An Upper Limb Exoskeleton for Physical Assistance

Advanced robotic

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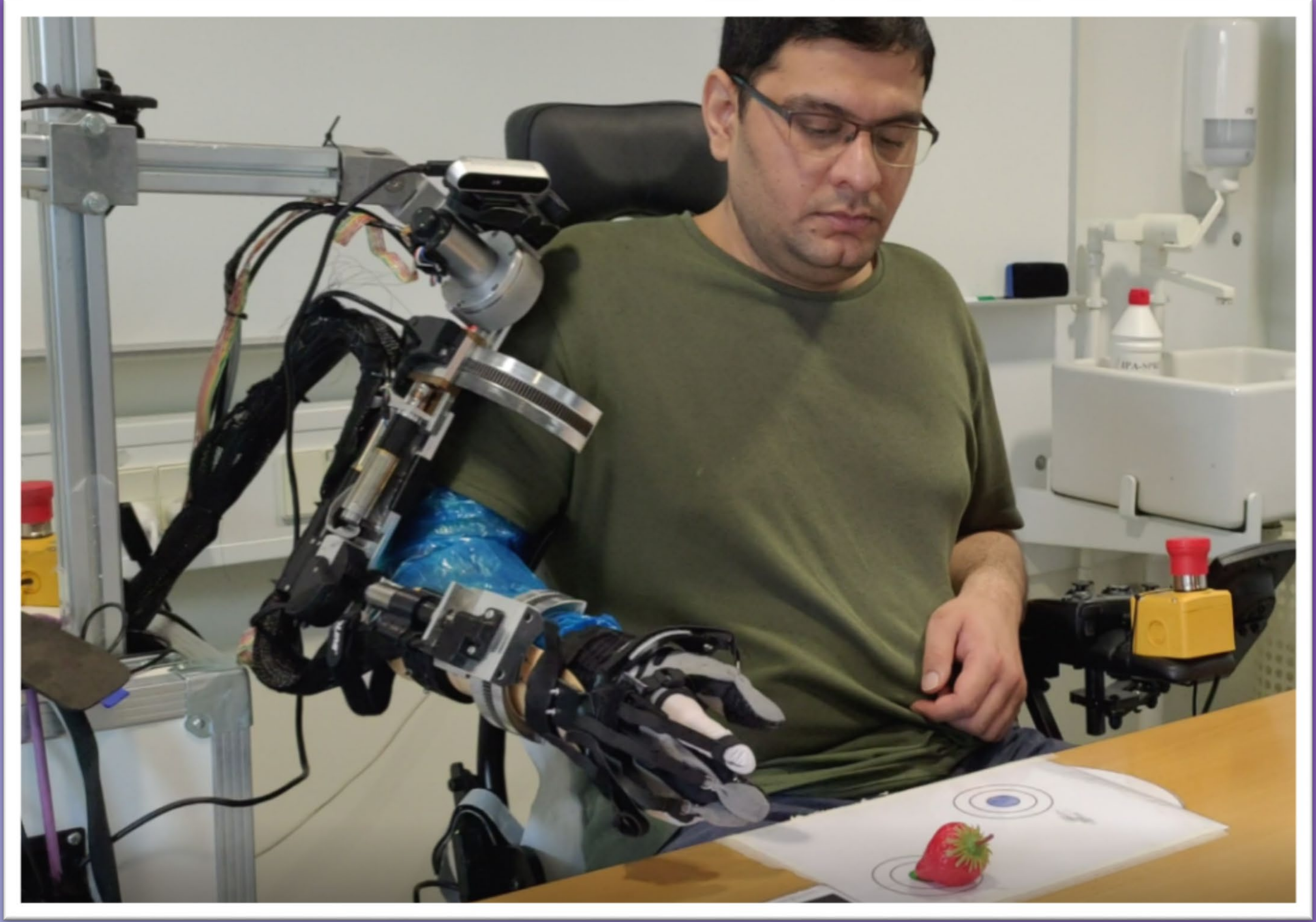


objectives

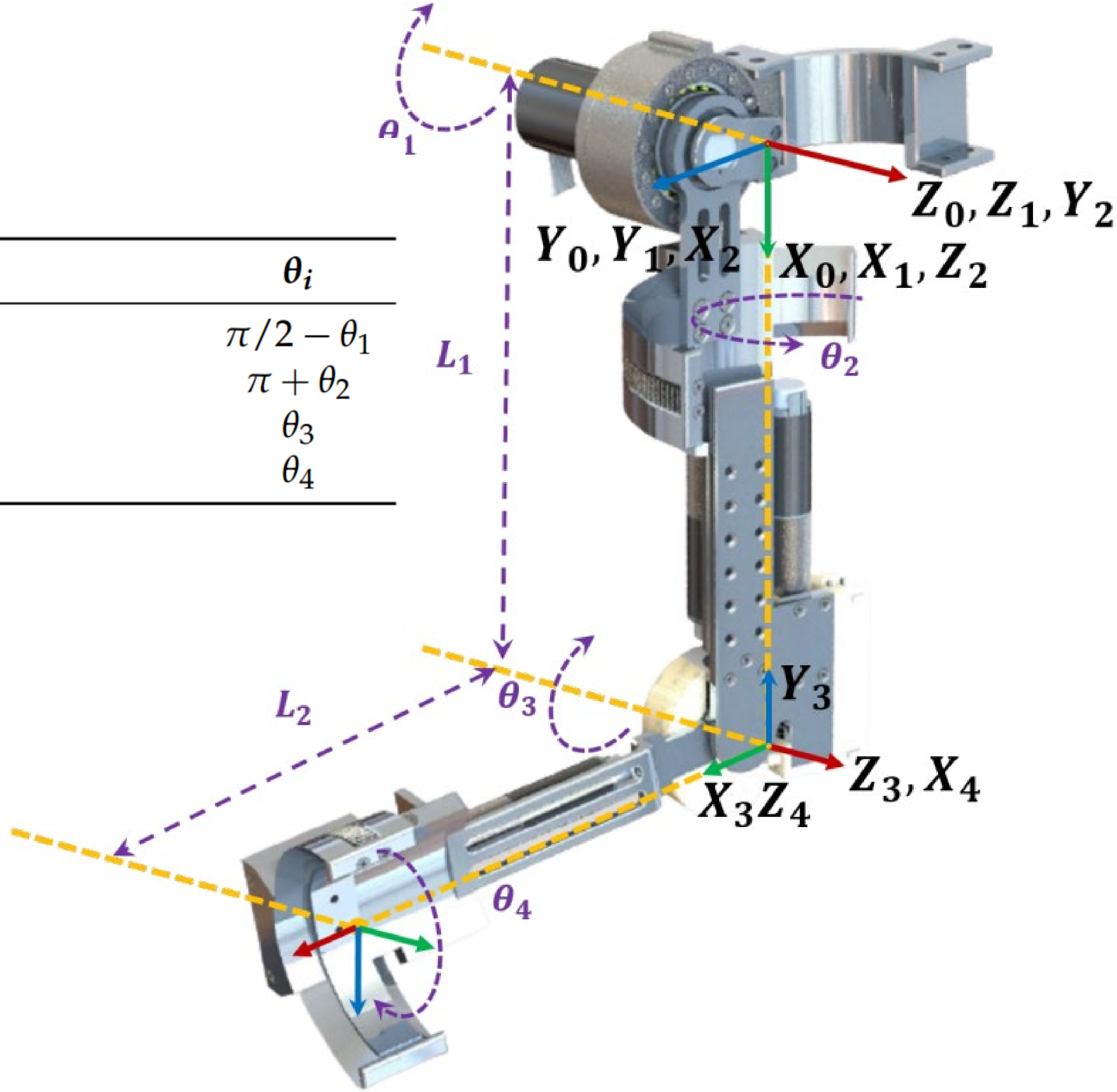
- inverse kinematics
- trajectory and jacobian
- inverse dynamics
- dynamics in presence of friction
- control of the manipulator

Kinematics

Table 1. Denavit–Hartenberg (DH) parameters.

