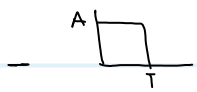
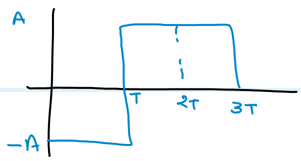
Part ①



1 was sent

"0" was sent

/ $g(t)$ input pulse , $r(t)$ received from channel

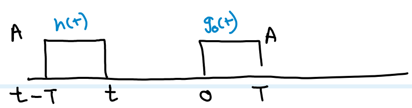
a) 

b) $h(t) = K\,g(T-t) = g(T-t)$ "let $K=1$" $= g_{"1"}(t) = \begin{cases} A & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} = A \cdot \text{rect}\left(\frac{t - T/2}{T}\right)$

$\therefore\ y(t) = g(t) * h(t) + w(t) * h(t)$

$= g_o(t) + n(t)$ due to 1 pulse

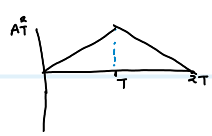
→ when "1" was sent



① for $0 < t < T$: $g_o(t) = \int_0^t A^2\,dt = A^2 t$

② for $T < t < 2T$: $g_o(t) = \int_{t-T}^{T} A^2\,dt = A^2\,[2T - t]$

$g_o(t) = \begin{cases} A^2 t & 0 < t < T \\ A^2\,[2T - t] & T < t < 2T \end{cases}$

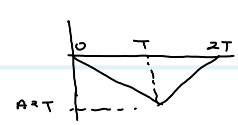


$y(t) = g_o(t)$ ignoring noise when "1" sent

→ when "0" sent

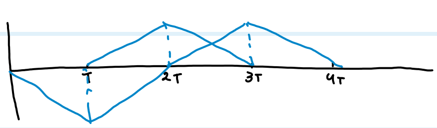
$\therefore\ g_{"0"}(t) = -g_{"1"}(t)$

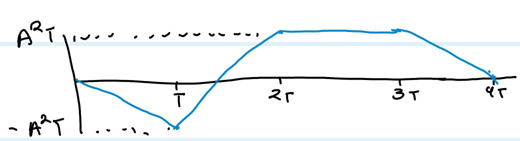
$\therefore\ g_{o_{"0"}}(t) = -g_{"1"}(t) * h(t) = -g_{o_{"1"}}(t) = \begin{cases} -A^2 t & 0 < t < T \\ -A^2[2T - t] & T < t < 2T \end{cases}$



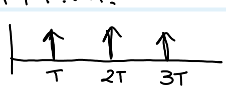
→ Final Out put Signal ignoring noise

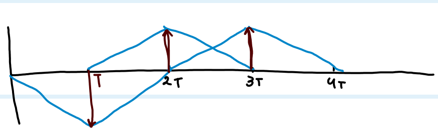
$y(t)$

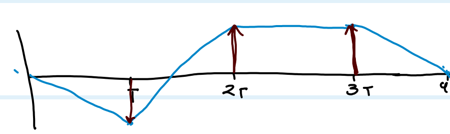




c) will sample at $T, 2T, 3T$







Output after sampling

d) more general

→ Transmitter ⇒

→ Formatter → Source encoder → Channel encoder → Modulator → Pulse Shapping →

could be encryption

Dr. hala

$g(t)$

Formatter ⇒

→ Sampler → Quantizer → encoder → pulse code Modulation signal $g(t)$

in assignment

Binary data "1" "0" .. → Pulse Amplitude modulation → Transmit Filter → channel → Σ

clk

$g(t)$

eg:

AWGN

e) general:

→ Receiver ⇒

→ Receiver Filter → demodulator → Channel decoder → Source decoder → Formatter →

decryption

→ Sampler → decoder → $y(t)$

assignment

$r(t)$ → Receiver Filter → Sampler → Decision Device → "1" or "0"

received signal

$t = iT_s$

Part II

$\therefore T = 1$, $w(t) \sim N(0, \frac{N_0}{2})$ $g(t) =$ ... or ...

a) Matched Filter

– $h(t) = K g(T-t)$ / Energy of filter $= 1 = \frac{N_0}{2}$ $\Rightarrow N_0 = 2$

