70068 Scheduling and Resource Allocation CW

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1 Question 1

Everything in this section will use 0-based indexing.

1.1 LCL Proof

The *Least Cost Last* (LCL) rule solves the scheduling problem $1|prec|f_{\max}$ optimally, in $O(n^2)$ time, by constructing an optimal schedule from back to front.

Definitions

- $N = \{1, 2, \ldots, n\}$ is the index set of all jobs
- $L \subseteq N$ is the subset of jobs without successors (which can be scheduled last)
- $p(S) = \sum_{j \in S} p_j$ is the total processing time of a subset S
- $f^*_{\max}(S)$ is the cost of the optimal schedule for subset S
- $f_{\max}$ denotes the maximum (not necessarily optimal) cost across all jobs in a schedule
- $f_j(p(N))$ is a cost function which assigns a penalty/cost depending on job j and the time at which the machine has finished processing, $p(N)$
  - In the notation of the coursework specification $g_j(\cdot) = f_j(\cdot)$
  - In the next section we'll use $g_j(C_j) = T_j = \max(0, C_j - d_j)$ (tardiness)

Proof and Discussion

1. One job in L must be scheduled last. A job can be selected for the final job which minimises $f_j(p(N))$, but it can never result in a cost lower than the cost of the optimal schedule, otherwise that would be the optimal schedule. This is expressed as such:

   $$f^*_{\max}(N) \geq \min_{j \in L} f_j(p(N))$$

2. And removing a job $j \in N$ can't increase the optimal cost any more, since the cost of scheduling a job last is ≥ 0, so omitting it cannot result in a more expensive schedule:

   $$f^*_{\max}(N) \geq f^*_{\max}(N - \{j\}),\ \forall j \in N$$

3. We can then select a $J_l \in L$ that minimises $f_j(p(N))$:

   $$f_l(p(N)) = \min_{j \in L} f_j(p(N))$$

   Which gives us:

   $$f^*_{\max}(N) \geq \max\{f_l(p(N)), f^*_{\max}(N - \{l\})\}$$

4. The right hand side of the above inequality is the cost of an optimal schedule, where J_l is processed last, so you can recursively apply the LCL rule to $N - \{J_l\}$ and construct a schedule in reverse order

   - Since J_l is found in $O(n)$ time, then with n, repeated applications of the LCL rule yields the optimal schedule in $O(n^2)$ time

You can also show that LCL is optimal by using the *adjacent pairwise interchange argument*:

- You take an optimal schedule S and assume it isn't an LCL schedule

- Since it's not an LCL schedule, there must exist at least one pair where a lower cost job precedes a higher cost one
- It can then be shown that swapping these jobs yields a lower cost schedule, which is a contradiction since we took an optimal schedule
- So by virtue of the fact that a lower cost job can precede a higher cost one leads to a contradiction
- And so such pairs cannot exist for an optimal schedule, so we can reject the assumption that this isn't an LCL schedule
- And by contradiction then we have that LCL is optimal for $1|prec|f_{\max}$

Small Example

Example Setup:
Jobs: J0, J1, J2, J3, J4
Processing Times: p=[2,3,1,2,3]
Dude Dates: d=[6,5,7,4,9]
Precedence Constraints (DAG): J0 $\rightarrow$ J1, J0 $\rightarrow$ J2, J3 $\rightarrow$ J4
Cost Function: $f_j(C_j) = T_j = max(0, C_j - d_j)$

Iterations We can schedule jobs in reverse order using the *Least Cost Last* (LCL) rule:

0. Available jobs: $V = \{1, 2, 4\}$ (no successors)
   $p(N) = 11$

   $$f_1(p(N)) = \max(0, 11 - 5) = 6, \quad f_2(p(N)) = \max(0, 11 - 4) = 7, \quad f_4(p(N)) = \max(0, 11 - 9) = 2$$

   Select J_4 (minimises $f_j(p(N))$)
   Partial schedule cost: 2
   Updated schedule: [4]

1. Available jobs: $V = \{1, 2, 3\}$ (no successors)
   $p(N) = 8$

   $$f_1(p(N)) = \max(0, 8 - 5) = 3, \quad f_2(p(N)) = \max(0, 8 - 4) = 4, \quad f_3(p(N)) = \max(0, 8 - 7) = 1$$

   Select J_3 (minimises $f_j(p(N))$)
   Partial schedule cost: 2
   Updated schedule: $[3, 4]$

2. Available jobs: $V = \{1, 2\}$ (no successors)
   $p(N) = 6$

   $$f_1(p(N)) = \max(0, 6 - 5) = 1, \quad f_2(p(N)) = \max(0, 6 - 4) = 2$$

   Select J_1 (minimises $f_j(p(N))$)
   Partial schedule cost: 2
   Updated schedule: $[1, 3, 4]$

3. Available jobs: $V = \{2\}$ (no successors)
   $p(N) = 3$

   $$f_2(p(N)) = \max(0, 3 - 4) = 0$$

   Select J_2
   Partial schedule cost: 2
   Updated schedule: $[2, 1, 3, 4]$

4. Available jobs: $V = \{0\}$ (no successors)
   $p(N) = 2$

   $$f_0(p(N)) = \max(0, 2 - 6) = 0$$

   Select J_0
   Partial schedule cost: 2
   Updated schedule: $[0, 2, 1, 3, 4]$

Results

- Final schedule: $[J_0, J_2, J_1, J_3, J_4]$
- Completion times:

  $C_0 = 2$, $C_2 = 3$, $C_1 = 6$, $C_3 = 8$, $C_4 = 11$

- Tardiness:

  $T_0 = 0$, $T_2 = 0$, $T_1 = 1$, $T_3 = 1$, $T_4 = 2$

- Maximum cost:

  $f_{\max} = \max(T_0, T_2, T_1, T_3, T_4) = 2$

1.2 LCL Implementation

Iterations

0. Available jobs: $V = \{0, 30\}$ (no successors)
   $p(N) = 170$

   $f_0(p(N)) = \max(0, 170 - 172) = 0, \quad f_{30}(p(N)) = \max(0, 170 - 269) = 0$

   Select J_{30} (minimises $f_j(p(N))$)
   Partial schedule cost: 0

   Updated schedule: [30]

1. Available jobs: $V = \{0, 1, 10\}$ (no successors)
   $p(N) = 160$

   $f_0(p(N)) = \max(0, 160 - 172) = 0, \quad f_1(p(N)) = \max(0, 160 - 82) = 78, \quad f_{10}(p(N)) = \max(0, 160 - 253) = 0$

   Select J_0 (minimises $f_j(p(N))$)
   Partial schedule cost: 0

   Updated schedule: $[0, 30]$

...