Joints	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	0	$\pi/2 - \theta_1$
2	$\pi/2$	0	L_1	$\pi + \theta_2$
3	$-\pi/2$	0	0	θ_3
4	0	0	L_2	θ_4

$$
T_{i-1,i} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



the overall matrix of transformation from the base frame to the wrist:

$$\mathrm{T}_{0,4} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & n_{14} \\ m_{21} & m_{22} & m_{23} & n_{24} \\ m_{31} & m_{32} & m_{33} & n_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$m_{11} = c\theta_4(s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) - c\theta_1 s\theta_2 s\theta_4$$

$$m_{12} = -s\theta_4(s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) - c\theta_1 c\theta_4 s\theta_2$$

$$m_{13} = c\theta_3 s\theta_1 - c\theta_1 c\theta_2 s\theta_3$$

$$m_{21} = -c\theta_4(c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) - s\theta_1 s\theta_2 s\theta_4$$

$$m_{22} = s\theta_4(c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) - c\theta_4 s\theta_1 s\theta_2$$

$$m_{23} = -c\theta_1 c\theta_3 - c\theta_2 s\theta_1 s\theta_3$$

$$m_{31} = c\theta_2 s\theta_4 + c\theta_3 c\theta_4 s\theta_2$$

$$m_{32} = c\theta_2 c\theta_4 - c\theta_3 s\theta_2 s\theta_4$$

$$n_{14} = L_2(c\theta_3 s\theta_1 - c\theta_1 c\theta_2 s\theta_3) + L_1 s\theta_1$$

$$n_{24} = -L_2(c\theta_1 c\theta_3 + c\theta_2 s\theta_1 s\theta_3) - L_1 c\theta_1$$

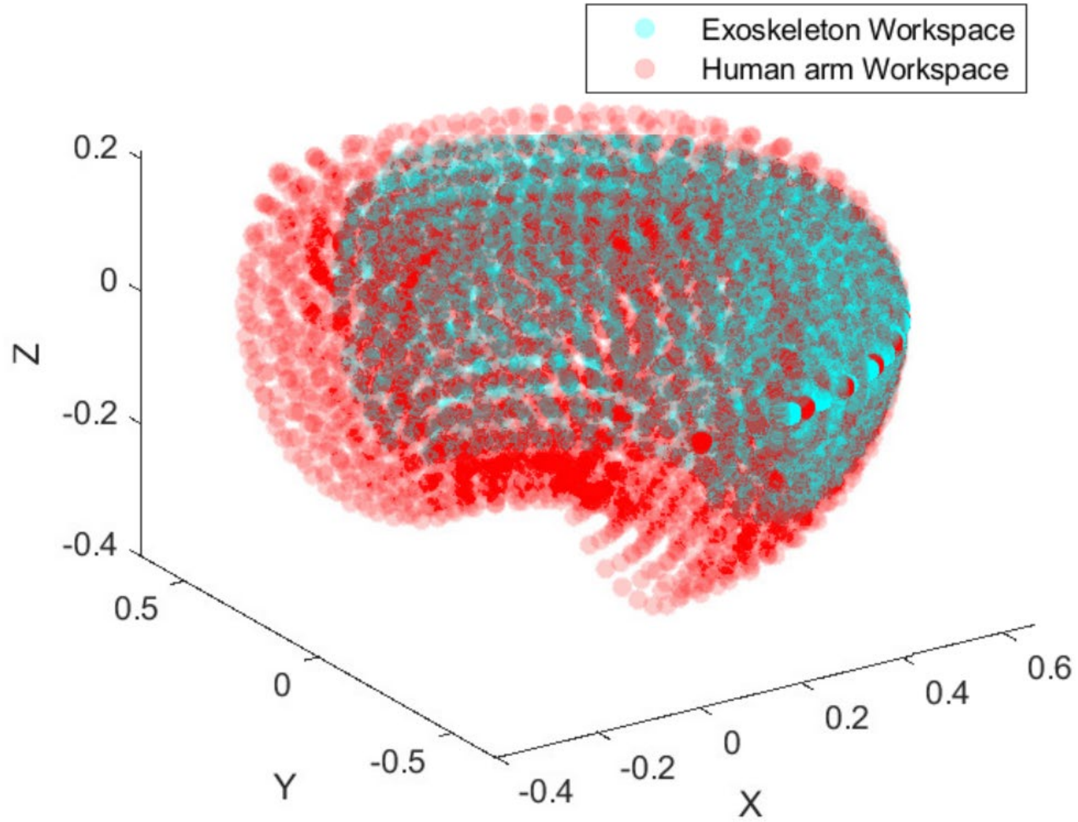
$$n_{34} = -L_2 s\theta_2 s\theta_3$$

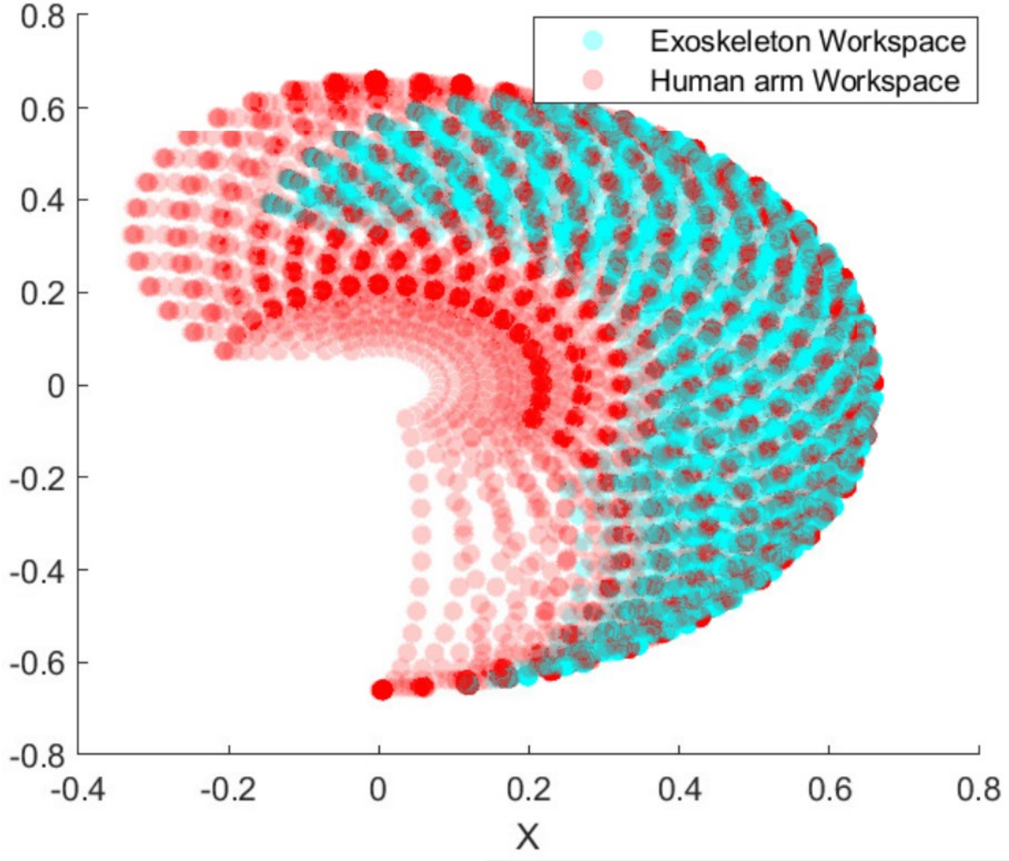
The inverse kinematics is derived from the transformation matrix (2). The joint angles can be obtained as:

$$\begin{aligned}
\theta_2 &= \pi + \arctan 2(n_{34}, n_{14}) \\
\theta_3 &= \pm \arcsin\left(\frac{n_{34}}{L_2 \sin\theta_2}\right) \\
\theta_1 &= \frac{\pi}{2} + \frac{1}{2} \arcsin \frac{n_{14} - n_{24}}{L_1 + L_2 \cos\theta_3 + L_2 \cos\theta_2 \sin\theta_3} \\
\theta_4 &= \arccos\left(\frac{m_{32}\cos\theta_2 + m_{31}\cos\theta_3 \sin\theta_2}{\cos^2\theta_2 - \cos\theta_3 \sin\theta_2}\right)
\end{aligned} \tag{3}$$

