**9.1** A sinusoidal voltage is given by the expression

$$v = 100\cos(240\pi t + 45^\circ)\ \text{mV}.$$

Find (a) $f$ in hertz; (b) $T$ in milliseconds; (c) $V_m$; (d) $v(0)$; (e) $\phi$ in degrees and radians; (f) the smallest positive value of $t$ at which $v = 0$; and (g) the smallest positive value of $t$ at which $dv/dt = 0$.

Solution :

**[a]** $\omega = 2\pi f = 240\pi\ \text{rad/s}, \qquad f = \dfrac{\omega}{2\pi} = 120\ \text{Hz}$

**[b]** $T = 1/f = 8.33\ \text{ms}$

**[c]** $V_m = 100\ \text{V}$

**[d]** $v(0) = 100\cos(45^\circ) = 70.71\ \text{V}$

**[e]** $\phi = 45^\circ; \qquad \phi = \dfrac{45^\circ(2\pi)}{360^\circ} = \dfrac{\pi}{4} = 0.7854\ \text{rad}$

**[f]** $V = 0$ when $240\pi t + 45^\circ = 90^\circ$. Now resolve the units:

$$(240\pi\ \text{rad/s})t = \frac{45^\circ}{57.3^\circ/\text{rad}} = \frac{\pi}{4}\ \text{rad}, \qquad t = 1.042\ \text{ms}$$

**[g]** $(dv/dt) = (-100)240\pi\sin(240\pi t + 45^\circ)$

$$(dv/dt) = 0 \quad \text{when} \quad 240\pi t + 45^\circ = 180^\circ$$

or $$240\pi t = \frac{135^\circ}{57.3^\circ/\text{rad}} = \frac{3\pi}{4}\ \text{rad}$$

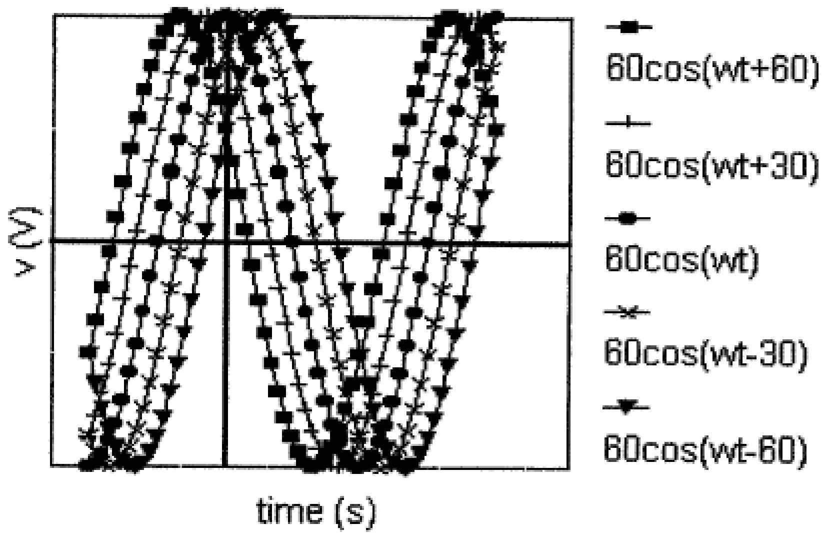
Therefore $t = 3.125\ \text{ms}$

**9.2** In a single graph, sketch $v = 60\cos(\omega t + \phi)$ versus $\omega t$ for $\phi = -60^\circ, -30^\circ, 0^\circ, 30^\circ$, and $60^\circ$.

a) State whether the voltage function is shifting to the right or left as $\phi$ becomes more positive.

b) What is the direction of shift if $\phi$ changes from 0 to $-30^\circ$?

Solution :




[a] Left as $\phi$ becomes more positive

[b] Right

**9.3** At $t = -250/6\ \mu s$, a sinusoidal voltage is known to be zero and going positive. The voltage is next zero at $t = 1250/6\ \mu s$. It is also known that the voltage is 75 V at $t = 0$.

a) What is the frequency of $v$ in hertz?

b) What is the expression for $v$?

Solution:

**[a]** $$\frac{T}{2} = \frac{1250}{6} + \frac{250}{6} = 250\ \mu s; \qquad T = 500\ \mu s$$

$$f = \frac{1}{T} = \frac{10^6}{500} = 2000\text{Hz}$$

**[b]** $v = V_m \sin(\omega t + \theta)$

$$\omega = 2\pi f = 4000\pi \text{ rad/s}$$

$$4000\pi\left(\frac{-250}{6} \times 10^{-6}\right) + \theta = 0; \qquad \therefore\ \theta = \frac{\pi}{6}\text{ rad} = 30^\circ$$

$$v = V_m \sin[4000\pi t + 30^\circ]$$

$$75 = V_m \sin 30^\circ; \qquad V_m = 150\,\text{V}$$

$$v = 150 \sin[4000\pi t + 30^\circ] = 150 \cos[4000\pi t - 60^\circ]\,\text{V}$$

**9.4** A sinusoidal current is zero at $t = 150\ \mu s$ and increasing at a rate of $2 \times 10^4\pi$ A/s. The maximum amplitude of the voltage is 10 A.

a) What is the frequency of $v$ in radians per second?

b) What is the expression for $v$?

Solution:

[a] By hypothesis

$$i = 10\cos(\omega t + \theta)$$

$$\frac{di}{dt} = -10\omega\sin(\omega t + \theta)$$

$$\therefore \quad 10\omega = 20{,}000\pi; \qquad \omega = 2000\pi \text{ rad/s}$$

[b] $f = \dfrac{\omega}{2\pi} = 1000$ Hz; $\qquad T = \dfrac{1}{f} = 1$ ms $= 1000\ \mu$s

$$\frac{150}{1000} = \frac{3}{20}, \qquad \therefore \quad \theta = -90 - \frac{3}{20}(360) = -144^\circ$$

$$\therefore \quad i = 10\cos(2000\pi t - 144^\circ)\text{ A}$$

**9.5** Consider the sinusoidal voltage

$$v(t) = 170 \cos(120\pi t - 60^\circ) \text{ V}.$$

a) What is the maximum amplitude of the voltage?
b) What is the frequency in hertz?
c) What is the frequency in radians per second?
d) What is the phase angle in radians?
e) What is the phase angle in degrees?
f) What is the period in milliseconds?
g) What is the first time after $t = 0$ that $v = 170$ V?
h) The sinusoidal function is shifted 125/18 ms to the right along the time axis. What is the expression for $v(t)$?
i) What is the minimum number of milliseconds that the function must be shifted to the right if the expression for $v(t)$ is $170 \sin 120\pi t$ V?
j) What is the minimum number of milliseconds that the function must be shifted to the left if the expression for $v(t)$ is $170 \cos 120\pi t$ V?

Solution:

[a] 170 V

[b] $2\pi f = 120\pi; \qquad f = 60\text{Hz}$

[c] $\omega = 120\pi = 376.99 \text{ rad/s}$

[d] $\theta(\text{rad}) = \dfrac{-\pi}{180}(60) = \dfrac{-\pi}{3} = -1.05 \text{ rad}$

[e] $\theta = -60^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67 \text{ ms}$

[g] $120\pi t - \dfrac{\pi}{3} = 0; \qquad \therefore \ t = \dfrac{1}{360} = 2.78$ m

[h] $$v = 170 \cos\left[120\pi\left(t + \frac{0.125}{18}\right) - \frac{\pi}{3}\right]$$

$$= 170 \cos[120\pi t + (15\pi/18) - (\pi/3)]$$

$$= 170 \cos[120\pi t + (\pi/2)]$$

$$= -170 \sin 120\pi t \text{ V}$$

[i] $120\pi(t - t_o) - (\pi/3) = 120\pi t - (\pi/2)$

$$\therefore \ 120\pi t_o = \frac{\pi}{6}; \qquad t_o = \frac{25}{18} \text{ ms}$$

[j] $120\pi(t - t_o) - (\pi/3) = 120\pi t$

$$\therefore \ 120\pi t_o = \frac{\pi}{3}; \qquad t_o = \frac{25}{9} \text{ ms}$$

**9.6** Show that

$$\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi)dt = \frac{V_m^2 T}{2}$$

Solution:

$$\begin{aligned}
u &= \int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\, dt \\
&= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\, dt \\
&= \frac{V_m^2}{2}\left\{ \int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\, dt \right\} \\
&= \frac{V_m^2}{2}\left\{ T + \frac{1}{2\omega}\left[ \sin(2\omega t + 2\phi)\, \Big|_{t_o}^{t_o+T} \right] \right\} \\
&= \frac{V_m^2}{2}\left\{ T + \frac{1}{2\omega}\left[ \sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi) \right] \right\} \\
&= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right)
\end{aligned}$$

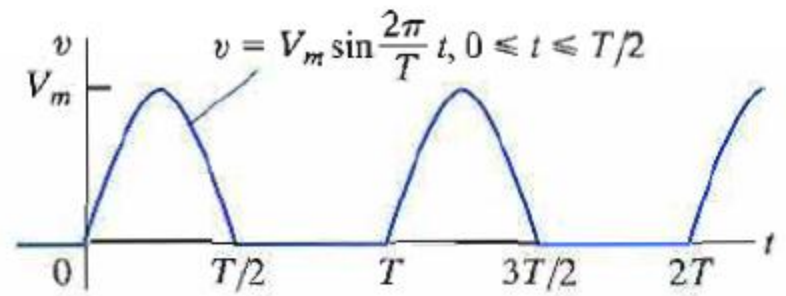
**9.7** The rms value of the sinusoidal voltage supplied to the convenience outlet of a U.S. home is 120 V. What is the maximum value of the voltage at the outlet?

Solution

$$V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(120) = 169.71\text{ V}$$

**9.8** Find the rms value of the half-wave rectified sinusoidal voltage shown.

Figure P9.8




Solution:

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t \, dt}$$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}t\right) dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right) dt = \frac{V_m^2 T}{4}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T}\frac{V_m^2 T}{4}} = \frac{V_m}{2}$

**9.9** The voltage applied to the circuit shown in Fig. 9.5 at $t = 0$ is $100 \cos(400t + 60^\circ)$ V. The circuit resistance is 40 Ω and the initial current in the 75 mH inductor is zero.

a) Find $i(t)$ for $t \geq 0$.

b) Write the expressions for the transient and steady-state components of $i(t)$.

c) Find the numerical value of $i$ after the switch has been closed for 1.875 ms.

d) What are the maximum amplitude, frequency (in radians per second), and phase angle of the steady-state current?

e) By how many degrees are the voltage and the steady-state current out of phase?

Solution

[a] The numerical values of the terms in Eq. 9.8 are

$$V_m = 100, \qquad R/L = 533.33, \qquad \omega L = 30$$

$$\sqrt{R^2 + \omega^2 L^2} = 50$$

$$\phi = 60^\circ, \qquad \theta = \tan^{-1} 30/40, \qquad \theta = 36.87^\circ$$

$$i = \left[-1.84e^{-533.33t} + 2\cos(400t + 23.13^\circ)\right] \text{ A}, \qquad t \geq 0$$

[b] Transient component $= -1.84e^{-533.33t}$ A
Steady-state component $= 2\cos(400t + 23.13^\circ)$ A

[c] By direct substitution into Eq 9.9, $i(1.875\,\text{ms}) = 133.61\,\text{mA}$

[d] 2 A, 400 rad/s, 23.13°

[e] The current lags the voltage by 36.87°.

**9.10** a) Verify that Eq. 9.9 is the solution of Eq. 9.8. This can be done by substituting Eq. 9.9 into the left-hand side of Eq. 9.8 and then noting that it equals the right-hand side for all values of $t > 0$. At $t = 0$, Eq. 9.9 should reduce to the initial value of the current.

b) Because the transient component vanishes as time elapses and because our solution must satisfy the differential equation for all values of $t$, the steady-state component, by itself, must also satisfy the differential equation. Verify this observation by showing that the steady-state component of Eq. 9.9 satisfies Eq. 9.8.

Solution

Note:

$$L\frac{di}{dt} + Ri = V_m \cos(\omega t + \phi), \quad (9.8)$$

$$i = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta), \quad (9.9)$$

[a] From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi - \theta) e^{-(R/L)t}}{\sqrt{R^2 + \omega^2 L^2}} + \frac{V_m R\cos(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$L\frac{di}{dt} + Ri = V_m \left[\frac{R\cos(\omega t + \phi - \theta) - \omega L\sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$V_m \cos(\omega t + \phi)$

At $t = 0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m \cos(\phi - \theta)}{\sqrt{R^2 - \omega^2 L^2}} + \frac{V_m \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} = 0$$

[b] $i_{ss} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$

Therefore

$$L \frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi - \theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

$$L \frac{di_{ss}}{dt} + Ri_{ss} = V_m \left[ \frac{R \cos(\omega t + \phi - \theta) - \omega L \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} \right]$$

$$= V_m \cos(\omega t + \phi)$$

**9.11** Use the concept of the phasor to combine the following sinusoidal functions into a single trigonometric expression:

a) $y = 100\cos(300t + 45°) + 500\cos(300t - 60°)$,

b) $y = 250\cos(377t + 30°) - 150\sin(377t + 140°)$,

c) $y = 60\cos(100t + 60°) - 120\sin(100t - 125°) + 100\cos(100t + 90°)$, and

d) $y = 100\cos(\omega t + 40°) + 100\cos(\omega t + 160°) + 100\cos(\omega t - 80°)$.

solution

[a] $\mathbf{Y} = 100\underline{/45^\circ} + 500\underline{/-60^\circ} = 483.86\underline{/-48.48^\circ}$

$y = 483.86\cos(300t - 48.48^\circ)$

[b] $\mathbf{Y} = 250\underline{/30^\circ} - 150\underline{/50^\circ} = 120.51\underline{/4.8^\circ}$

$y = 120.51\cos(377t + 4.8^\circ)$

[c] $\mathbf{Y} = 60\underline{/60^\circ} - 120\underline{/-215^\circ} + 100\underline{/90^\circ} = 152.88\underline{/32.94^\circ}$

$y = 152.88\cos(100t + 32.94^\circ)$

[d] $\mathbf{Y} = 100\underline{/40^\circ} + 100\underline{/160^\circ} + 100\underline{/-80^\circ} = 0$

$y = 0$

**9.12** A 50 Hz sinusoidal voltage with a maximum amplitude of 340 V at $t = 0$ is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 8.5 A.

a) What is the frequency of the inductor current?

b) If the phase angle of the voltage is zero, what is the phase angle of the current?

c) What is the inductive reactance of the inductor?

d) What is the inductance of the inductor in millihenrys?

e) What is the impedance of the inductor?

Solution

[a] 50Hz

[b] $\theta_v = 0°$

$$\mathbf{I} = \frac{340\underline{/0°}}{j\omega L} = \frac{340}{\omega L}\underline{/-90°} = 8.5\underline{/-90°}; \qquad \theta_i = -90°$$

[c] $\dfrac{340}{\omega L} = 8.5; \qquad \omega L = 40\,\Omega$

[d] $L = \dfrac{40}{100\pi} = \dfrac{400}{\pi}\,\text{mH} = 127.32\,\text{mH}$

[e] $Z_L = j\omega L = j40\,\Omega$

**9.13** A 40 kHz sinusoidal voltage has zero phase angle and a maximum amplitude of 2.5 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of 125.67 $\mu$A.

a) What is the frequency of the current in radians per second?

b) What is the phase angle of the current?

c) What is the capacitive reactance of the capacitor?

d) What is the capacitance of the capacitor in microfarads?

e) What is the impedance of the capacitor?

