Z-transform using MATLAB

?

we need to know:-

(1) creating TF & its frequency response.

(2) Applying signal To LTI in Z-domain. &

(3) Evaluating Impulse response from TF.

(4) stability of TF.

(5) Inverse of Z-transform

(I) creating TF:-

MATLAB works with TF

$$\boxed{H(Z) = \frac{B(Z)}{A(Z)}}$$

$B(Z)$ → numerator written as vector.

$A(Z)$ → den. written as vector.

<u>note</u>: the vectors contain <u>the coff's</u> of $B(Z)$ & $A(Z)$ in <u>descending powers of -ve Z</u>

ex.

$$H(z) = \frac{z^2}{z^2+2z+3}$$

$$= \frac{1}{1+2z^{-1}+3z^{-2}}$$

$\therefore$ num = [1]   den = [1 2 3].

TF:-

syntax:-

```
H = tf(num, den, 'variable', 'z', Ts)
```

- num: vector of num. coff^s
- den: vector of den. coff^s.
- 'z': variable (can be z^-1) or s (laplace)
- Ts: Sampl Time neglec if con

ex.

Generate TF in ex.(1) with $T_s = 0.1$

freq. response of TF:-

To draw freq. response of LTI obtained from TF we use "freqz"

⟹ This will evaluate $H(z)\big|_{z=e^{j\omega}}$

دوا تشغيل FFT مباشرة وبيرسم مع f.

syntax:-

$\boxed{\text{freqz}(\underbrace{\text{num, den}}_{\text{TF}}, \underbrace{\text{number of test points}}_{\text{عدد النقط اللي بيحتاجها كل ما تزيد بتحسن}}, \underbrace{f_s}_{\text{sample freq.}})}$

بترسم mag. & phase.

$0 \rightarrow f_s$ (single sided)

ex. draw freq. response of the above TF.

(II) FILTER:-

↳ revise previous papers

deconvolution: the inverse of convolution process

$$x[n] \longrightarrow \boxed{\begin{matrix}\text{LTI} \\ h[n]\end{matrix}} \longrightarrow y[n] = x[n] * h[n]$$

$$x[n] = y[n] \text{ deconv. } h[n]$$

$\triangle$ deconvolution in MATLAB do a polynomial division in Z-domain

$$\therefore \boxed{x[n] = Z^{-1}\left\{\frac{Y(Z)}{H(Z)}\right\}}$$

ex.

```
x = [1 2 3 4]
h = [4 3 2 1]
∴ y = conv(x,h);
x = deconv(y,h)
```

4) Obtaining Impulse & step response from TF:-

Impulse response $h[n] \triangleq$ o/p of system when i/p signal is $\delta[n]$

$\therefore$

```
h = filter(num, den, [1 zeros(1,N-1)])
```

Impulse response of N points

TF

$\delta[n]$ of N points

Also, step response.

```
s = filter(num, den, [ones(1,N)]);
```

ex-

find Impulse response of system shown, Is it stable?!

IV) STABILITY of H(z):-

To discuss stability of H(z), we must calculate poles & zeros zplane

zero-pole plot

$\boxed{\text{zplane([num],[den.])}}$

pzmap

calculate pole-zero map of LTI systems

$\boxed{\text{[P,Z] = pzmap(syst)}}$

لا ترسم مقابلة TF

discrete-cont بين

To identify stability

stable

All poles inside unit circle

$|P| < 1$.

$\boxed{\text{abs(P)} < 1}$

marginally stable.

1 pole on unit circle.

$|P| = 1$.

$\boxed{\text{abs(P)} = 1}$

unstable.

$|P| > 1$

$\boxed{\text{abs(P)} > 1}$

NOTE:-

we can transform from poles & zeros to TF as

$\boxed{\text{[b,a] = zp2tf(zeros,poles,gain)}}$

Inverse z-transform :-

→ using partial fraction expansion

$$H(z) = \frac{r(1)}{1-p(1)z^{-1}} + \cdots + \frac{r(n)}{1-p(n)z^{-1}} + k(1) + k(2)z^{-1} + \cdots$$

$$\therefore h[n] = r(1)\left(p(1)\right)^n + r(2)\left(p(2)\right)^n + \cdots + k(1)\delta[n] + k(2)\delta[n-1]$$

syntax :-

$$\boxed{[\underbrace{r}_{\text{num of each Term}},\ \underbrace{p}_{\text{den. of each Term}},\ \underbrace{k}_{\delta\text{'s}}] = \text{residuez}(\underbrace{b,a}_{\text{Transfer } f^n})}$$

لاحظ: إن الأمر عكسه سالب بالعكس

-- [b, a] = residuez[r, p, k].