Digital Logic Design:

$\boxed{\text{Lecture } 4}$

Rules of Boolean Algebra:

There are 12 basic rules useful in manipulating and simplifying Boolean expressions.

1) $A + 0 = A$
2) $A + 1 = 1$
3) $A \cdot 0 = 0$
4) $A \cdot 1 = A$
5) $A + A = A$
6) $A + \overline{A} = 1$
7) $A \cdot A = A$
8) $A \cdot \overline{A} = 0$
9) $\overline{\overline{A}} = A$
10) $A + AB = A$: $A + AB = A(1+B) = A.1 = A$
11) $A + \overline{A}B = A + B$: $A + \overline{A}B = A + AB + \overline{A}B$ [$\because A = A + AB$ rule 10]

$= AA + AB + \overline{A}B$ [$\because A = AA$ rule 7]

$= AA + AB + A\overline{A} + \overline{A}B$ [$\because A\overline{A} = 0$, $A + 0 = A$ rule 8 and 1]

$= A + B$

$A + \overline{A}B = (A + \overline{A})(A + B)$ [Factorizing]

$= 1.(A + B)$ [$A + \overline{A} = 1$ Rule 6]

$= A + B$ [illegible]

12) $(A+B)(A+C) = A+BC$

Proof : $(A+B)(A+C) = AA + AC + AB + BC$ Distributive law

$= A + AC + AB + BC$ [$A \cdot A = A$ rule 7]

$= A(1+C) + AB + BC$ Distributive law

$= A \cdot 1 + AB + BC$ [$A+1 = A$ rule 2]

$= A + AB + BC$ [$A \cdot 1 = A$ rule 4]

$= A(1+B) + BC$

$= A \cdot 1 + BC$ [$A+1 = A$ rule 2]

$= A+BC$

Demorgan's Theorems :

first theorem :

The complement of a product of variables is equal to the sum of the complements of the variables.

$\overline{AB} = \overline{A} + \overline{B}$

second theorem :

The complement of a sum of variables is equal to the product of the complements of the variables.

$\overline{A+B} = \overline{A}\,\overline{B}$

Apply De morgan's theorems to the followin expressions:

$$\overline{A+B+C+D} = \bar{A}.\bar{B}.\bar{C}.\bar{D}$$

$$\overline{ABCD} = \bar{A}+\bar{B}+\bar{C}+\bar{D}$$

$$\overline{\bar{A}\bar{B}\bar{C}\bar{D}} = \bar{\bar{A}}+\bar{\bar{B}}+\bar{\bar{C}}+\bar{\bar{D}}$$

$$= A+B+C+D$$

$$\overline{(AB+C)(A+BC)} = (\overline{AB+C}) + (\overline{A+BC})$$

$$= (\overline{AB})\bar{C} + \bar{A}(\overline{BC})$$

$$= (\bar{A}+\bar{B})\bar{C} + \bar{A}(\bar{B}+\bar{C})$$

$$\overline{(A+B+C)D} = \overline{A+B+C} + \bar{D}$$

$$= \bar{A}\,\bar{B}\,\bar{C} + \bar{D}$$

$$\overline{A\bar{B} + \bar{C}D + EF} = (\overline{A\bar{B}})(\overline{\bar{C}D})(\overline{EF})$$

$$= (\bar{A}+\bar{\bar{B}})(\bar{\bar{C}}+\bar{D})(\bar{E}+\bar{F})$$

$$= (\bar{A}+B)(C+\bar{D})(\bar{E}+\bar{F})$$

$$\overline{\overline{(A+B)}+\bar{C}} = (\overline{\overline{A+B}})(\bar{\bar{C}}) = (A+B)C$$

$$\overline{(A+B)\bar{C}\bar{D}+E+\bar{F}} = (\overline{(A+B)\bar{C}\bar{D}})\bar{E}\bar{\bar{F}}$$