4. Available jobs: $V = \{1, 4\}$ (no successors)
   $p(N) = 147$

   $f_1(p(N)) = \max(0, 147 - 82) = 65, \quad f_4(p(N)) = \max(0, 147 - 93) = 54$

   Select J_1 (minimises $f_j(p(N))$) Partial schedule cost: 65

   Updated schedule: $[1, 14, 10, 0, 30]$

   This iteration was noteable since it incurred the maximum tardiness incurred for this problem.

...

17. Available jobs: $V = \{17, 18, 20, 5\}$ (no successors)
    $p(N) = 68$

    $f_{17}(p(N)) = \max(0, 68 - 77) = 0, \quad f_{18}(p(N)) = \max(0, 68 - 88) = 0,$

    $f_{20}(p(N)) = \max(0, 68 - 71) = 0, \quad f_5(p(N)) = \max(0, 68 - 71) = 0$

    Select J_{17} (arbitrarily breaking the tie)
    Partial schedule cost: 65

    Updated schedule: $[17, 16, 15, 13, 28, 12, 27, 26, 25, 24, 23, 11, 4, 1, 14, 10, 0, 30]$

...

30. Available jobs: $V = \{29\}$ (no successors)
    $p(N) = 2$

    $f_{29}(p(N)) = \max(0, 2 - 329) = 0$

    Select J_{29} (minimises $f_j(p(N))$)
    Final schedule cost: 65

    Final schedule: $[29, 9, 8, 3, 2, 7, 6, 22, 21, 5, 20, 19, 18, 17, 16, 15, 13, 28, 12, 27, 26, 25, 24, 23, 11, 4, 1, 14, 10, 0, 30]$

Note: the tie breaking doesn't specify any ordering in the current implementation (since the set of available jobs is backed by a hash table).

2 Question 2

2.1 Tabu Search

Throughout this section, I used $\gamma = 10$ and $L = 20$. I forced the initial solution to be the one specified in the coursework specification.

I also included the first few iterations and all noteable intermediate solutions for each value of K.

2.1.1 K=10

Iterations

0. **Candidate schedule costs**: [341, 341, 341, 341, 341, 341, 338, 343, 341, 341, 341, 339, 351, 331, 359, 341, 336, 346, 336, 346, 336]
   Tabu list: { (17, 25) }
   Best cost so far: 331
   Tabu list used?: No
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 16, 14, 5, 23, 15, 4, 10, 1, 0, 30]
   Selected neighbour cost: 331

1. **Candidate schedule costs**: [331, 331, 331, 331, 331, 331, 328, 333, 331, 331, 331, 329, 341, 331, 326, 336, 326, 336, 326]
   Tabu list: { (17, 25), (5, 14) }
   Best cost so far: 326
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 16, 5, 14, 23, 15, 4, 10, 1, 0, 30]
   Selected neighbour cost: 326

2. **Candidate schedule costs**: [326, 326, 326, 326, 326, 323, 328, 326, 326, 326, 324, 336, 326, 320, 331, 326, 321, 331, 321, 326]
   Tabu list: { (17, 25), (5, 14), (5, 16) }
   Best cost so far: 320
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 5, 16, 14, 23, 15, 4, 10, 1, 0, 30]
   Selected neighbour cost: 320

3. **Candidate schedule costs**: [320, 320, 320, 320, 317, 322, 320, 320, 320, 318, 330, 302, 326, 320, 315, 325, 315, 320, 320]
   Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24) }
   Best cost so far: 302
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 5, 24, 16, 14, 23, 15, 4, 10, 1, 0, 30]
   Selected neighbour cost: 302

4. **Candidate schedule costs**: [302, 302, 302, 302, 299, 304, 302, 302, 302, 300, 312, 292, 320, 302, 302, 297, 307, 297, 302, 302]
   Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25) }
   Best cost so far: 292
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 14, 23, 15, 4, 10, 1, 0, 30]
   Selected neighbour cost: 292

5. **Candidate schedule costs**: [292, 292, 292, 289, 294, 292, 292, 292, 290, 288, 302, 292, 292, 287, 297, 287, 292, 292, 292]
   Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23) }
   Best cost so far: 287
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 14, 15, 23, 4, 10, 1, 0, 30]
   Selected neighbour cost: 287

6. **Candidate schedule costs**: [287, 287, 287, 284, 289, 287, 287, 287, 285, 283, 297, 287, 282, 292, 292, 282, 287, 287, 287]
   Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15) }
   Best cost so far: 282
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 15, 14, 23, 4, 10, 1, 0, 30]
   Selected neighbour cost: 282

7. **Candidate schedule costs**: [282, 282, 282, 279, 284, 282, 282, 282, 280, 278, 292, 282, 287, 282, 287, 277, 282, 282, 282]
   Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10) }
   Best cost so far: 277
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
   Selected neighbour cost: 277

8. **Candidate schedule costs**: [277, 277, 274, 279, 277, 277, 277, 275, 273, 287, 277, 282, 277, 282, 277, 277, 277, 277]
   Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17) }
   Best cost so far: 273
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
   Selected neighbour cost: 273

9. **Candidate schedule costs**: [273, 270, 275, 273, 273, 273, 271, 273, 277, 283, 273, 278, 273, 278, 273, 273, 273, 273, 273]
   Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21) }
   Best cost so far: 270
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 2, 21, 26, 27, 7, 6, 18, 20, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
   Selected neighbour cost: 270

Results There weren't enough iterations to reach an optimal solution, evidenced by the fact that every iteration yielded a more optimal solution.