Solution

[a] $\omega = 2\pi f = 80\pi \times 10^3 = 251.33\,\text{krad/s} = 251{,}327.41\,\text{rad/s}$

[b] $$\mathbf{I} = \frac{2.5 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(2.5 \times 10^{-3})\underline{/0^\circ} = 2.5 \times 10^{-3}\omega C\underline{/90^\circ}$$

$$\therefore \quad \theta_i = 90^\circ$$

[c] $125.66 \times 10^{-6} = 2.5 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{2.5 \times 10^{-3}}{125.66 \times 10^{-6}} = 19.89\,\Omega, \quad \therefore \quad X_C = -19.89\,\Omega$$

[d] $$C = \frac{1}{19.89(\omega)} = \frac{1}{(19.89)(80\pi \times 10^3)}$$

$$C = 0.2 \times 10^{-6} = 0.2\,\mu\text{F}$$

[e] $$Z_c = j\left(\frac{-1}{\omega C}\right) = -j19.89\,\Omega$$

**9.14** The expressions for the steady-state voltage and current at the terminals of the circuit seen in Fig. P9.14 are

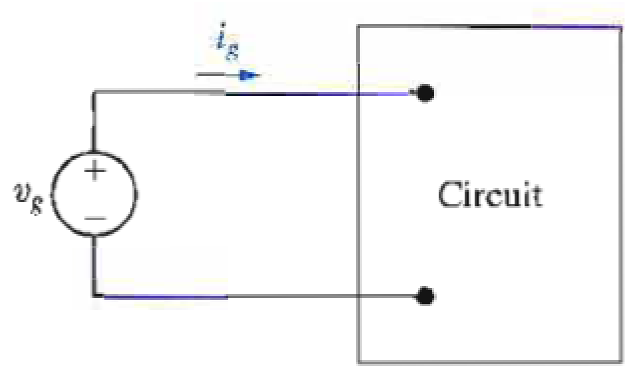
$$v_g = 150 \cos(8000\pi t + 20^\circ)\ \text{V},$$

$$i_g = 30 \sin(8000\pi t + 38^\circ)\ \text{A}$$

a) What is the impedance seen by the source?

b) By how many microseconds is the current out of phase with the voltage?

**Figure P9.14**




Solution

[a] $\mathbf{V}_g = 150\underline{/20^\circ}$; $\quad \mathbf{I}_g = 30\underline{/-52^\circ}$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 5\underline{/72^\circ}\,\Omega$$

[b] $i_g$ lags $v_g$ by 72°:

$$2\pi f = 8000\pi; \qquad f = 4000\ \text{Hz}; \qquad T = 1/f = 250\ \mu\text{s}$$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{72}{360}(250) = 50\ \mu\text{s}$$

**9.15** A 20 Ω resistor and a 1 μF capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of a 1 Ω resistor and a 40 μH inductor. These three parallel branches are driven by a sinusoidal current source whose current is 20 cos(50,000$t$ − 20°) A.

PSPICE

a) Draw the frequency-domain equivalent circuit.

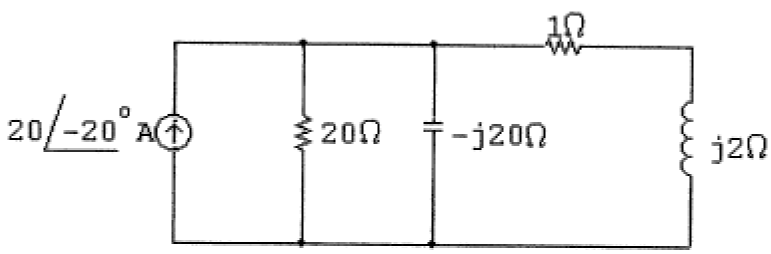
b) Reference the voltage across the current source as a rise in the direction of the source current, and find the phasor voltage.

c) Find the steady-state expression for $v(t)$.

Solution

[a] $j\omega L = j(5\times10^4)(40\times10^{-6}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{5\times10^4} = -j20\,\Omega; \qquad \mathbf{I}_g = 20\underline{/-20^\circ}\text{ A}$$




[b] $\mathbf{V}_o = 20\underline{/-20^\circ} Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{20} + j\frac{1}{20} + \frac{1}{1+j2}$$

$$Y_e = 0.05 + j0.05 + 0.20 - j0.40 = 0.25 - j0.35\text{ S}$$

$$Z_e = \frac{1}{0.25 - j0.35} = 2.32\underline{/54.46^\circ}\,\Omega$$

$$\mathbf{V}_o = (20\underline{/-20^\circ})(2.32\underline{/54.46^\circ}) = 46.4\underline{/34.46^\circ}\text{ V}$$

[c] $v_o = 46.4\cos(5\times10^4 t + 34.46^\circ)$ V

**9.16** A 400 Ω resistor, a 87.5 mH inductor, and a 312.5 nF capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is 500 cos (8000*t* + 60°) V.

PSPICE

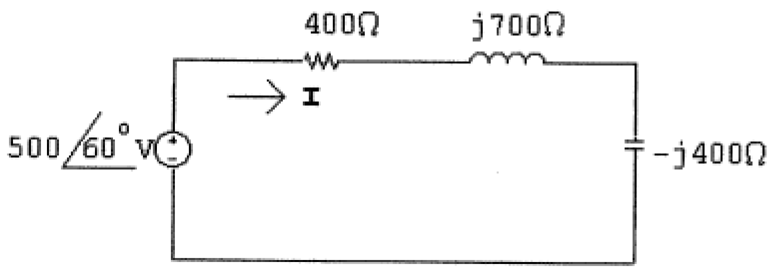
a) Draw the frequency-domain equivalent circuit.

b) Reference the current in the direction of the voltage rise across the source, and find the phasor current.

c) Find the steady-state expression for *i*(*t*).

Solution

[a]




[b] $\mathbf{I} = \dfrac{500\underline{/60^\circ}}{400 + j700 - j400} = 1\underline{/23.13^\circ}\ \text{A}$

[c] $i = 1\cos(8000t + 23.13^\circ)\ \text{A}$

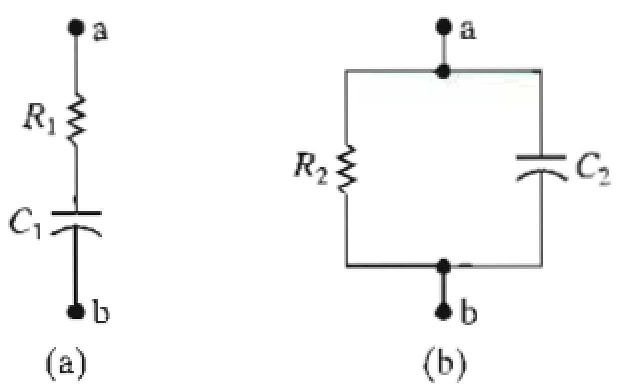
**9.17** a) Show that at a given frequency $\omega$, the circuits in Fig. P9.17(a) and (b) will have the same impedance between the terminals a,b if

$$R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2},$$

$$C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}.$$

b) Find the values of resistance and capacitance that when connected in series will have the same impedance at 80 krad/s as that of a 500 Ω resistor connected in parallel with a 25 nF capacitor.

**Figure P9.17**




## Solution

[a] $Z_1 = R_1 - j\dfrac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $$R_1 = \frac{500}{1 + (64 \times 10^8)(25 \times 10^4)(625 \times 10^{-18})} = 250\,\Omega$$

$$C_1 = \frac{2}{(64 \times 10^8)(25 \times 10^4)(25 \times 10^{-9})} = 50\,\text{nF}$$

**9.20** a) Show that at a given frequency $\omega$, the circuits in Fig. P9.19(a) and (b) will have the same impedance between the terminals a,b if

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1}, \qquad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}.$$

(*Hint:* The two circuits will have the same impedance if they have the same admittance.)

b) Find the values of resistance and inductance that when connected in parallel will have the same impedance at 10 krad/s as a 5 k$\Omega$ resistor connected in series with a 500 mH inductor.

Solution

[a] $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

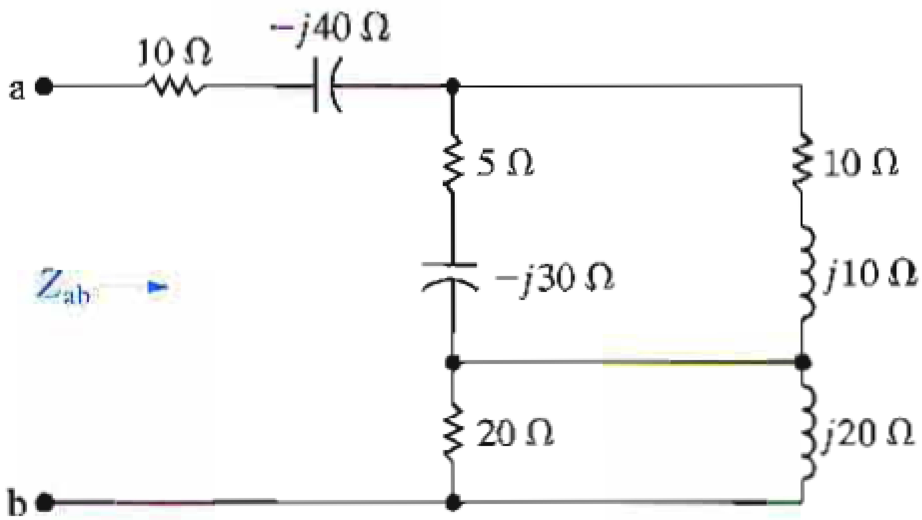
[b] $R_2 = \dfrac{25 \times 10^6 + 10^8(0.25)}{5 \times 10^3} = 10 \times 10^3$

$\therefore \quad R_2 = 10\,\text{k}\Omega$

$$L_2 = \frac{50 \times 10^6}{10^8(0.5)} = 1\,\text{H}$$

**9.23** Find the impedance $Z_{ab}$ in the circuit seen in Fig. P9.23. Express $Z_{ab}$ in both polar and rectangular form.

Figure P9.23




Solution

$Z_1 = 10 - j40\,\Omega$

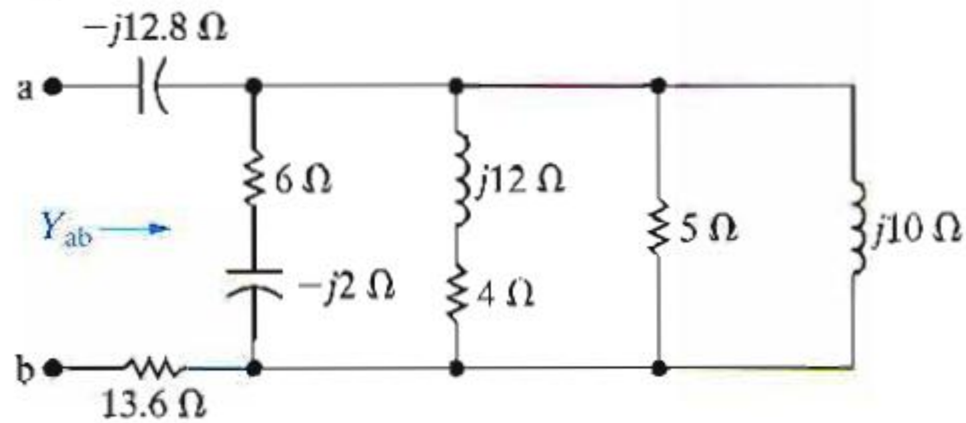
$$Z_2 = \frac{(5 - j10)(10 + j30)}{15 + j20} = 10 - j10\,\Omega$$

$$Z_3 = \frac{20(j20)}{20 + j20} = 10 + j10\,\Omega$$

$\therefore\quad Z_{ab} = Z_1 + Z_2 + Z_3 = 30 - j40\,\Omega = 50\underline{/-53.13^\circ}\,\Omega$

**9.24** Find the admittance $Y_{ab}$ in the circuit seen in Fig. P9.24. Express $Y_{ab}$ in both polar and rectangular form. Give the value of $Y_{ab}$ in millisiemens.

Figure P9.24




Solution

First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{ab} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

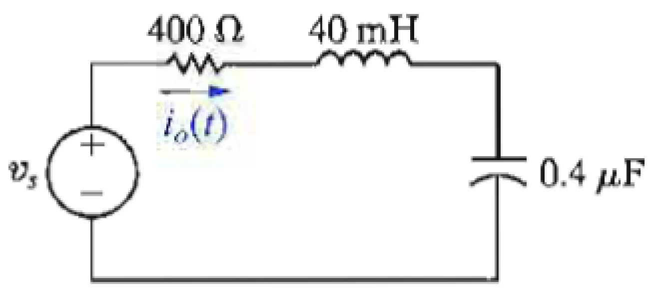
$$Y_{ab} = \frac{1}{Z_{ab}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

**9.25** Find the steady-state expression for $i_o(t)$ in the circuit in Fig. P9.25 if $v_s = 750 \cos 5000t$ mV.

PSPICE

Figure P9.25




Solution

$$Z = 400 + j(5)(40) - j\frac{1000}{(5)(0.4)} = 500\underline{/-36.87^\circ}\,\Omega$$

$$\mathbf{I}_o = \frac{750\underline{/0^\circ} \times 10^{-3}}{500\underline{/-36.87^\circ}} = 1.5\underline{/36.87^\circ}\,\text{mA}$$

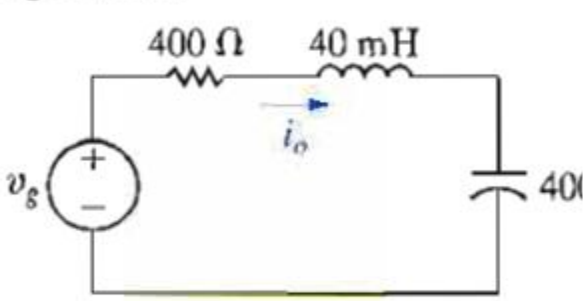
$$i_o(t) = 1.5\cos(5000t + 36.87^\circ)\,\text{mA}$$

**9.26** The circuit shown in Fig. P9.26 is operating in the sinusoidal steady state. Find the value of $\omega$ if

$$i_o = 100 \sin(\omega t + 81.87^\circ)\,\text{mA},$$

$$v_g = 50 \cos(\omega t - 45^\circ)\,\text{V}.$$

Figure P9.26




Solution

$$\mathbf{V}_g = 50\underline{/-45^\circ}\,\text{V}; \qquad \mathbf{I}_g = 100\underline{/-8.13^\circ}\,\text{mA}$$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 500\underline{/-36.87^\circ}\,\Omega = 400 - j300\,\Omega$$

$$Z = 400 + j\left(0.04\omega - \frac{2.5 \times 10^6}{\omega}\right)$$

$$\therefore\ 0.04\omega - \frac{2.5 \times 10^6}{\omega} = -300$$

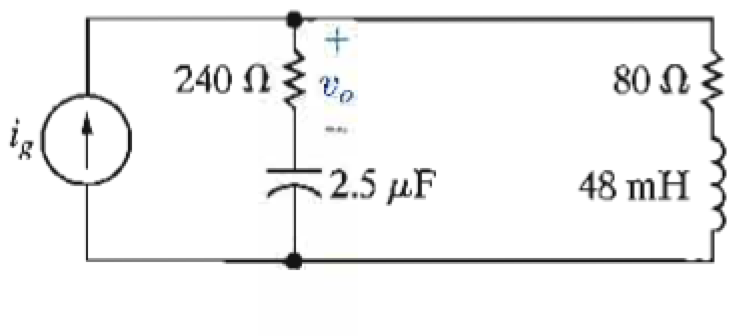
$$\therefore\ \omega^2 + 7500\omega - 62.5 \times 10^6 = 0$$

$$\therefore\ \omega = -3750 \pm \sqrt{(3750)^2 + 62.5 \times 10^6} = -3750 \pm 8750$$

$$\omega > 0, \qquad \therefore\ \omega = 5000\,\text{rad/s}$$

Find the steady-state expression for $v_o$ in the circuit of Fig. P9.27 if $i_g = 200 \cos 5000t$ mA.