Work space analysis

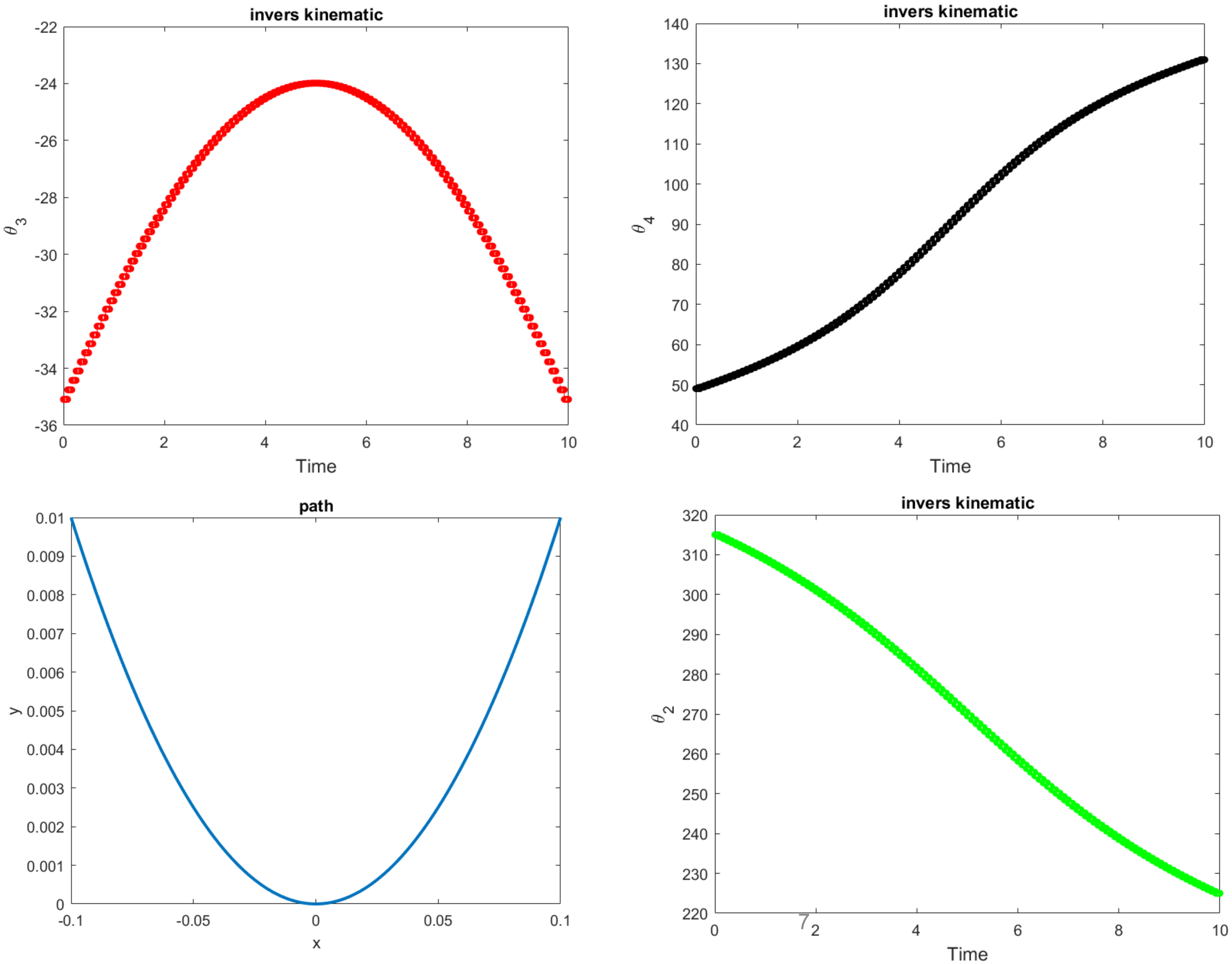
Using forward kinematics:



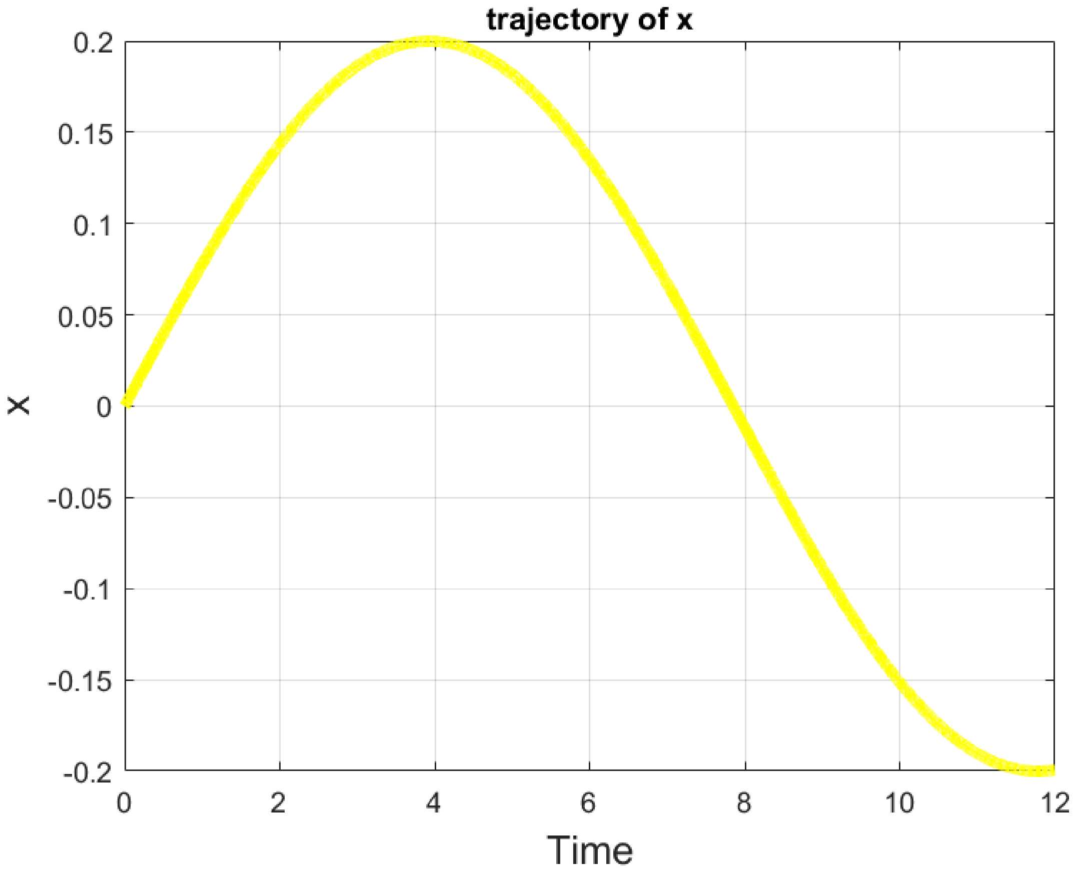


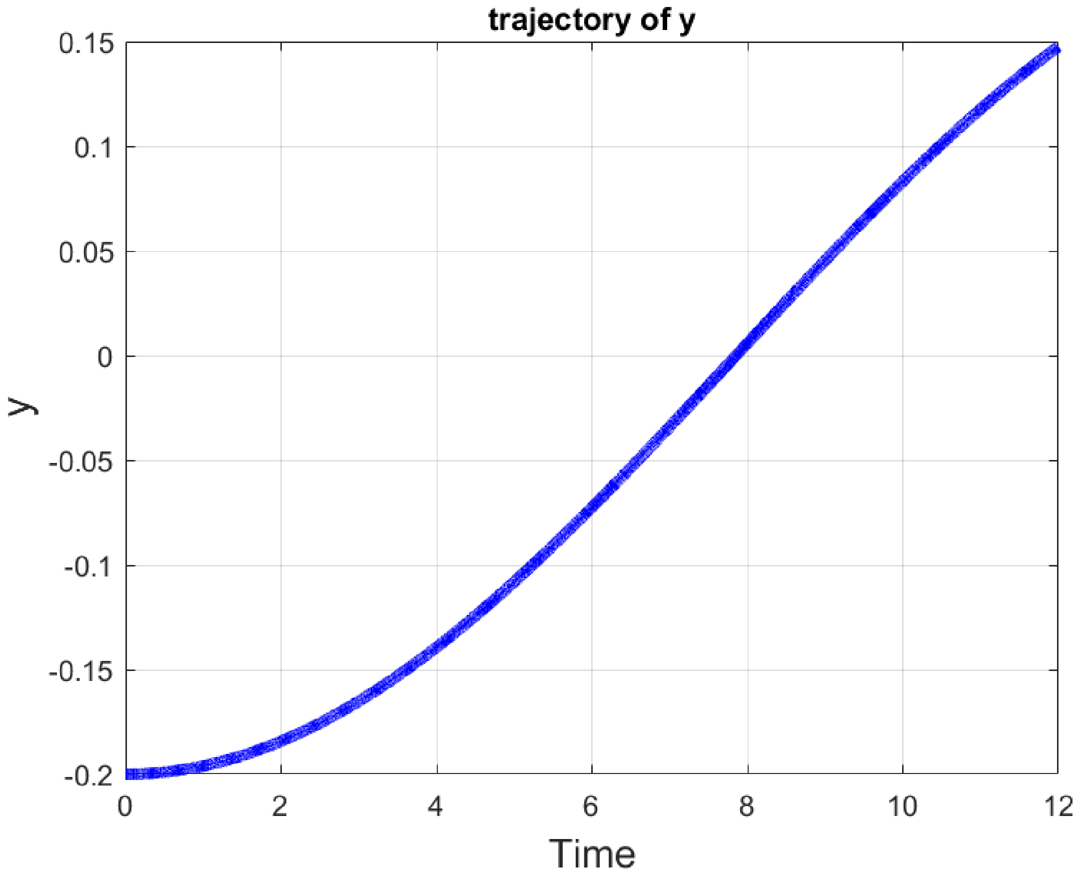
isotropic view of upper limb exoskeleton and human arm workspace

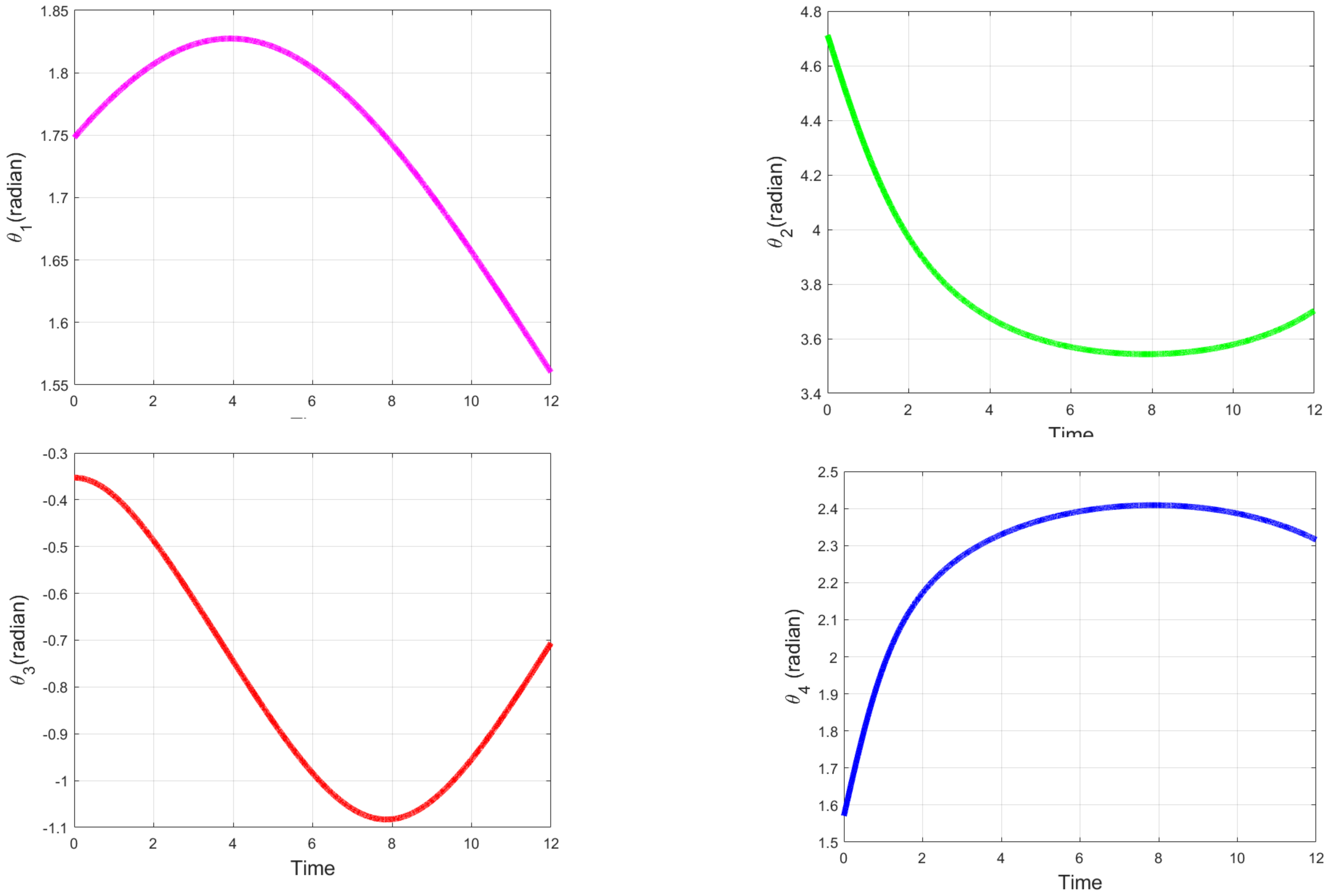
Simulations for path



Trajectory Simulation





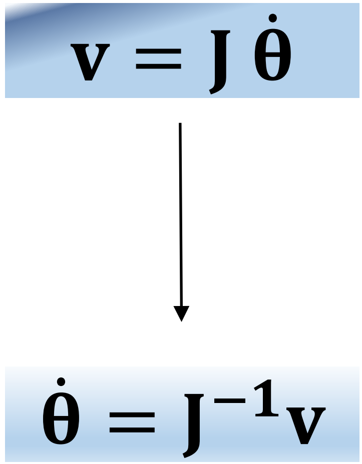


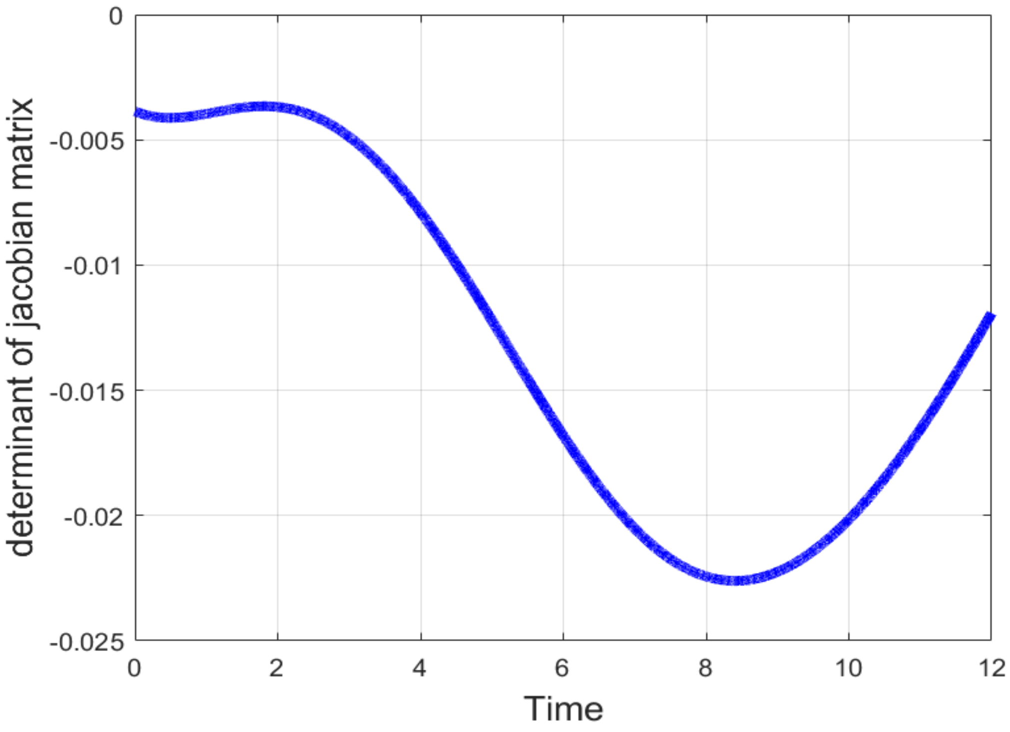
θ1(radian)
1.85
1.8
1.75
1.7
1.65
1.6
1.55
0 2 4 6 8 10 12
θ2(radian)
4.8
4.6
4.4
4.2
4
3.8
3.6
3.4
0 2 4 6 8 10 12
Time
θ3(radian)
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
0 2 4 6 8 10 12
Time
θ4 (radian)
2.5
2.4
2.3
2.2
2.1
2
1.9
1.8
1.7
1.6
1.5
0 2 4 6 8 10 12
Time