2.1.2 K=100

Iterations

0. **Candidate schedule costs**: [341, 341, 341, 341, 341, 341, 338, 343, 341, 341, 341, 339, 351, 331, 359, 341, 336, 346, 336, 346, 336]
   Tabu list: { (17, 25) }
   Best cost so far: 331
   Tabu list used?: No
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 16, 14, 5, 23, 15, 4, 10, 1, 0, 30]
   Selected neighbour cost: 331

1. **Candidate schedule costs**: [331, 331, 331, 331, 331, 331, 328, 333, 331, 331, 331, 329, 341, 331, 326, 336, 326, 336, 326]
   Tabu list: { (17, 25), (5, 14) }
   Best cost so far: 326
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 16, 5, 14, 23, 15, 4, 10, 1, 0, 30]
   Selected neighbour cost: 326

2. **Candidate schedule costs**: [326, 326, 326, 326, 326, 323, 328, 326, 326, 326, 324, 336, 326, 320, 331, 326, 321, 331, 321, 326]
   Tabu list: { (17, 25), (5, 14), (5, 16) }
   Best cost so far: 320
   Tabu list used?: Yes
   Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 5, 16, 14, 23, 15, 4,

10, 1, 0, 30]
Selected neighbour cost: 320

3. **Candidate schedule costs**: [320, 320, 320, 320, 317, 322, 320, 320, 320, 318, 330, 302, 326, 320, 315, 325, 315, 320, 320]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24) }
Best cost so far: 302
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 5, 24, 16, 14, 23, 15, 4, 10, 1, 0, 30]
Selected neighbour cost: 302

4. **Candidate schedule costs**: [302, 302, 302, 302, 299, 304, 302, 302, 302, 300, 312, 292, 320, 302, 302, 297, 307, 297, 302, 302]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25) }
Best cost so far: 292
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 14, 23, 15, 4, 10, 1, 0, 30]
Selected neighbour cost: 292

5. **Candidate schedule costs**: [292, 292, 292, 289, 294, 292, 292, 292, 290, 288, 302, 292, 292, 287, 297, 287, 292, 292, 292]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23) }
Best cost so far: 287
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 14, 15, 23, 4, 10, 1, 0, 30]
Selected neighbour cost: 287

6. **Candidate schedule costs**: [287, 287, 287, 284, 289, 287, 287, 287, 285, 283, 297, 287, 282, 292, 292, 282, 287, 287, 287]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15) }
Best cost so far: 282
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 15, 14, 23, 4, 10, 1, 0, 30]
Selected neighbour cost: 282

7. **Candidate schedule costs**: [282, 282, 282, 279, 284, 282, 282, 282, 280, 278, 292, 282, 287, 282, 287, 277, 282, 282, 282]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10) }
Best cost so far: 277
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 277

8. **Candidate schedule costs**: [277, 277, 274, 279, 277, 277, 277, 275, 273, 287, 277, 282, 277, 282, 277, 277, 277, 277]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17) }
Best cost so far: 273
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 273

9. **Candidate schedule costs**: [273, 270, 275, 273, 273, 273, 271, 273, 277, 283, 273, 278, 273, 278, 273, 273, 273, 273, 273]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21) }
Best cost so far: 270
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 2, 21, 26, 27, 7, 6, 18, 20, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 270

10. **Candidate schedule costs**: [273, 270, 270, 270, 270, 268, 270, 274, 280, 270, 275, 270, 275, 270, 270, 270, 270, 270, 267]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19) }
Best cost so far: 267
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 7, 6, 18, 20, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 267

11. **Candidate schedule costs**: [267, 267, 267, 267, 265, 267, 271, 277, 267, 272, 267, 272, 267, 267, 267, 267, 270, 267]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20) }
Best cost so far: 265
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 7, 6, 20, 18, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 265

12. **Candidate schedule costs**: [265, 265, 259, 267, 263, 269, 275, 265, 270, 265, 270, 265, 265, 265, 265, 268, 265, 265]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20), (6, 20) }
Best cost so far: 259
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 7, 20, 6, 18, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 259

13. **Candidate schedule costs**: [259, 259, 265, 259, 257, 263, 269, 259, 264, 259, 264, 259, 259, 259, 259, 262, 259, 259, 259]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20), (6, 20), (5, 18) }
Best cost so far: 257
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 7, 20, 6, 5, 18, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 257

16. **Candidate schedule costs**: [259, 267, 257, 257, 239, 262, 257, 262, 257, 257, 257, 257, 260, 257, 257, 257, 257, 257]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20), (6, 20), (5, 18), (7, 20), (16, 24), (15, 24) }
Best cost so far: 239
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 20, 7, 6, 5, 18, 17, 25, 16, 15, 24, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 239

18. **Candidate schedule costs**: [239, 229, 257, 239, 239, 244, 239, 239, 239, 239, 242, 239, 239, 239, 239, 239, 241]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20), (6, 20), (5, 18), (7, 20), (16, 24), (15, 24), (16, 25), (15, 25) }
Best cost so far: 229
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 20, 7, 6, 5, 18, 17, 16, 15, 25, 24, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 229

36. **Candidate schedule costs**: [229, 229, 223, 232, 229, 229, 229, 229, 229, 231, 234, 229, 234, 229, 229, 229, 229, 229]
Tabu list: { (16, 25), (15, 25), (14, 24), (14, 25), (22, 28), (9, 28), (9, 22), (8, 28), (8, 22), (13, 28), (13, 22), (8, 13), (9, 13), (12, 28), (12, 22), (8, 12), (11, 28), (11, 22), (3, 28), (2, 28) }
Best cost so far: 223
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 12, 8, 11, 22, 3, 2, 28, 19, 21, 26, 27, 20, 7, 6, 5, 18, 17, 16, 15, 14, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 223

45. **Candidate schedule costs**: [223, 221, 225, 228, 223, 228, 223, 223, 223, 223, 223, 223, 226, 223, 223, 223, 223, 223]
Tabu list: { (13, 28), (13, 22), (8, 13), (9, 13), (12, 28), (12, 22), (8, 12), (11, 28), (11, 22), (3, 28), (2, 28), (19, 28), (21, 28), (19, 21), (26, 27), (20, 26), (20, 27), (7, 26), (6, 26), (5, 26) }
Best cost so far: 221
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 12, 8, 11, 22, 3, 2, 21, 19, 28, 20, 27, 7, 6, 5, 26, 18, 17, 16, 15, 14, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 221