**Figure P9.27**

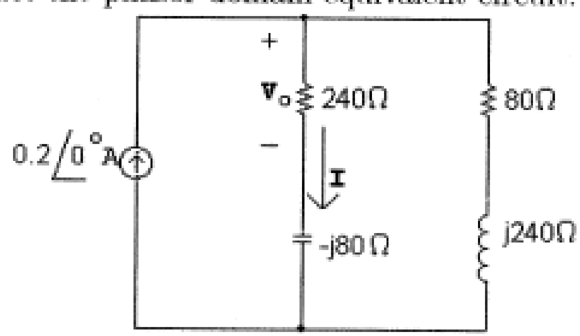



Solution

$$Z_L = j(5000)(48 \times 10^{-3}) = j240\,\Omega$$

$$Z_C = \frac{-j}{(5000)(2.5 \times 10^{-6})} = -j80\,\Omega$$

Construct the phasor domain equivalent circuit:




Using current division:

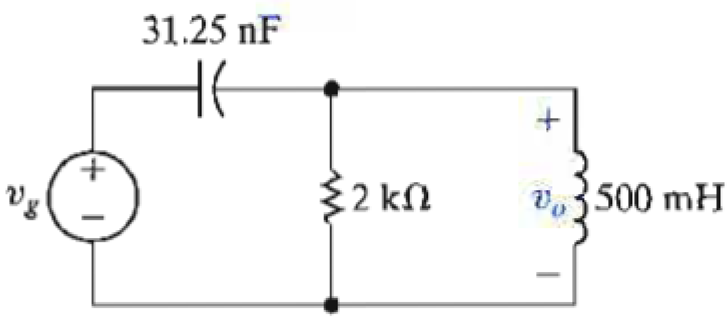
$$\mathbf{I} = \frac{(80 + j240)}{240 - j80 + 80 + j240}(0.2) = 0.1 + j0.1\text{ A}$$

$$\mathbf{V}_o = 240\mathbf{I} = 24 + j24 = 33.94\angle 45^\circ$$

$$v_o = 33.94 \cos(5000t + 45^\circ)\text{ V}$$

**9.28** The circuit in Fig. P9.28 is operating in the sinusoidal steady state. Find the steady-state expression for $v_o(t)$ if $v_g = 64 \cos 8000t$ V.
PSPICE

**Figure P9.28**

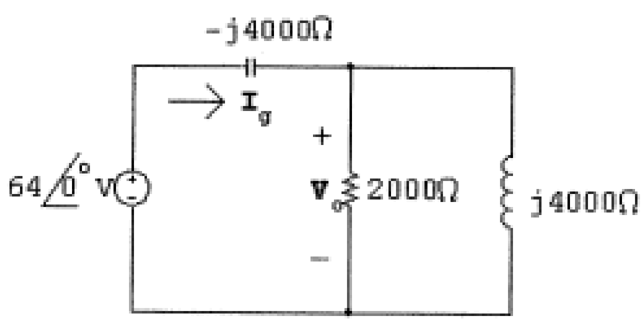



Solution

$$\frac{1}{j\omega C} = \frac{10^9}{(31.25)(8000)} = -j4000\,\Omega$$

$$j\omega L = j8000(500)10^{-3} = j4000\,\Omega$$

$$\mathbf{V}_g = 64\underline{/0^\circ}\text{ V}$$




$$Z_e = \frac{(2000)(j4000)}{2000 + j4000} = 1600 + j800\,\Omega$$

$$Z_T = 1600 + j800 - j4000 = 1600 - j3200\,\Omega$$

$$\mathbf{I}_g = \frac{64\underline{/0^\circ}}{1600 - j3200} = 8 + j16\text{ mA}$$

$$\mathbf{V}_o = Z_e\mathbf{I}_g = (1600 + j800)(0.008 + j0.016) = j32 = 32\underline{/90^\circ}\text{ V}$$

$$v_o = 32\cos(8000t + 90^\circ)\text{ V}$$

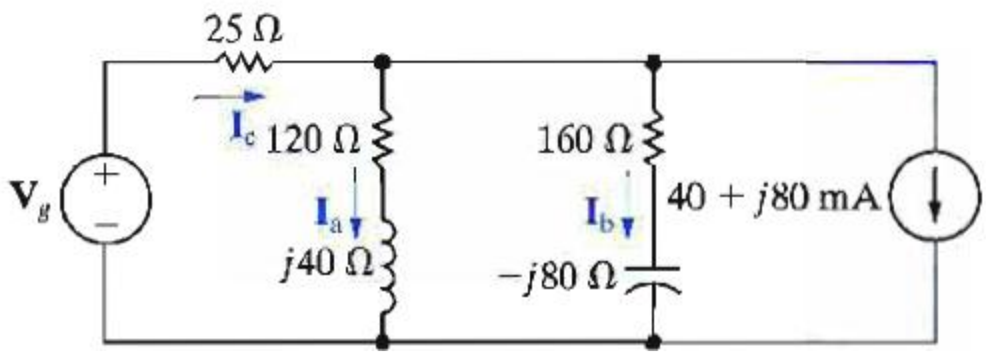
**9.29** The phasor current $\mathbf{I}_a$ in the circuit shown in Fig. P9.29 is $40 \angle 0^\circ$ mA.

PSPICE

a) Find $\mathbf{I}_b$, $\mathbf{I}_c$, and $\mathbf{V}_g$.

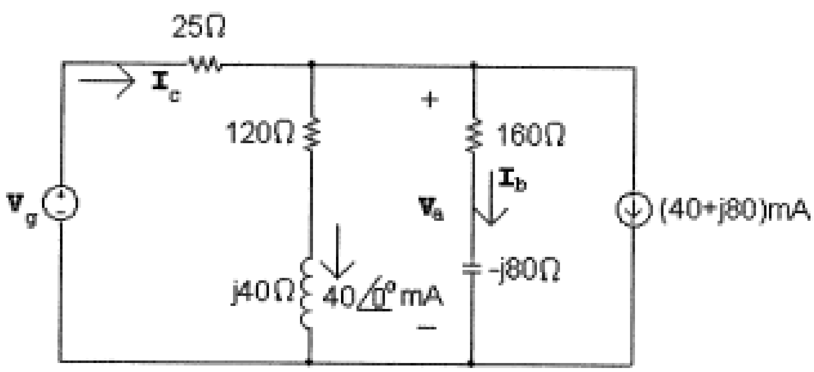
b) If $\omega = 800$ rad/s, write the expressions for $i_b(t)$, $i_c(t)$, and $v_g(t)$.

**Figure P9.29**




Solution

[a]




$$\mathbf{V}_a = (120 + j40)(0.04\underline{/0^\circ}) = 4.8 + j1.6\,\text{V}$$

$$\mathbf{I}_b = \frac{4.8 + j1.6}{160 - j80} = 20 + j20\,\text{mA}$$

$$\mathbf{I}_c = 40\underline{/0^\circ} + (20 + j20) + (40 + j80)\,\text{mA} = 100 + j100\,\text{mA}$$

$$\mathbf{V}_g = 25\mathbf{I}_c + \mathbf{V}_a = 25(0.100 + j0.100) + 4.8 + j1.6 = 7.3 + j4.1\,\text{V}$$

[b] $i_b = 28.28\cos(800t + 45^\circ)\,\text{mA}$

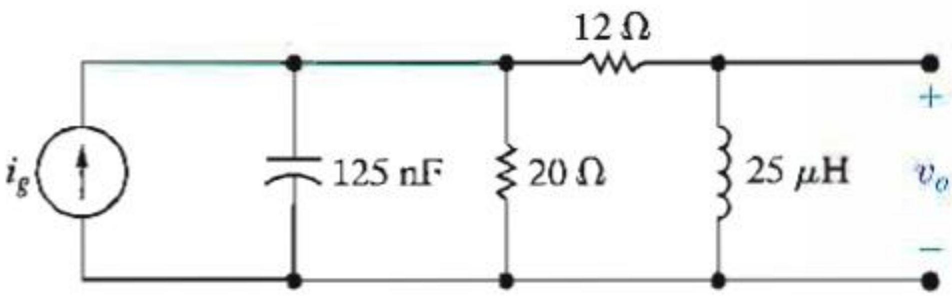
$$i_c = 141.42\cos(800t + 45^\circ)\,\text{mA}$$

$$v_g = 8.37\cos(800t + 29.32^\circ)\,\text{V}$$

**9.30** PSPICE a) For the circuit shown in Fig. P9.30, find the steady-state expression for $v_o$ if $i_g = 5\cos(8 \times 10^5 t)$ A.

b) By how many nanoseconds does $v_o$ lag $i_g$?

**Figure P9.30**




. Solution

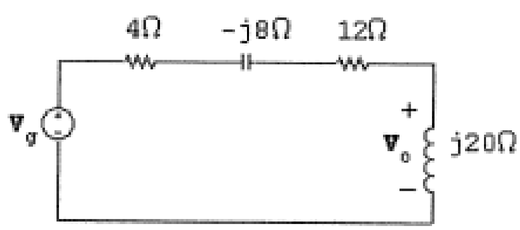
[a] $$\frac{1}{j\omega C} = \frac{10^9}{j8 \times 10^5(125)} = -j10\,\Omega$$

$$j\omega L = j8 \times 10^5(25 \times 10^{-6}) = j20\,\Omega$$

$$Z_e = \frac{(-j10)(20)}{20 - j10} = 4 - j8\,\Omega$$

$$\mathbf{I}_g = 5\underline{/0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 5(4 - j8) = 20 - j40\,\text{V}$$




$$\mathbf{V}_o = \frac{(20 - j40)(j20)}{(16 + j12)} = 44 - j8 = 44.72\underline{/-10.30^\circ}\,\text{V}$$

$$v_o = 44.72\cos(8 \times 10^5 t - 10.30^\circ)\,\text{V}$$

[b] $\omega = 2\pi f = 8 \times 10^5;\qquad f = \dfrac{4 \times 10^5}{\pi}$

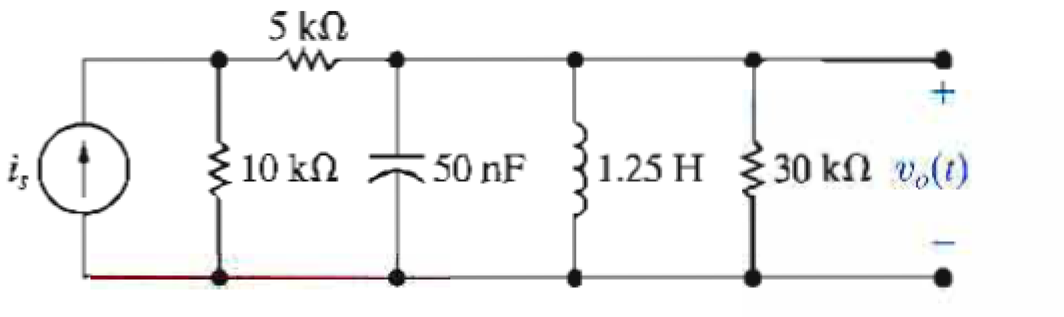
$$T = \frac{1}{f} = \frac{\pi}{4 \times 10^5} = 2.5\pi\,\mu\text{s}$$

$$\therefore\quad \frac{10.30}{360}(2.5\pi) = 224.82\,\text{ns}$$

$\therefore\quad v_o$ lags $i_g$ by 224.82 ns

**9.31** The circuit in Fig. P9.31 is operating in the sinusoidal steady state. Find $v_o(t)$ if $i_s(t) = 15 \cos 8000t$ mA.
PSPICE

**Figure P9.31**



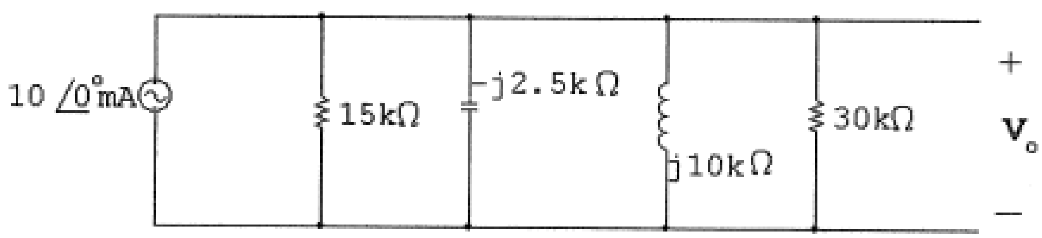

Solution

P 9.31 $\mathbf{I}_s = 15\underline{/0^\circ}$ mA

$$\frac{1}{j\omega C} = \frac{10^6}{j0.05(8000)} = -j2500\,\Omega$$

$$j\omega L = j8000(1.25) = j10{,}000\,\Omega$$

After two source transformations we have




$$15\,\text{k}\Omega \| 30\,\text{k}\Omega = 10\,\text{k}\Omega$$

$$Y_o = \frac{10^{-3}}{10} + \frac{1}{-j2500} + \frac{1}{j10^4} = 10^{-4}(1 + j3)$$

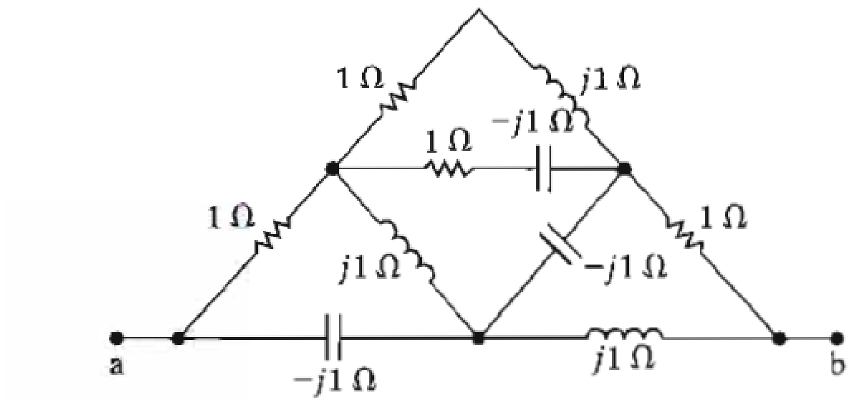
$$Z_o = \frac{10^4}{1 + j3} = (1 - j3)\,\text{k}\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z_o = (10)(1 - j3) = 10 - j30 = 31.62\underline{/-71.57^\circ}\,\text{V}$$

$$v_o = 31.62\cos(8000t - 71.57^\circ)\,\text{V}$$

**9.34** Find $Z_{ab}$ for the circuit shown in Fig P9.34.

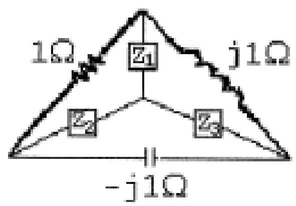
Figure P9.34




Simplify the top triangle using series and parallel combinations:

$(1 + j1)\|(1 - j1) = 1\,\Omega$

Convert the lower left delta to a wye:

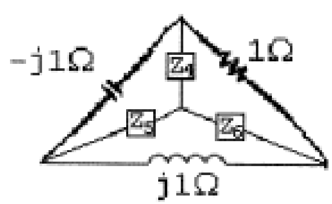



$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

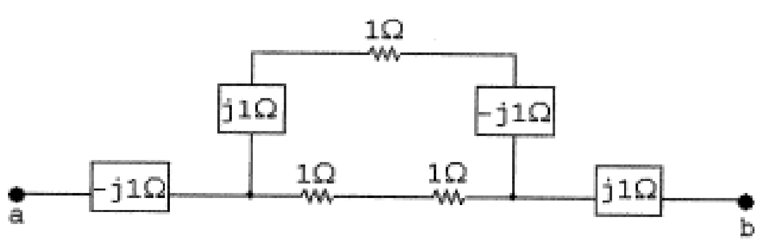



$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:




Simplify the middle portion of the circuit by making series and parallel combinations:

$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$

$Z_{ab} = -j1 + 2/3 + j1 = 2/3\,\Omega$

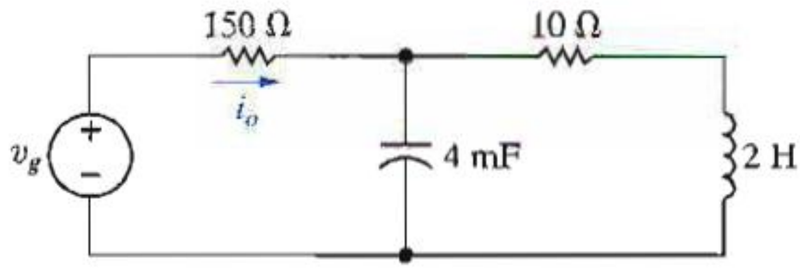
**9.35** The frequency of the sinusoidal voltage source in the circuit in Fig. P9.35 is adjusted until the current $i_o$ in phase with $v_g$.

