k map :

$\bar{A} + A\bar{B} + AB\bar{C}$

000 001 010 011 100 101 110

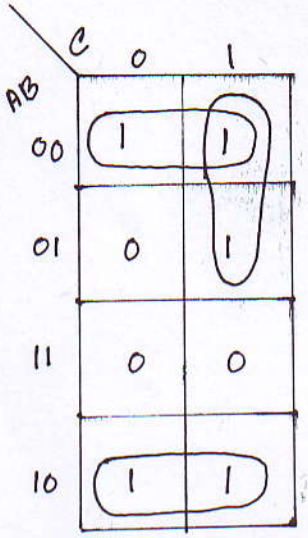
AB \ C	0	1
00	1	1
01	1	1
11	1	0
10	1	1

K map simplification of SOP expression :

Use a K map to minimize the following expression :

$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

1 0 1 0 1 1 0 0 1 0 0 0 1 0 0



Simplified SOP expression

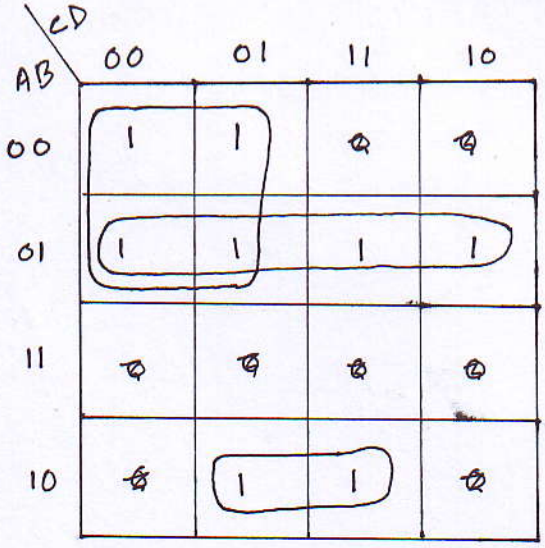
$\bar{B} + \bar{A}C$

1's in aajacent cells can be grouped. Each group can have 2,4,8,16 ..... ones.

Use K map to minimize the following expression :

$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D}$

0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 1 0 1 1 1 0 1 1 0



Simplified SOP expression :

$\bar{A}\bar{C} + \bar{A}B + A\bar{B}D$

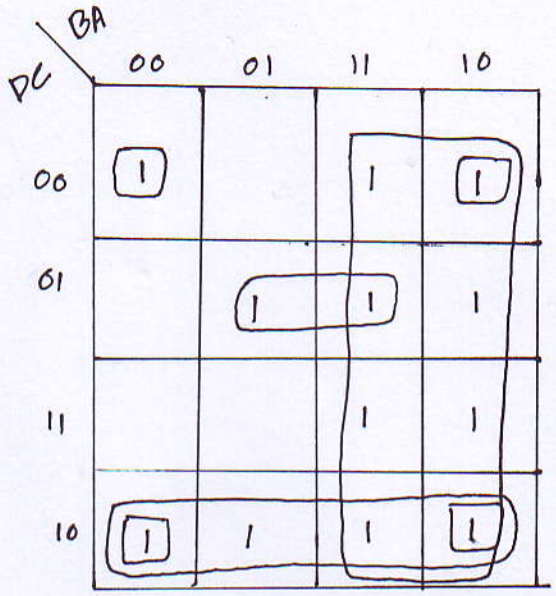
Simplify the following SOP expression using K map :

$\bar{D}\bar{C}\bar{B}\bar{A} + \bar{D}\bar{C}B\bar{A} + \bar{D}\bar{C}BA + \bar{D}C\bar{B}A + \bar{D}CB\bar{A} + \bar{D}CBA + D\bar{C}\bar{B}\bar{A}$

0000 0010 0011 0101 0110 0111 1000

$+ D\bar{C}B\bar{A} + DCBA + DCB\bar{A} + D\bar{C}BA + D\bar{C}\bar{B}A$

1001



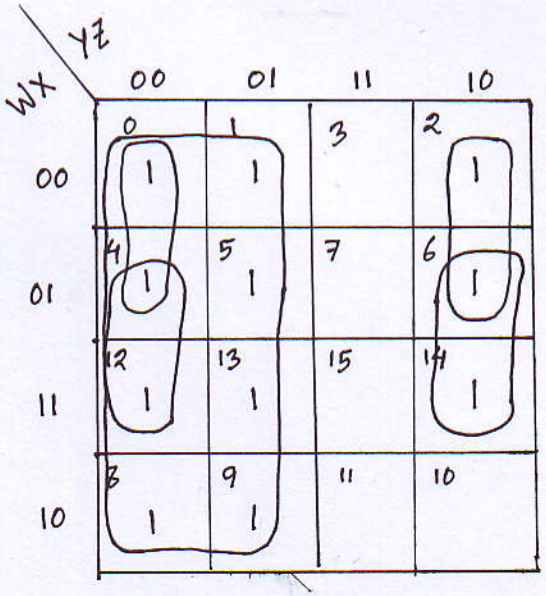
Simplified expression :

$B + D\bar{C} + \bar{C}\bar{A} + \bar{D}CA$

Simplify the function expressed in sum of minterms form

$F(W, X, Y, Z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

no. of minterms = 11 (minterms means standard product, ref. : morris mano pg 48)



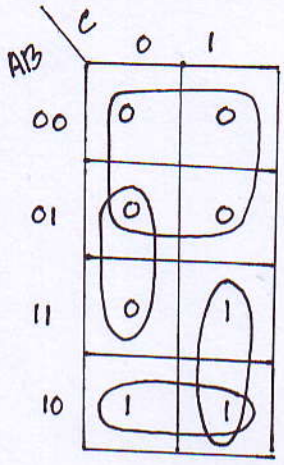
$\therefore F = \bar{Y} + \bar{W}\bar{Z} + X\bar{Z}$

Use a K map to minimize the following :

POS expression :

$(A+B+C)\ (A+B+\bar{C})\ (A+\bar{B}+C)\ (A+\bar{B}+\bar{C})\ (\bar{A}+\bar{B}+C)$

000 001 010 011 110



Simplified POS expression :

$\bar{F} = \bar{A} + B\bar{C}$

$F = \overline{\bar{A} + B\bar{C}}$

$= \bar{\bar{A}}\,(\overline{B\bar{C}})$

$= A\,(\bar{B}+C)$

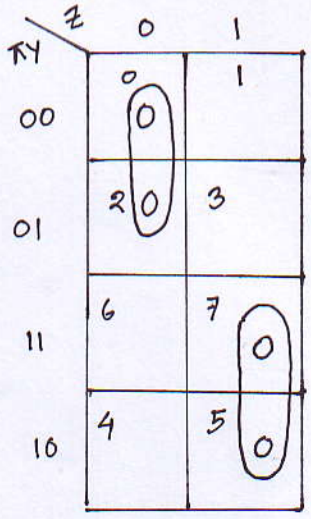
$F = A\bar{B} + AC$

$= A(\bar{B}+C)$

simplify the function expressed in product of maxterm form.

$F(x,y,z) = \Pi\,(0,2,5,7)$

no. of maxterm or sum = 4



$\bar{F} = \bar{x}\bar{z} + xz$

$F = \overline{\bar{x}\bar{z} + xz}$

$= \overline{\bar{x}\bar{z}} + \overline{xz}$

$= (\bar{\bar{x}} + \bar{\bar{z}})(\bar{x} + \bar{z})$

$= (x+z)(\bar{x}+\bar{z})$

The '1' functions represents the minterms and

The '0' functions represents the maxterms.