47. **Candidate schedule costs**: [221, 219, 226, 221, 226, 221, 221, 221, 221, 221, 221, 224, 221, 221, 221, 221, 221, 223]
Tabu list: { (8, 13), (9, 13), (12, 28), (12, 22), (8, 12), (11, 28), (11, 22), (3, 28), (2, 28), (19, 28), (21, 28), (19, 21), (26, 27), (20, 26), (20, 27), (7, 26), (6, 26), (5, 26), (18, 26), (17, 26) }
Best cost so far: 219
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 12, 8, 11, 22, 3, 2, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 26, 16, 15, 14, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 219

49. **Candidate schedule costs**: [219, 217, 224, 219, 224, 219, 219, 219, 219, 219, 219, 222, 219, 219, 219, 219, 219, 221]
Tabu list: { (12, 28), (12, 22), (8, 12), (11, 28), (11, 22), (3, 28), (2, 28), (19, 28), (21, 28), (19, 21), (26, 27), (20, 26), (20, 27), (7, 26), (6, 26), (5, 26), (18, 26), (17, 26), (16, 26), (15, 26) }
Best cost so far: 217
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 12, 8, 11, 22, 3, 2, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 16, 15, 26, 14, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 217

55. **Candidate schedule costs**: [222, 217, 217, 217, 217, 211, 220, 217, 217, 217, 217, 217, 219, 222, 217]
Tabu list: { (2, 28), (19, 28), (21, 28), (19, 21), (26, 27), (20, 26), (20, 27), (7, 26), (6, 26), (5, 26), (18, 26), (17, 26), (16, 26), (15, 26), (14, 26), (9, 13), (8, 11), (8, 22), (3, 8), (2, 8) }
Best cost so far: 211
Tabu list used?: Yes
Selected neighbour: [29, 9, 13, 12, 11, 22, 3, 2, 8, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 16, 15, 14, 26, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 211

61. **Candidate schedule costs**: [211, 211, 211, 211, 209, 217, 211, 211, 211, 211, 211, 211, 213, 216, 211, 216]
Tabu list: { (20, 27), (7, 26), (6, 26), (5, 26), (18, 26), (17, 26), (16, 26), (15, 26), (14, 26), (9, 13), (8, 11), (8, 22), (3, 8), (2, 8), (11, 22), (12, 22), (13, 22), (9, 22), (3, 11), (2, 11) }
Best cost so far: 209
Tabu list used?: Yes
Selected neighbour: [29, 22, 9, 13, 12, 3, 2, 11, 8, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 16, 15, 14, 26, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 209

63. **Candidate schedule costs**: [209, 209, 206, 211, 209, 209, 209, 209, 209, 209, 209, 211, 214, 209, 214, 209, 209]
Tabu list: { (6, 26), (5, 26), (18, 26), (17, 26), (16, 26), (15, 26), (14, 26), (9, 13), (8, 11), (8, 22), (3, 8), (2, 8), (11, 22), (12, 22), (13, 22), (9, 22), (3, 11), (2, 11), (3, 12), (2, 12) }
Best cost so far: 206
Tabu list used?: Yes
Selected neighbour: [29, 22, 9, 13, 3, 2, 12, 11, 8, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 16, 15, 14, 26, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 206

76. **Candidate schedule costs**: [208, 208, 206, 201, 211, 206, 206, 206, 209, 206, 206, 206, 206, 206, 206, 206]
Tabu list: { (12, 22), (13, 22), (9, 22), (3, 11), (2, 11), (3, 12), (2, 12), (8, 21), (11, 21), (12, 21), (8, 19), (11, 19), (8, 28), (8, 20), (8, 27), (14, 26), (14, 25), (14, 24), (14, 23), (4, 14) }
Best cost so far: 201
Tabu list used?: Yes
Selected neighbour: [29, 22, 9, 13, 3, 2, 21, 12, 19, 11, 28, 20, 27, 8, 7, 6, 5, 18, 17, 16, 15, 26, 25, 24, 23, 4, 14, 1, 10, 0, 30]
Selected neighbour cost: 201

77. **Candidate schedule costs**: [203, 206, 196, 206, 201, 201, 201, 204, 201, 201, 201, 201, 201, 201, 201, 203]
Tabu list: { (13, 22), (9, 22), (3, 11), (2, 11), (3, 12), (2, 12), (8, 21), (11, 21), (12, 21), (8, 19), (11, 19), (8, 28), (8, 20), (8, 27), (14, 26), (14, 25), (14, 24), (14, 23), (4, 14), (1, 14) }
Best cost so far: 196
Tabu list used?: Yes
Selected neighbour: [29, 22, 9, 13, 3, 2, 21, 12, 19, 11, 28, 20, 27, 8, 7, 6, 5, 18, 17, 16, 15, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 196

82. **Candidate schedule costs**: [201, 196, 196, 196, 186, 199, 196, 196, 196, 196, 196, 196, 196, 198, 198]
Tabu list: { (2, 12), (8, 21), (11, 21), (12, 21), (8, 19), (11, 19), (8, 28), (8, 20), (8, 27), (14, 26), (14, 25), (14, 24), (14, 23), (4, 14), (1, 14), (9, 13), (13, 22), (9, 22), (3, 22), (2, 22) }
Best cost so far: 186
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 3, 2, 22, 21, 12, 19, 11, 28, 20, 27, 8, 7, 6, 5, 18, 17, 16, 15, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 186

99. **Candidate schedule costs**: [186, 186, 188, 188, 191, 186, 186, 186, 186, 186, 186]
Tabu list: { (9, 22), (3, 22), (2, 22), (3, 9), (3, 13), (2, 9), (2, 13), (12, 21), (12, 22), (9, 12), (19, 21), (19, 22), (9, 19), (12, 19), (11, 21), (11, 22), (9, 11), (21, 28), (22, 28), (20, 27) }
Best cost so far: 186
Tabu list used?: Yes
Selected neighbour: [29, 3, 2, 13, 19, 12, 11, 9, 28, 22, 21, 27, 20, 8, 7, 6, 5, 18, 17, 16, 15, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 186

Results There were again not enough iterations to find the optimal solution. The best solution found was 186, which remained the same for more than 10 iterations, indicating that there was evidence of cycling for this value of K.