PSPICE

a) Find the frequency in hertz.

b) Find the steady-state expression for $i_o$ (at the frequency found in [a]) if $v_g = 10\cos\omega t$ V.

**Figure P9.35**




[a] $$Y_p = \frac{1}{10 + j2\omega} + j4 \times 10^{-3}\omega$$

$$= \frac{10 - j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

$$= \frac{10}{100 + 4\omega^2} - \frac{j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

$Y_p$ is real when

$$4 \times 10^{-3}\omega = \frac{2\omega}{100 + 4\omega^2}$$

or $\quad \omega^2 = 100; \quad \omega = 10$ rad/s; $\quad f = 5/\pi = 1.59$Hz

[b] $$Y_p(10\,\text{rad/s}) = \frac{10}{500} = 20\,\text{mS}$$

$$Z_p(10\,\text{rad/s}) = \frac{10^3}{20} = 50\,\Omega$$

$$Z(10\,\text{rad/s}) = 50 + 150 = 200\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_g}{200}\,\text{A} = \frac{10\angle 0^\circ}{200} = 50\angle 0^\circ\,\text{mA}$$

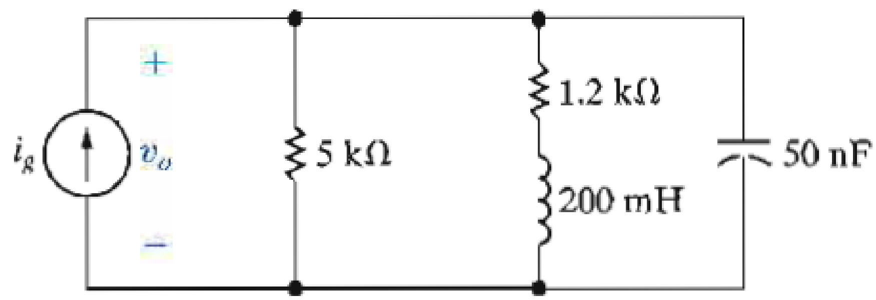
$$i_o = 50\cos 10t\,\text{mA}$$

**9.37** The frequency of the sinusoidal current source in
PSPICE the circuit in Fig. P9.37 is adjusted until $v_o$ is in phase with $i_g$.

a) What is the value of $\omega$ in radians per second?

b) If $i_g = 2.5 \cos \omega t$ mA (where $\omega$ is the frequency found in [a]), what is the steady-state expression for $v_o$?

**Figure P9.37**




[a] $Y_1 = \dfrac{1}{5000} = 0.2 \times 10^{-3}\,\mathrm{S}$

$$Y_2 = \frac{1}{1200 + j0.2\omega}$$

$$= \frac{1200}{1.44 \times 10^6 + 0.04\omega^2} - j\frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

$$Y_3 = j\omega 50 \times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For $i_g$ and $v_o$ to be in phase the $j$ component of $Y_T$ must be zero; thus,

$$\omega 50 \times 10^{-9} = \frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

or

$$0.04\omega^2 + 1.44 \times 10^6 = \frac{0.2 \times 10^9}{50} = 4 \times 10^6$$

$$\therefore\quad 0.04\omega^2 = 2.56 \times 10^6 \qquad \therefore\quad \omega = 8000\,\mathrm{rad/s} = 8\,\mathrm{krad/s}$$

[b] $Y_T = 0.2 \times 10^{-3} + \dfrac{1200}{1.44 \times 10^6 + 0.04(64) \times 10^6} = 0.5 \times 10^{-3}\,\mathrm{S}$

$$\therefore\quad Z_T = 2000\,\Omega$$

$$\mathbf{V}_o = (2.5 \times 10^{-3}\underline{/0^\circ})(2000) = 5\underline{/0^\circ}$$

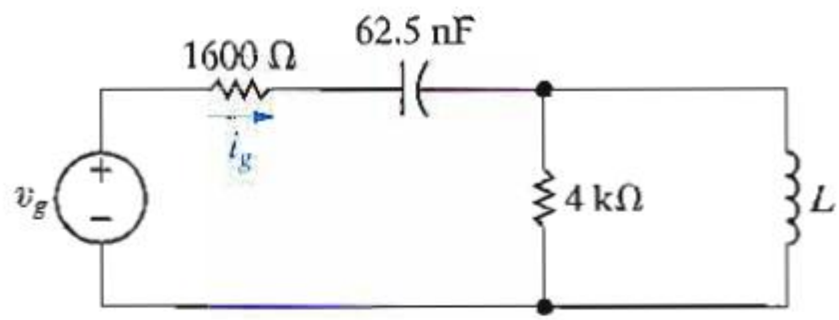
$$v_o = 5 \cos 8000t\ \mathrm{V}$$

**9.39** a) The source voltage in the circuit in Fig. P9.39 is $v_g = 96 \cos 10{,}000t$ V. Find the values of $L$ such that $i_g$ is in phase with $v_g$ when the circuit is operating in the steady state.

PSPICE

b) For the values of $L$ found in (a), find the steady-state expressions for $i_g$.

**Figure P9.39**




[a] $Z_1 = 1600 - j\dfrac{10^9}{10^4(62.5)} = 1600 - j1600\,\Omega$

$$Z_1 = \frac{4000(j10^4L)}{4000 + j10^4L} = \frac{4 \times 10^5L^2 + j16 \times 10^4L}{16 + 100L^2}$$

$$Z_T = Z_1 + Z_2 = 1600 + \frac{4 \times 10^5L^2}{16 + 100L^2} - j1600 + j\frac{16 \times 10^4L}{16 + 100L^2}$$

$Z_T$ is resistive when

$$\frac{16 \times 10^4L}{16 + 100L^2} = 1600 \qquad \text{or}$$

$$L^2 - L + 0.16 = 0$$

Solving, $L_1 = 0.8$ H and $L_2 = 0.2$ H.

[b] When $L = 0.8$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.64)}{16 + 64} = 4800\,\Omega$$

$$\mathbf{I}_g = \frac{96\underline{/0^\circ}}{4.8} \times 10^{-3} = 20\underline{/0^\circ}\text{ mA}$$

$$i_g = 20 \cos 10{,}000t \text{ mA}$$

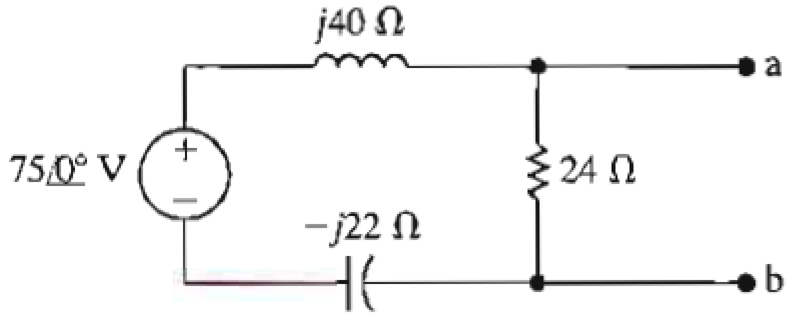
When $L = 0.2$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.04)}{16 + 4} = 2400\,\Omega$$

$$i_g = 40 \cos 10{,}000t \text{ mA}$$

**9.40** Use source transformations to find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.40.

**Figure P9.40**




Step 1 to Step 2:

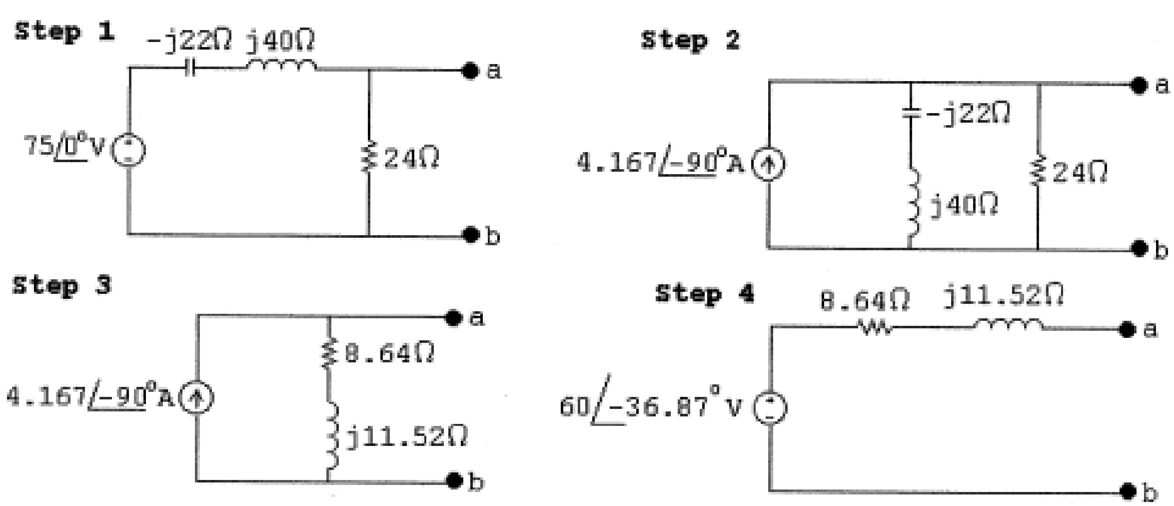
$$\frac{75\angle 0^\circ}{j18} = -j4.167 = 4.167\angle -90^\circ \text{ A}$$

Step 2 to Step 3:

$$(j18)\|24 = \frac{(j18)(24)}{24 + j18} = 8.64 + j11.52\,\Omega$$

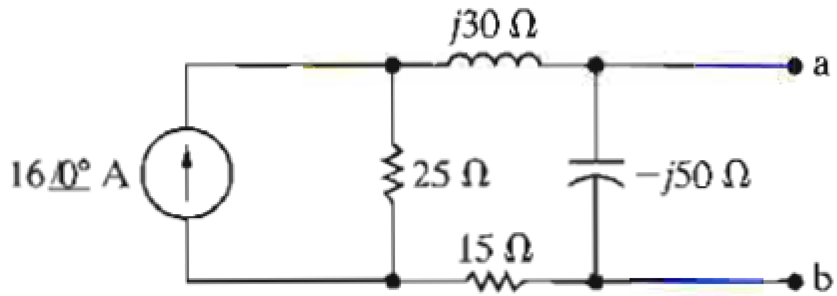
Step 3 to Step 4:

$$(4.167\angle -90^\circ)(8.64 + j11.52) = 60\angle -36.87^\circ \text{ V}$$



**9.41** Use source transformations to find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.41.

**Figure P9.41**




P 9.41 Step 1 to Step 2:

$(16\underline{/0^\circ})(25) = 400\underline{/0^\circ}\text{ V}$

Step 2 to Step 3:

$25 + 15 + j30 = (40 + j30)\,\Omega$

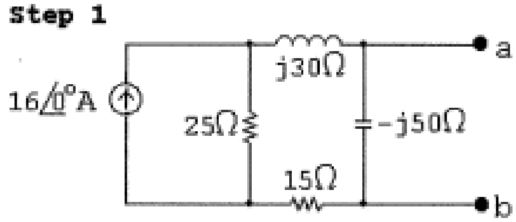
Step 3 to Step 4:

$$\frac{400\underline{/0^\circ}}{(40 + j30)} = 8\underline{/-36.87^\circ}\text{ A}$$

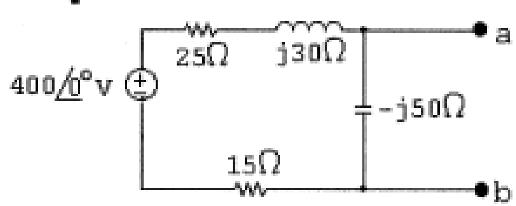
Step 4 to Step 5:

$$(40 + j30 \| -j50 = \frac{(-j50)(40 + j30)}{40 + j30 - j50} = 50 - j25\,\Omega$$

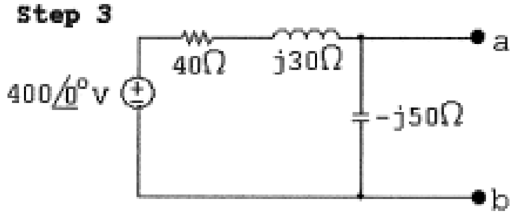
**Step 1**



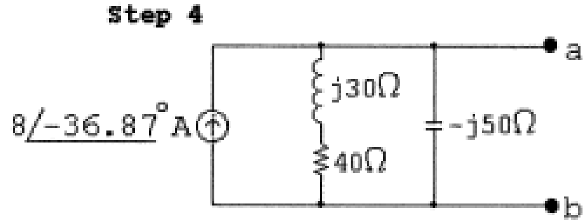

**Step 2**



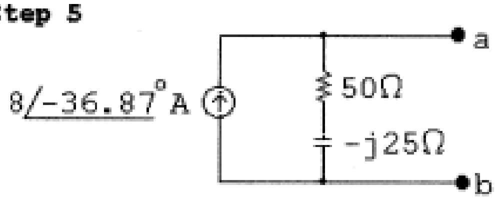

**Step 3**




**Step 4**




**Step 5**



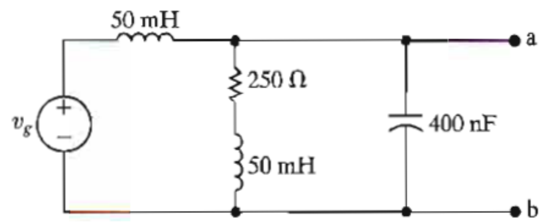
**9.42** The sinusoidal voltage source in the circuit in Fig. P9.42 is developing a voltage equal to $22.36 \cos(5000t + 26.565^\circ)$ V.

a) Find the Thévenin voltage with respect to the terminals a,b.

b) Find the Thévenin impedance with respect to the terminals a,b.

