

Q2) $f(x) = \cos(x)\ln(x) + \frac{e^x}{5} = 0 \rightarrow f'(x) = -\sin(x)\ln(x) + \frac{\cos(x)}{x} + \frac{e^x}{5}$

ITERATION	x_i	$f(x)$	$f'(x)$	x	ERROR (%)
1	2	1.18936021	7.57447427	0.14005728	1327.98166
2	0.14005728	-1.7163835	7.57447427	0.366659	61.801597
3	0.366659	-0.64180518	3.19431715	0.5695353	35.6214032
4	0.5695353	-0.120597	2.13571084	0.625998	9.01956935
5	0.62599759	-0.00556651	1.913006	0.6289625	0.455568

$\hookrightarrow \text{ROOT} = 0.628962485$
 $= 0.629 \text{ (3. s.f.)}$

Q1) $f(x) = \sin(x) + \frac{(1-x)e^x}{\cos(x) + 1.2} = 0$

ITERATION	a	b	$a(\text{root})$	$f(a)$	$f(b)$	$f(a)f(b)$	ERROR (%)
1	0	2	1	0.451545	0.941470	0.39240681	100
2	1	2	1.5	0.94147099	-0.765929	-0.04415041	33.33
3	1	1.5	1.25	0.841147099	-0.3731420	0.313999976	20
4	1.25	1.5	1.375	0.37314296	-0.082644	-0.03083919	9.0404
5	1.25	1.375	1.3125	0.37314296	0.16907239	0.06308217	4.762

$\hookrightarrow \text{ROOT} = 1.3125$

Q3) $A = \begin{bmatrix} 5 & 1 & 2 \\ 4 & 1 & 6 \\ 1 & 8 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$

$\hookrightarrow -\frac{4}{5}R_1 + R_2 \rightarrow R_2$ $\begin{bmatrix} 5 & 1 & 2 \\ 0 & 0.2 & 4.4 \\ 1 & 8 & 2 \end{bmatrix} \xrightarrow{-\frac{1}{5}R_1 + R_3 \rightarrow R_3} \begin{bmatrix} 5 & 1 & 2 \\ 0 & 0.2 & 4.4 \\ 0 & 7.9 & 1.6 \end{bmatrix} \xrightarrow{\text{SWITCH } R_2/R_3} \begin{bmatrix} 5 & 1 & 2 \\ 0 & 7.9 & 1.6 \\ 0 & 0.2 & 4.4 \end{bmatrix}$

$\hookrightarrow -\frac{1}{39}R_2 + R_3 \rightarrow R_3$ $\begin{bmatrix} 5 & 1 & 2 \\ 0 & 7.9 & 1.6 \\ 0 & 0 & 4.3589 \end{bmatrix}$

\downarrow
 $*B = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}$

$\hookrightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 0.8 & 1 & 0 \\ 0.2 & 0.256 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 5 & 1 & 2 \\ 0 & 7.9 & 1.6 \\ 0 & 0 & 4.3589 \end{bmatrix}$

$\hookrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 0.2 & 0.0256 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}$

$\boxed{d_1 = 3}$

$0.5(3) + d_2 = 2$

$\hookrightarrow \boxed{d_2 = -0.4}$

$(0.2)3 + 0.0256(-0.4) + d_3 = 6$
 $\boxed{d_3 = 5.41024}$

$\begin{bmatrix} 5 & 1 & 2 \\ 0 & 7.9 & 1.6 \\ 0 & 0 & 4.3589 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -0.4 \\ 5.41024 \end{bmatrix}$

$4.3589x_3 = 5.41024$

$\boxed{x_3 = 1.241}$

$7.9x_2 + 1.6(1.241) = -0.4$

$\boxed{x_2 = -2.3841}$

$5x_1 + 1(-0.3056) + 2(1.241) = 3$

$\boxed{x_1 = 0.16512}$

$\begin{bmatrix} x_1 = 0.16512 \\ x_2 = -2.3841 \\ x_3 = 1.241 \end{bmatrix}$

Q4) $\begin{bmatrix} 5 & 1 & 2 \\ 1 & 8 & 2 \\ 4 & 1 & 6 \end{bmatrix}$ (DOMINANT DIAGONAL OF A)