2.1.3 K=1000

Iterations

0. **Candidate schedule costs**: [341, 341, 341, 341, 341, 341, 338, 343, 341, 341, 341, 339, 351, 331, 359, 341, 336, 346, 336, 346, 336]
Tabu list: { (17, 25) }
Best cost so far: 331
Tabu list used?: No
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 16, 14, 5, 23, 15, 4, 10, 1, 0, 30]
Selected neighbour cost: 331

1. **Candidate schedule costs**: [331, 331, 331, 331, 331, 331, 328, 333, 331, 331, 331, 329, 341, 331, 326, 336, 326, 336, 326]
Tabu list: { (17, 25), (5, 14) }
Best cost so far: 326
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 16, 5, 14, 23, 15, 4, 10, 1, 0, 30]
Selected neighbour cost: 326

2. **Candidate schedule costs**: [326, 326, 326, 326, 326, 323, 328, 326, 326, 326, 324, 336, 326, 320, 331, 326, 321, 331, 321, 326]
Tabu list: { (17, 25), (5, 14), (5, 16) }
Best cost so far: 320
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 24, 5, 16, 14, 23, 15, 4, 10, 1, 0, 30]
Selected neighbour cost: 320

3. **Candidate schedule costs**: [320, 320, 320, 320, 317, 322, 320, 320, 320, 318, 330, 302, 326, 320, 315, 325, 315, 320, 320]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24) }
Best cost so far: 302
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 25, 5, 24, 16, 14, 23, 15, 4,

10, 1, 0, 30]
Selected neighbour cost: 302

4. **Candidate schedule costs**: [302, 302, 302, 302, 299, 304, 302, 302, 302, 300, 312, 292, 320, 302, 302, 297, 307, 297, 302, 302]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25) }
Best cost so far: 292
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 14, 23, 15, 4, 10, 1, 0, 30]
Selected neighbour cost: 292

5. **Candidate schedule costs**: [292, 292, 292, 289, 294, 292, 292, 292, 290, 288, 302, 292, 292, 287, 297, 287, 292, 292, 292]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23) }
Best cost so far: 287
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 14, 15, 23, 4, 10, 1, 0, 30]
Selected neighbour cost: 287

6. **Candidate schedule costs**: [287, 287, 287, 284, 289, 287, 287, 287, 285, 283, 297, 287, 282, 292, 292, 282, 287, 287, 287]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15) }
Best cost so far: 282
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 15, 14, 23, 4, 10, 1, 0, 30]
Selected neighbour cost: 282

7. **Candidate schedule costs**: [282, 282, 282, 279, 284, 282, 282, 282, 280, 278, 292, 282, 287, 282, 287, 277, 282, 282, 282]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10) }
Best cost so far: 277
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 17, 5, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 277

8. **Candidate schedule costs**: [277, 277, 274, 279, 277, 277, 277, 275, 273, 287, 277, 282, 277, 282, 277, 277, 277, 277]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17) }
Best cost so far: 273
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 21, 2, 26, 27, 7, 6, 18, 20, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 273

9. **Candidate schedule costs**: [273, 270, 275, 273, 273, 273, 271, 273, 277, 283, 273, 278, 273, 278, 273, 273, 273, 273, 273]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21) }
Best cost so far: 270
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 19, 2, 21, 26, 27, 7, 6, 18, 20, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 270

10. **Candidate schedule costs**: [273, 270, 270, 270, 270, 268, 270, 274, 280, 270, 275, 270, 275, 270, 270, 270, 270, 270, 267]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19) }
Best cost so far: 267
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 7, 6, 18, 20, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 267

11. **Candidate schedule costs**: [267, 267, 267, 267, 265, 267, 271, 277, 267, 272, 267, 272, 267, 267, 267, 267, 270, 267]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20) }
Best cost so far: 265
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 7, 6, 20, 18, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 265

12. **Candidate schedule costs**: [265, 265, 259, 267, 263, 269, 275, 265, 270, 265, 270, 265, 265, 265, 265, 268, 265, 265]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20), (6, 20) }
Best cost so far: 259
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 7, 20, 6, 18, 5, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 259

13. **Candidate schedule costs**: [259, 259, 265, 259, 257, 263, 269, 259, 264, 259, 264, 259, 259, 259, 259, 262, 259, 259, 259]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20), (6, 20), (5, 18) }
Best cost so far: 257
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 7, 20, 6, 5, 18, 17, 25, 24, 16, 15, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 257

16. **Candidate schedule costs**: [259, 267, 257, 257, 239, 262, 257, 262, 257, 257, 257, 257, 260, 257, 257, 257, 257, 257]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20), (6, 20), (5, 18), (7, 20), (16, 24), (15, 24) }
Best cost so far: 239
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 20, 7, 6, 5, 18, 17, 25, 16, 15, 24, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 239

18. **Candidate schedule costs**: [239, 229, 257, 239, 239, 244, 239, 239, 239, 239, 242, 239, 239, 239, 239, 239, 241]
Tabu list: { (17, 25), (5, 14), (5, 16), (5, 24), (5, 25), (15, 23), (14, 15), (1, 10), (5, 17), (2, 21), (2, 19), (18, 20), (6, 20), (5, 18), (7, 20), (16, 24), (15, 24), (16, 25), (15, 25) }
Best cost so far: 229
Tabu list used?: Yes
Selected neighbour: [29, 28, 22, 9, 8, 13, 12, 11, 3, 2, 19, 21, 26, 27, 20, 7, 6, 5, 18, 17, 16, 15, 25, 24, 14, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 229