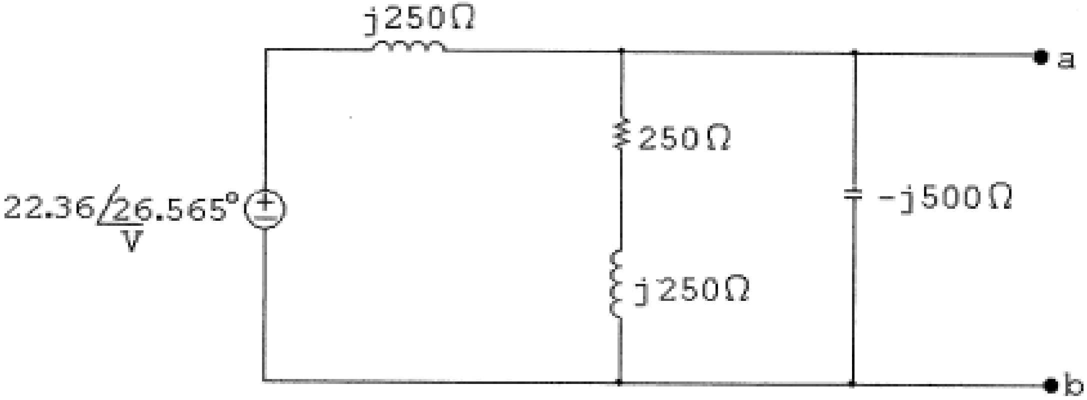
c) Draw the Thévenin equivalent.

Figure P9.42




P 9.42 [a] $j\omega L = j(5000)(50) \times 10^{-3} = j250\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(5000)(400 \times 10^{-9})} = -j500\,\Omega$$




Using voltage division,

$$\mathbf{V}_{ab} = \frac{(250 + j250)\|(-j500)}{j250 + (250 + j250)\|(-j500)}(23.36\underline{/26.565^\circ}) = 20\underline{/0^\circ}$$

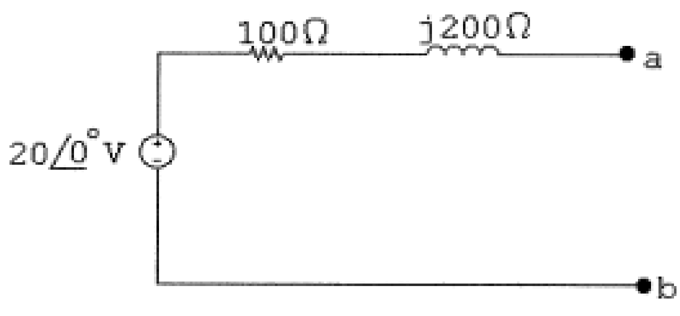
$$\mathbf{V}_{Th} = \mathbf{V}_{ab} = 20\underline{/0^\circ}\text{ V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{Th} = Z_{ab}$:

$$Y_{ab} = \frac{1}{j250} + \frac{1}{250 + j250} + \frac{1}{-j500} = 2 - j4\text{ mS}$$

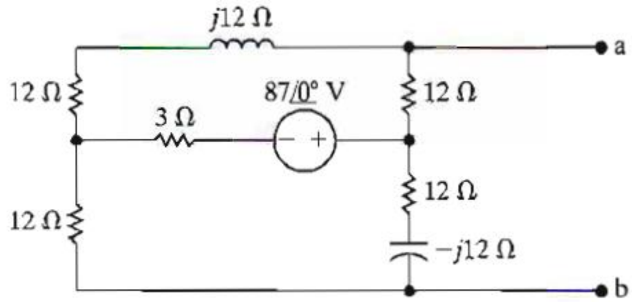
$$Z_{Th} = Z_{ab} = \frac{1}{Y_{ab}} = 100 + j200\,\Omega$$

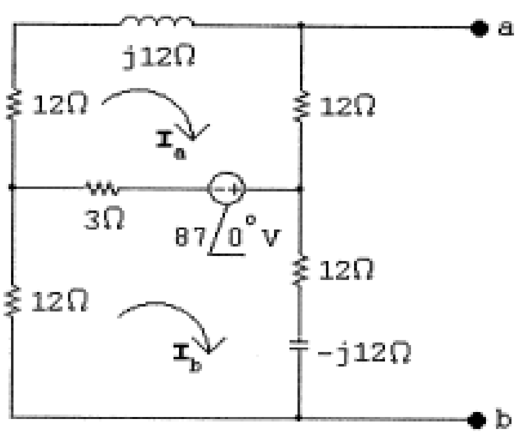
[c]



**9.43** Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.43.

Figure P9.43




$$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b = -87\underline{/0^\circ}$$

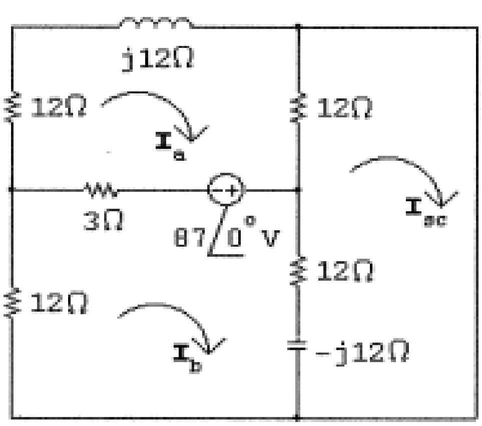
$$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b = 87\underline{/0^\circ}$$

Solving,

$$\mathbf{I}_a = -2.4167 + j1.21; \qquad \mathbf{I}_b = 2.4167 + j1.21$$

$$\mathbf{V}_{\mathrm{Th}} = 12\mathbf{I}_a + (12 - j12)\mathbf{I}_b = 14.5\underline{/0^\circ}\text{ V}$$

Short Circuit Test:




$$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b - 12\mathbf{I}_{sc} = -87$$

$$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b - (12 - j12)\mathbf{I}_{sc} = 87$$

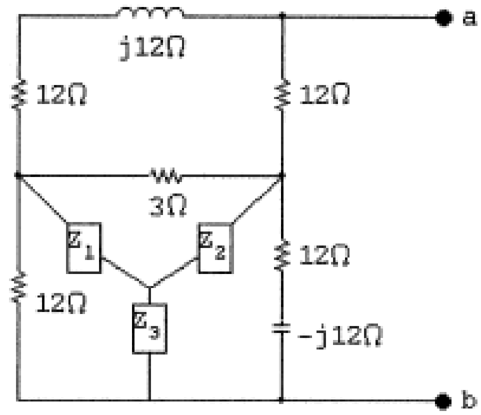
$$-12\mathbf{I}_a - (12 - j12)\mathbf{I}_b + (24 - j12)\mathbf{I}_{sc} = 0$$

Solving,

$$\mathbf{I}_{sc} = 1\underline{/0^\circ}$$

$$Z_{\rm Th} = \frac{\mathbf{V}_{\rm Th}}{\mathbf{I}_{\rm sc}} = \frac{14.5/0^\circ}{1/0^\circ} = 14.5\,\Omega$$

Alternate calculation for $Z_{\rm Th}$:

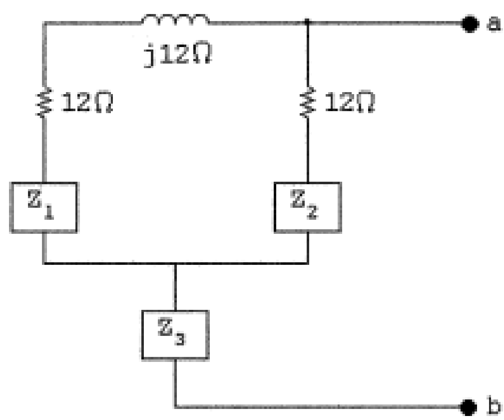



$$\sum Z = 12 + 3 + 12 - j12 = 27 - j12$$

$$Z_1 = \frac{36}{27 - j12} = \frac{12}{9 - j4}$$

$$Z_2 = \frac{36 - j36}{27 - j12} = \frac{12 - j12}{9 - j4}$$

$$Z_3 = \frac{12(12 - j12)}{27 - j12} = \frac{48 - j48}{9 - j4}$$




$$Z_{\rm a} = 12 + j12 + \frac{12}{9 - j4} = \frac{12(14 + j5)}{9 - j4}$$

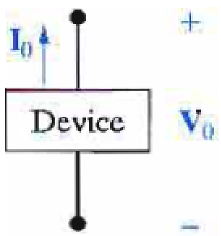
$$Z_{\rm b} = 12 + \frac{12 - j12}{9 - j4} = \frac{12(10 - j5)}{9 - j4}$$

$$Z_{\rm a}\|Z_{\rm b} = \frac{165 - j20}{18 - j8}$$

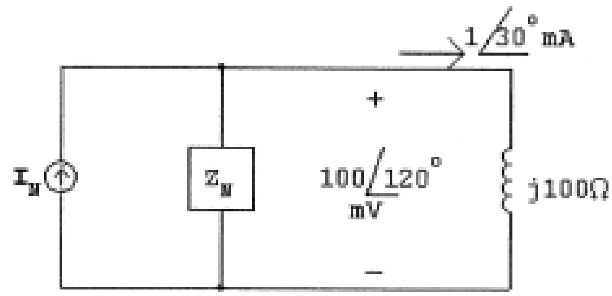
$$Z_3 + Z_{\rm a}\|Z_{\rm b} = \frac{48 - j48}{9 - j4} + \frac{165 - j20}{18 - j8} = 14.5\,\Omega$$

**9.44** The device in Fig. P9.44 is represented in the frequency domain by a Norton equivalent. When an inductor having an impedance of $j100\ \Omega$ is connected across the device, the value of $\mathbf{V}_0$ is $100\underline{/120^\circ}$ mV. When a capacitor having an impedance of $-j100\ \Omega$ is connected across the device, the value of $\mathbf{I}_0$ is $-3\underline{/210^\circ}$ mA. Find the Norton current $\mathbf{I}_N$ and the Norton impedance $Z_N$.

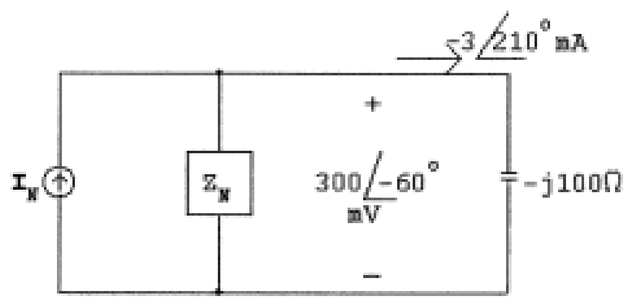
**Figure P9.44**




P 9.44




$$\mathbf{I}_N = \frac{0.1\underline{/120^\circ}}{Z_N} + 1\underline{/30^\circ}\,\text{mA}, \quad Z_N \text{ in k}\Omega$$



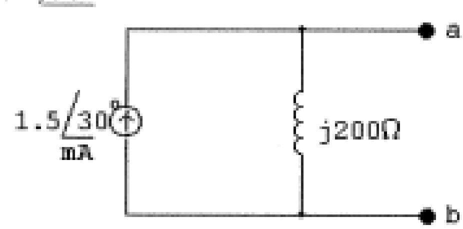

$$\mathbf{I}_N = \frac{0.3\underline{/-60^\circ}}{Z_N} + (-3\underline{/210^\circ})\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

$$\frac{0.1\underline{/120^\circ}}{Z_N} + 1\underline{/30^\circ} = \frac{0.3\underline{/-60^\circ}}{Z_N} + (-3\underline{/210^\circ})$$

$$\frac{0.3\underline{/-60^\circ} - 0.1\underline{/120^\circ}}{Z_N} = 1\underline{/30^\circ} + 3\underline{/210^\circ}$$

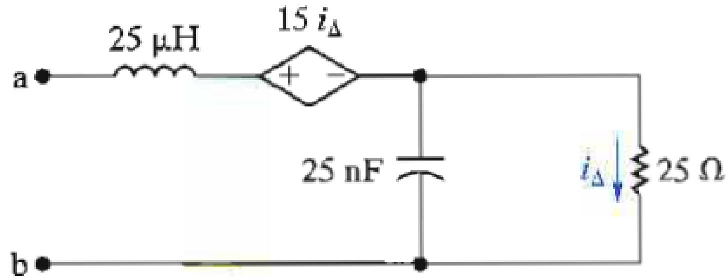
$$Z_N = \frac{0.3\underline{/-60^\circ} - 0.1\underline{/120^\circ}}{1\underline{/30^\circ} + 3\underline{/210^\circ}} = 0.2\underline{/90^\circ} = j0.2\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{0.1\underline{/120^\circ}}{0.2\underline{/90^\circ}} + 1\underline{/30^\circ} = 1.5\underline{/30^\circ}\,\text{mA}$$



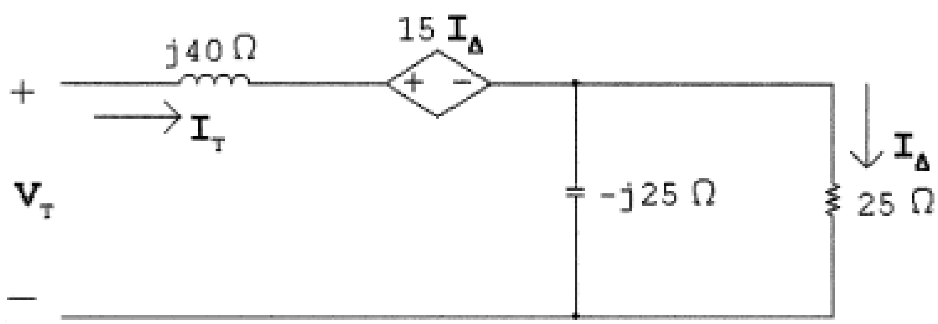
**9.45** Find $Z_{ab}$ in the circuit shown in Fig. P9.45 when the circuit is operating at a frequency of 1.6 Mrad/s.

**Figure P9.45**




$$j\omega L = j1.6 \times 10^6(25 \times 10^{-6}) = j40\,\Omega$$

$$\frac{1}{j\omega C} = \frac{10^{-6} \times 10^9}{j1.6(25)} = -j25\,\Omega$$




$$\mathbf{V}_T = j40\mathbf{I}_T + 15\mathbf{I}_\Delta + 25\mathbf{I}_\Delta$$

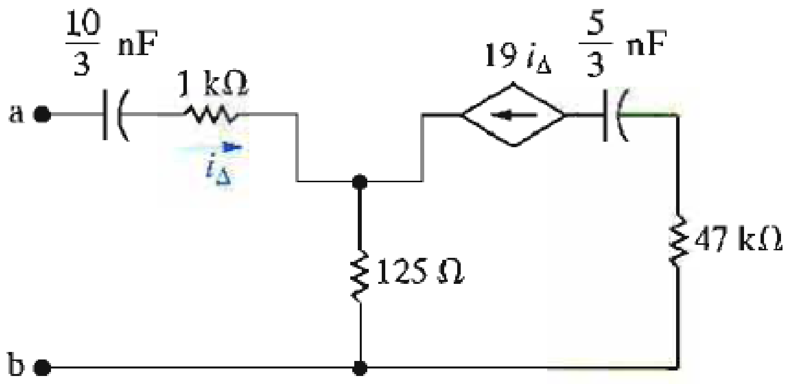
$$\mathbf{I}_\Delta = \frac{\mathbf{I}_T(-j25)}{25 - j25} = \frac{-j\mathbf{I}_T}{1 - j1}$$

$$\mathbf{V}_T = j40\mathbf{I}_T + 40\frac{(-j\mathbf{I}_T)}{1 - j1}$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{ab} = j40 + 20(-j)(1 + j) = 20 + j20\,\Omega = 28.28\underline{/45^\circ}\,\Omega$$

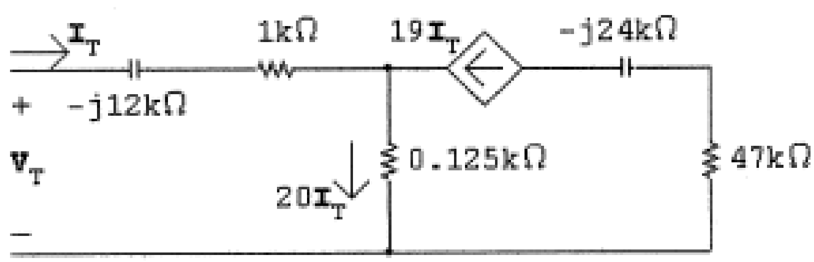
**9.46** Find the Thévenin impedance seen looking into the terminals a,b of the circuit in Fig. P9.46 if the frequency of operation is 25 krad/s.

