# Problem Solving by Search

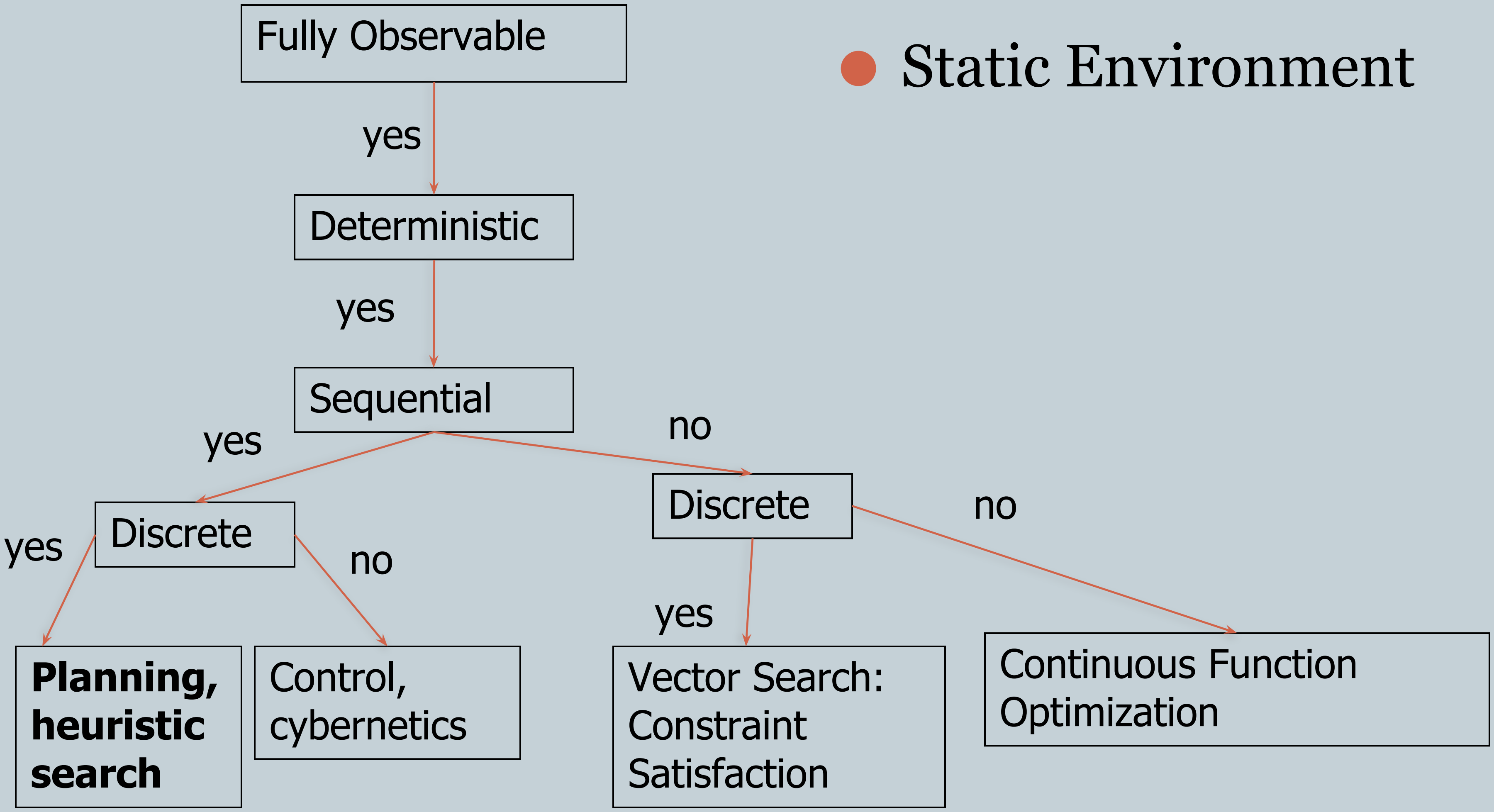
## CHAPTER 3

Stuart Russell and Peter Norvig, Artificial Intelligence: A Modern Approach, Global Edition 3/E

## Outline

- Problem formulation: representing sequential problems.
- Example problems.
- Planning for solving sequential problems without uncertainty.
- Basic search algorithms

# Environment Type Discussed In this Lecture


Fully Observable
yes
Deterministic
yes
Sequential
yes
no
Discrete
yes
no
Discrete
yes
no
**Planning, heuristic search**
Control, cybernetics
Vector Search: Constraint Satisfaction
Continuous Function Optimization

- Static Environment

## Choice in a Deterministic Known Environment

- Without uncertainty, choice is trivial in principle: choose what you know to be the best option.
- Trivial if the problem is represented in a look-up table.

| Option | Value |
|---|---|
| Chocolate | 10 |
| Coffee | 20 |
| Book | 15 |

This is the standard problem representation in decision theory (economics).

## Computational Choice Under Certainty

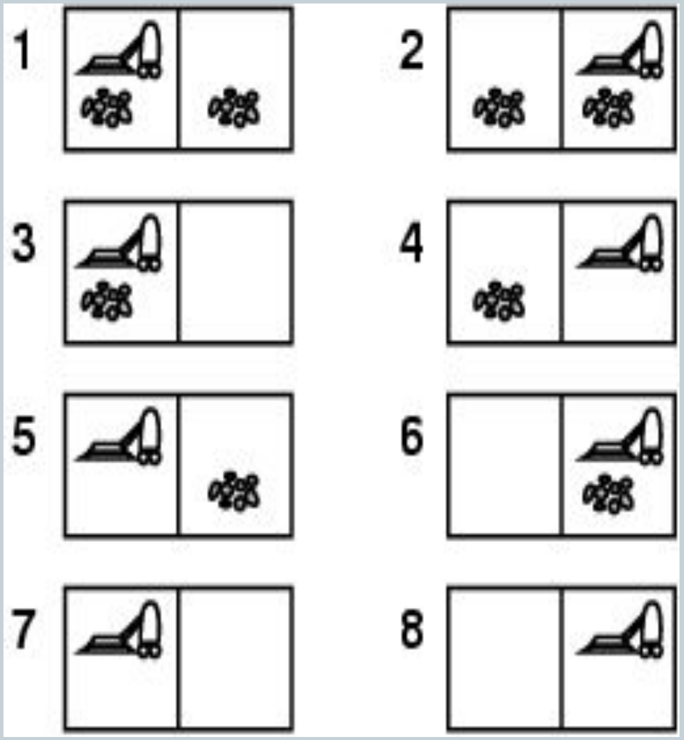
- But choice can be *computationally* hard if the problem information is represented differently.
- Options may be **structured** and the best option needs to be constructed.
  - E.g., an option may consist of a path, sequence of actions, plan, or strategy.
- The value of options may be given **implicitly** rather than explicitly.
  - E.g., cost of paths need to be computed from map.

## Problem Types

- **Deterministic**, fully observable -> single-state problem
  - Agent knows exactly which state it will be in; solution is a **sequence**
- **Non-observable** -> conformant problem
  - Agent may have no idea where it is; solution (if any) is a **sequence**
- **Nondeterministic** and/or partially observable -> contingency problem
  - percepts provide new information about current state
    solution is a contingent plan or a policy
    often **interleave search, execution**
- **Unknown state space** -> exploration problem ("online")

# Sequential Action Example

- Deterministic, fully observable: single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
  - Vacuum world: everything observed
  - Romania: The full map is observed



- Single-state:

Start in #5. Solution??

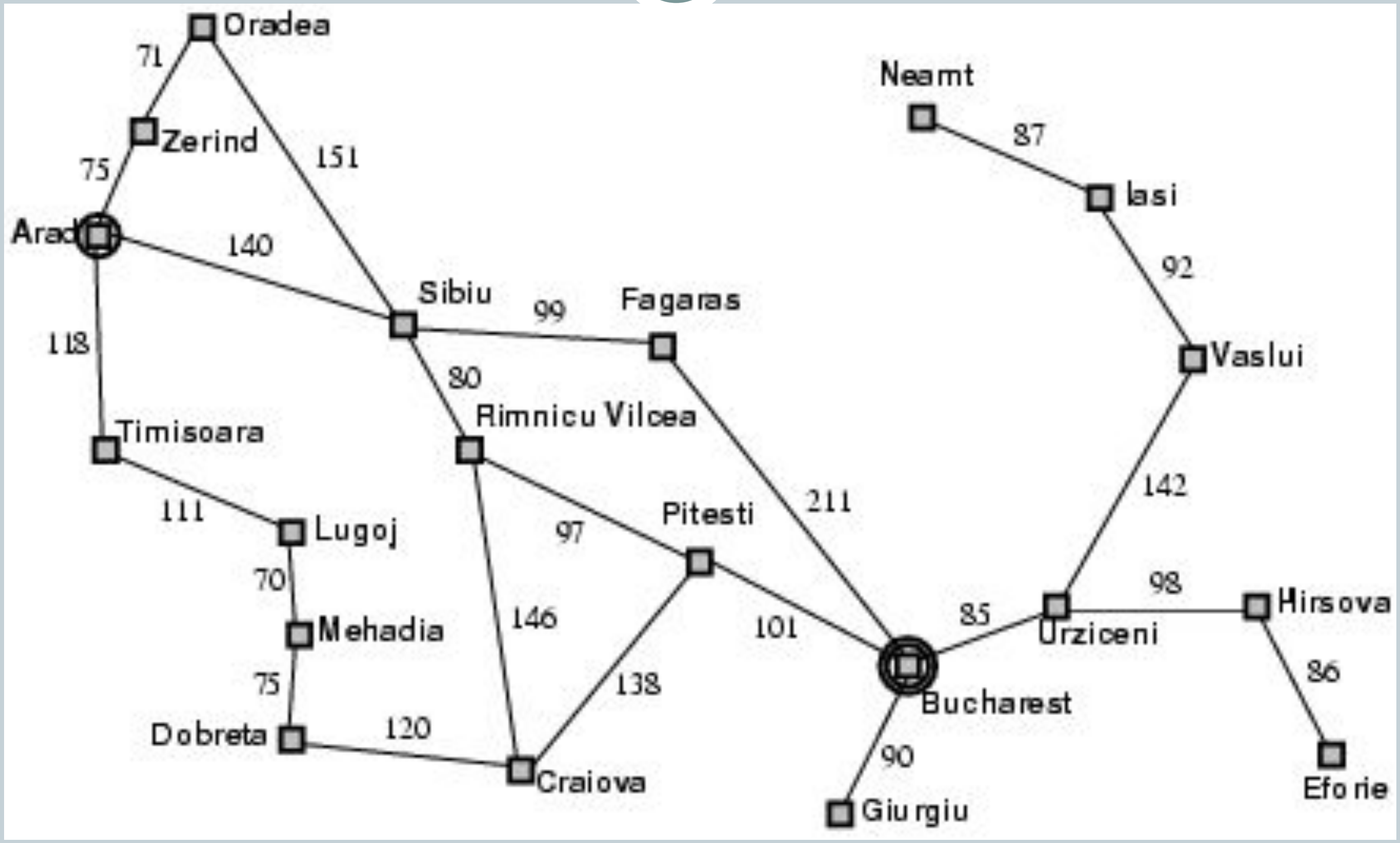
  - [Right, Suck]

## Example: Romania

- On holiday in Romania; currently in Arad.

- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

# Example: Romania



Abstraction: The process of removing details from a representation
Is the map a good representation of the problem? What is a good replacement?

# Single-state problem formulation

A problem is defined by 4 items:

- **initial state** e.g., "at Arad"
- **Successor function** S(x)= set of action–state
- **Goal test**, can be
  - explicit, e.g., x = "at Bucharest", or "checkmate" in chess
  - implicit, e.g., NoDirt(x)
- **Path cost** (additive) e.g., sum of distances, number of actions executed, etc. c(x, a, y) is the step cost, assumed to be ≥ 0

A solution is a sequence of actions leading from the initial state to a goal state

## The successor function

- Successor function: for a given state, returns a set of action/new-state pairs.
- Vacuum-cleaner world: (A, dirty, clean) → (’Left’, (A, dirty, clean)),(’Right’, (B, dirty, clean)), (’Suck’, (A, clean, dirty)), (’NoOp, (A, dirty, clean))
- Romania: In(Arad) → ((Go(Timisoara), In(Timisoara), (Go(Sibiu), In(Sibiu)), (Go(Zerind), In(Zerind))

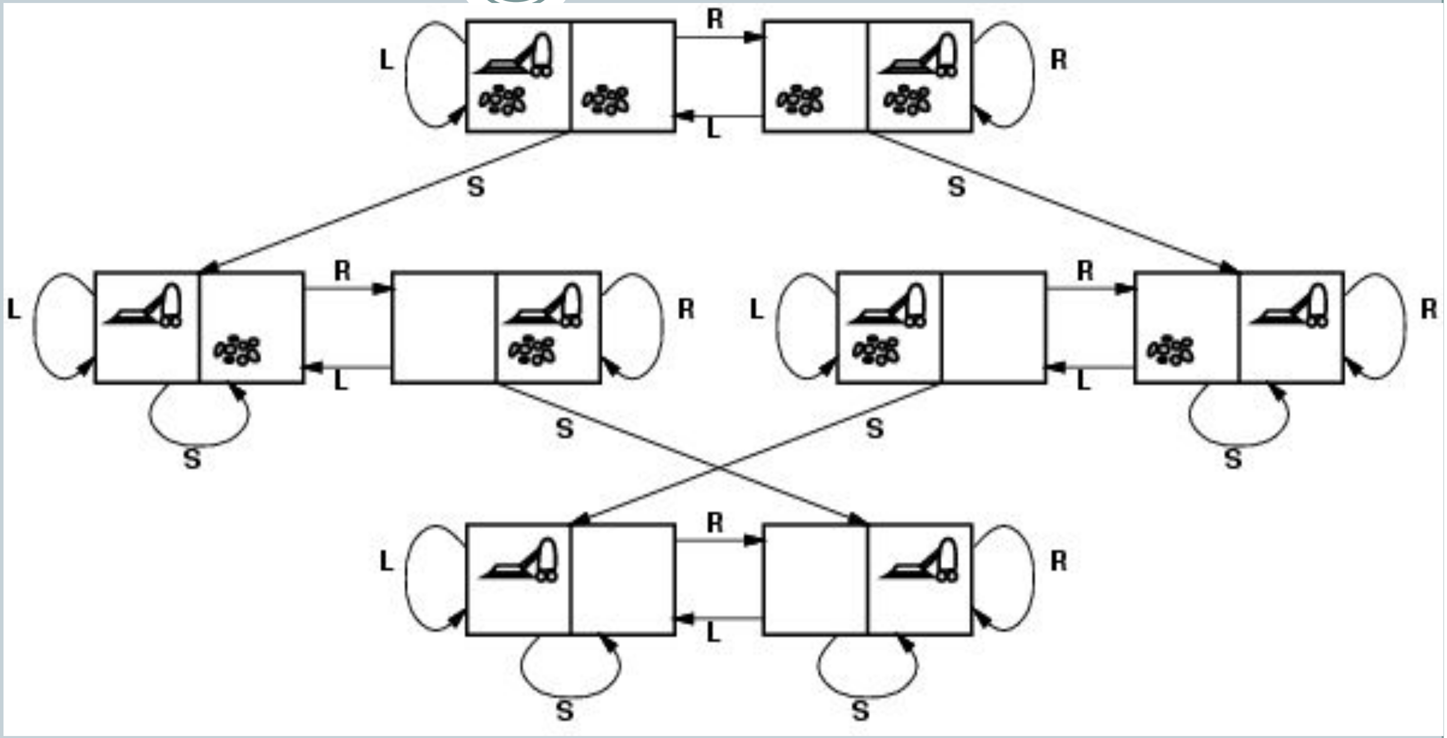
# Size of space

- 8-puzzle: 9!/2 = 181, 000 states (easy)
- 15-puzzle: ~ 1.3 trillion states (pretty easy)
- 24-puzzle: ~ 1025 states (hard)
- TSP, 20 cities: 20! = 2.43 × 1018 states (hard)