$$= ((\overline{A+B}) + \bar{\bar{C}} + \bar{\bar{D}}))\bar{E}F$$

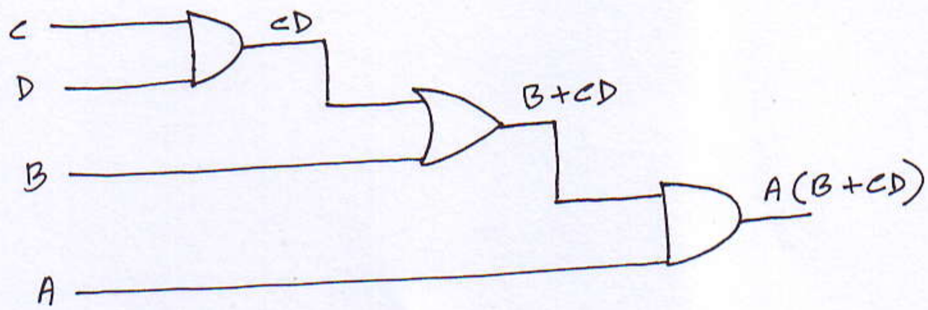
$$= (\bar{A}\bar{B} + C + D)\bar{E}F$$

$$\overline{\overline{A+B\bar{C}} + D\left(\overline{E+\bar{F}}\right)} = \left(\overline{\overline{A+B\bar{C}}}\right)\left(\overline{D\left(\overline{E+\bar{F}}\right)}\right)$$

$$= \left(A+B\bar{C}\right)\left(\bar{D} + \overline{\overline{E+\bar{F}}}\right)$$

$$= \left(A+B\bar{C}\right)\left(\bar{D} + E + \bar{F}\right)$$

Boolean Expression for a Logic circuit :



Simplification using Boolen Algebra :

a) $AB + A(B+C) + B(B+C) = AB + AB + AC + BB + BC$

$$= AB + AC + B + BC$$

$$= AB + AC + B(1+C)$$

$$= AB + AC + B$$

$$= B(A+1) + AC$$

$$= B + AC$$

b) $\overline{AB} + \overline{AC} + \bar{A}\bar{B}\bar{C} = \bar{A} + \bar{B} + \bar{A} + \bar{C} + \bar{A}\bar{B}\bar{C}$

$$= \bar{A} + \bar{B} + \bar{C} + \bar{A}\bar{B}\bar{C} \quad \left[\bar{A} + \bar{A} = \bar{A}\right]$$

$$= \bar{A} + \bar{B} + \bar{C}\left(1 + \bar{A}\bar{B}\right)$$

$$= \bar{A} + \bar{B} + \bar{C}$$

$$= \overline{ABC}$$

c) $\left[A\bar{B}(C+BD) + \bar{A}\bar{B}\right]C = \left(A\bar{B}C + A\bar{B}BD + \bar{A}\bar{B}\right)C$

$= (A\bar{B}C + A.0.D + \bar{A}\bar{B})C \quad [B\bar{B} = 0]$

$= (A\bar{B}C + 0 + \bar{A}\bar{B})C \quad [A.0 = 0]$

$= (A\bar{B}C + \bar{A}\bar{B})C \quad [A + 0 = 0]$

$= A\bar{B}CC + \bar{A}\bar{B}C$

$= A\bar{B}C + \bar{A}\bar{B}C \quad [C\cdot C = C]$

$= \bar{B}C(A+\bar{A}) \quad [A+\bar{A} = 1]$

$= \bar{B}C\cdot 1$

$= \bar{B}C \quad [A\cdot 1 = A]$

Standard forms of Boolean Expressions :

All Boolean expressions can be expressed in two standard forms :

1) The Sum of Product (SOP) form,

2) The Product of Sum (POS) form.

1) The SOP form :

examples : $AB + B\bar{C}$

$ABC + CDE + \bar{B}C\bar{D}$

convert the following expressions into SOP form :

a) $AB + B(CD + EF) = AB + BCD + BEF$

b) $\overline{\overline{A+B} + C} = (\overline{\overline{A+B}})\bar{C} = (A+B)\bar{C} = A\bar{C} + B\bar{C}$

The POS Form :

examples : $(\bar{A}+B)(A+\bar{B}+C)$

$(A+B)(A+\bar{B}+C)(\bar{A}+C)$

The standard SOP form :

A standard SOP expression is one in which all the variables in the domain appear in each product term in the expression.

example : $A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + AB\bar{C}\bar{D}$

convert the following Boolean expression into standard SOP form :

$$A\bar{B}\bar{C} + \bar{A}\bar{B} + AB\bar{C}D$$

$$A\bar{B}C = A\bar{B}C(D+\bar{D}) = A\bar{B}CD + A\bar{B}C\bar{D}$$

$$\bar{A}\bar{B} = \bar{A}\bar{B}(C+\bar{C}) = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

$$= \bar{A}\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}\bar{C}(D+\bar{D})$$

$$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$\therefore\ A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D = A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD$$

$$+ \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D.$$