36. **Candidate schedule costs**: [229, 229, 223, 232, 229, 229, 229, 229, 229, 231, 234, 229, 234, 229, 229, 229, 229, 229]
Tabu list: { (16, 25), (15, 25), (14, 24), (14, 25), (22, 28), (9, 28), (9, 22), (8, 28), (8, 22), (13, 28), (13, 22), (8, 13), (9, 13), (12, 28), (12, 22), (8, 12), (11, 28), (11, 22), (3, 28), (2, 28) }
Best cost so far: 223
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 12, 8, 11, 22, 3, 2, 28, 19, 21, 26, 27, 20, 7, 6, 5, 18, 17, 16, 15, 14, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 223

45. **Candidate schedule costs**: [223, 221, 225, 228, 223, 228, 223, 223, 223, 223, 223, 223, 226, 223, 223, 223, 223, 223]
Tabu list: { (13, 28), (13, 22), (8, 13), (9, 13), (12, 28), (12, 22), (8, 12), (11, 28), (11, 22), (3, 28), (2, 28), (19, 28), (21, 28), (19, 21), (26, 27), (20, 26), (20, 27), (7, 26), (6, 26), (5, 26) }
Best cost so far: 221
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 12, 8, 11, 22, 3, 2, 21, 19, 28, 20, 27, 7, 6, 5, 26, 18, 17, 16, 15, 14, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 221

47. **Candidate schedule costs**: [221, 219, 226, 221, 226, 221, 221, 221, 221, 221, 221, 224, 221, 221, 221, 221, 221, 223]
Tabu list: { (8, 13), (9, 13), (12, 28), (12, 22), (8, 12), (11, 28), (11, 22), (3, 28), (2, 28), (19, 28), (21, 28), (19, 21), (26, 27), (20, 26), (20, 27), (7, 26), (6, 26), (5, 26), (18, 26), (17, 26) }
Best cost so far: 219
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 12, 8, 11, 22, 3, 2, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 26, 16, 15, 14, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 219

49. **Candidate schedule costs**: [219, 217, 224, 219, 224, 219, 219, 219, 219, 219, 219, 222, 219, 219, 219, 219, 219, 221]
Tabu list: { (12, 28), (12, 22), (8, 12), (11, 28), (11, 22), (3, 28), (2, 28), (19, 28), (21, 28), (19, 21), (26, 27), (20, 26), (20, 27), (7, 26), (6, 26), (5, 26), (18, 26), (17, 26), (16, 26), (15, 26) }
Best cost so far: 217
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 12, 8, 11, 22, 3, 2, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 16, 15, 26, 14, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 217

55. **Candidate schedule costs**: [222, 217, 217, 217, 217, 211, 220, 217, 217, 217, 217, 217, 219, 222, 217]
Tabu list: { (2, 28), (19, 28), (21, 28), (19, 21), (26, 27), (20, 26), (20, 27), (7, 26), (6, 26), (5, 26), (18, 26), (17, 26), (16, 26), (15, 26), (14, 26), (9, 13), (8, 11), (8, 22), (3, 8), (2, 8) }
Best cost so far: 211
Tabu list used?: Yes
Selected neighbour: [29, 9, 13, 12, 11, 22, 3, 2, 8, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 16, 15, 14, 26, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 211

61. **Candidate schedule costs**: [211, 211, 211, 211, 209, 217, 211, 211, 211, 211, 211, 211, 213, 216, 211, 216]
Tabu list: { (20, 27), (7, 26), (6, 26), (5, 26), (18, 26), (17, 26), (16, 26), (15, 26), (14, 26), (9, 13), (8, 11), (8, 22), (3, 8), (2, 8), (11, 22), (12, 22), (13, 22), (9, 22), (3, 11), (2, 11) }
Best cost so far: 209
Tabu list used?: Yes
Selected neighbour: [29, 22, 9, 13, 12, 3, 2, 11, 8, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 16, 15, 14, 26, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 209

63. **Candidate schedule costs**: [209, 209, 206, 211, 209, 209, 209, 209, 209, 209, 209, 211, 214, 209, 214, 209, 209]
Tabu list: { (6, 26), (5, 26), (18, 26), (17, 26), (16, 26), (15, 26), (14, 26), (9, 13), (8, 11), (8, 22), (3, 8), (2, 8), (11, 22), (12, 22), (13, 22), (9, 22), (3, 11), (2, 11), (3, 12), (2, 12) }
Best cost so far: 206
Tabu list used?: Yes
Selected neighbour: [29, 22, 9, 13, 3, 2, 12, 11, 8, 21, 19, 28, 20, 27, 7, 6, 5, 18, 17, 16, 15, 14, 26, 25, 24, 23, 4, 1, 10, 0, 30]
Selected neighbour cost: 206

76. **Candidate schedule costs**: [208, 208, 206, 201, 211, 206, 206, 206, 209, 206, 206, 206, 206, 206, 206, 206]
Tabu list: { (12, 22), (13, 22), (9, 22), (3, 11), (2, 11), (3, 12), (2, 12), (8, 21), (11, 21), (12, 21), (8, 19), (11, 19), (8, 28), (8, 20), (8, 27), (14, 26), (14, 25), (14, 24), (14, 23), (4, 14) }
Best cost so far: 201
Tabu list used?: Yes
Selected neighbour: [29, 22, 9, 13, 3, 2, 21, 12, 19, 11, 28, 20, 27, 8, 7, 6, 5, 18, 17, 16, 15, 26, 25, 24, 23, 4, 14, 1, 10, 0, 30]
Selected neighbour cost: 201

77. **Candidate schedule costs**: [203, 206, 196, 206, 201, 201, 201, 204, 201, 201, 201, 201, 201, 201, 201, 203]
Tabu list: { (13, 22), (9, 22), (3, 11), (2, 11), (3, 12), (2, 12), (8, 21), (11, 21), (12, 21), (8, 19), (11, 19), (8, 28), (8, 20), (8, 27), (14, 26), (14, 25), (14, 24), (14, 23), (4, 14), (1, 14) }
Best cost so far: 196
Tabu list used?: Yes
Selected neighbour: [29, 22, 9, 13, 3, 2, 21, 12, 19, 11, 28, 20, 27, 8, 7, 6, 5, 18, 17, 16, 15, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 196

82. **Candidate schedule costs**: [201, 196, 196, 196, 186, 199, 196, 196, 196, 196, 196, 196, 196, 198, 198]
Tabu list: { (2, 12), (8, 21), (11, 21), (12, 21), (8, 19), (11, 19), (8, 28), (8, 20), (8, 27), (14, 26), (14, 25), (14, 24),