## Selecting a state space

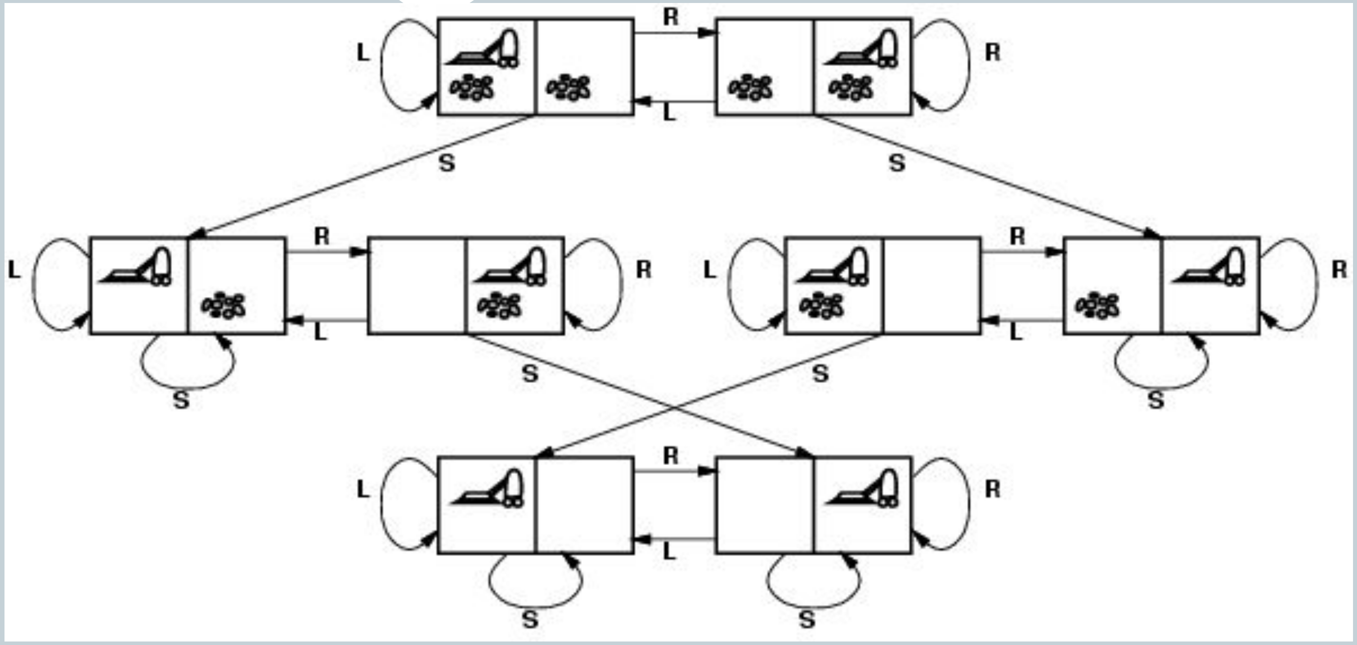
- Real world is complex
  - state space must be abstracted for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  - e.g., "Arad -> Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- (Abstract) solution =
  - set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

# Vacuum world state space graph



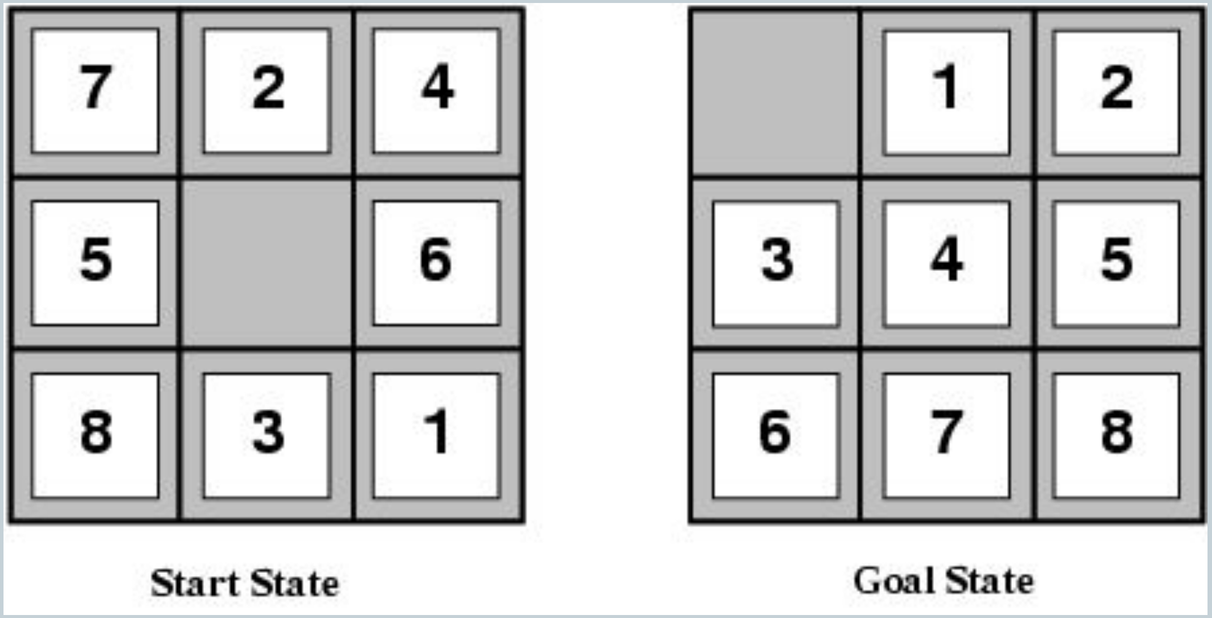
- states?
- actions?
- goal test?
- path cost?

# Vacuum world state space graph



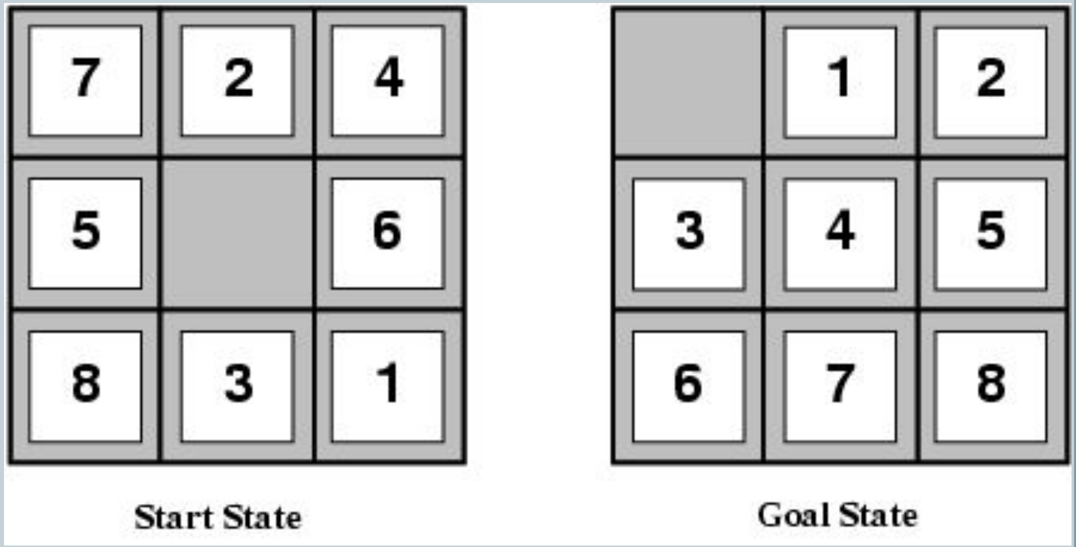
- states? integer dirt and robot location
- actions? *Left*, *Right*, *Suck*
- goal test? no dirt at all locations
- path cost? 1 per action

## Example: The 8-puzzle



- states?
- actions?
- goal test?
- path cost?

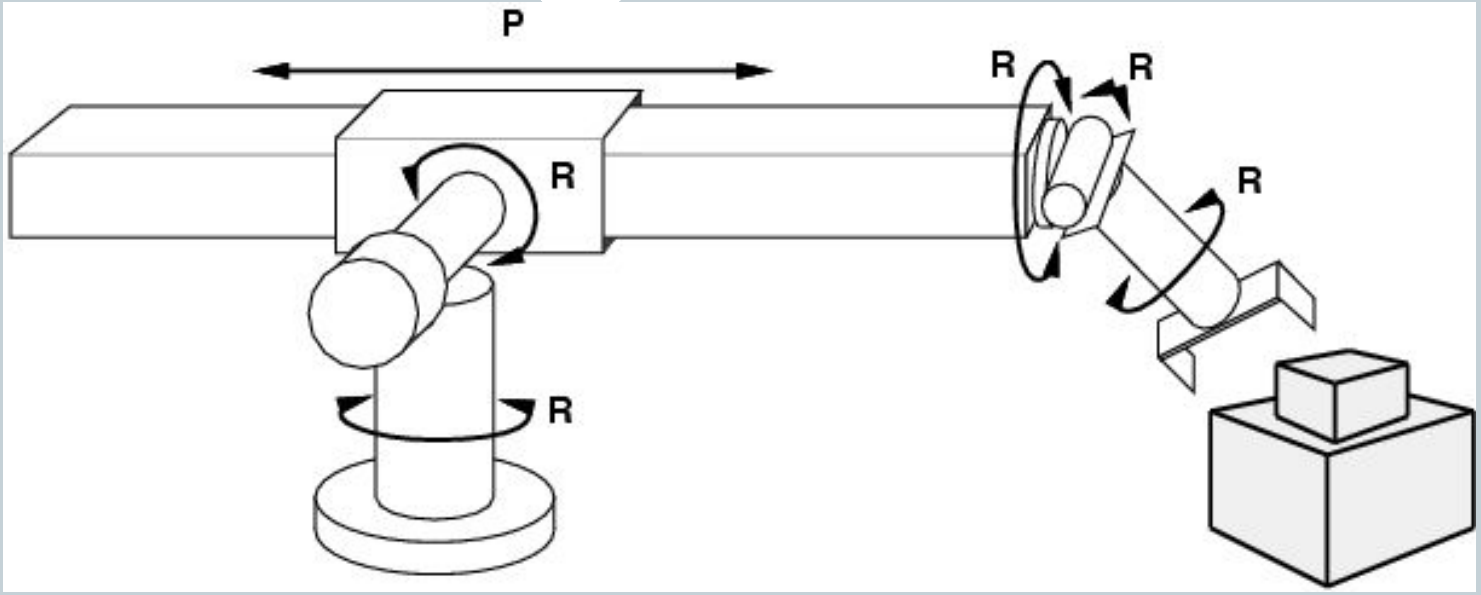
# Example: The 8-puzzle



- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move

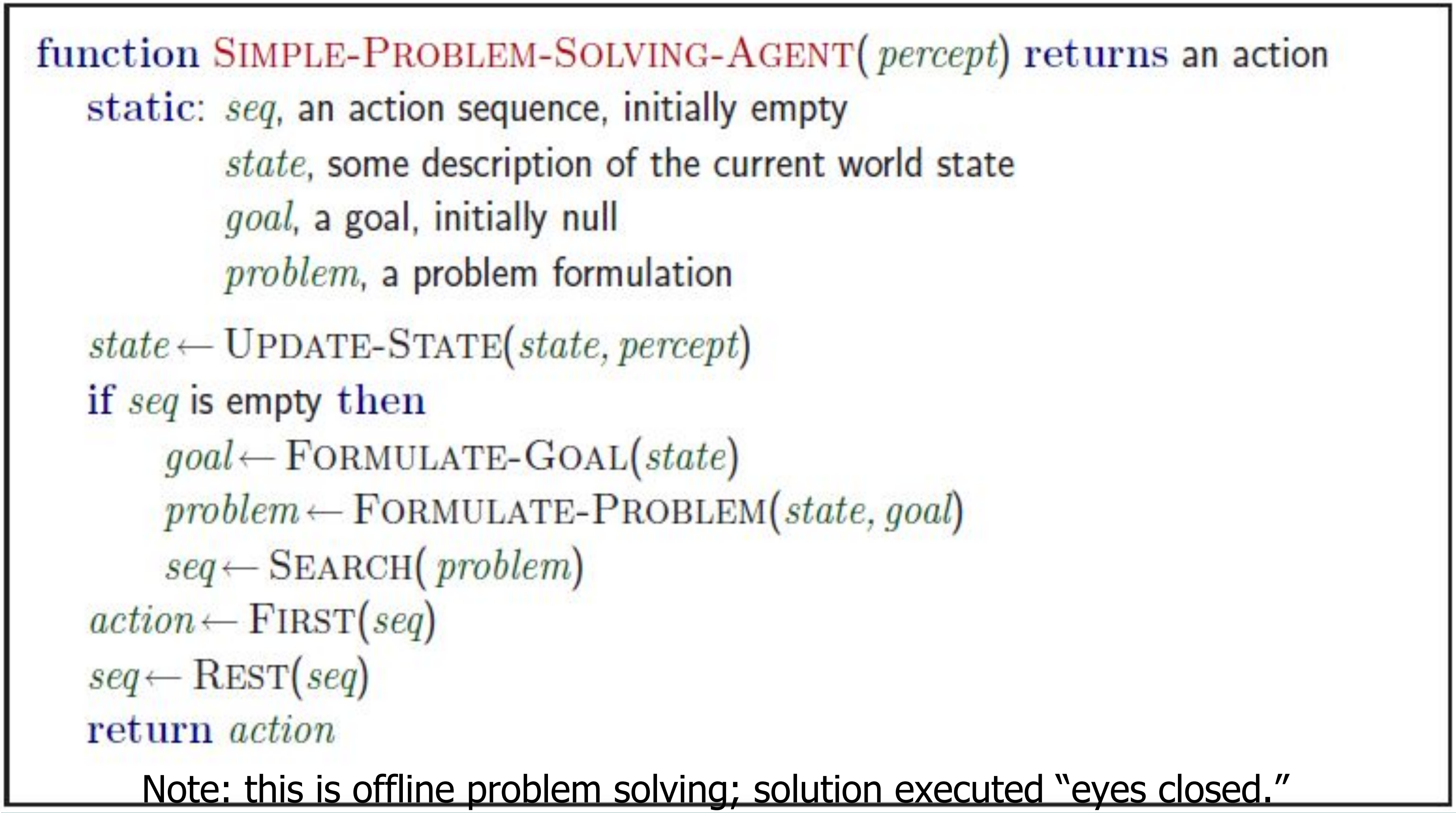
[Note: optimal solution of *n*-Puzzle family is NP-hard]