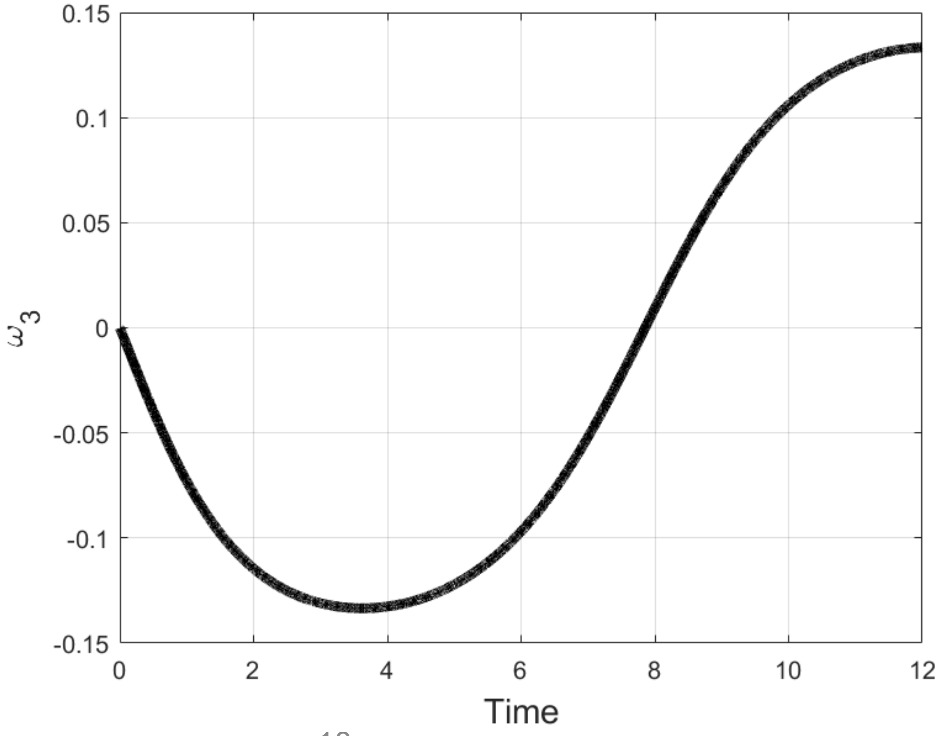
Singularity Analysis

We take Jacobian in the form of:

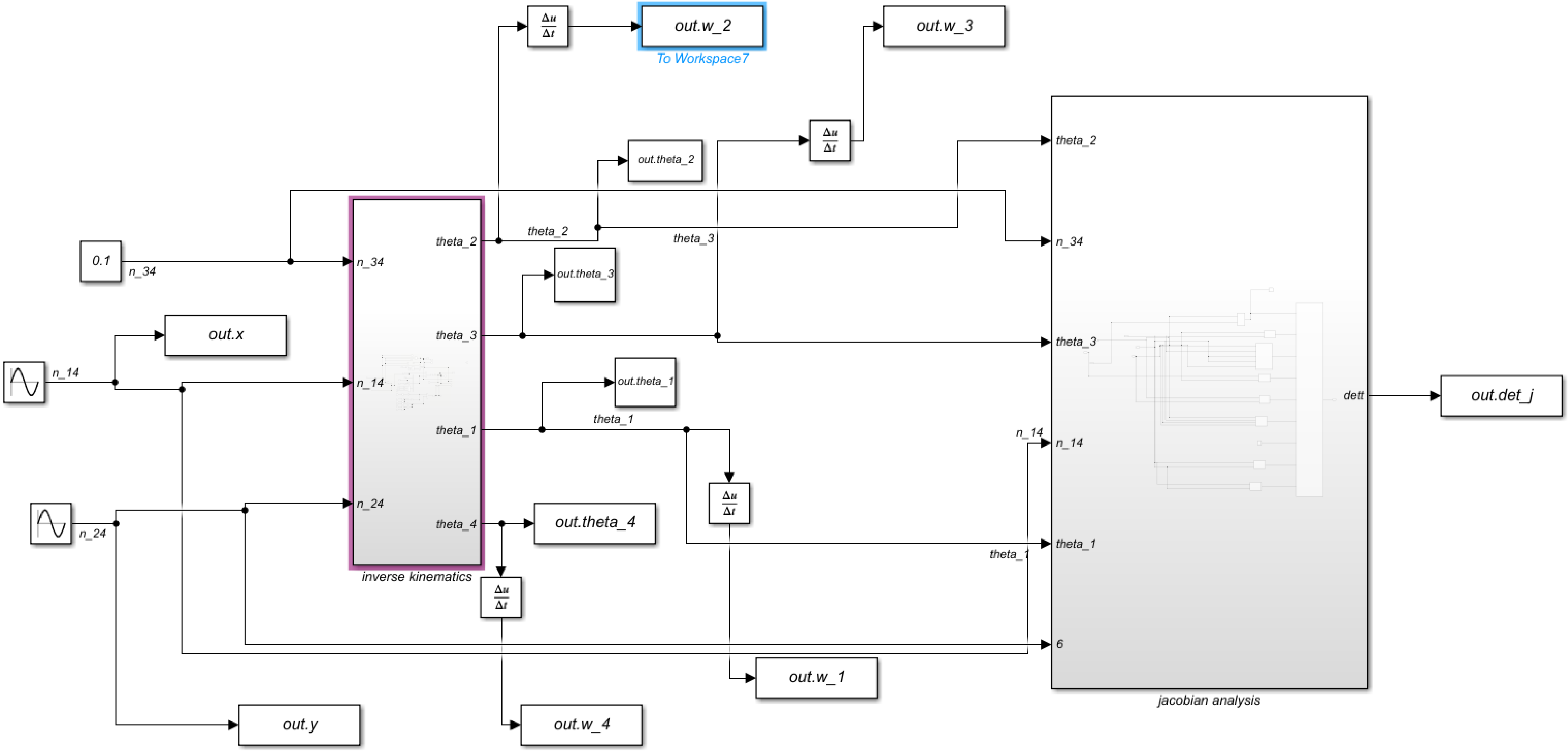
$$\mathbf{J} = \frac{\partial \boldsymbol{n}}{\partial \boldsymbol{\theta}_i} = \begin{bmatrix} -n_{24} + L_1 c\theta_1 & -n_{34} c\theta_1 & -L_2(s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) \\ n_{14} + L_1 s\theta_1 & -n_{34} s\theta_1 & L_2(c\theta_1 s\theta_3 - s\theta_1 c\theta_2 c\theta_3) \\ 0 & -L_2 c\theta_2 s\theta_3 & -L_2 s\theta_2 c\theta_3 \end{bmatrix} \tag{4}$$







Practical review



Dynamics of manipulator

Using Lagrange formulation :

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + \tau_g = \tau \quad (6)$$

Table A1. Mechanical properties of the exoskeleton, and the average estimated anthropomorphic parameters for human subjects.