**Figure P9.46**




$$\frac{1}{\omega C_1} = \frac{(10^{-3})(10^9)}{25(10/3)} = 12\,\text{k}\Omega$$

$$\frac{1}{\omega C_2} = \frac{(10^{-3})(10^9)}{25(5/3)} = 24\,\text{k}\Omega$$

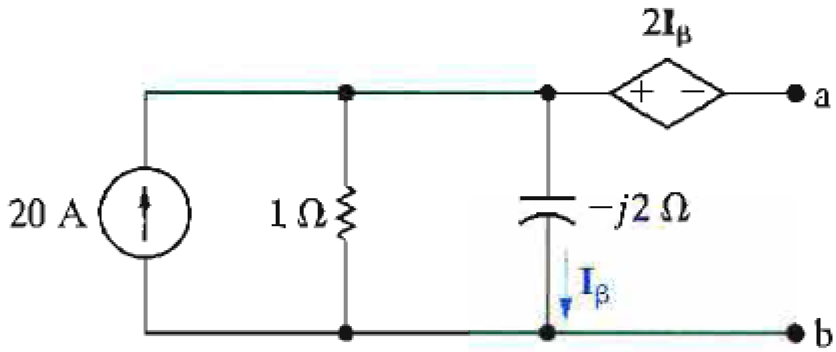



$$\mathbf{V}_T = (1 - j12)\mathbf{I}_T + 20\mathbf{I}_T(0.125)$$

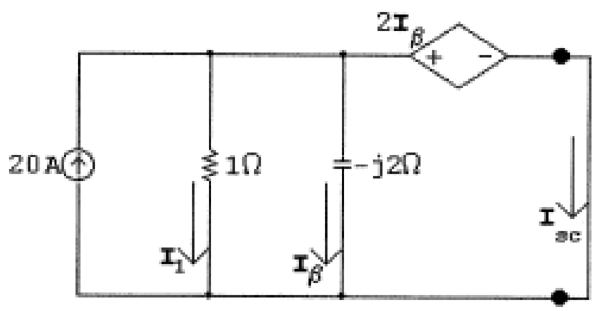
$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 3.5 - j12\,\text{k}\Omega$$

**9.47** Find the Norton equivalent with respect to terminals a,b in the circuit of Fig. P9.47.

**Figure P9.47**




Short circuit current

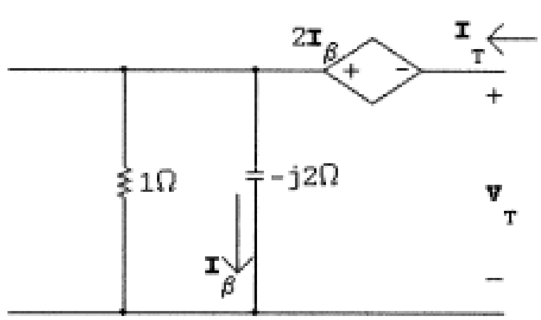



$$\mathbf{I}_\beta = \frac{2\mathbf{I}_\beta}{-j2}$$

$-j2\mathbf{I}_\beta = 2\mathbf{I}_\beta; \qquad \therefore\ \mathbf{I}_\beta = 0$

$\mathbf{I}_1 = 0; \qquad \therefore\ \mathbf{I}_{sc} = 20\,\text{A} = \mathbf{I}_N$

The Norton impedance is the same as the Thévenin impedance. Find it using a test source

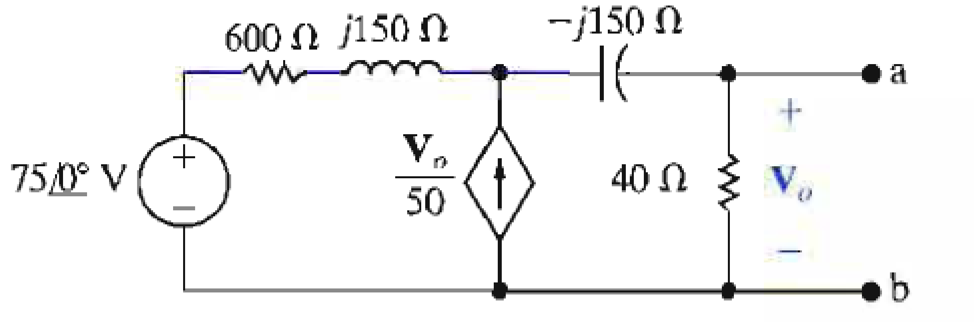


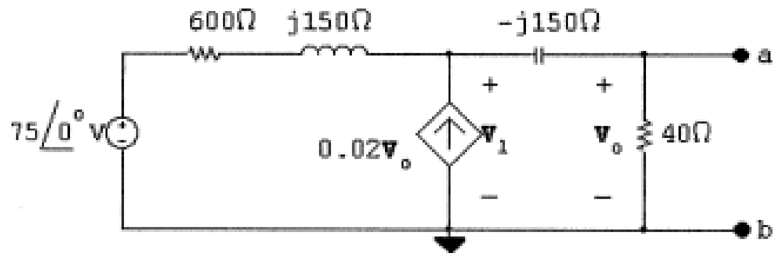

$$\mathbf{V}_T = -2\mathbf{I}_\beta - j2\mathbf{I}_\beta = (-2 - j2)\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{1}{1 - j2}\mathbf{I}_T$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{(-2 - j2)\mathbf{I}_\beta}{[(1 - j2)/1]\mathbf{I}_\beta} = \frac{-2 - j2}{1 - j2} = 0.4 - j1.2\,\Omega$$

**9.48** Find the Thévenin equivalent circuit with respect to the terminals a,b of the circuit shown in Fig. P9.48.

**Figure P9.48**




$$\frac{\mathbf{V}_1 - 75}{150(4 + j1)} - \frac{0.02\mathbf{V}_1(40)}{40 - j150} + \frac{\mathbf{V}_1}{40 - j150} = 0$$

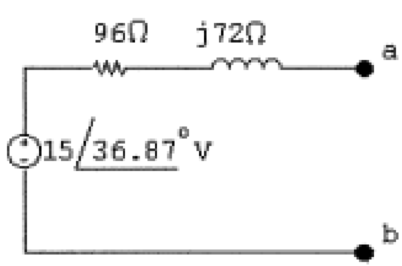
$$\therefore \quad \mathbf{V}_1 = \frac{75(4 - j15)}{16 - j12}$$

$$\mathbf{V}_{\text{Th}} = \frac{40\mathbf{V}_1}{40 - j150} = \frac{4}{4 - j15} \cdot \frac{75(4 - j15)}{16 - j12}$$

$$= \frac{75}{4 - j3} = 15\underline{/36.87^\circ}\text{ V}$$

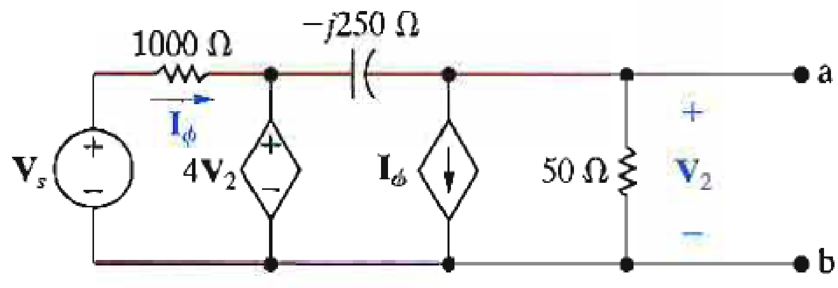
$$\mathbf{I}_{sc} = \frac{75}{600} = \frac{1}{8}\text{ A}$$

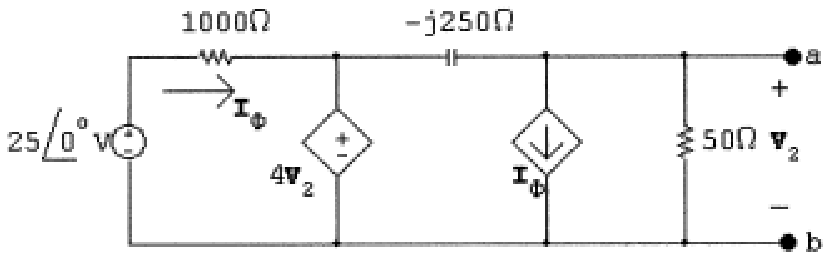
$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{sc}} = 120\underline{/36.87^\circ} = 96 + j72\ \Omega$$



**9.49** Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.49 when $\mathbf{V}_s = 25\angle 0^\circ$ V.

**Figure P9.49**




$$\frac{\mathbf{V}_2}{50} + \frac{25 - 4\mathbf{V}_2}{1000} + \frac{\mathbf{V}_2 - 4\mathbf{V}_2}{-j250} = 0$$

Solving,

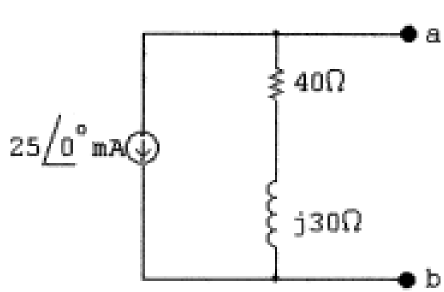
$$\mathbf{V}_2 = -1 - j0.75\text{ V} = 1.25\angle 216.87^\circ\text{ V}$$

$$\mathbf{I}_{sc} = -\mathbf{I}_\phi = \frac{-25\angle 0^\circ}{1000} = -25\angle 0^\circ\text{ mA}$$

$$Z_{\text{Th}} = \frac{1.25\angle 216.87^\circ}{-25\times 10^{-3}\angle 0^\circ} = 50\angle 36.87^\circ\ \Omega = 40 + j30\ \Omega$$

$$\mathbf{I}_N = \mathbf{I}_{sc} = -25\angle 0^\circ\text{ mA}$$

$$Z_N = Z_{\text{Th}} = 50\angle 36.87^\circ = 40 + j30\ \Omega$$



**9.50** The circuit shown in Fig. P9.50 is operating at a frequency of 10 krad/s. Assume $\alpha$ is real and lies between $-50$ and $+50$, that is, $-50 \leq \alpha \leq 50$.

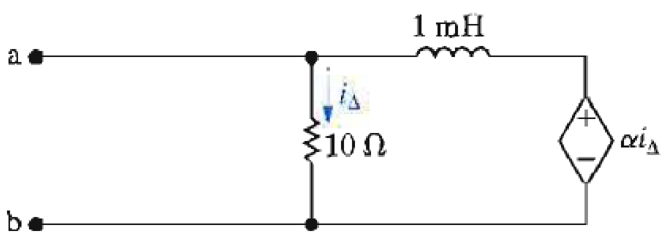
a) Find the value of $\alpha$ so that the Thévenin impedance looking into the terminals a,b is purely resistive.

b) What is the value of the Thévenin impedance for the $\alpha$ found in (a)?

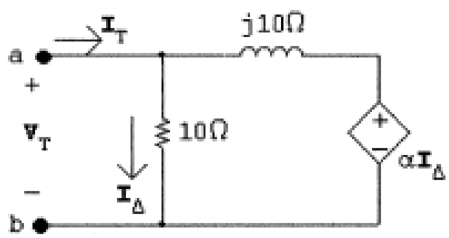
c) Can $\alpha$ be adjusted so that the Thévenin impedance equals $5 + j5\ \Omega$? If so, what is the value of $\alpha$?

d) For what values of $\alpha$ will the Thévenin impedance be inductive?

**Figure P9.50**




[a]




$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + \frac{\mathbf{V}_T - \alpha \mathbf{V}_T/10}{j10}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{10} + \frac{(1 - \alpha/10)}{j10} = \frac{(10-\alpha) + j10}{j100}$$

$$\therefore \quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{1000 + j100(10-\alpha)}{(10-\alpha)^2 + 100}$$

$Z_{\text{Th}}$ is real when $\alpha = 10$.

[b] $Z_{\text{Th}} = \dfrac{1000}{100} = 10\,\Omega$

[c] $Z_{\text{Th}} = 5 + j5$

$$\frac{1000}{(10-\alpha)^2 + 100} = 5; \qquad (10-\alpha)^2 = 100$$

$$\therefore \quad 10 - \alpha = \pm 10; \qquad \alpha = 10 \mp 10$$

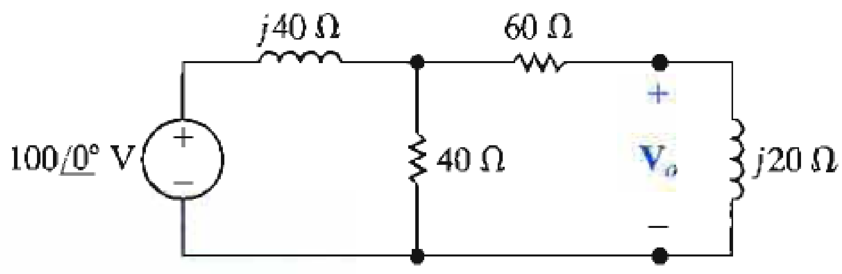
$$\alpha = 0; \qquad \alpha = 20$$

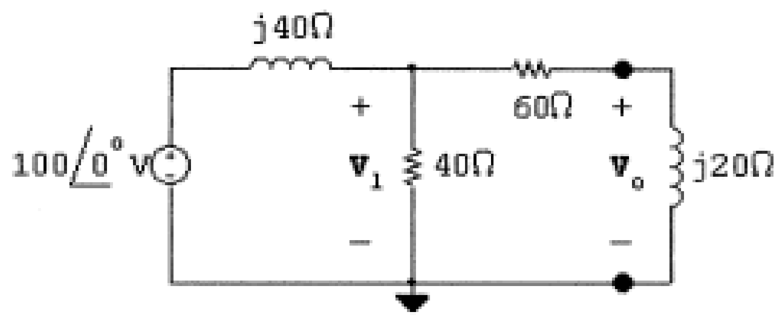
But the $j$ term can only equal the real term with $\alpha = 0$. Thus, $\alpha = 0$.

[d] $Z_{\text{Th}}$ will be inductive when $\alpha < 10$.