# Example: robotic assembly



- states?: real-valued coordinates of robot joint angles parts of the object to be assembled
- actions?: continuous motions of robot joints
- goal test?: complete assembly
- path cost?: time to execute

# Problem-solving agents



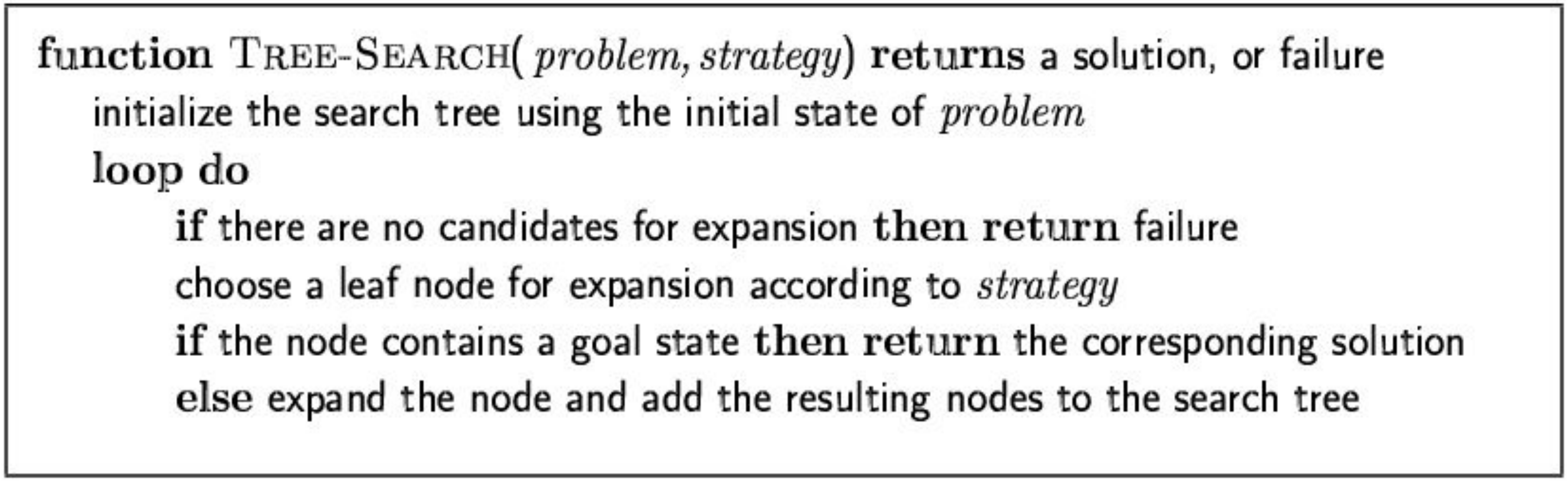
```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH(problem)
    action ← FIRST(seq)
    seq ← REST(seq)
    return action
```

Note: this is offline problem solving; solution executed “eyes closed.”