Link	Exoskeleton		Human Subject	
	Length (m)	**Weight (kg)**	**Length (m)**	**Weight (kg)**
Upper arm	0.33	2.5	0.33	1.386
Forearm	0.246	1.5	0.37	0.886

All entries of the dynamic Equation:

inertia matrix:

$$M_{11} = I_1 + L_1^2 m_3 + L_{c1}^2 m_1 + L_{c1}^2 m_2 + L_{c2}^2 m_3 c\theta_2{}^2 + L_{c2}^2 m_3 c\theta_3{}^2 + 2L_1 L_{c2} m_3 c\theta_3 - L_{c2}^2 m_3 c\theta_2{}^2 c\theta_3{}^2$$

$$M_{12} = L_{c2} m_3 s\theta_2 s\theta_3 (L_1 + L_{c2} c\theta_3)$$

$$M_{13} = -L_{c2} m_3 c\theta_2 (L_{c2} + L_1 c\theta_3)$$

$$M_{21} = L_{c2} m_3 s\theta_2 s\theta_3 (L_1 + L_{c2} c\theta_3)$$

$$M_{22} = -m_3 L_{c2}^2 c\theta_3{}^2 + m_3 L_{c2}^2 + I_2$$

$$M_{23} = 0$$

$$M_{31} = -L_{c2} m_3 c\theta_2 (L_{c2} + L_1 c\theta_3)$$

$$M_{32} = 0$$

$$M_{33} = m_3 L_{c2}^2 + I_3$$

Coriolis forces:

$$C_1 = L_{c2}m_3(L_1\dot{\theta}_2^2 c\theta_2 s\theta_3 + L_1\dot{\theta}_3^2 c\theta_2 s\theta_3 - 2L_1\dot{\theta}_1\dot{\theta}_3 s\theta_3 - L_{c2}\dot{\theta}_1\dot{\theta}_2 s2\theta_2 - L_{c2}\dot{\theta}_1\dot{\theta}_3 s2\theta_3$$
$$+L_{c2}\dot{\theta}_2^2 c\theta_2 c\theta_3 s\theta_3 + 2L_1\dot{\theta}_2\dot{\theta}_3 c\theta_3 s\theta_2 + 2L_{c2}\dot{\theta}_2\dot{\theta}_3 c\theta_3{}^2 s\theta_2 + 2L_{c2}\dot{\theta}_1\dot{\theta}_2 c\theta_2 c\theta_3{}^2 s\theta_2 + 2L_{c2}\dot{\theta}_1\dot{\theta}_3 c\theta_2{}^2 c\theta_3 s\theta_3)$$
$$C_2 = (L_{c2}^2 m_3(-2c\theta_2 s\theta_2\dot{\theta}_1^2 c\theta_3{}^2 + s2\theta_2\dot{\theta}_1^2 + 4\dot{\theta}_3 s\theta_2\dot{\theta}_1 c\theta_3{}^2 - 4\dot{\theta}_3 sin\theta_2\dot{\theta}_1 + 2\dot{\theta}_2\dot{\theta}_3 sin2\theta_3))/2$$
$$C_3 = (L_{c2}m_3(2L_1\dot{\theta}_1^2 s\theta_3 + L_{c2}\dot{\theta}_1^2 sin2\theta_3 - L_{c2}\dot{\theta}_2^2 s2\theta_3 + 4L_{c2}\dot{\theta}_1\dot{\theta}_2 s\theta_2 - 2L_{c2}\dot{\theta}_1^2 c\theta_2{}^2 c\theta_3 s\theta_3 - 4L_{c2}\dot{\theta}_1\dot{\theta}_2 c\theta_3{}^2 s\theta_2))/2$$

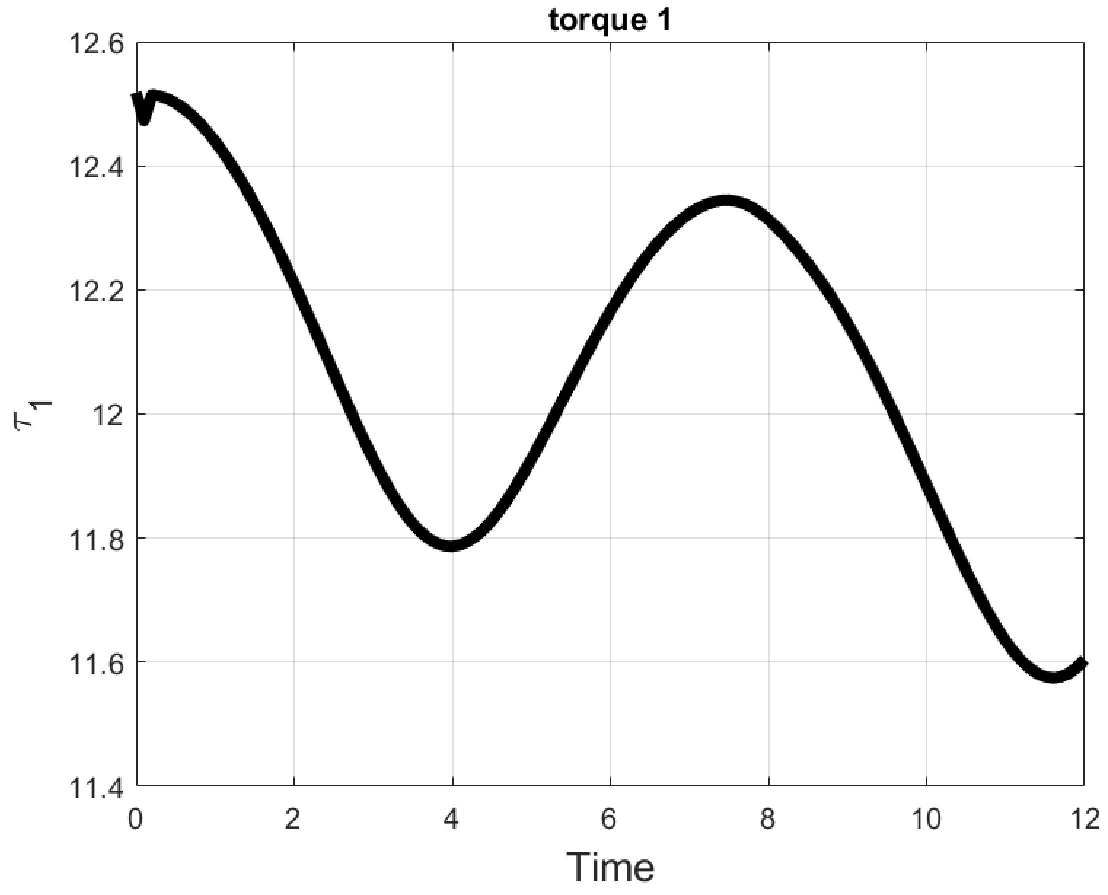
The torque due to gravity:

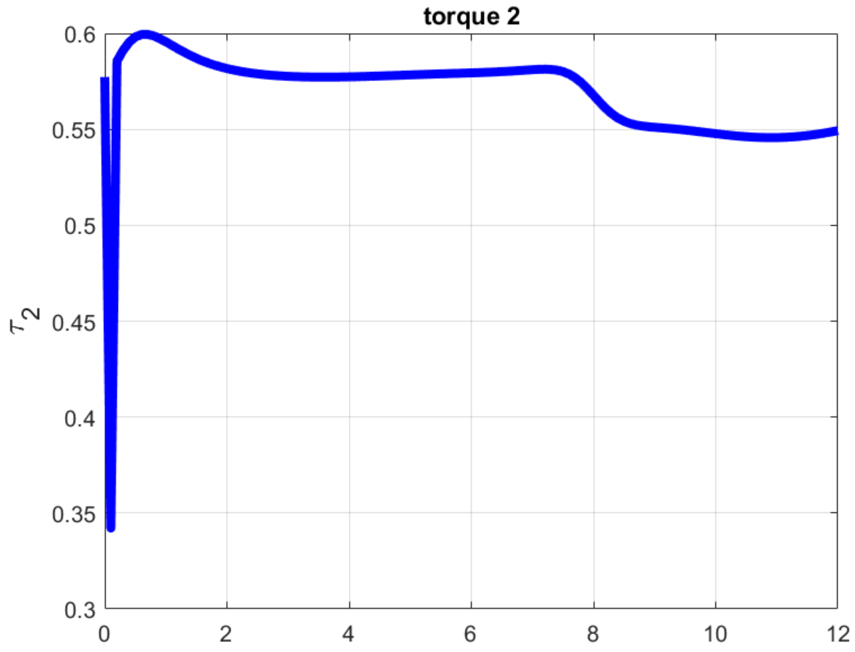
$$G_1 = L_1 m_3 s\theta_1 + L_{c1}m_1 s\theta_1 + L_{c1}m_2 s\theta_1 + L_{c2}m_3 c\theta_3 s\theta_1 - L_{c2}m_3 c\theta_1 c\theta_2 s\theta_3$$

