THE HONG KONG POLYTECHNIC UNIVERSITY

Department of Mechanical Engineering

Programme	:	BEng(Hons) in Mechanical Engineering (43460)			
Subject Title	:	Numerical Methods for Engineers	**Subject Code**	:	ME46002
Session	:	Semester 1, 2018/2019			
Date	:	Dec. 07, 2018	**Time**	:	19:00-22:00
Time Allowed	:	3 hours	**Subject Examiner(s)**	:	Dr. G. P. Zheng

This question paper has 5 pages (attachments included).

Instructions to Candidates :
This paper has 4 questions.
Answer all 4 questions.
All questions carry equal marks.

Constants : **NIL**

Others : **NIL**

Available from Invigilator : **NIL**

DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO

Question 1

(a) Determine the root of the following equation

$$f(x) = 2 + \frac{\cos(x)}{x} - \frac{x}{\sin(x)} = 0$$

using bisection method with two initial guesses of a=1 and b=3. Perform the computation until the percentage relative error is less than 2%. x is in unit of radian.

(7 marks)

(b) Employ the Newton-Raphson method to determine the root for

$$f(x) = x^3 + \cos(\frac{x}{2}) \cdot e^{-x} - 1.5 = 0$$

using an initial guess of x_0= 0.55. Perform the computation until the percentage relative error is less than 0.1%. x is in unit of radian.

(8 marks)

(c) Find all the eigenvalues of a matrix

$$A = \begin{bmatrix} -0.25 & -2.25 & 0.25 \\ 1.75 & 4.75 & 0.25 \\ -1 & 1 & 3 \end{bmatrix}.$$

(10 marks)

Question 2

(a) Solve the simultaneous linear equations $A\,\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 2 & 6 & 1 \\ 7 & 1 & 4 \\ 8 & 5 & 2 \end{bmatrix} \quad \textit{and} \quad b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix},$$

using the Gaussian elimination *with pivoting*.

(7 marks)

(b) Solve the following simultaneous equations using LU decomposition

$$\begin{bmatrix} 6 & 1 & 3 \\ 5 & 8 & 2 \\ 1 & 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}.$$

(7 marks)

(c) Solve the following equations using the Gauss-Seidel method until the percent relative error falls below $\varepsilon_s = 1\%$.

$$6x_1 + 9x_2 + x_3 = 40$$
$$6x_1 - x_2 - x_3 = 3$$
$$-3x_1 + x_2 + 12x_3 = 50$$

(7 marks)

(d) Find all the eigenvalues of a matrix

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 1 & 0.8 & 0 & 0 \\ 6 & 2.6 & 0 & 0 \\ 4.8 & 3 & 2 & 2.4 \end{bmatrix}.$$

Describe the procedure of finding the eigenvalues in details.

(4 marks)

Question 3

(a) For an nth-order Newton's divided difference interpolating polynomial $f_n(x)$, the error of interpolation can be estimated by $R_n = \left| g[x_{n+1}, x_n, x_{n-1},..., x_0] \cdot (x - x_0)(x - x_1) \cdots (x - x_n) \right|$, where $(x_0, f(x_0))$, $(x_1, f(x_1))$,..., $(x_{n+1}, f(x_{n+1}))$ are data points; $g[x_{n+1}, x_n, x_{n-1},..., x_0]$ is the (n+1)-th finite divided difference. To minimize R_n, if there are more than n+1 data points available for calculating $f_n(x)$ using the nth-order Newton's interpolating polynomial, n+1 data points $(x_0, f(x_0))$, $(x_1, f(x_1))$,..., $(x_n, f(x_n))$ should be chosen from the available data points such that $\left| (x - x_0)(x - x_1) \cdots (x - x_n) \right|$ is the smallest. Given the data

x_i	3.2	5.6	6.2	7.3	8.0	8.5
$f(x_i)$	2.4	7.8	12.5	16.6	12.5	6.5

Calculate $f_n(7.0)$ using Newton's divided difference interpolating polynomials with n = 3. Estimate the error R_3 of $f_3(7.0)$.

(7 marks)

(b) The relation between the moving distance h of an object and the time t is $h = a(\sqrt{t} - 0.4)^b$, where a and b are coefficients. It is measured that h=2.0 m, 4.2 m, 6.4 m, 8.3 m, 10.0 m at t=0.2 s, 0.4 s, 0.6 s, 0.8 s and 1.0 s, respectively. Describe how to fit the data using least-squares regression to find the coefficients a and b. Find the values of a and b.

(8 marks)

(c) A set of three data points (x_i, y_i), i = 0, 1, 2, with x_i = -2, 0, 2 and y_i = -8, 0, 8, are fitted with cubic splines $f(x) = a + bx + cx^2 + dx^3$.

(i) Demonstrate the analytical procedure to obtain the coefficients of the cubic splines by showing that the number of unknown coefficients matches with the number of equations that you would establish. Show the procedures of establishing the equations.

(4 marks)

(ii) If the data are fitted with cubic splines with the constrains at the end points, as follows: $f'(2) = 6$, $f'(-2) = 6$, determine the cubic functions.

(6 marks)

Question 4

(a) Based on the Taylor's series

$$f(x+a)=\sum_{n=0}^{\infty}\frac{x^n}{n!}f^{(n)}(a),$$

where $f^{(n)}(a)$ is the n-th derivative of function $f(x)$ at $x=a$, $0!=1$ and $n!=1\times2\times\ldots\times n$, derive the finite difference formula with the second order accuracy to calculate the first derivative at $x=x_0$.

(6 marks)

(b) The temperatures of a thin rod are tabulated below. The temperature $T(x,t)$ is in unit of °C; the positions x_1 to x_6 are in unit of cm; time t is in unit of second.

	$x_1=0$	$x_2=0.2$	$x_3=0.4$	$x_4=0.6$	$x_5=0.8$	$x_6=1.0$
t=0	0.0	16	24	24	16	0.0
t=0.01	0.0	12	20	20	12	0.0
t=0.02	0.0	10	16	16	10	0.0

Verify by calculating $\partial T/\partial t$ using the forward finite difference and $\partial^2 T/\partial x^2$ using the central finite difference that the temperatures $T(x,t)$ listed in the table are solutions to the heat conduction equation $\frac{\partial T}{\partial t}=2\frac{\partial^2 T}{\partial x^2}$ at x_2 to x_5 and at time t=0.01 s. The partial differentiation is defined as

$$\frac{\partial T}{\partial t}=\frac{dT(x,t)}{dt}\bigg|_{x\ \ fixed},$$

$$\frac{\partial^2 T}{\partial x^2}=\frac{d^2T(x,t)}{dx^2}\bigg|_{t\ \ fixed}.$$

Given: the central finite difference for second derivatives

$$f''(x_0)=\frac{f(x_0-h)-2f(x_0)+f(x_0+h)}{h^2}.$$

(10 marks)

(c) Assume the integration

$$I=\int_{x_0-h}^{x_0+h}f(x)dx$$

is computed using Simpson's 1/3 rule, as follows:

$$I=\int_{x_0-h}^{x_0+h}f(x)dx=\frac{h}{3}[f(x_0-h)+4f(x_0)+f(x_0+h)].$$

Show that the numerical integration is of the 5-th order accuracy, i.e., the error of the numerical integration is proportional to h^5.

(9 marks)

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