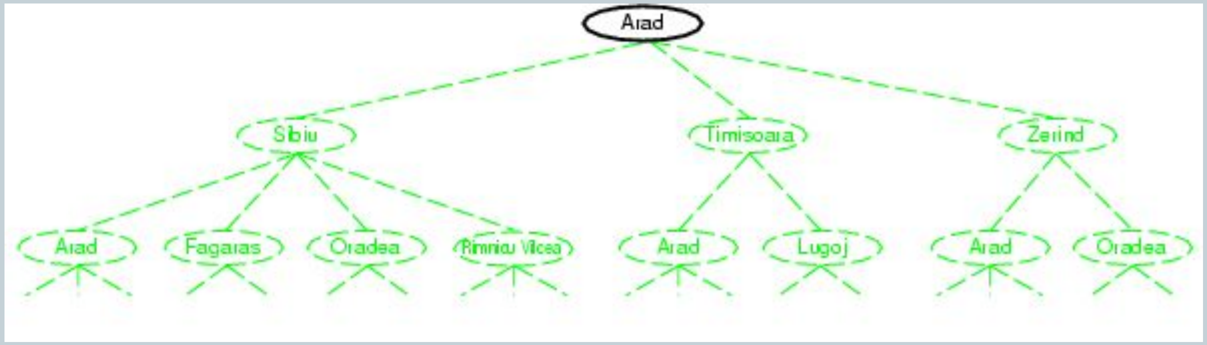
## Tree search algorithms

- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)



```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

# Tree search example



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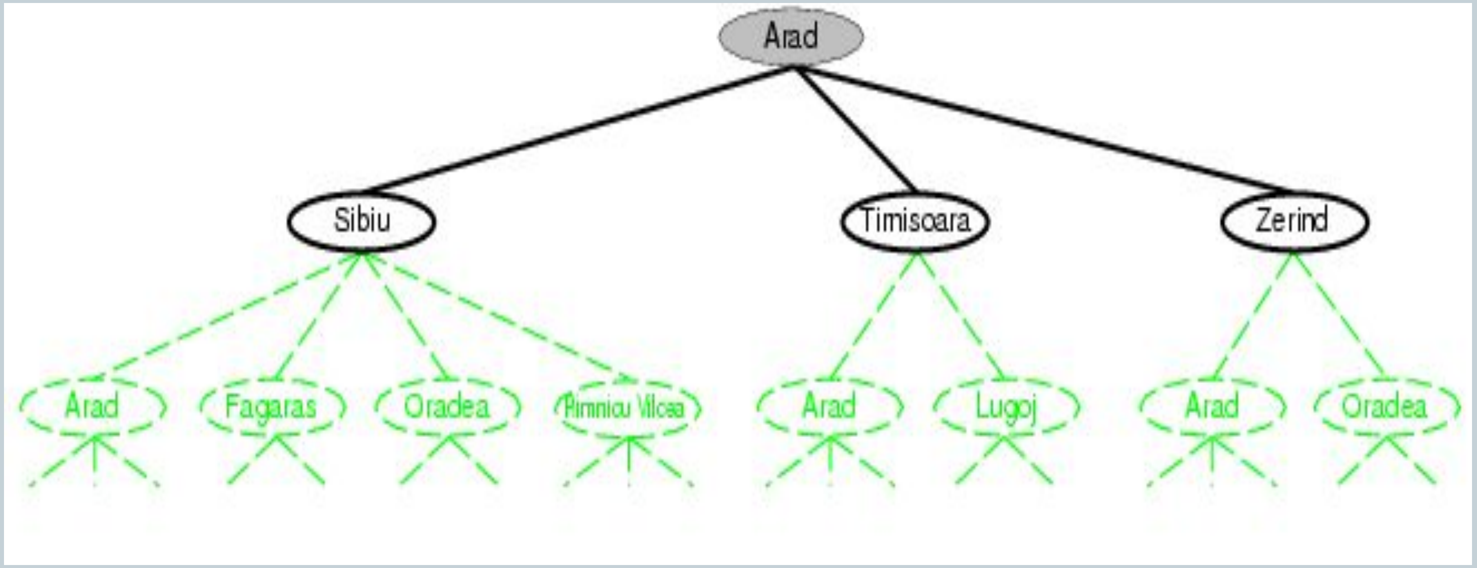
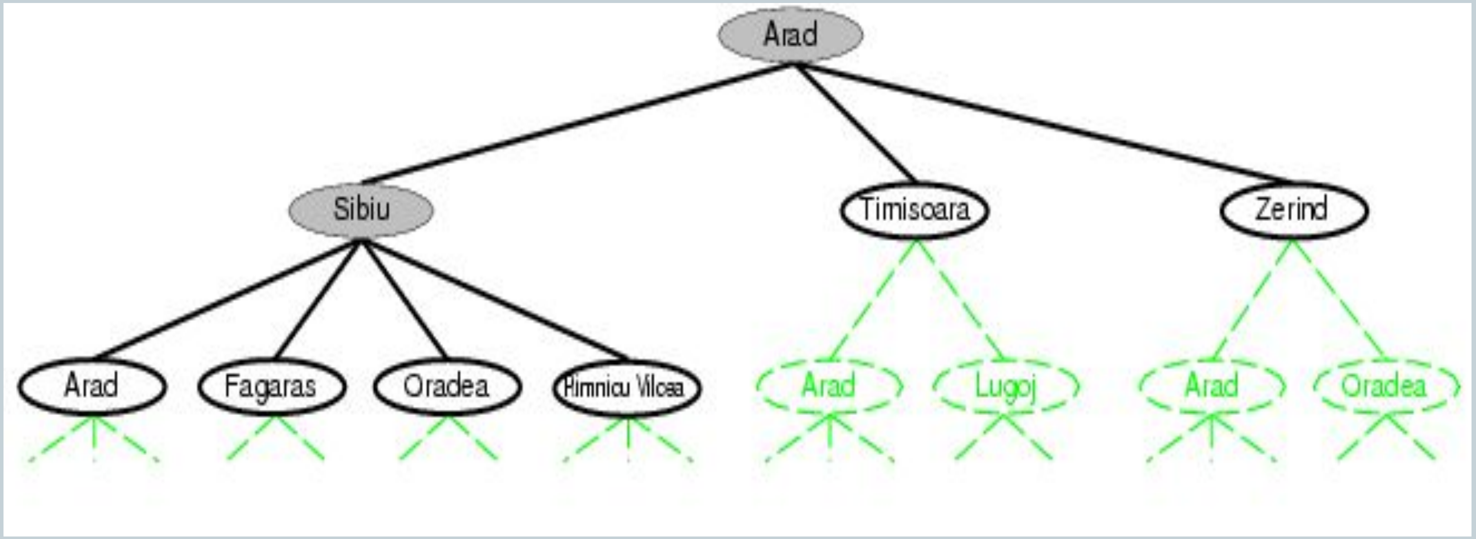


## Tree search example



## Search Graph vs. State Graph

- Be careful to distinguish
  - Search tree: nodes are **sequences of actions.**
  - State Graph: Nodes are states of the environment.
  - We will also consider soon **search graphs**.
- Demo: http://aispace.org/search/

## Search strategies

- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - $b$: maximum branching factor of the search tree
  - $d$: depth of the least-cost solution
  - $m$: maximum depth of the state space (may be $\infty$)

## Search Strategies

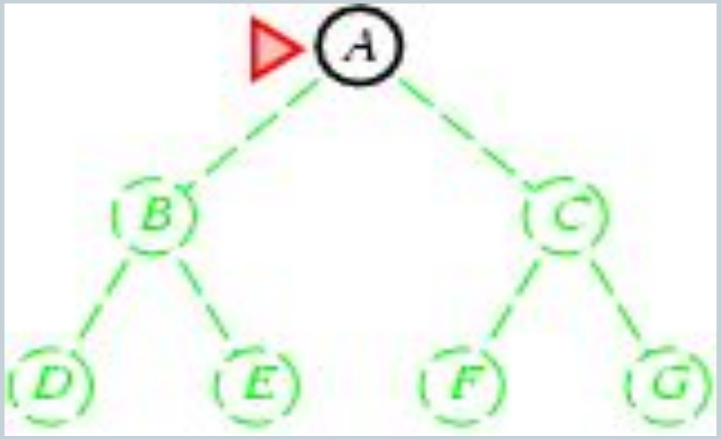
- Uninformed (blind) search
- Informed Search
- Adversarial Search (Game Theory)

## Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
- Uninformed search (blind search)
  - Breadth-first search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search

## Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
    - is a FIFO queue, i.e., new successors go at end



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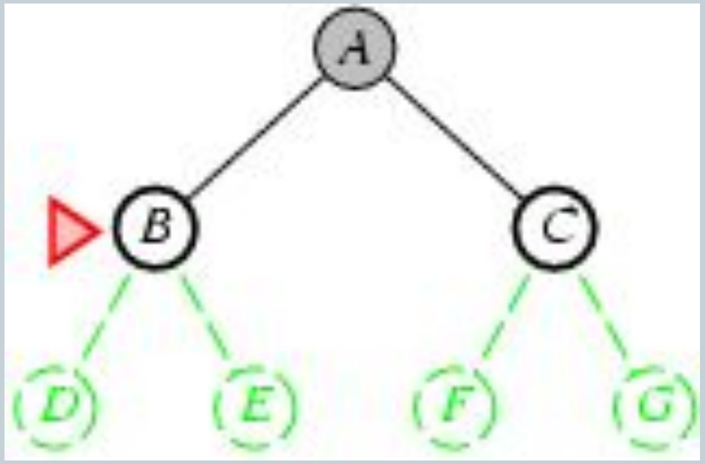

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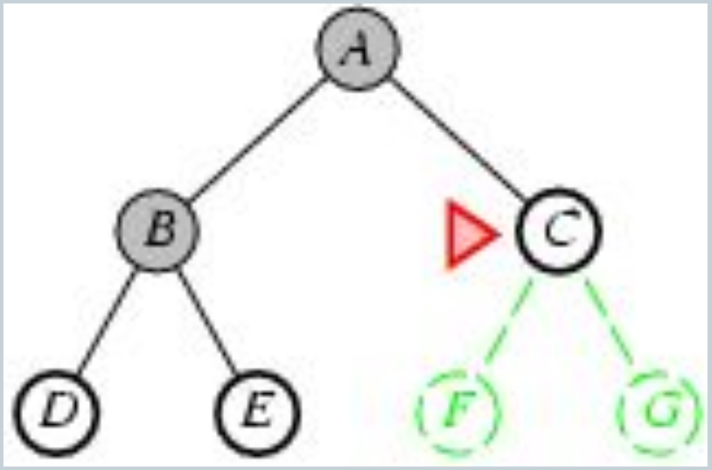

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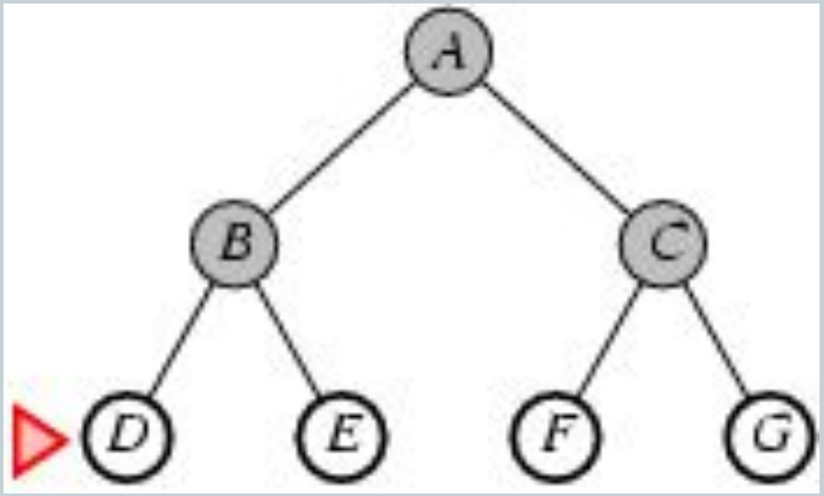
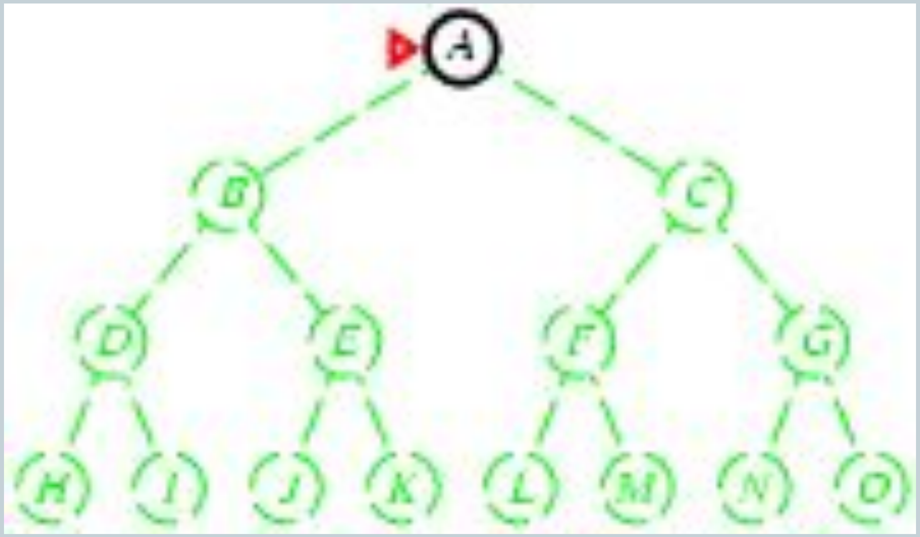


## Properties of breadth-first search

- Complete? Time? Space? Optimal?
- Complete? Yes (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots+b^d + b(b^d-1) = O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)
- Space is the bigger problem (more than time)

# Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - LIFO queue, i.e., put successors at front



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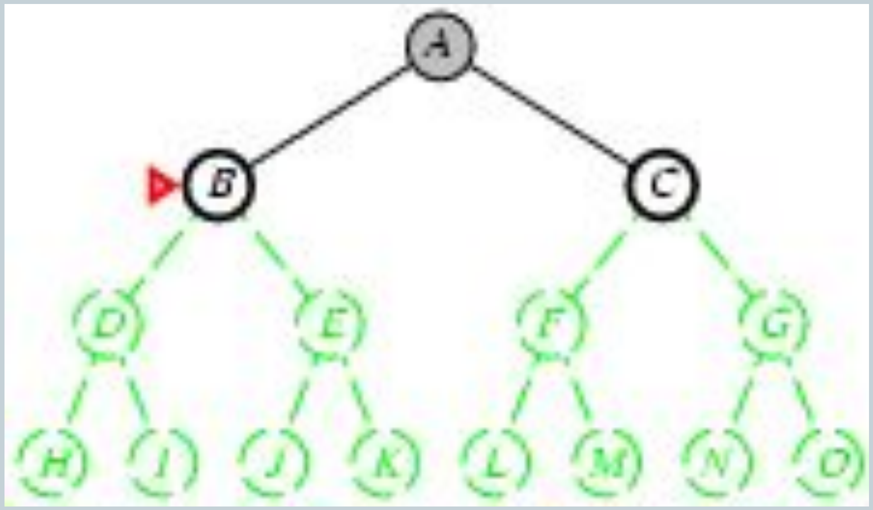

# Depth-first search

- Expand deepest unexpanded node

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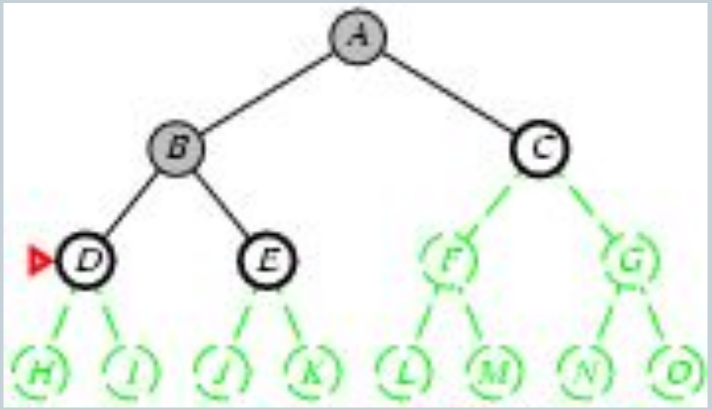
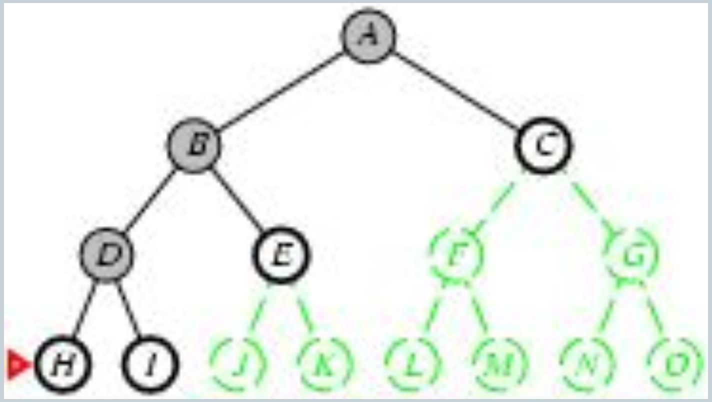


# Depth-first search

- Expand deepest unexpanded node

- Implementation:
  - *frontier* = LIFO queue, i.e., put successors at front



# Depth-first search

- Expand deepest unexpanded node

- Implementation:
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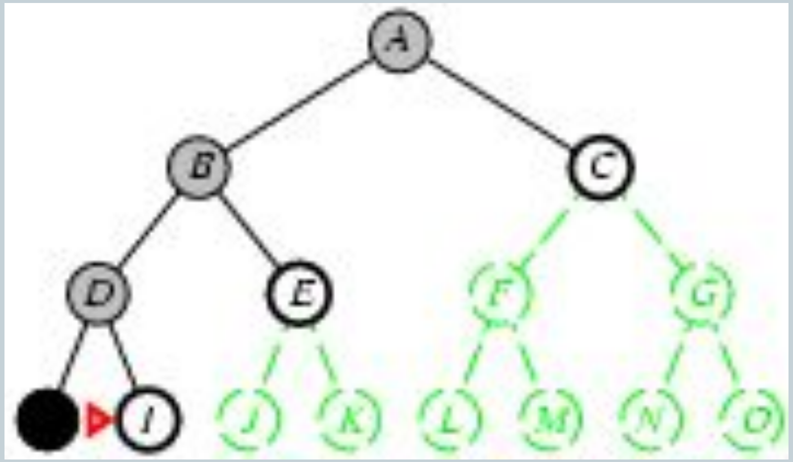

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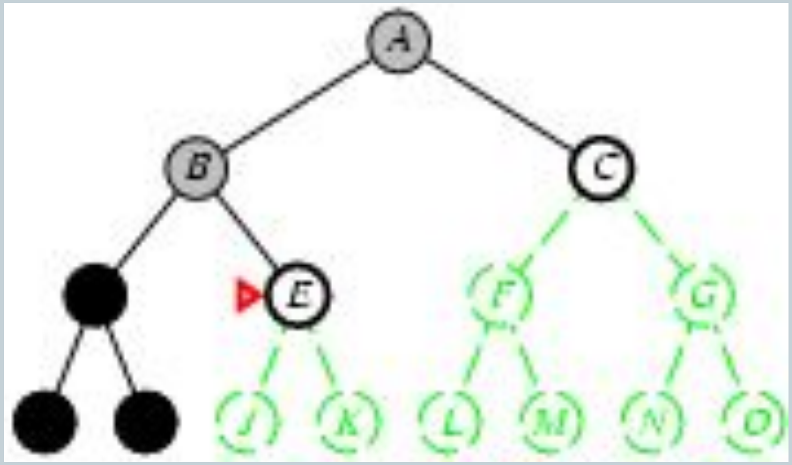

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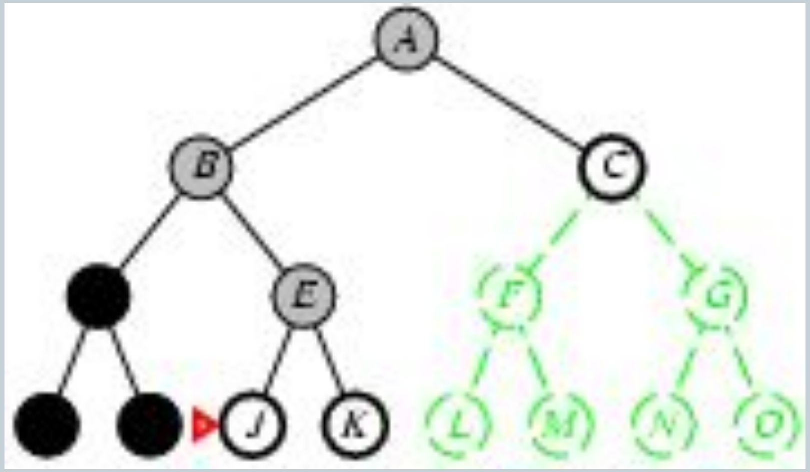

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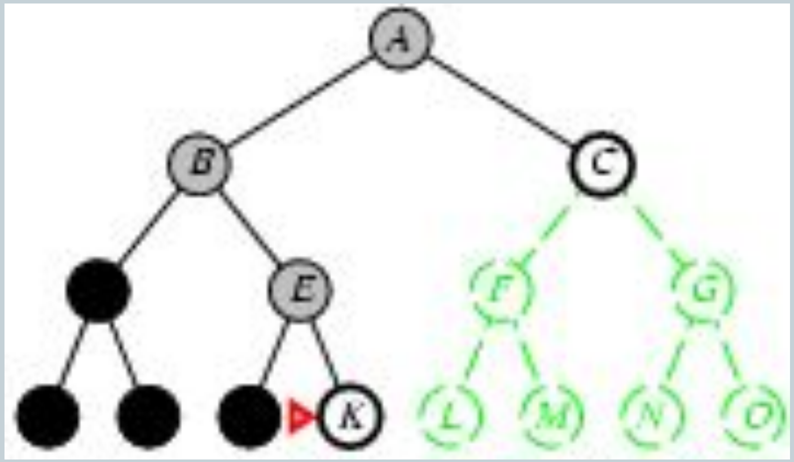

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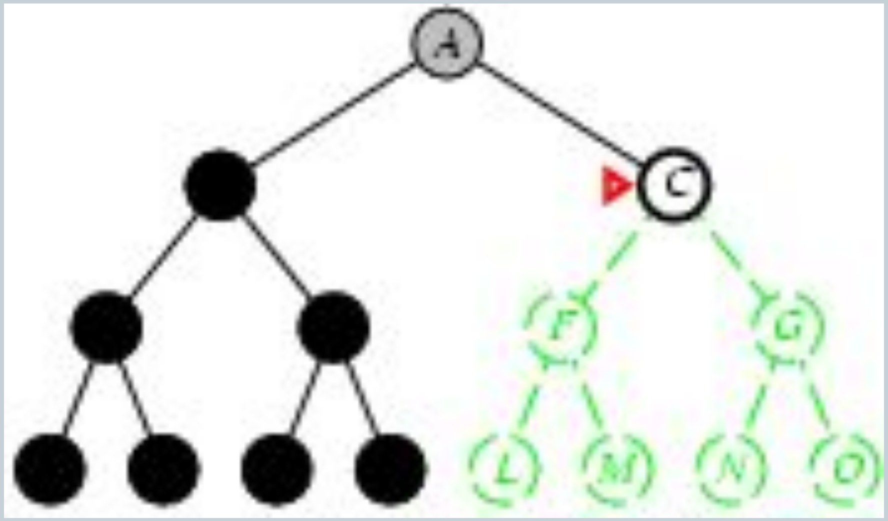

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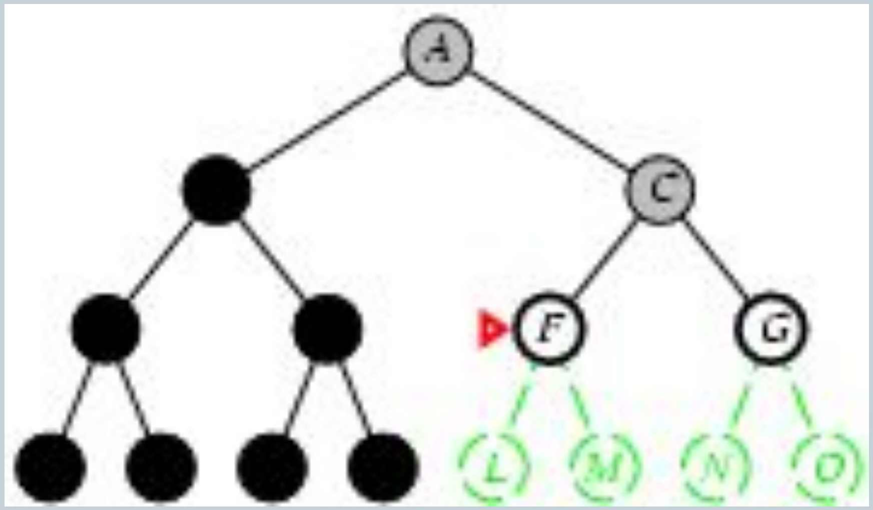

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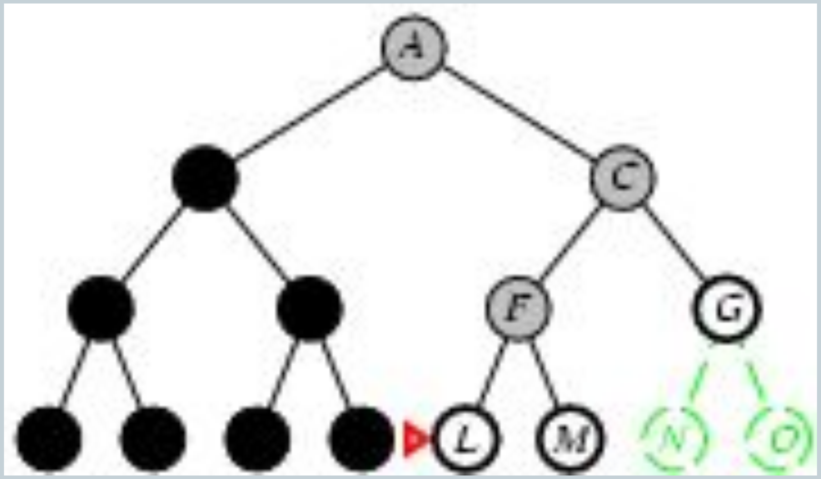

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- Expand deepest unexpanded node [http://aispace.org/search/](http://aispace.org/search/)
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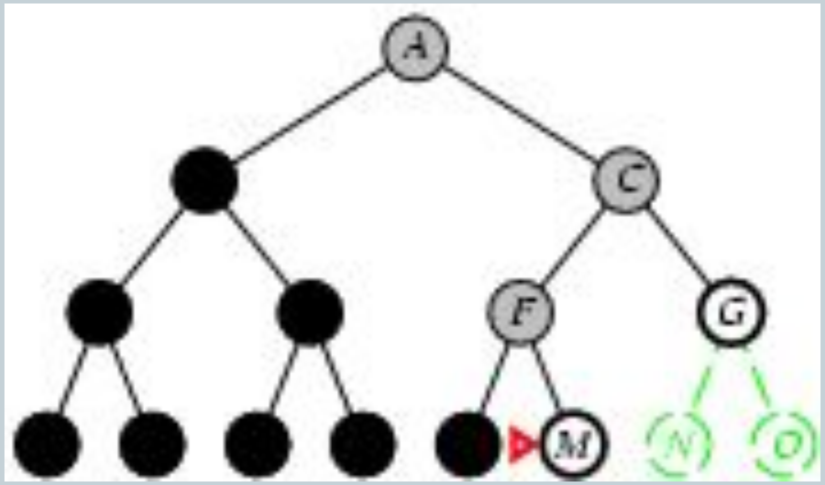


# Properties of depth-first search

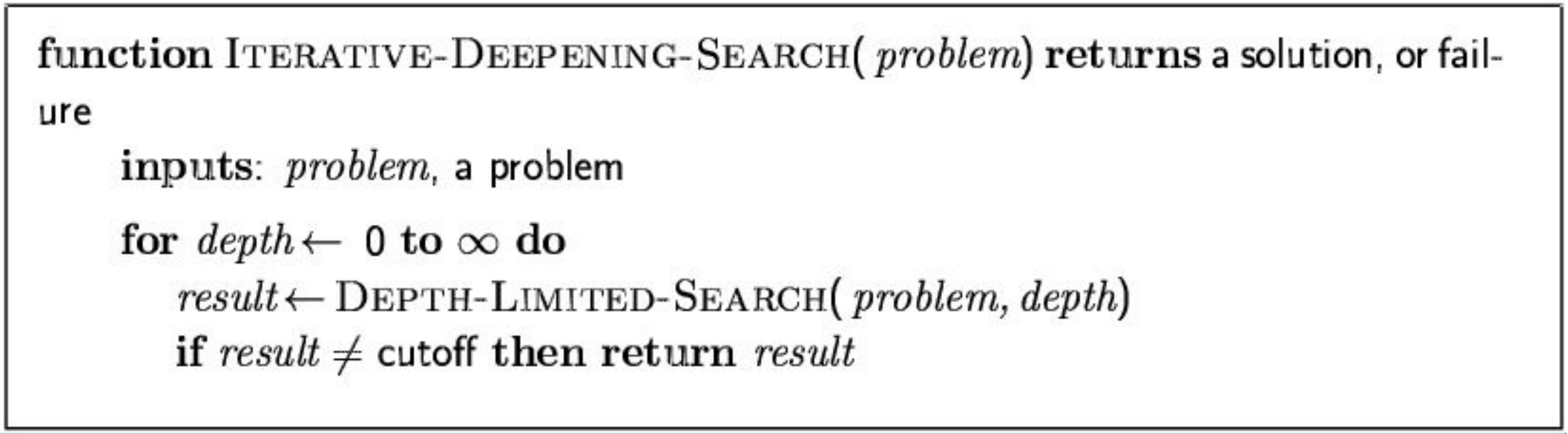
- Complete? Time? Space? Optimal?
- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path (graph search)

    ☐ complete in finite spaces
- Time? $O(b^m)$: terrible if maximum depth $m$ is much larger than solution depth $d$
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space! Store single path with unexpanded siblings.
  - Seems to be common in animals and humans.
- Optimal? No.
- Important for exploration (on-line search).

# Depth-limited search

- depth-first search with depth limit *l*,
  - i.e., nodes at depth *l* have no successors
  - Solves infinite loop problem
- Common AI strategy: let user choose search/resource bound.
  Complete? No if l < d:
- Time? $O(b^l)$:
- Space? $O(bl)$, i.e., linear space!
- Optimal? No if l > b

# Iterative deepening search

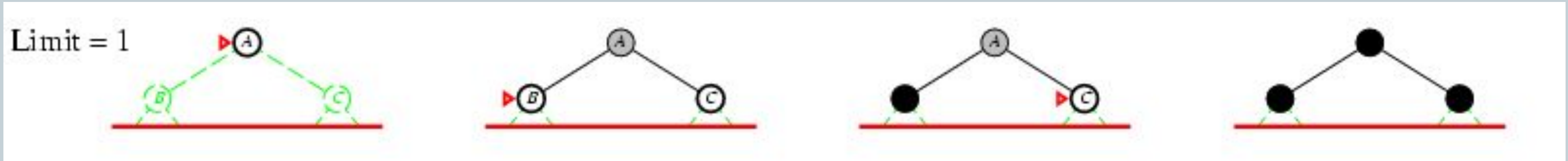