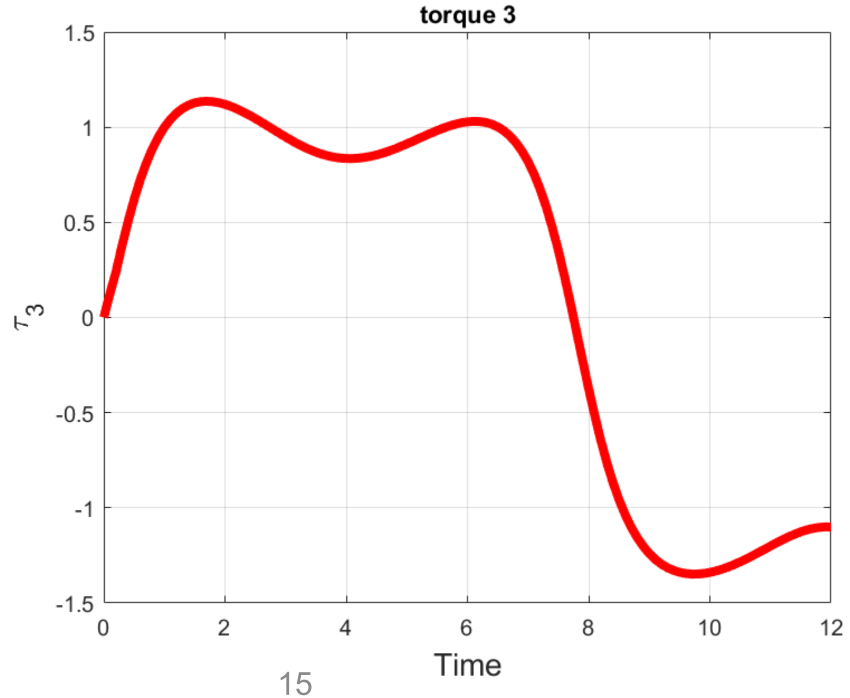
$$G_2 = L_{c2}m_3 s\theta_1 s\theta_2 s\theta_3 \qquad \times g$$

$$G_3 = L_{c2}m_3(c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1)$$

Inverse dynamic simulations







Inclusion of non-rigid body effects

- ✓ all mechanisms are affected by frictional forces.
- ✓ the forces due to friction can actually be quite large perhaps equaling 25% of the torque required to move the manipulator in typical situations.
- ✓ so it is important to model (at least approximately) these forces of friction.
- ✓ a very simple model for friction is viscous friction, in which the torque due to friction is proportional to the velocity of joint motion.

$$\tau_{friction} = \vartheta\dot{\theta}$$

✓ Another possible simple model for friction, Coulomb friction, is sometimes used.

$$\tau_{friction} = c\ sgn(\dot{\theta})$$

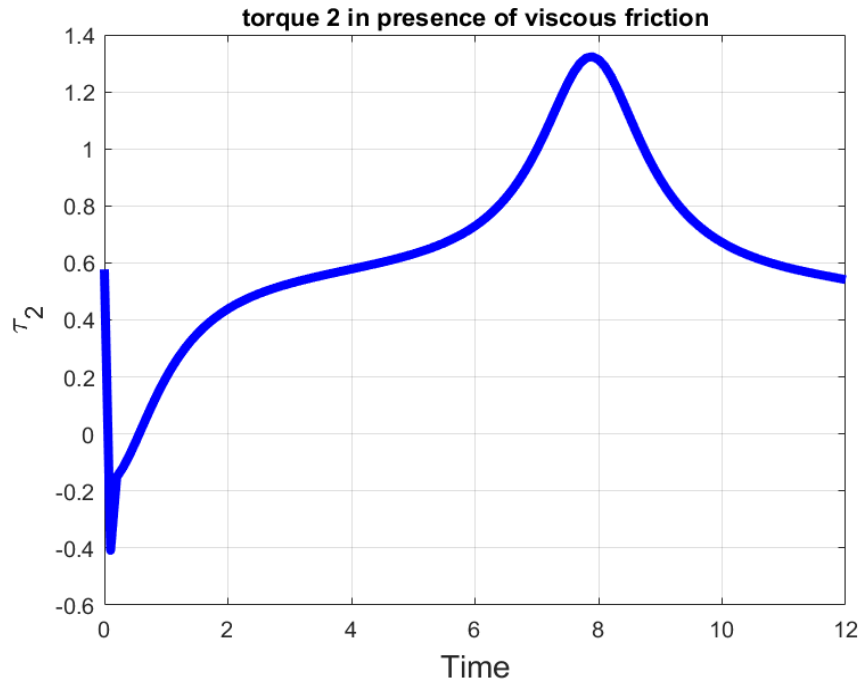
in general:

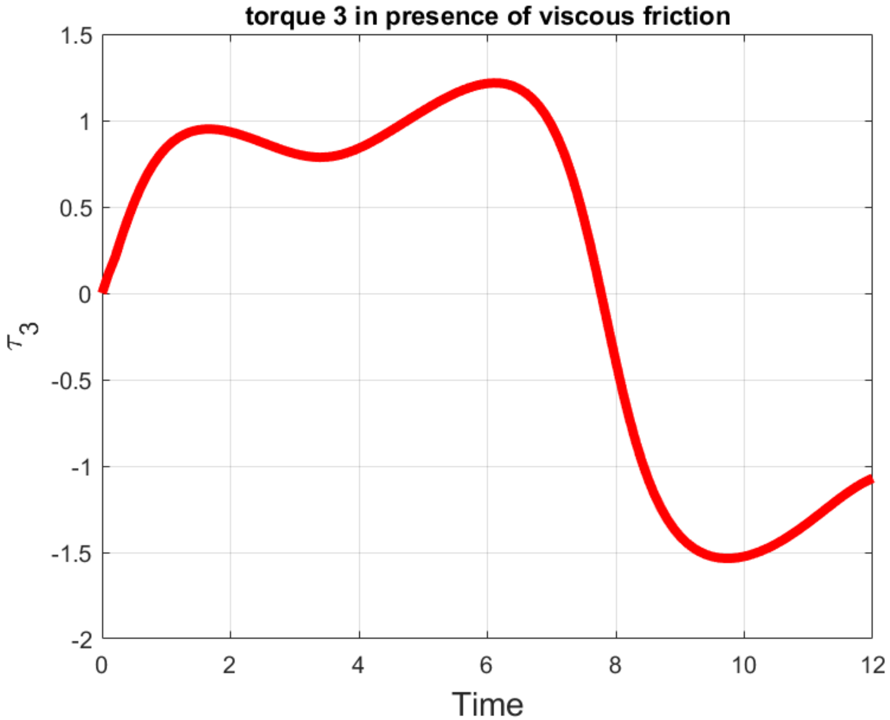
$$\tau_{friction} = \vartheta\dot{\theta} + c\ sgn(\dot{\theta}) = F(\theta, \dot{\theta})$$

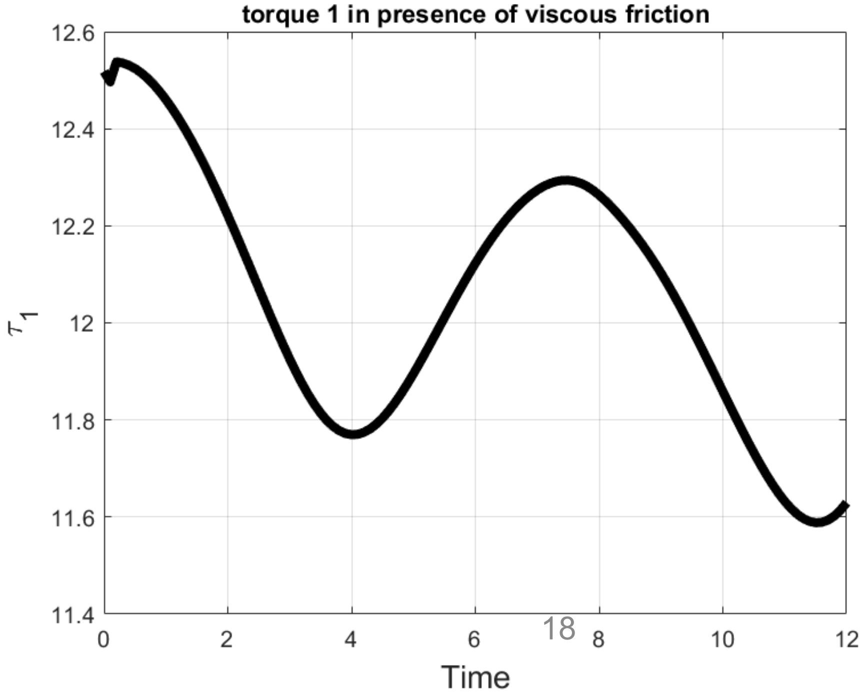
$$\boldsymbol{\tau = M\ddot{\theta} + C\dot{\theta} + \tau_g + F(\theta, \dot{\theta})}$$