**9.51** Use the node-voltage method to find $\mathbf{V}_o$ in the circuit in Fig. P9.51.

**Figure P9.51**




$$\frac{\mathbf{V}_1 - 100}{j40} + \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{60 + j20} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 30 - j40\,\text{V}$$

$$\mathbf{V}_o = \frac{\mathbf{V}_1}{60 + j20}(j20) = \left(\frac{j}{3 + j}\right)\mathbf{V}_1$$

$$\mathbf{V}_o = 15 + j5\,\text{V} = 15.81\underline{/18.43^\circ}\,\text{V}$$

**9.52** Use the node-voltage method to find the steady-state expression for $v_o(t)$ in the circuit in Fig. P9.52 if

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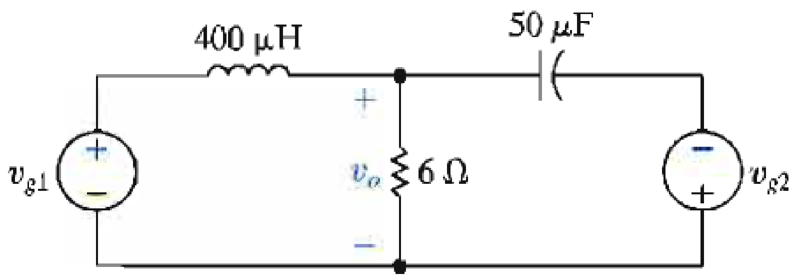
$$v_{g1} = 10\cos(5000t + 53.13^\circ)\ \text{V},$$

$$v_{g2} = 8\sin 5000t\ \text{V}.$$

**Figure P9.52**

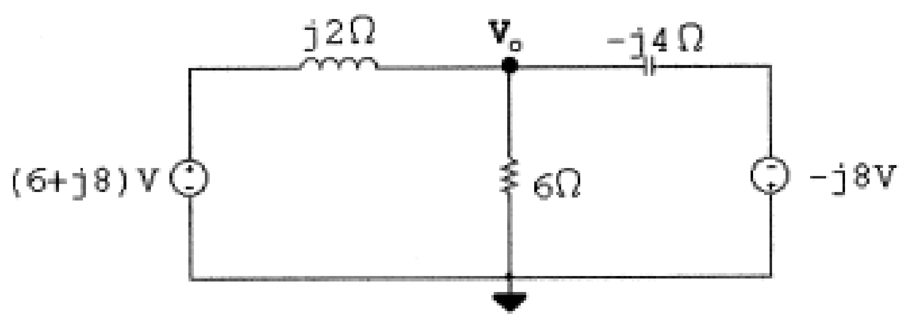



$$j\omega L = j(5000)(0.4\times10^{-3}) = j2\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(5000)(50)} = -j4\,\Omega$$

$$\mathbf{V}_{g1} = 10\underline{/53.13^\circ} = 6 + j8\ \text{V}$$

$$\mathbf{V}_{g2} = 8\underline{/-90^\circ} = -j8\ \text{V}$$




$$\frac{\mathbf{V}_o - 6 - j8}{j2} + \frac{\mathbf{V}_o}{6} + \frac{\mathbf{V}_o + (-j8)}{-j4} = 0$$

Solving,

$$\mathbf{V}_o = 12\underline{/0^\circ}$$

$$v_o(t) = 12\cos 5000t\ \text{V}$$

**9.53** Use the node voltage method to find the steady-state expressions for the branch currents $i_a$ and $i_b$ in the circuit seen in Fig. P9.53 if $v_a = 100 \sin 10{,}000t$ V and $v_b = 500 \cos 10{,}000t$ V.

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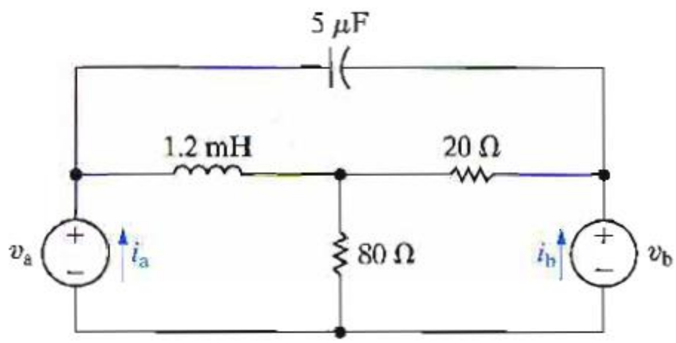
**Figure P9.53**

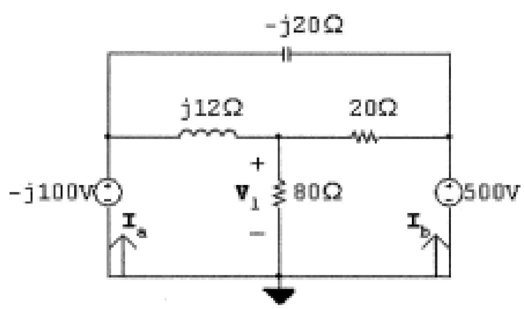



$$j\omega L = j10^4(1.2 \times 10^{-3}) = j12\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j10^6}{5 \times 10^4} = -j20\,\Omega$$

$$\mathbf{V}_a = 100\underline{/-90^\circ} = -j100\text{ V}$$

$$\mathbf{V}_b = 500\underline{/0^\circ} = 500\text{ V}$$




$$\frac{\mathbf{V}_1}{80} + \frac{\mathbf{V}_1 - 500}{20} + \frac{\mathbf{V}_1 + j100}{j12} = 0$$

Solving,

$$\mathbf{V}_1 = 160\underline{/53.13^\circ}\text{ V} = 96 + j128\text{ V}$$

$$\mathbf{I}_a = \frac{-j100 - 96 - j128}{j12} + \frac{-j100 - 500}{-j20}$$

$$= -14 - j17 = 22.02\underline{/-129.47^\circ}\text{ A}$$

$$i_a = 22.02\cos(10{,}000t - 129.47^\circ)\text{ A}$$

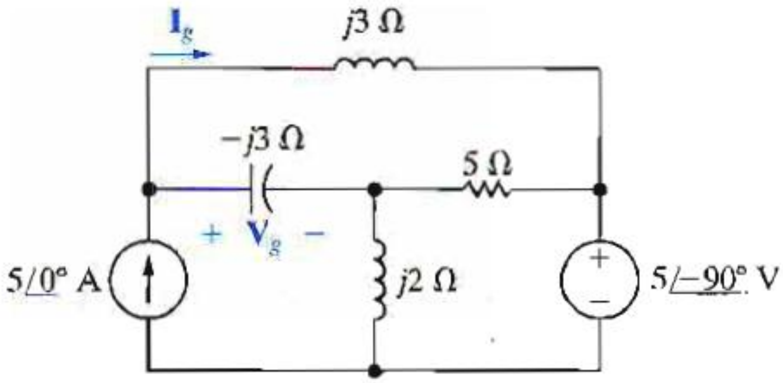
$$\mathbf{I}_b = \frac{500 - 96 - j128}{20} + \frac{500 + j100}{-j20}$$

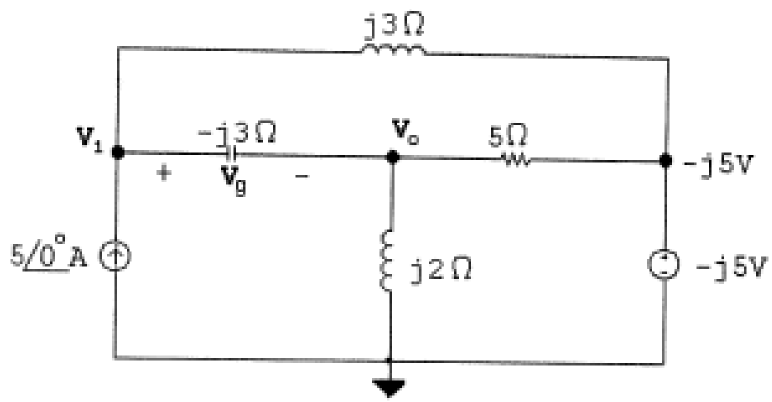
$$= 15.2 + j18.6 = 24.02\underline{/50.74^\circ}\text{ A}$$

$$i_b = 24.02\cos(10{,}000t + 50.74^\circ)\text{ A}$$

**9.54** Use the node-voltage method to find the phasor voltage $\mathbf{V}_g$ in the circuit shown in Fig. P9.54.

**Figure P9.54**




$$\frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o + j5}{5} + \frac{\mathbf{V}_o - \mathbf{V}_1}{-j3} = 0$$

$$(5 + j6)\mathbf{V}_o + 10\mathbf{V}_1 = 30$$

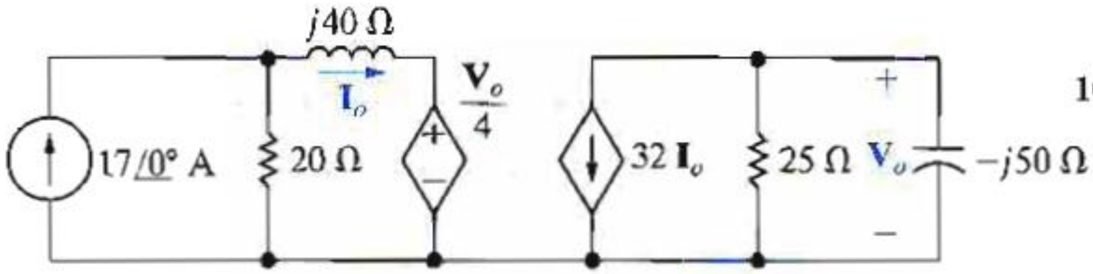
$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j3} + \frac{\mathbf{V}_1 + j5}{j3} = 0$$

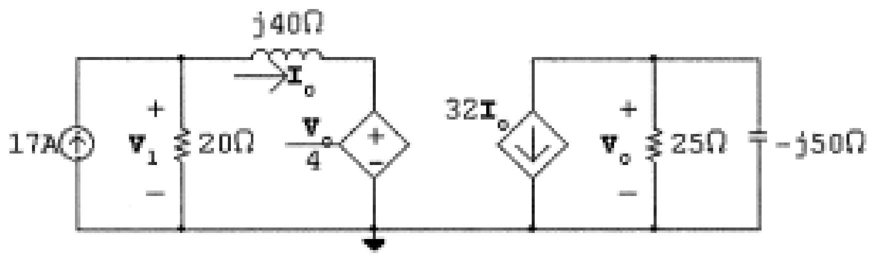
$$\mathbf{V}_o = j10; \qquad \mathbf{V}_1 = 9 - j5$$

$$\mathbf{V}_g = \mathbf{V}_1 - \mathbf{V}_o = 9 - j5 - j10 = 9 - j15 = 17.49/\underline{-59.04^\circ}\ \text{V}$$

**9.55** Use the node-voltage method to find $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit seen in Fig. P9.55.

Figure P9.55




$$\frac{\mathbf{V}_o}{25} + \frac{\mathbf{V}_o}{-j50} + 32\mathbf{I}_o = 0$$

$$(2+j)\mathbf{V}_o = -1600\mathbf{I}_o$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/4)}{j40}$$

$$\therefore \quad \mathbf{V}_1 = (-160 + j120)\mathbf{I}_o$$

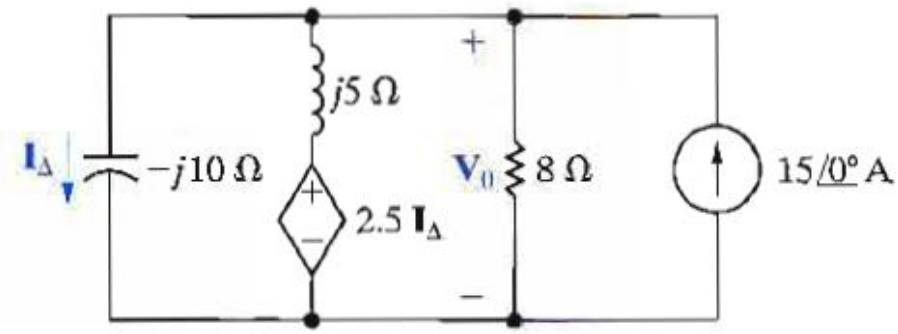
$$17 = \frac{\mathbf{V}_1}{20} + \mathbf{I}_o = (-8 + j6)\mathbf{I}_o + \mathbf{I}_o = (-7 + j6)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{17}{(-7 + j6)} = -1.4 - j1.2 \text{ A} = 1.84\underline{/-139.40^\circ} \text{ A}$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o = 1280 + j320 = 1319.39\underline{/14.04^\circ} \text{ V}$$

**9.56** Use the node-voltage method to find the phasor voltage $\mathbf{V}_o$ in the circuit shown in Fig. P9.56. Express the voltage in both polar and rectangular form.

**Figure P9.56**




$$-15/\!\underline{0^\circ} + \frac{\mathbf{V}_o}{8} + \frac{\mathbf{V}_o - 2.5\mathbf{I}_\Delta}{j5} + \frac{\mathbf{V}_o}{-j10} = 0$$

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j10}$$

Solving,

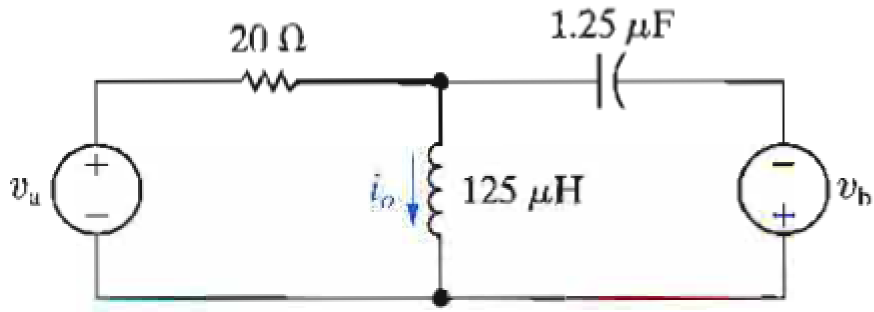
$$\mathbf{V}_o = 72 + j96 = 120/\!\underline{53.13^\circ}\ \text{V}$$

**9.57** Use the mesh-current method to find the steady-state expression for $i_o(t)$ in the circuit in Fig. P9.57 if

$$v_a = 60 \cos 40{,}000t \text{ V},$$

$$v_b = 90 \sin (40{,}000t + 180°) \text{ V}.$$

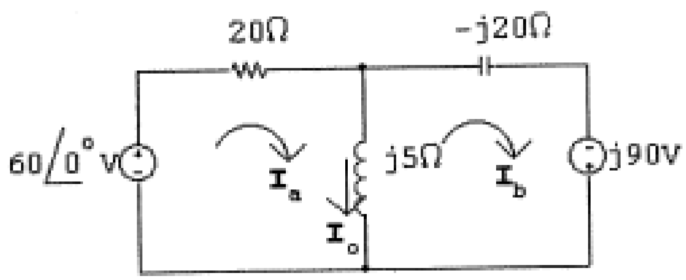
**Figure P9.57**




$$\mathbf{V}_a = 60\angle 0° \text{ V}; \qquad \mathbf{V}_b = 90\angle 90° \text{ V}$$

$$j\omega L = j(4 \times 10^4)(125 \times 10^{-6}) = j5\Omega$$

$$\frac{-j}{\omega C} = \frac{-j10^6}{40{,}000(1.25)} = -j20\,\Omega$$




$$60 = (20 + j5)\mathbf{I}_a - j5\mathbf{I}_b$$

$$j90 = -j5\mathbf{I}_a - j15\mathbf{I}_b$$

Solving,

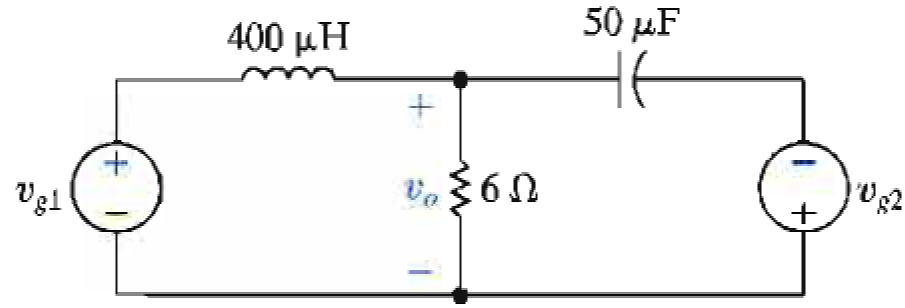
$$\mathbf{I}_a = 2.25 - j2.25 \text{ A}; \qquad \mathbf{I}_b = -6.75 + j0.75 \text{ A}$$

$$\mathbf{I}_o = \mathbf{I}_a - \mathbf{I}_b = 9 - j3 = 9.49\angle -18.43° \text{ A}$$