```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or fail-
ure
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

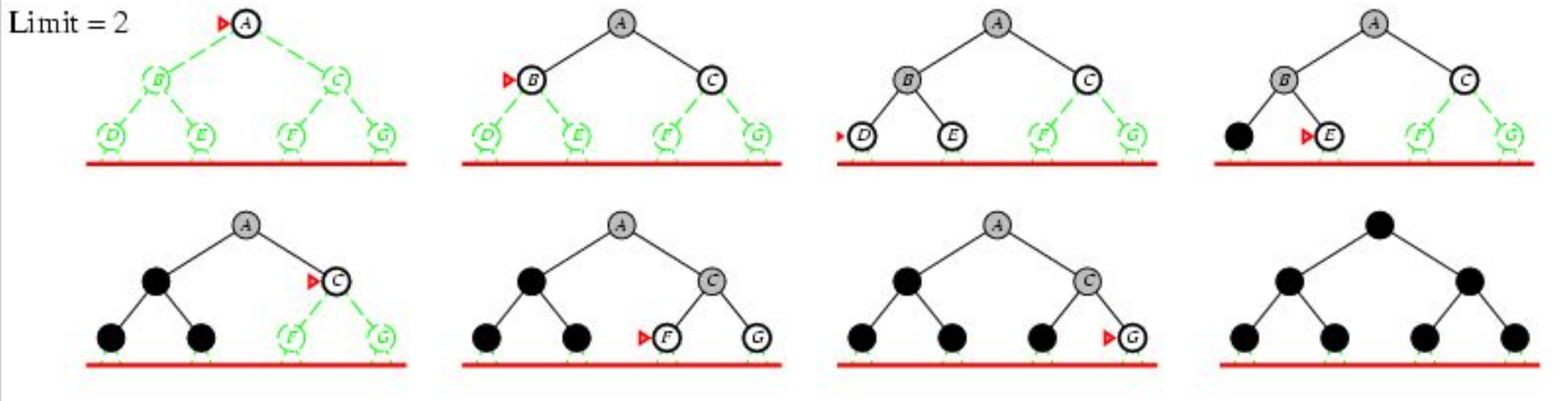
# Iterative deepening search $l = 0$

Limit = 0

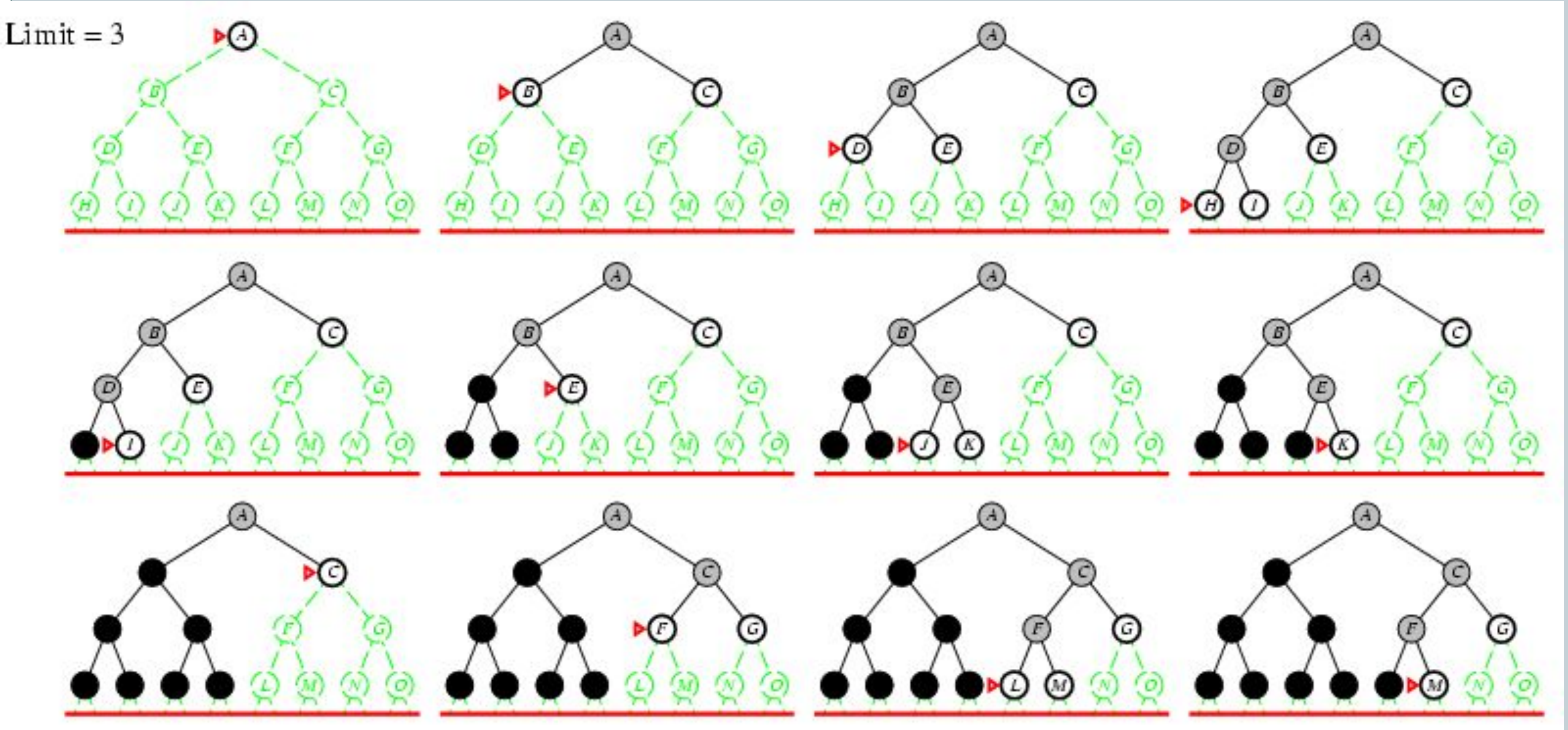
## Iterative deepening search *l* =1



# Iterative deepening search $l$ =2



# Iterative deepening search $l$ =3



# Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \dots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\, b^1 + (d-1)b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10$, $d = 5$,