(14, 23), (4, 14), (1, 14), (9, 13), (13, 22), (9, 22), (3, 22), (2, 22) }
Best cost so far: 186
Tabu list used?: Yes
Selected neighbour: [29, 13, 9, 3, 2, 22, 21, 12, 19, 11, 28, 20, 27, 8, 7, 6, 5, 18, 17, 16, 15, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 186

166. **Candidate schedule costs**: [188, 184, 188, 191, 188, 186, 186, 186, 186, 186, 186, 186]
Tabu list: { (11, 22), (12, 22), (13, 22), (18, 21), (19, 21), (11, 21), (12, 21), (13, 21), (11, 19), (12, 19), (11, 18), (11, 28), (11, 27), (11, 20), (9, 11), (8, 11), (7, 11), (6, 11), (5, 11), (11, 17) }
Best cost so far: 184
Tabu list used?: Yes
Selected neighbour: [29, 3, 2, 22, 21, 13, 19, 12, 18, 28, 27, 20, 9, 8, 7, 6, 5, 17, 11, 16, 15, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 184

168. **Candidate schedule costs**: [184, 182, 186, 189, 186, 184, 184, 184, 184, 184, 184, 184, 188]
Tabu list: { (13, 22), (18, 21), (19, 21), (11, 21), (12, 21), (13, 21), (11, 19), (12, 19), (11, 18), (11, 28), (11, 27), (11, 20), (9, 11), (8, 11), (7, 11), (6, 11), (5, 11), (11, 17), (11, 16), (11, 15) }
Best cost so far: 182
Tabu list used?: Yes
Selected neighbour: [29, 3, 2, 22, 21, 13, 19, 12, 18, 28, 27, 20, 9, 8, 7, 6, 5, 17, 16, 15, 11, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 182

224. **Candidate schedule costs**: [182, 180, 186, 184, 182, 187, 182, 182, 185, 182, 182, 182, 182, 182, 182, 184]
Tabu list: { (2, 28), (3, 28), (12, 28), (13, 28), (18, 22), (19, 22), (18, 27), (19, 27), (22, 27), (9, 18), (9, 19), (9, 22), (18, 21), (19, 21), (8, 18), (8, 19), (7, 18), (18, 20), (6, 18), (5, 18) }
Best cost so far: 180
Tabu list used?: Yes
Selected neighbour: [29, 28, 13, 12, 3, 2, 27, 9, 22, 21, 8, 19, 7, 20, 6, 5, 18, 17, 16, 15, 11, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 180

253. **Candidate schedule costs**: [180, 177, 182, 182, 180, 185, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180]
Tabu list: { (2, 28), (2, 12), (2, 13), (9, 27), (9, 28), (9, 12), (9, 13), (2, 9), (22, 27), (22, 28), (12, 22), (21, 27), (21, 28), (8, 27), (8, 28), (19, 27), (7, 27), (20, 27), (6, 27), (5, 27) }
Best cost so far: 177
Tabu list used?: Yes
Selected neighbour: [29, 3, 9, 2, 13, 22, 12, 21, 8, 28, 19, 7, 20, 6, 5, 27, 18, 17, 16, 15, 26, 25, 24, 23, 11, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 177

255. **Candidate schedule costs**: [177, 174, 179, 177, 182, 177, 177, 177, 177, 177, 177, 177, 177, 177, 177, 177, 177, 179]
Tabu list: { (2, 13), (9, 27), (9, 28), (9, 12), (9, 13), (2, 9), (22, 27), (22, 28), (12, 22), (21, 27), (21, 28), (8, 27), (8, 28), (19, 27), (7, 27), (20, 27), (6, 27), (5, 27), (18, 27), (17, 27) }
Best cost so far: 174
Tabu list used?: Yes
Selected neighbour: [29, 3, 9, 2, 13, 22, 12, 21, 8, 28, 19, 7, 20, 6, 5, 18, 17, 27, 16, 15, 26, 25, 24, 23, 11, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 174

257. **Candidate schedule costs**: [174, 171, 176, 174, 179, 174, 174, 174, 174, 174, 174, 174, 174, 174, 174, 174, 174, 176]
Tabu list: { (9, 28), (9, 12), (9, 13), (2, 9), (22, 27), (22, 28), (12, 22), (21, 27), (21, 28), (8, 27), (8, 28), (19, 27), (7, 27), (20, 27), (6, 27), (5, 27), (18, 27), (17, 27), (16, 27), (15, 27) }
Best cost so far: 171
Tabu list used?: Yes
Selected neighbour: [29, 3, 9, 2, 13, 22, 12, 21, 8, 28, 19, 7, 20, 6, 5, 18, 17, 16, 15, 27, 26, 25, 24, 23, 11, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 171

312. **Candidate schedule costs**: [171, 170, 173, 173, 171, 171, 176, 171, 171, 171, 173, 171, 171, 171, 171, 171]
Tabu list: { (12, 28), (13, 28), (2, 28), (3, 28), (9, 12), (9, 13), (12, 19), (13, 19), (9, 19), (2, 19), (8, 12), (8, 13), (7, 12), (7, 13), (12, 22), (13, 22), (6, 12), (12, 21), (5, 12), (12, 20) }

Best cost so far: 170
Tabu list used?: Yes
Selected neighbour: [29, 28, 3, 19, 2, 9, 8, 7, 22, 13, 6, 21, 5, 20, 12, 18, 17, 16, 15, 11, 27, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 170

314. **Candidate schedule costs**: [170, 167, 172, 170, 170, 175, 170, 170, 170, 172, 170, 170, 170, 170, 170, 170, 170]
Tabu list: { (2, 28), (3, 28), (9, 12), (9, 13), (12, 19), (13, 19), (9, 19), (2, 19), (8, 12), (8, 13), (7, 12), (7, 13), (12, 22), (13, 22), (6, 12), (12, 21), (5, 12), (12, 20), (12, 18), (12, 17) }
Best cost so far: 167
Tabu list used?: Yes
Selected neighbour: [29, 28, 3, 19, 2, 9, 8, 7, 22, 13, 6, 21, 5, 20, 18, 17, 12, 16, 15, 11, 27, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 167