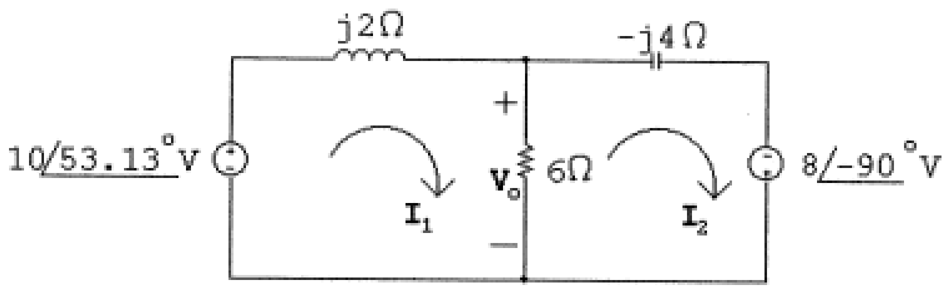
$$i_o(t) = 9.49 \cos(40{,}000t - 18.43°) \text{ A}$$

**9.58** Use the mesh-current method to find the steady-state expression for $v_o(t)$ in the circuit in Fig. P9.52.

**Figure P9.52**




From the solution to Problem 9.52 the phasor-domain circuit is




$$10\angle 53.13^\circ = (6 + j2)\mathbf{I}_1 - 6\mathbf{I}_2$$

$$8\angle -90^\circ = -6\mathbf{I}_1 + (6 - j4)\mathbf{I}_2$$

$$\mathbf{V}_o = (\mathbf{I}_1 - \mathbf{I}_2)6$$

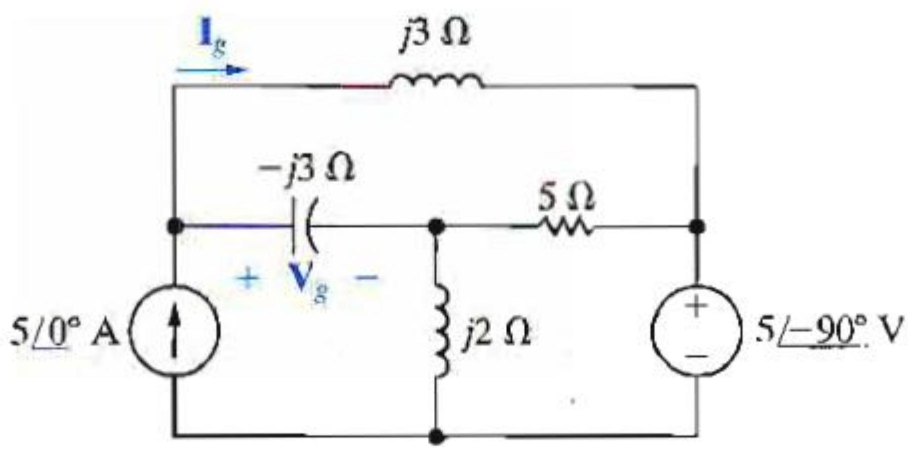
Solving,

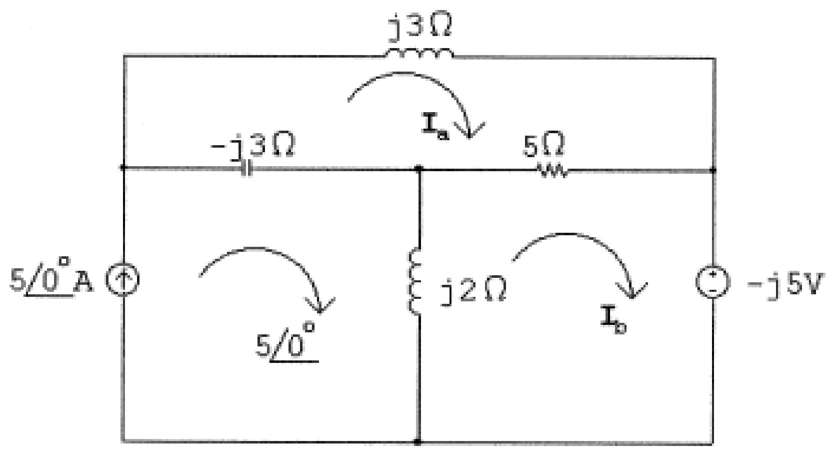
$$\mathbf{V}_o = 12\angle 0^\circ \text{ V}$$

$$v_o(t) = 12\cos 5000t \text{ V}$$

**9.59** Use the mesh-current method to find the phasor current $\mathbf{I}_g$ in the circuit in Fig. P9.54.

**Figure P9.54**




$$j3\mathbf{I}_a + 5(\mathbf{I}_a - \mathbf{I}_b) - j3(\mathbf{I}_a - 5) = 0$$

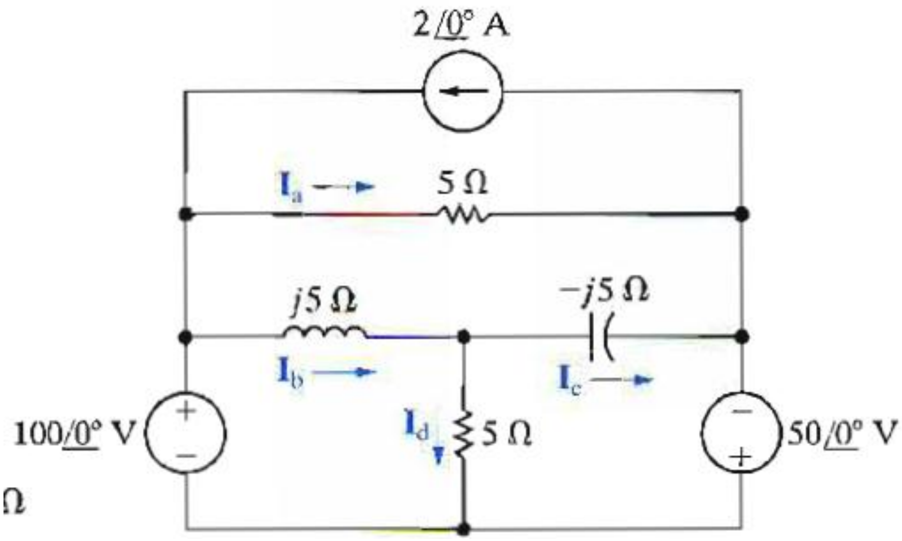
$$j2(\mathbf{I}_b - 5) + 5(\mathbf{I}_b - \mathbf{I}_a) - j5 = 0$$

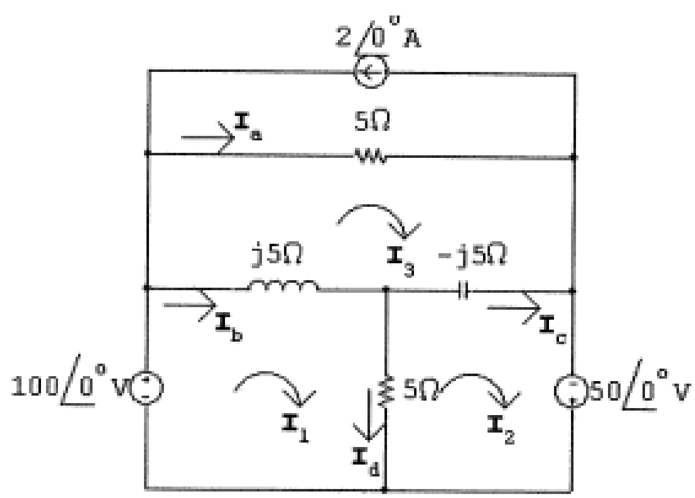
Solving,

$\mathbf{I}_a = -j3; \qquad \mathbf{I}_g = -j3 = 3\underline{/-90^\circ}$ A

**9.60** Use the mesh-current method to find the branch currents $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$, and $\mathbf{I}_d$ in the circuit shown in Fig. P9.60.

**Figure P9.60**




$$100\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - 5\mathbf{I}_2 - j5\mathbf{I}_3$$

$$50\underline{/0^\circ} = -5\mathbf{I}_1 + (5 - j5)\mathbf{I}_2 + j5\mathbf{I}_3$$

$$-10\underline{/0^\circ} = -j5\mathbf{I}_1 + j5\mathbf{I}_2 + 5\mathbf{I}_3$$

Solving,

$\mathbf{I}_1 = 58 - j20$ A; $\quad \mathbf{I}_2 = 58 + j10$ A; $\quad \mathbf{I}_3 = 28 + j0$ A

$$\mathbf{I}_a = \mathbf{I}_3 + 2 = 30 + j0\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_3 = 58 - j20 - 28 = 30 - j20\ \text{A}$$

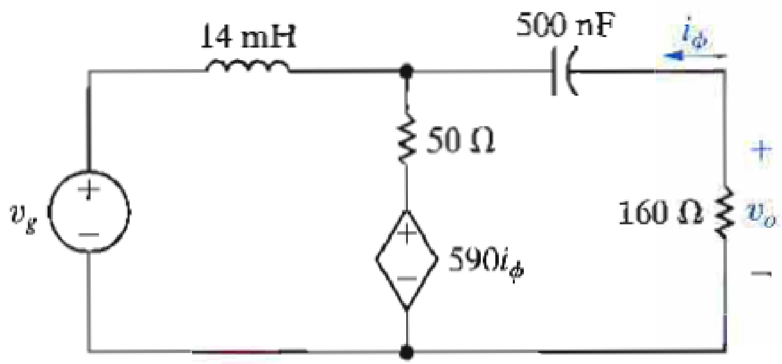
$$\mathbf{I}_c = \mathbf{I}_2 - \mathbf{I}_3 = 58 + j10 - 28 = 30 + j10\ \text{A}$$

$$\mathbf{I}_d = \mathbf{I}_1 - \mathbf{I}_2 = 58 - j20 - 58 - j10 = -j30\ \text{A}$$

**9.61** Use the mesh-current method to find the steady-state expression for $v_o$ in the circuit seen in Fig. P9.61, if $v_g$ equals $72 \cos 5000t$ V.

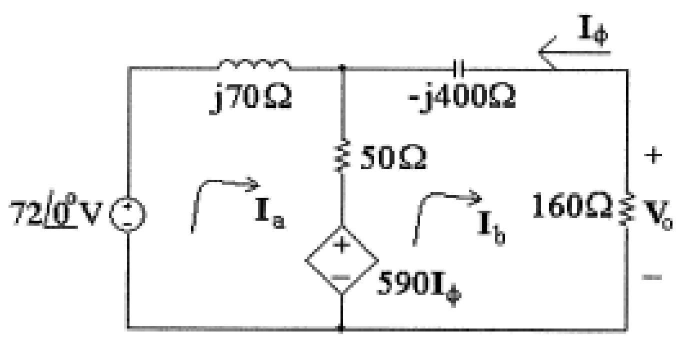
PSPICE

Figure P9.61




$$j\omega L = j5000(14 \times 10^{-3}) = j70\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(0.5 \times 10^{-6})} = -j400\,\Omega$$




$$72\angle 0^\circ = (50 + j70)\mathbf{I}_a - 50\mathbf{I}_b + 590(-\mathbf{I}_b)$$

$$0 = -50\mathbf{I}_a - 590(-\mathbf{I}_b) + (210 - j400)\mathbf{I}_b$$

Solving,

$$\mathbf{I}_b = (50 - j50)\text{ mA}$$

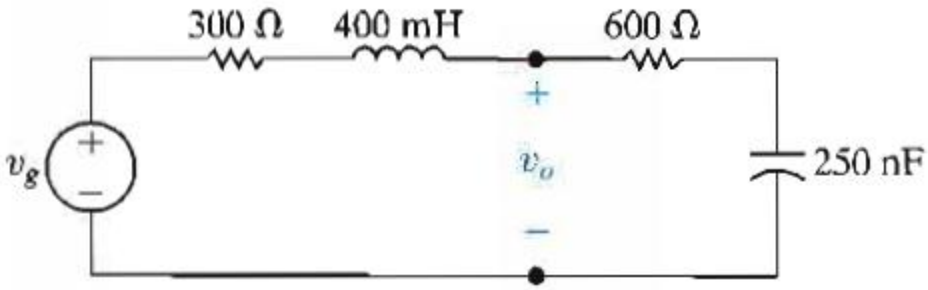
$$\mathbf{V}_o = 160\mathbf{I}_b = 8 - j8 = 11.31\angle -45^\circ$$

$$v_o = 11.31 \cos(5000t - 45^\circ)\text{ V}$$

**9.62** Use the concept of voltage division to find the steady-state expression for $v_o(t)$ in the circuit in Fig. P9.62 if $v_g = 75\cos 5000t$ V.

PSPICE

Figure P9.62




$$Z_o = 600 - j\frac{10^6}{(5000)(0.25)} = 600 - j800\,\Omega$$

$$Z_T = 300 + j2000 + 600 - j800 = 900 + j1200\,\Omega = 1500\underline{/53.13^\circ}\,\Omega$$

$$\mathbf{V}_o = \mathbf{V}_g\frac{Z_o}{Z_T} = \frac{(75\underline{/0^\circ})(1000\underline{/-53.13^\circ})}{1500\underline{/53.13^\circ}} = 50\underline{/-106.26^\circ}\,\text{V}$$

$$v_o = 50\cos(5000t - 106.26^\circ)\,\text{V}$$

**9.64** The sinusoidal voltage source in the circuit shown in Fig. P9.64 is generating the voltage $v_g = 1.2\cos 100t$ V. If the op amp is ideal, what is the steady-state expression for $v_o(t)$?

PSPICE

$$\mathbf{V}_g = 1.2\underline{/0^\circ}\,\text{V}; \qquad \frac{1}{j\omega C} = \frac{10^6}{j100} = -j10\,\text{k}\Omega$$

Let $\mathbf{V}_a$ = voltage across 1 $\mu$F capacitor, positive at upper terminal

Then:

$$\frac{\mathbf{V}_a - 1.2\underline{/0^\circ}}{10} + \frac{\mathbf{V}_a}{-j10} + \frac{\mathbf{V}_a}{10} = 0; \qquad \therefore\ \mathbf{V}_a = (0.48 - j0.24)\,\text{V}$$

$$\frac{0 - \mathbf{V}_a}{10} + \frac{0 - \mathbf{V}_o}{200} = 0; \qquad \mathbf{V}_o = -20\mathbf{V}_a$$

$$\therefore\ \mathbf{V}_o = -9.6 + j4.8 = 10.73\underline{/153.43^\circ}\,\text{V}$$

$$v_o = 10.73\cos(100t + 153.43^\circ)\,\text{V}$$