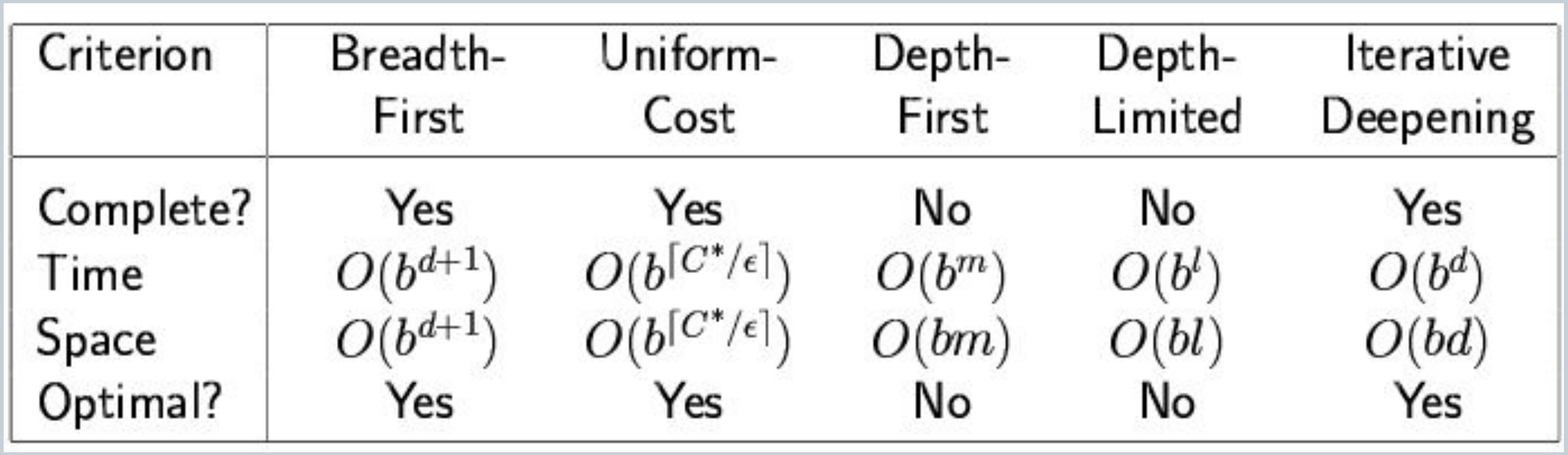
$N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
$N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$

- Overhead = (123,456 - 111,111)/111,111 = 11%

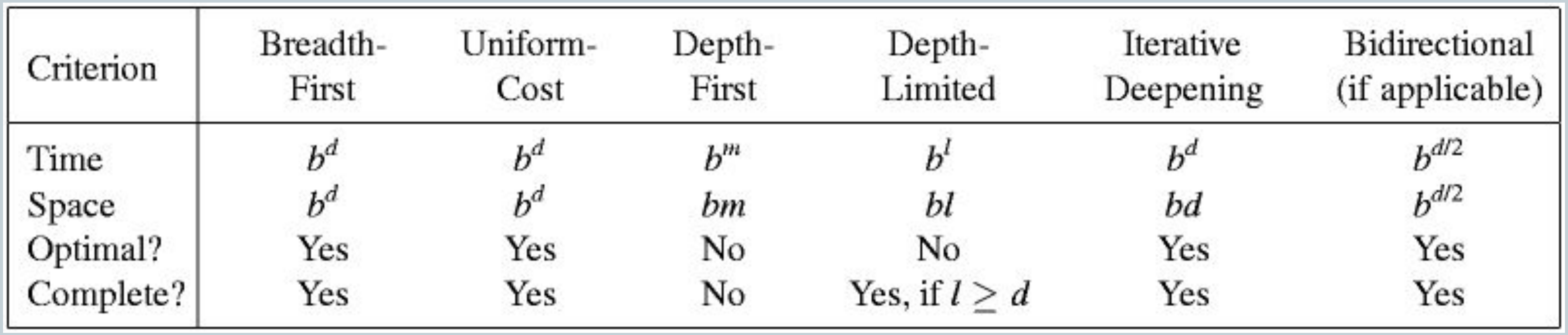
# Properties of iterative deepening search

- Complete? Yes
- Time? $(d+1)b^0 + d\,b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

# Summary of algorithms

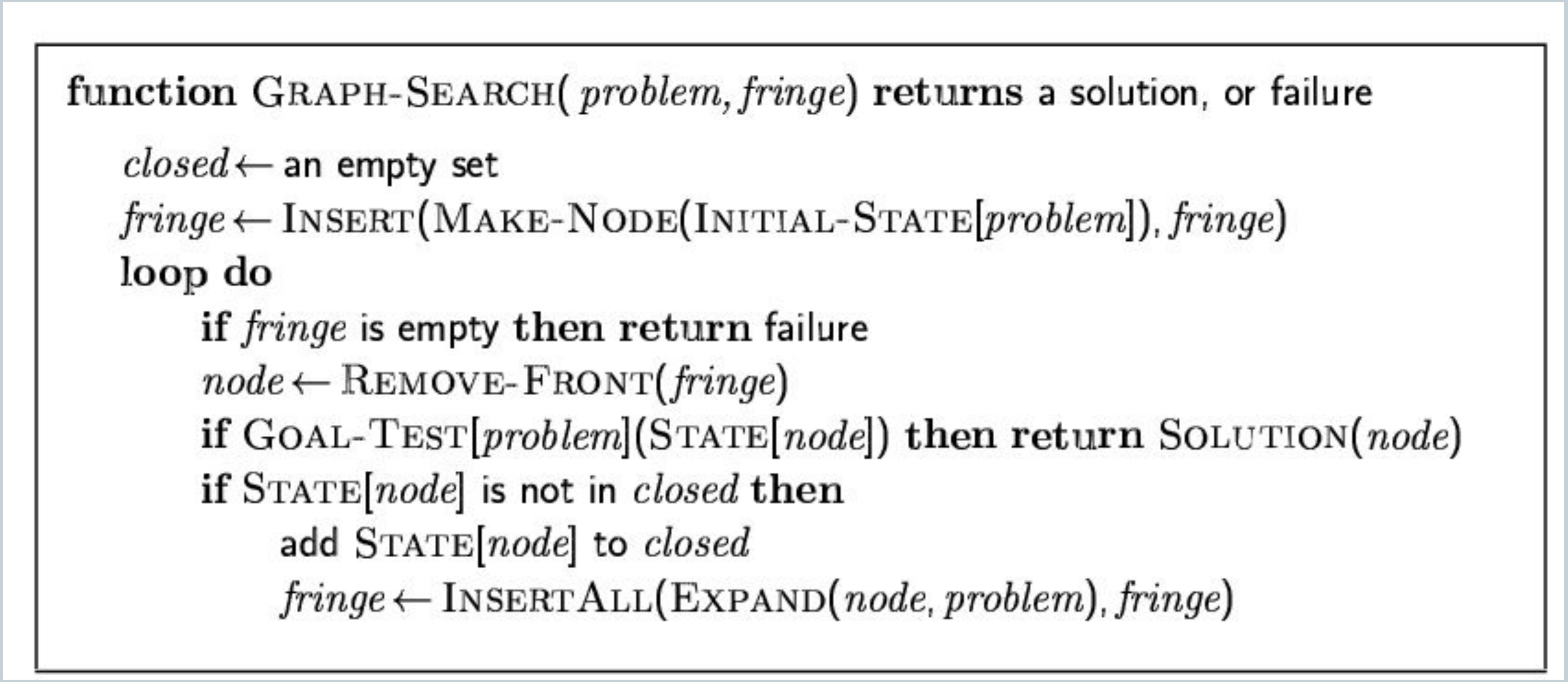


| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening |
|---|---|---|---|---|---|
| Complete? | Yes | Yes | No | No | Yes |
| Time | $O(b^{d+1})$ | $O(b^{\lceil C^*/\epsilon \rceil})$ | $O(b^m)$ | $O(b^l)$ | $O(b^d)$ |
| Space | $O(b^{d+1})$ | $O(b^{\lceil C^*/\epsilon \rceil})$ | $O(bm)$ | $O(bl)$ | $O(bd)$ |
| Optimal? | Yes | Yes | No | No | Yes |



| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening | Bidirectional (if applicable) |
|---|---|---|---|---|---|---|
| Time | $b^d$ | $b^d$ | $b^m$ | $b^l$ | $b^d$ | $b^{d/2}$ |
| Space | $b^d$ | $b^d$ | $bm$ | $bl$ | $bd$ | $b^{d/2}$ |
| Optimal? | Yes | Yes | No | No | Yes | Yes |
| Complete? | Yes | Yes | No | Yes, if $l \geq d$ | Yes | Yes |

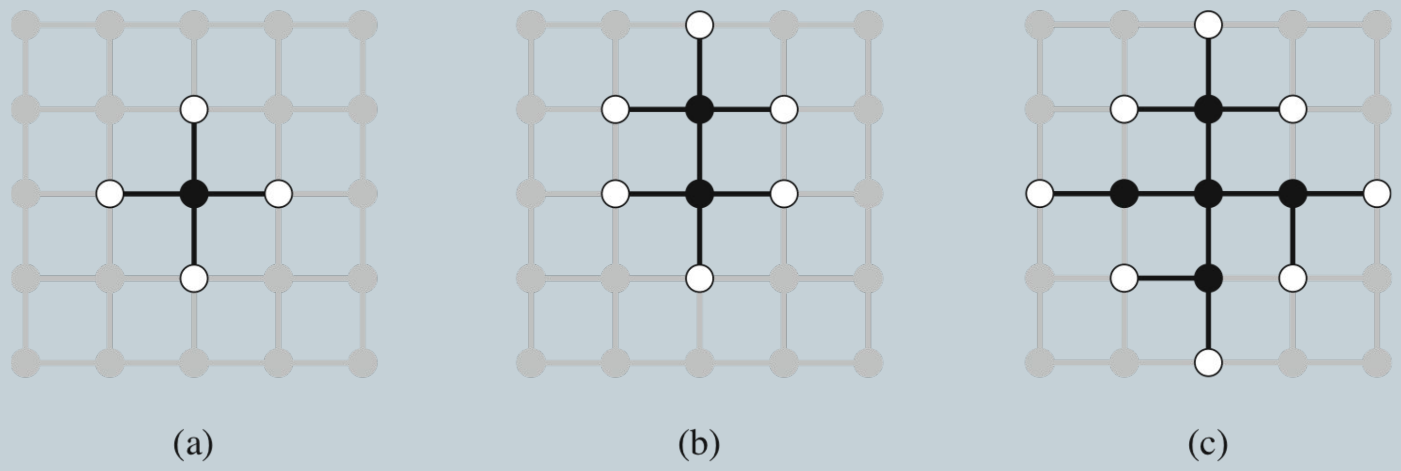
# Graph search



```
function GRAPH-SEARCH(problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

- Simple solution: just keep track of which states you have visited.
- Usually easy to implement in modern computers.

# The Separation Property of Graph Search



- Black: expanded nodes.
- White: frontier nodes.
- Grey: unexplored nodes.

## Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

End of Chapter 3

# Informed search algorithms

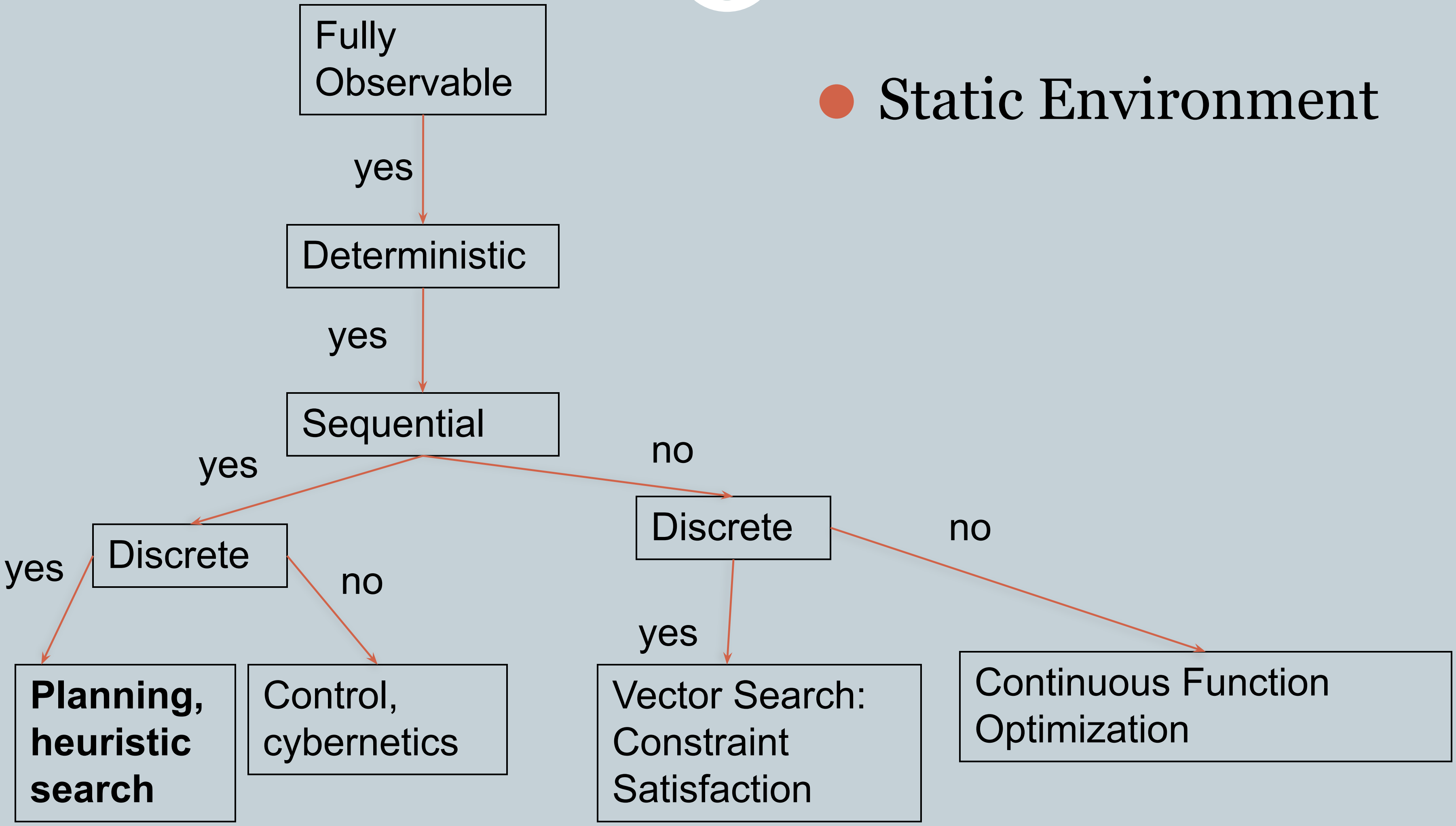
CHAPTER 4

Stuart Russell and Peter Norvig, Artificial Intelligence: A Modern Approach, Global Edition 3/E

# Outline

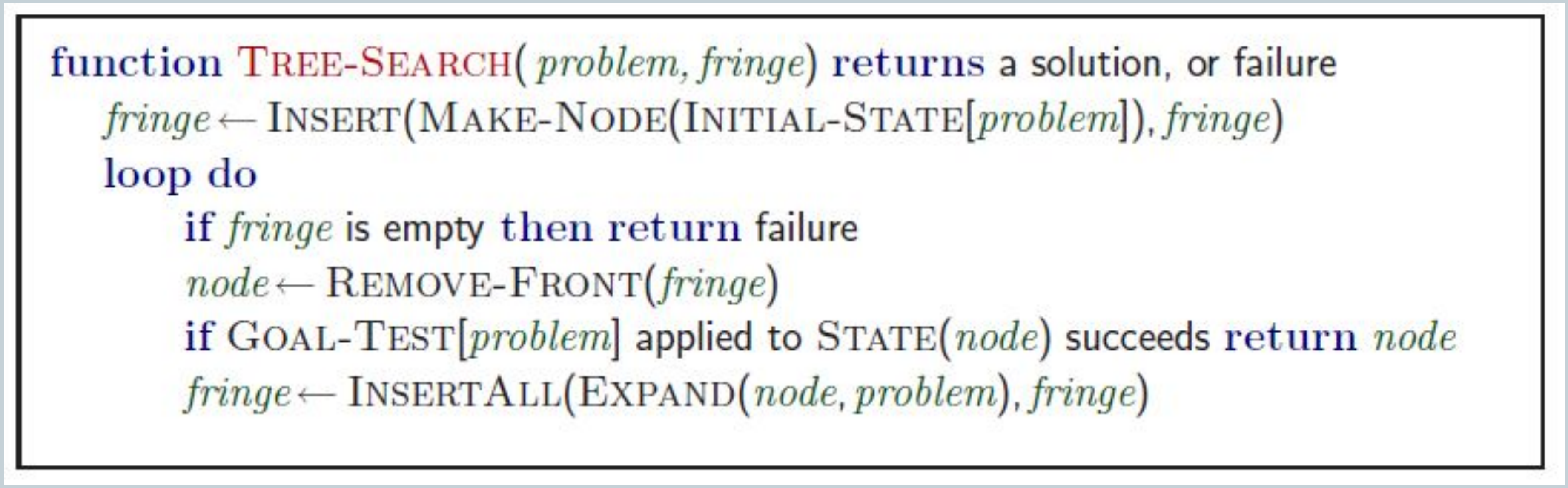
- Best-first search
- $A^*$ search
- Heuristics
- Local search algorithms
- Hill-climbing search
- Simulated annealing search
- Local beam search

# Environment Type Discussed In this Lecture


Fully Observable
yes
Deterministic
yes
Sequential
yes
no
Discrete
yes
no
Discrete
no
yes
Planning, heuristic search
Control, cybernetics
Vector Search: Constraint Satisfaction
Continuous Function Optimization

- Static Environment

## Review: Tree search