$= g_{\text{"1"}}(t) = 1$ for $0 < t < 1$

– $y(t) = r(t) * h(t) = \underbrace{g(t) * h(t)}_{g_0(t)} + \underbrace{w(t) * h(t)}_{n(t)}$

$$g_0(T) = \int_{-\infty}^{\infty} g(\tau) \cdot h(T-\tau)\, d\tau = \int_0^T g(\tau) \cdot g_{\text{"1"}}(\tau)\, d\tau = \begin{cases} 1 & \text{"1" was sent} \\ -1 & \text{"0" was sent} \end{cases} = \begin{cases} A^2 T \\ -A^2 T \end{cases}$$

$$\therefore y(t) = \begin{cases} \overset{A^2T}{\cancel{1}} + n(t) & \text{"1" was sent} \\ -\overset{A^2T}{\cancel{1}} + n(t) & \text{"0" was sent} \end{cases}$$

– $E(n(T)) = E\left(\int_{-\infty}^{\infty} w(\tau)\, h(T-t)\, d\tau\right) = \int_{-\infty}^{\infty} E[\underbrace{w(\tau)}_{0}]\, E[h(T-\tau)]\, d\tau = 0$

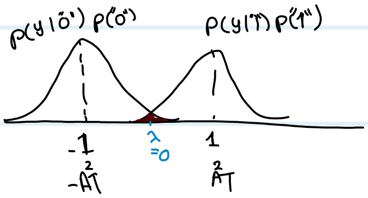
$$Var[n(T)] = E[n^2(T)] - (E[\underbrace{n(t)}_{0}])^2 = R_n(0) = \int_{-\infty}^{\infty} Q_n(f)\, df = \int_{-\infty}^{\infty} Q_w(f)\, |H(f)|^2\, df$$

$$= \frac{N_0}{2} \int_0^T |h(t)|^2\, dt$$

$$= \frac{N_0}{2} A^2 . T$$

$p(y|\text{"1"}) \sim N(A^2T, \frac{N_0}{2}A^2T)$ "1" was sen

$p(y|\text{"0"}) \sim N(-A^2T, \frac{N_0}{2}A^2T)$ "0" was sent



By symmetry $\lambda = 0$
same variance

$-\,p(\text{error}) = p(\text{err}\,|\,\text{"1"})\,p(\text{"1"}) + p(\text{err}\,|\,\text{"0"})\,p(\text{"0"})$

$\because\ p(\text{"1"}) = p(\text{"0"}) = 0.5$

$\because\ p(y=\text{"1"}\,|\,\text{"0"}) = p(\text{error}\,|\,\text{"0"}) = p(y>0\,|\,\text{"0"})$

$\because$ By symmetry $\quad p(\text{error}\,|\,\text{"1"}) = p(\text{error}\,|\,\text{"0"})$

$\therefore\ p(\text{error}) = p(\text{error}\,|\,\text{"0"}) = P_r(y>0\,|\,0) = P_r\left(z > \dfrac{0+A^2T}{\sqrt{\frac{N_0}{2}A^2T}}\right)$

$\because\ E = g_0(T)$

$$= Q\left(\frac{\sqrt{2}.A.\sqrt{T}}{\sqrt{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(A\sqrt{\frac{T}{N_0}}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E}{N_0}}\right)$$

$\text{erfc}(x) = 2\,Q(\sqrt{2}\,x)$

$Q(x) = \frac{1}{2}\,\text{erfc}\left(\frac{1}{\sqrt{2}}x\right)$

---

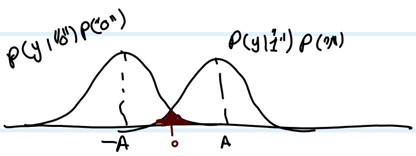
b) $h(t) = \delta(t)$

$-\ \because\ y(t) = \underbrace{g(t) * h(t)}_{g_0(t)} + \underbrace{w(t) * h(t)}_{n(t)}$

$-\ g_0(t) = g(t) * h(t) = g(t) * \delta(t) = g(t) = \begin{cases} A & \text{"1" was sent} \\ -A & \text{"0" was sent} \end{cases}$ for $0<t<1$

$-\ n(t) = w(t) * \delta(t) = w(t) \sim N\left(0, \frac{N_0}{2}\right)$

$-\ \begin{aligned} p(y\,|\,\text{"1"}) &\sim N\left(A, \tfrac{N_0}{2}\right) && \text{"1" was sent} \\ p(y\,|\,\text{"0"}) &\sim N\left(-A, \tfrac{N_0}{2}\right) && \text{"0" was sent} \end{aligned}$