simulations

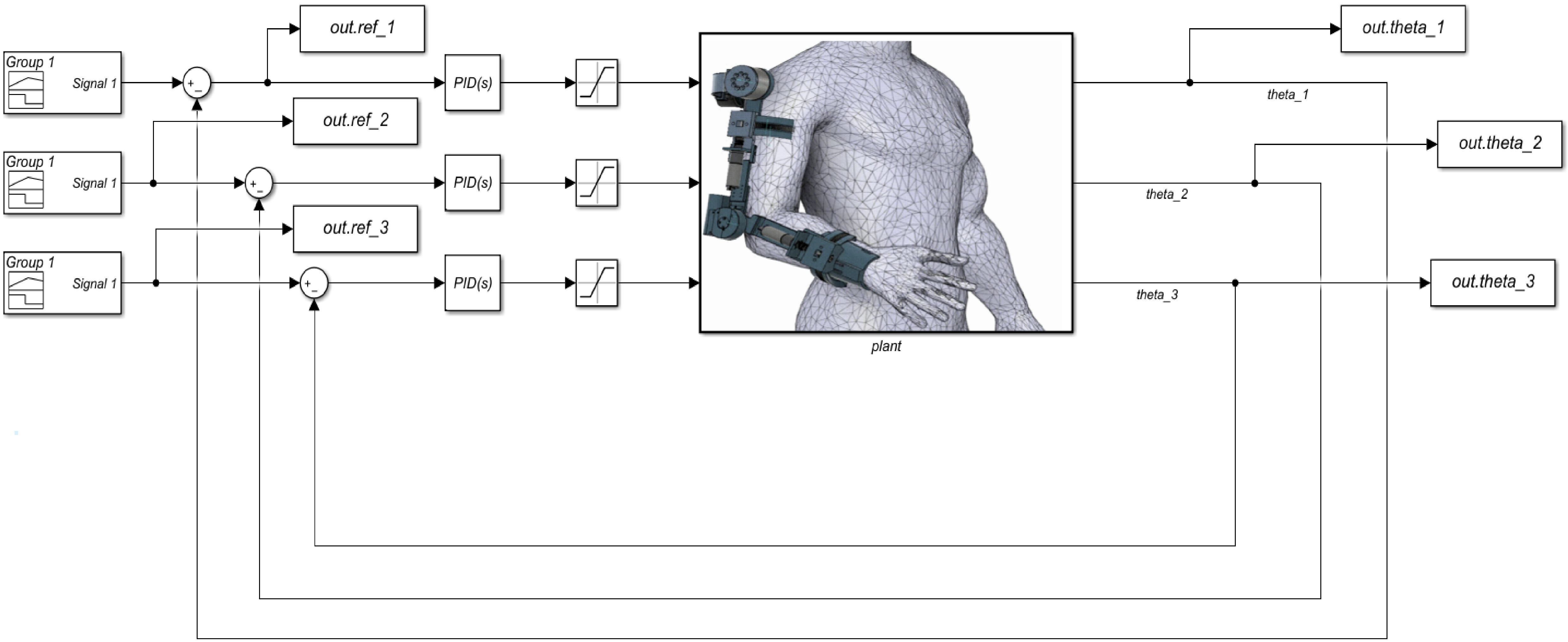
v_1	**0.68**
v_2	0.8
v_3	0.7

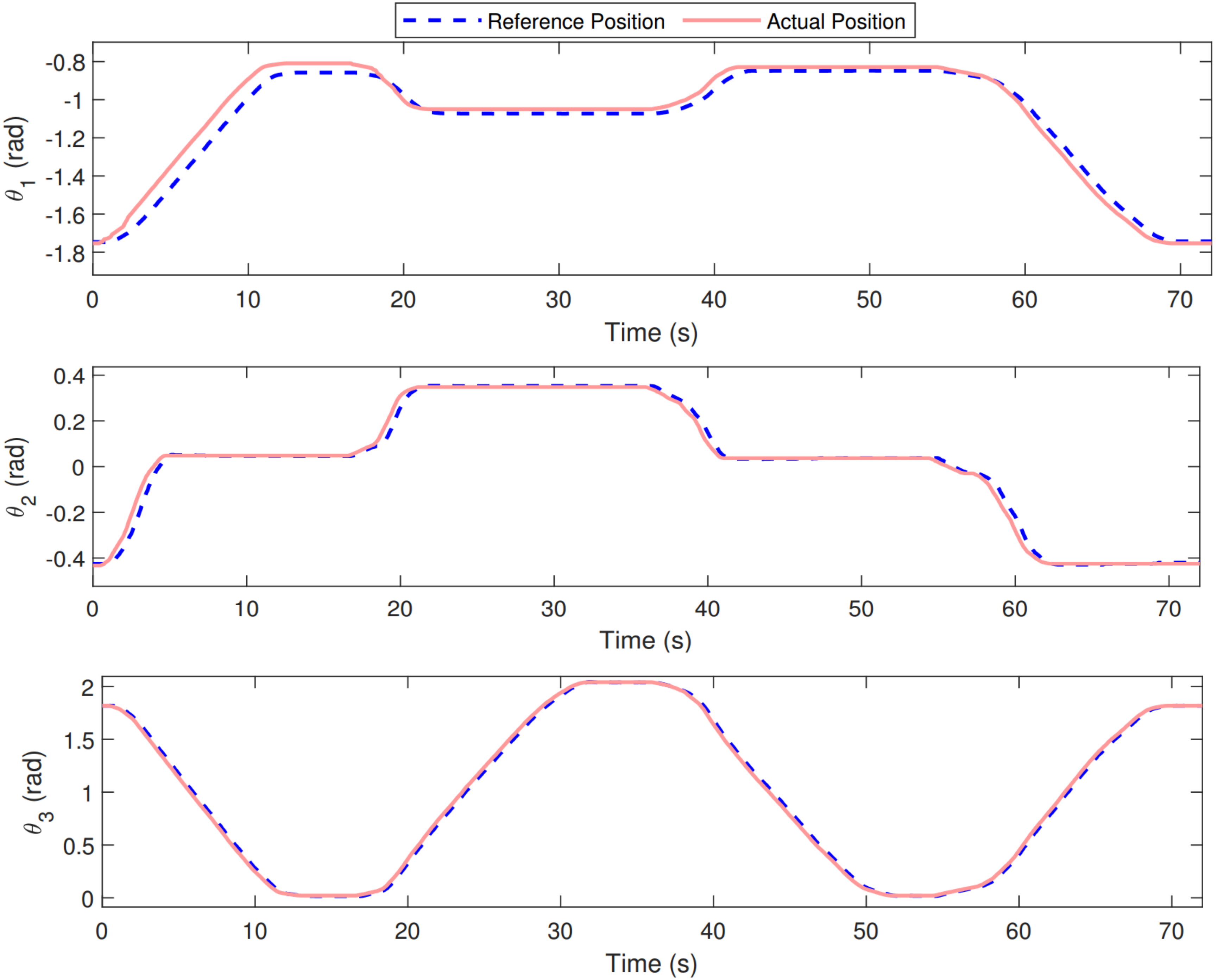






Position control of manipulator




Reference Position
Actual Position
θ1 (rad)
-0.8
-1
-1.2
-1.4
-1.6
-1.8
0
10
20
30
40
50
60
70
Time (s)
θ2 (rad)
0.4
0.2
0
-0.2
-0.4
0
10
20
30
40
50
60
70
Time (s)
θ3 (rad)
2
1.5
1
0.5
0
0
10
20
30
40
50
60
70
Time (s)

Refferences

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