```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem] applied to STATE(node) succeeds return node
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

- A search strategy is defined by picking the order of node expansion
- Which nodes to check first?

## Knowledge and Heuristics

- Simon and Newell, *Human Problem Solving*, 1972.
- S&N: intelligence comes from **heuristics** that help find promising states fast.

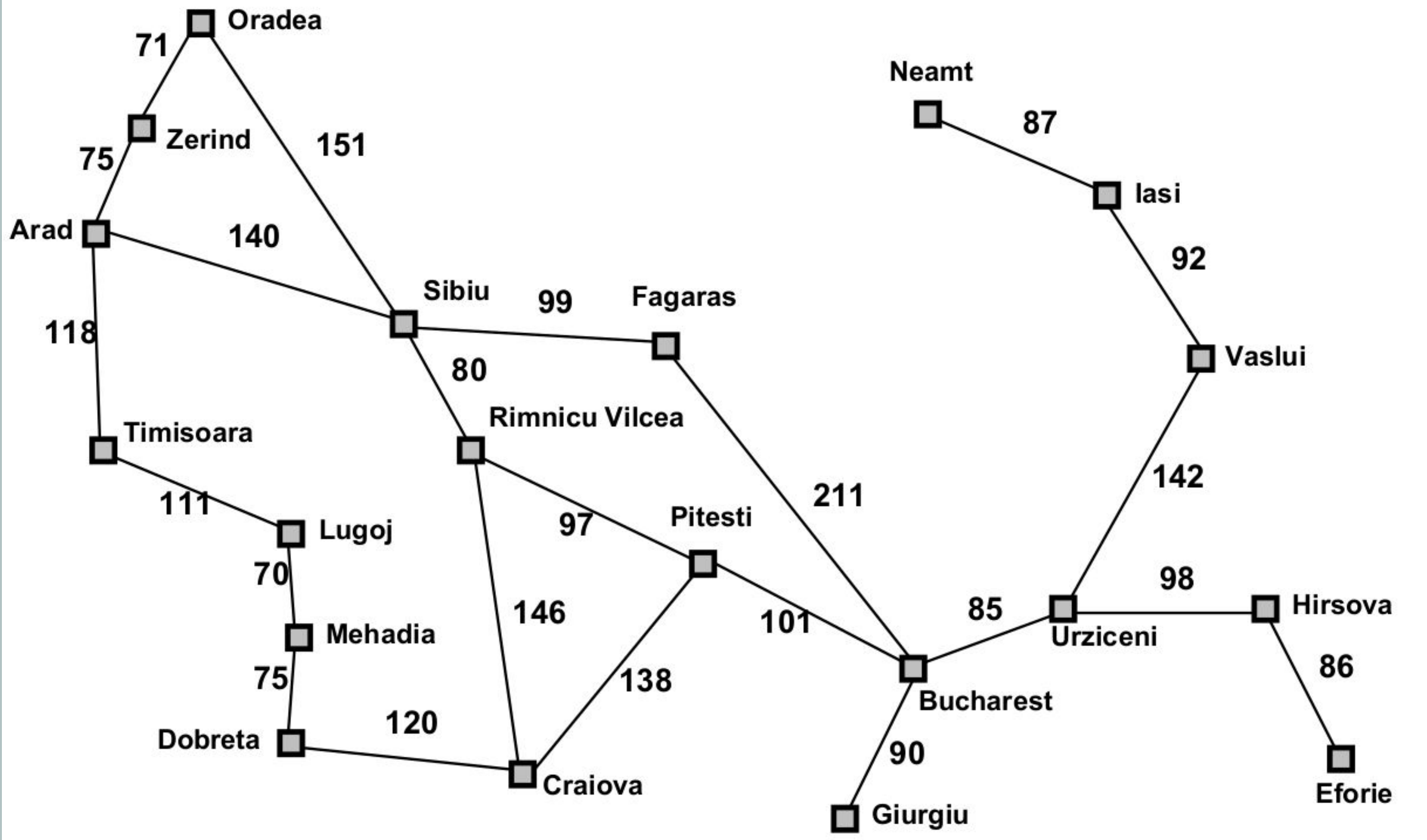
## Best-first search

- Idea: use an evaluation function *f(n)* for each node
  - estimate of "desirability"
  - Expand most desirable unexpanded node

- Implementation:
  Order the nodes in frontier in decreasing order of desirability

- Special cases:
  - greedy best-first search
  - $A^*$ search

# Romania with step costs in km



Straight−line distance to Bucharest

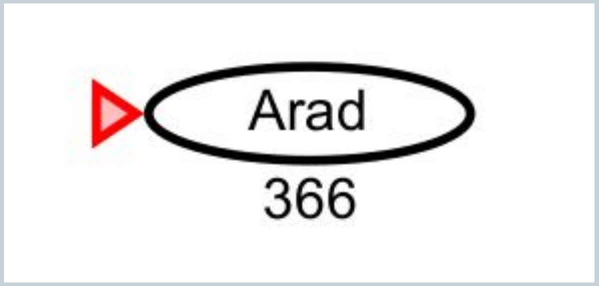
| | |
|---|---|
| **Arad** | 366 |
| **Bucharest** | 0 |
| **Craiova** | 160 |
| **Dobreta** | 242 |
| **Eforie** | 161 |
| **Fagaras** | 178 |
| **Giurgiu** | 77 |
| **Hirsova** | 151 |
| **Iasi** | 226 |
| **Lugoj** | 244 |
| **Mehadia** | 241 |
| **Neamt** | 234 |
| **Oradea** | 380 |
| **Pitesti** | 98 |
| **Rimnicu Vilcea** | 193 |
| **Sibiu** | 253 |
| **Timisoara** | 329 |
| **Urziceni** | 80 |
| **Vaslui** | 199 |
| **Zerind** | 374 |

# Greedy best-first search

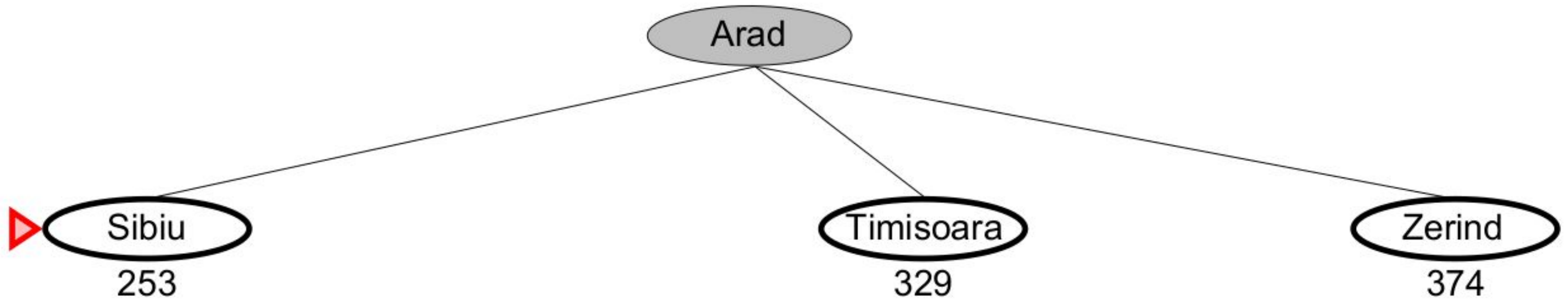
- Evaluation function
  - $f(n) = h(n)$ (heuristic)
  - = estimate of cost from $n$ to *goal*

- e.g., $h_{SLD}(n)$ = straight-line distance from $n$ to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal

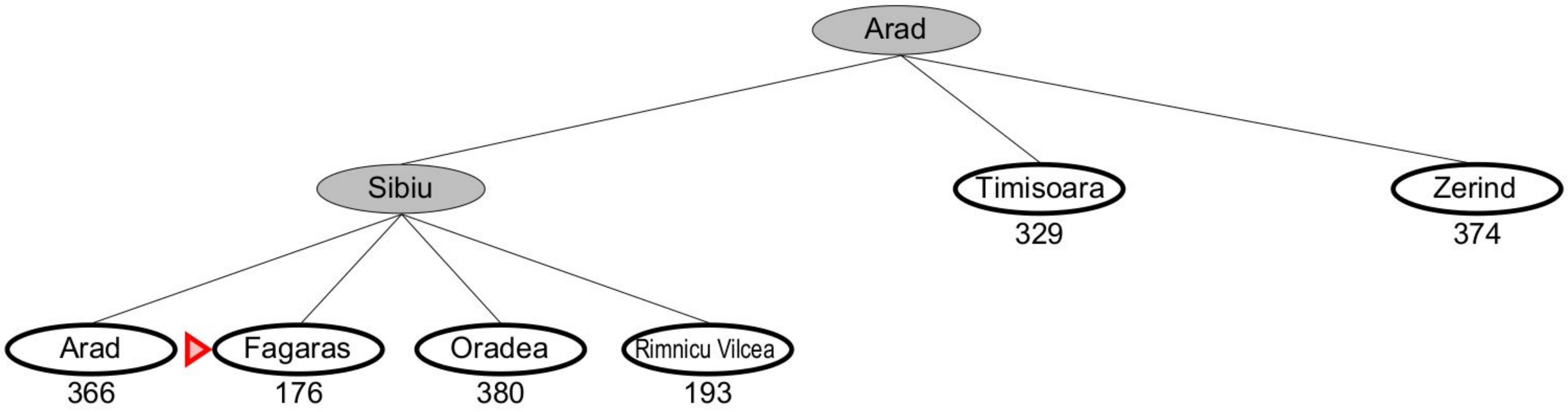
## Greedy best-first search example



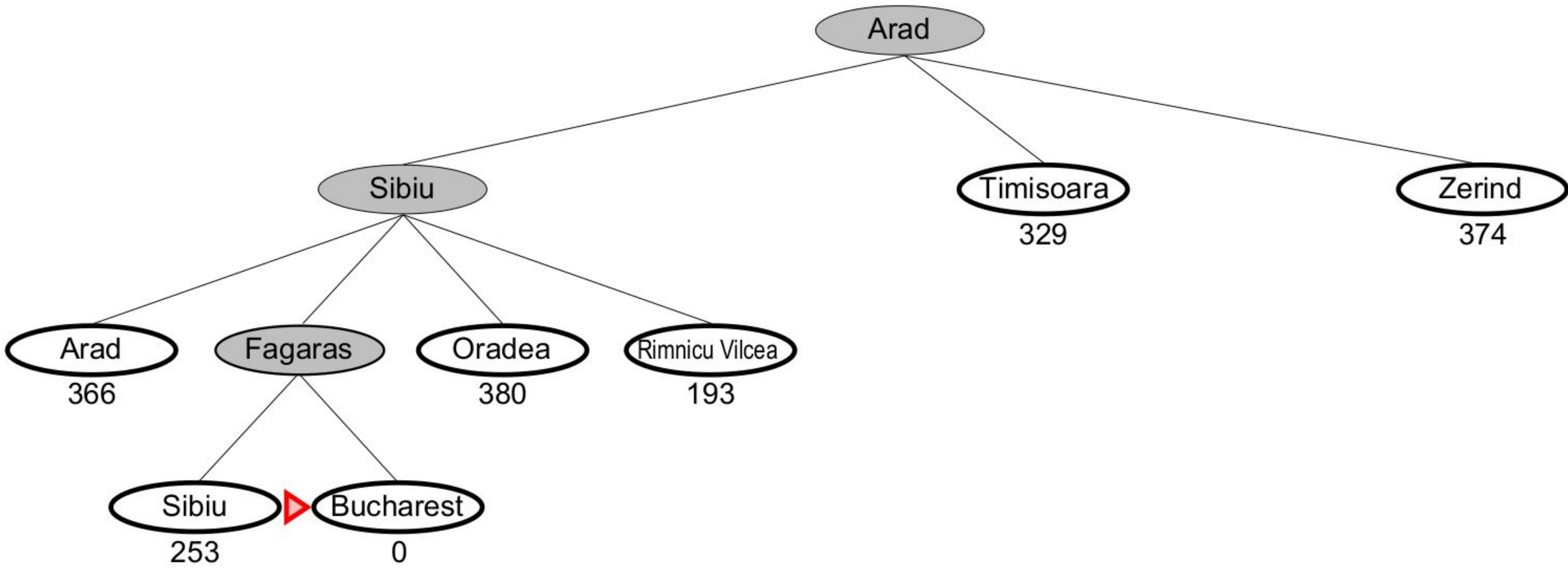
# Greedy best-first search example



## Greedy best-first search example



# Greedy best-first search example



http://aispace.org/search/

# Properties of greedy best-first search

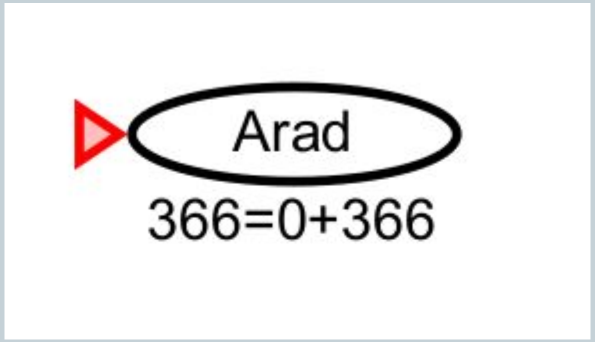
- Complete? No – can get stuck in loops,
  - e.g. as Oradea as goal
    - Iasi ☐ Neamt ☐ Iasi ☐ Neamt ☐
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

