ME46002 Chapter 5: Tutorial questions

Question 1:

For a function $f(x) = \ln(1+x)\cdot\sin(x)$ at x_0=0.1, use forward finite difference, backward finite difference and center finite difference to compute the first derivative $f'(x)$ using an interval of h=0.02, respectively.

Solution:

$$Forward \quad F.D.: \qquad f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$$

$$Backward \quad F.D.: \qquad f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h}$$

$$Center \quad F.D.: \qquad f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

Thus at x_0=0.1 and h=0.02

$f'(x)$ =0.202584, 0.168241, 0.185425 respectively.

Question 2:

(i) Use central difference approximations to estimate the 1st derivative of y=e^x at x=2 for h=0.1.

(ii) Employ the following formulas to estimate the 1st derivative of y=f(x)=e^x at x=2 for h=0.1.

$$f'(x) = \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)}{12h}$$

(iii) Compare the error of numerical result in (i) and (ii)

(iv) Use central difference approximations to estimate the 2nd derivative of y=e^x at x=2 for h=0.1.

Solution:

(i-iii)

	x	$f(x)=e^x$
x_{i-2}	1.8	6.049647464
x_{i-1}	1.9	6.685894442
x_i	2	7.389056099
x_{i+1}	2.1	8.166169913
x_{i+2}	2.2	9.025013499

Both the first and second derivatives have the same value,

$$\text{exact} = e^2 = 7.389056099$$

The results are summarized as

	(i)	(ii)
First derivative	7.401377351	7.389031439
Relative error	-0.166750%	0.000334%

(iv)

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

	x	*f*(*x*)=e^x
x_{i-1}	1.9	6.685894442
x_i	2	7.389056099
x_{i+1}	2.1	8.166169913

Second derivative	7.395215699
Relative error	-0.083361%

Question 3:

Taking h=0.1, compare the results of integration by using trapezoidal rules and Simpson's 1/3 rule and the exact value of the following integral.

$$I = \int_0^1 \frac{1}{1+x}\,dx$$

Solution:

Exact Integral

$$I = \int_0^1 \frac{1}{1+x}\,dx = \ln(1+x)\Big|_0^1 = \ln(2) - \ln(1) = 0.6931471806$$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f(x)	1	1/1.1	1/1.2	1/1.3	1/1.4	1/1.5	1/1.6	1/1.7	1/1.8	1/1.9	1/2

By trapezoidal rule with h=1/10

$$I \cong \frac{h}{2}\left(f_0 + 2(f_{0.1} + f_{0.2} + f_{0.3} + f_{0.4} + f_{0.5} + f_{0.6} + f_{0.7} + f_{0.8} + f_{0.9}) + f_1\right)$$

$$= \left(\frac{0.1}{2}\right)\left(1 + 2\left(\frac{1}{1.1} + \frac{1}{1.2} + \frac{1}{1.3} + \frac{1}{1.4} + \frac{1}{1.5} + \frac{1}{1.6} + \frac{1}{1.7} + \frac{1}{1.8} + \frac{1}{1.9}\right) + \frac{1}{2}\right) = 0.6937714$$

By Simpson’s 1/3 rule with h=1/10

$$I = \int_0^1 \frac{1}{1+x}dx \cong \frac{h}{3}\left(f_0 + 4f_{0.1} + 2f_{0.2} + 4f_{0.3} + 2f_{0.4} + 4f_{0.5} + 2f_{0.6} + 4f_{0.7} + 2f_{0.8} + 4f_{0.9} + f_1\right)$$

$$= \frac{0.1}{3}\left(1 + \frac{4}{1.1} + \frac{2}{1.2} + \frac{4}{1.3} + \frac{2}{1.4} + \frac{4}{1.5} + \frac{2}{1.6} + \frac{4}{1.7} + \frac{2}{1.8} + \frac{4}{1.9} + 0.5\right) = 0.69315023$$