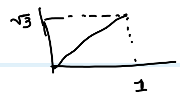
By symmetry $\quad \lambda = 0$

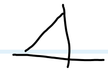
$\because\ p(\text{"1"}) = p(\text{"0"}) = \frac{1}{2}$, & By symmetry $\quad p(\text{err}\,|\,\text{"1"}) = p(\text{err}\,|\,\text{"0"})$

$\therefore\ p(\text{error}) = p(\text{err}\,|\,1)\,p(1) + p(\text{err}\,|\,0)\,p(0)$

$$= p(\text{err}\,|\,0) = p(y > \overset{\lambda}{0}\,|\,0) = p\left(z > \frac{0+A}{\sqrt{N_0/2}}\right) = Q\left(\frac{A\sqrt{2}}{\sqrt{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\frac{A}{\sqrt{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\frac{E}{\sqrt{N_0}}\right)$$

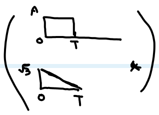
c) $h(t) = \begin{cases} \frac{\sqrt{3}}{T} t & 0 < t < T \\ 0 & \text{otherwise} \end{cases}$





$$y(t) = \underbrace{g(t) * h(t)}_{g_0(t)} + \underbrace{w(t) * h(t)}_{n(t)}$$

$$- g_{0\text{"1"}}(T) = \int_{-\infty}^{\infty} g_{\text{"1"}}(\tau)\, h(T-\tau)\, d\tau = \text{Area}\left(\right)$$



$$= \frac{\sqrt{3}}{2} TA \quad \Rightarrow \quad \text{"1" was sent}$$

$$g_0(T) = \begin{cases} \frac{\sqrt{3}}{2} AT & \text{"1" was sent} \\ -\frac{\sqrt{3}}{2} AT & \text{"0" was sent} \end{cases}$$

since $g_{\text{"0"}}(T) = -g_{\text{"1"}}(T) = -A$

so $g_{0\text{"0"}}(T) = g_{0\text{"1"}}(t)$

$$- n(T) = \int_{-\infty}^{\infty} w(\tau)\, h(T-\tau)\, d\tau$$

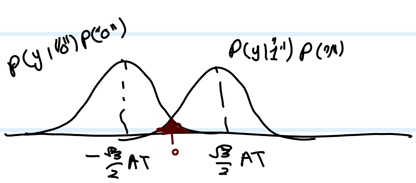
$$E(n(T)) = E\left(\int_{-\infty}^{\infty} w(\tau)\, h(T-\tau)\, d\tau \right) = \int_{-\infty}^{\infty} \underbrace{E(w(\tau))}_{\to 0} E(h(T-\tau))\, d\tau = 0$$

$$\text{Var}(n(t)) = E(n(t)^2) - \underbrace{E(n(t))^2}_{\to 0} = E(n(t)^2) = R_n(0) =$$

$$= \int_{-\infty}^{\infty} G_n(f)\, df = \int_{-\infty}^{\infty} G_w(f)\, |H(f)|^2 df$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} |h(t)|^2 dt = \frac{N_0}{2} \frac{3}{T^2} \int_0^T t^2 dt = \frac{N_0}{2} T$$

so $P(y \mid \text{"1"}) \sim N\left(\frac{\sqrt{3}}{2} A.T, \frac{N_0 T}{2}\right)$, $P(y \mid \text{"0"}) \sim N\left(-\frac{\sqrt{3}}{2} A.T, \frac{N_0 T}{2}\right)$



By symmetry $\lambda = 0$

so $p(\text{"1"}) = p(\text{"0"}) = \frac{1}{2}$, & By symmetry $p(\text{err} \mid \text{"1"}) = p(\text{err} \mid \text{"0"})$

so $p(\text{error}) = p(\text{err} \mid 1)\, p(1) + p(\text{err} \mid 0)\, p(0)$

$$= p(\text{err} \mid 0) = p(y > \overset{\lambda}{0} \mid 0) = p\left(z > \frac{0 + \frac{\sqrt{3}}{2} AT}{\sqrt{\frac{N_0 T}{2}}}\right) = Q\left(\frac{\sqrt{3} A \sqrt{T}}{\sqrt{2}\sqrt{N_0}}\right) = \frac{1}{2} \text{erfc}\left(\frac{\sqrt{3} A \sqrt{T}}{2} \cdot \frac{1}{\sqrt{N_0}}\right)$$

$$= \frac{1}{2} \text{erfc}\left(\frac{E}{\sqrt{T N_0}}\right)$$