## $A^*$ search

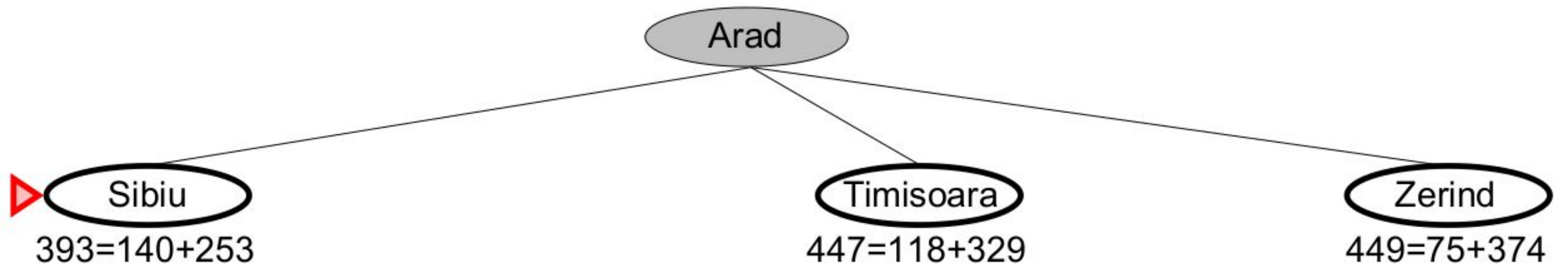
- Idea: avoid expanding paths that are already expensive.
- Very important!

- Evaluation function $f(n) = g(n) + h(n)$
  $g(n)$ = cost so far to reach $n$
- $h(n)$ = estimated cost from $n$ to goal
- **$f(n)$ = estimated total cost of path through $n$ to goal**

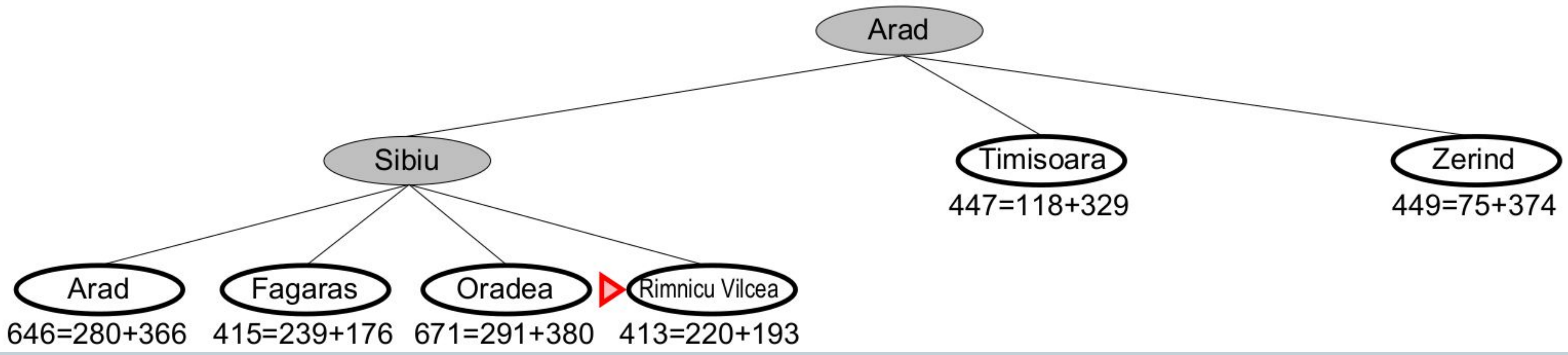
## $A^*$ search example



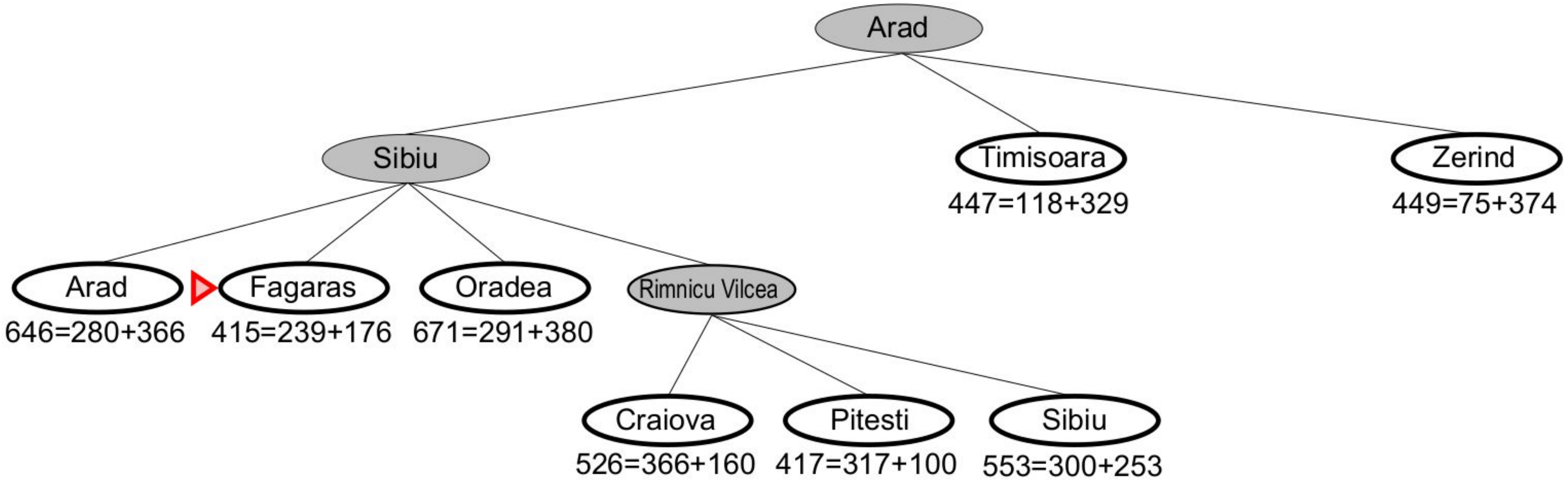
## A$^*$ search example



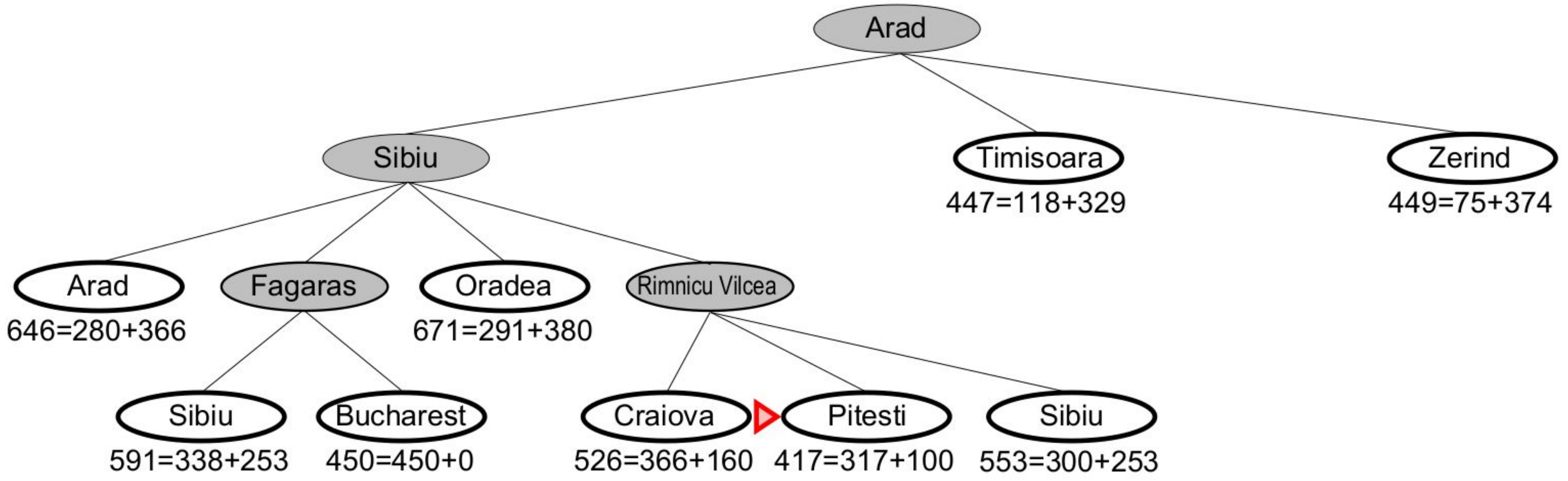
## A* search example



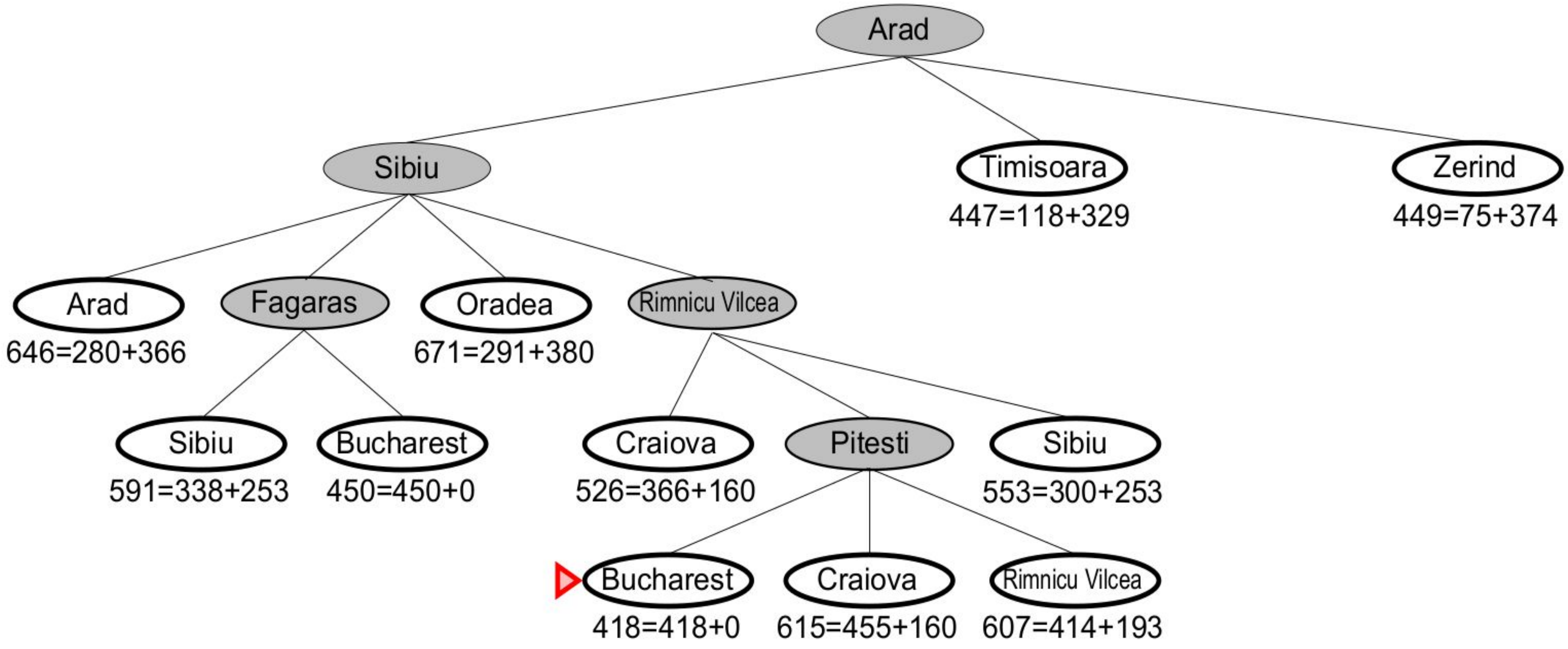
## A$^*$ search example



## A* search example



# A* search example



http://aispace.org/search/

- We stop when the node with the lowest f-value is a goal state.
- Is this guaranteed to find the shortest path?

## Properties of A*

- Complete? Yes (unless there are infinitely many nodes with f ≤ $f(G)$ )
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

## Summary

- Heuristic functions estimate costs of shortest paths
- Good heuristics can dramatically reduce search cost
- Greedy best-first search expands lowest h
  - incomplete and not always optimal
- A* search expands lowest g + h
  - complete and optimal
  - also optimally efficient (up to tie-breaks)