316. **Candidate schedule costs**: [167, 164, 169, 167, 167, 172, 167, 167, 167, 169, 167, 167, 167, 167, 167, 167, 167]
Tabu list: { (9, 12), (9, 13), (12, 19), (13, 19), (9, 19), (2, 19), (8, 12), (8, 13), (7, 12), (7, 13), (12, 22), (13, 22), (6, 12), (12, 21), (5, 12), (12, 20), (12, 18), (12, 17), (12, 16), (12, 15) }
Best cost so far: 164
Tabu list used?: Yes
Selected neighbour: [29, 28, 3, 19, 2, 9, 8, 7, 22, 13, 6, 21, 5, 20, 18, 17, 16, 15, 12, 11, 27, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 164

345. **Candidate schedule costs**: [164, 158, 167, 164, 164, 164, 169, 164, 164, 174, 164, 164, 164, 164, 164, 164, 164]
Tabu list: { (2, 28), (9, 28), (2, 9), (3, 9), (9, 19), (8, 28), (7, 28), (22, 28), (7, 22), (8, 22), (13, 28), (7, 13), (6, 28), (21, 28), (5, 28), (20, 28), (18, 28), (17, 28), (16, 28), (15, 28) }
Best cost so far: 158
Tabu list used?: Yes
Selected neighbour: [29, 9, 19, 3, 2, 22, 8, 13, 7, 6, 21, 5, 20, 18, 17, 16, 15, 28, 27, 12, 26, 25, 24, 23, 11, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 158

999. **Candidate schedule costs**: [158, 158, 158, 161, 158, 158, 158, 163, 158, 158, 158, 158, 158]
Tabu list: { (11, 26), (3, 18), (9, 19), (3, 9), (3, 19), (9, 18), (2, 9), (2, 18), (2, 19), (13, 21), (5, 22), (6, 22), (7, 22), (8, 22), (9, 22), (5, 13), (6, 13), (7, 13), (5, 21), (6, 21) }
Best cost so far: 158
Tabu list used?: Yes
Selected neighbour: [29, 3, 2, 19, 18, 22, 9, 8, 13, 7, 21, 6, 5, 20, 17, 16, 15, 12, 28, 27, 11, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]
Selected neighbour cost: 158

Results The best schedule x_{TS} found was:

$$[29, 3, 2, 19, 18, 22, 9, 8, 13, 7, 21, 6, 5, 20, 17, 16, 15, 12, 28, 27, 11, 26, 25, 24, 23, 4, 1, 14, 10, 0, 30]$$

Which had a total tardiness of **158**. Here it seems that the algorithm did effectively converge but the tabu list was not large enough to prevent cycling, as the best solution was found more than once. The algorithm could be improved by increasing the tabu list length to prevent revisiting recent states and cycling.

2.2 Parameter Exploration and Results

The best schedule x_{TS} found was:

$$[29, 3, 2, 9, 8, 7, 6, 22, 19, 5, 18, 21, 13, 20, 17, 16, 15, 12, 28, 27, 26, 25, 24, 23, 11, 4, 1, 14, 10, 0, 30]$$

with a total tardiness of **158**.

Parameters Used

- **Tabu List Length (L)**: 60 - this tabu list length was set to 60 to allow sufficient history to avoid cycling while maintaining exploration flexibility in the neighborhood (up to 29 adjacent swaps)
- **Maximum Iterations (K)**: 500 - this value was chosen to ensure the algorithm had enough time to converge to a good solution
- **Threshold for Accepting Worse Solutions (γ)**: 20 - this value balanced allowing worse solutions to escape local optima and converging efficiently

Parameter Effects

1. Tabu List Length (L):

- For smaller values of L (e.g., $L = 20$), the tabu list wasn't large enough to prevent revisiting recent states, leading to slower improvement, cycling and suboptimal solutions
- Increasing L to 60 was good for avoiding cycles and also allowing sufficient exploration of the neighborhood
- Larger values of L (e.g., $L = 80$) restricted the search too much and occasionally skipped good moves, resulting in worse solutions

2. Maximum Iterations (K):

- Setting K to 500 allowed the algorithm to converge fully, in fact it only needed 374 iterations
- Reducing K (e.g., $K = 200$) often resulted in suboptimal solutions, as the search terminated prematurely before the algorithm could escape local optima
- Increasing K beyond 500 did not result in significant improvements but increased runtime unnecessarily, and for cases where the tabu list was small, it just cycled

3. Threshold for Accepting Worse Solutions (γ):

- Smaller values of γ (e.g., $\gamma = 10$) caused the algorithm to become stuck in local optima, as fewer worse solutions were accepted, reducing exploration
- Larger values of γ (e.g., $\gamma = 30$) allowed too much exploration, which slowed convergence and resulted in noisy search behavior (i.e. frequent fluctuations), but did sometimes find better solutions
- $\gamma = 20$ provided a good tradeoff between exploration and exploitation, letting the algorithm escape local optima while still converging efficiently

Edge Case Analysis

When $L < 30$, the tabu list is too small to avoid revisits, which lead to slower improvement and cycling. For $L > 80$, excessive restrictions on moves resulted in skipped promising neighbors and worse solutions. Setting $K < 300$ often terminated the search prematurely before escaping local optima, while increasing K beyond 500 provided diminishing returns with no significant improvement in tardiness.

Limitations

The tabu search implementation performs well for the given problem size, but its efficiency may degrade for significantly larger DAGs or job counts. The parameter tuning process relied on manual exploration and could benefit from automated methods, such as Bayesian optimisation using a tool like Optuna. Additionally, revisiting solutions and cycling occasionally occurred when L was not large enough.

Conclusion

The best schedule was found with $L = 60$, $K = 500$, and $\gamma = 20$, achieving a total tardiness of **158**. These parameters provided a good balance between avoiding cycling, allowing sufficient exploration, and converging efficiently. Parameter tuning revealed that values for L and γ which were too small or too large degraded performance, and K needed to be just large enough to allow full convergence without unnecessary runtime overhead.