$$\rightarrow 5x_1 + x_2 + 2x_3 = 3$$

$$x_1 + 8x_2 + 2x_3 = 2$$

$$4x_1 + x_2 + 6x_3 = 6$$

\Rightarrow

$$x_1 = \frac{3 - 2x_3 - x_2}{5}$$

$$x_2 = \frac{2 - x_1 - 2x_3}{8}$$

$$x_3 = \frac{6 - 5x_1 - x_2}{6}$$

ITERATION ①

$$x_1 = \frac{3}{5}$$

$$x_2 = \frac{1.4}{8} = 0.175$$

$$x_3 = 0.570833$$

ITERATION ②

$$x_1 = \frac{3 - 0.175 - 2(0.570833)}{5}$$

$$= 0.33666$$

$$x_2 = \frac{2 - 0.33666 - 2(0.570833)}{8}$$

$$= 0.0652083$$

$$x_3 = \frac{6 - 4(0.33666) - 0.0652083}{6}$$

$$= 0.76466$$

ERROR

MAX ERROR

$$E_1 = 78.2\%$$

$$E_2 = 166.4\%$$

$$E_{23} = 25.35\%$$

$$E_{max} = 166.4\%$$

ITERATION ③

$$x_1 = \frac{3 - 10.0652083 - 2(0.76466)}{5}$$

$$= 0.2810833$$

$$x_2 = \frac{2 - 10.2810833 - 2(0.76466)}{8} = 0.0236927$$

$$x_3 = \frac{6 - 4(0.2810833) - 0.0236927}{6}$$

$$= 0.905667$$

$$E_3 = 19.77\%$$

$$E_4 = 175.77\%$$

$$E_{23} = 5.44\%$$

$$E_{max} = 175.77\%$$

ITERATION (4)

ERROR

MAX ERROR

$$x_1 = \frac{3(6.0236927) - 2(0.9001662)}{5}$$

$$E_{x1} = 3.4165\%$$

$$= 0.271796$$

$$x_2 = \frac{2(0.271796) - 2(0.9001662)}{6}$$

$$E_{x2} = 70.945\%$$

$$E_{\max} = 70.945\%$$

$$x_3 = \frac{6 - 4(0.271796) - 10.0139545}{6}$$

$$E_{x3} = 0.95102\%$$

$$= 0.916492$$

ITERATION (5)

$$x_1 = 0.2706310$$

$$E_{x1} = 0.4306\%$$

$$x_2 = 0.01204529$$

$$E_{x2} = 15.038\%$$

$$E_{\max} = 15.038\%$$

$$x_3 = 0.917571$$

$$E_{x3} = 0.13197\%$$

ITERATION (6)

$$x_1 = 0.27056189$$

$$E_{x1} = 0.0255799$$

$$x_2 = 0.0011786947$$

$$E_{x2} = 2.215\%$$

$$E_{\max} = 2.215\%$$

$$x_3 = 0.91760917$$

$$E_{x3} = 0.010961\%$$

ITERATION (7)

$$x_1 = 0.2705792$$

$$E_{x1} = 0.00601\%$$

$$x_2 = 0.01176249$$

$$E_{x2} = 0.207\%$$

$$E_{\max} = 0.207\%$$

$$x_3 = 0.917654$$

$$E_{x3} = 0.00935\%$$

$$E_{\max} = 0.207\% < 1\% \quad \text{AT ITERATION 7}$$

$$\rightarrow x_1 = 0.2705792$$

$$x_2 = 0.01176249$$

$$x_3 = 0.917654$$

Q5 $(A - \lambda I)(x) = 0$

$$\hookrightarrow \begin{bmatrix} 4-\lambda & 2 & 3 \\ 2 & -\lambda & 1 \\ 2 & 1 & 4-\lambda \end{bmatrix}$$

$$\rightarrow (4-\lambda)(-\lambda + \lambda^2 - 1) - 2(8 - 2\lambda - 2) + 3(2 + 2\lambda) = 0$$

$$\hookrightarrow -\lambda^3 + 8\lambda^2 - 5\lambda - 10 = 0$$

$$\boxed{\lambda_1 = -0.919416} \quad \boxed{\lambda_2 = 1.72155} \quad \boxed{\lambda_3 = 7.09622}$$

* WHEN $\lambda_1 = -0.919416$

$$\hookrightarrow \begin{bmatrix} 4-\lambda & 2 & 3 \\ 2 & -\lambda & 1 \\ 2 & 1 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 4.919416 & 2 & 3 \\ 2 & 0.919416 & 1 \\ 2 & 1 & 4.919416 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 4.919416 & 2 & 3 \\ 0 & 0.16936 & 3.57323 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -0.10997 \\ 0 & 1 & 21.03623 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 8.10997x_3$$

$$x_2 = -21.03623x_3$$

$$x_3 = x_3$$

$$\Rightarrow V_1 = \begin{bmatrix} 8.10997 \\ -21.03623 \\ 1 \end{bmatrix}$$

* WHEN $\lambda_2 = 1.7216$

$$\begin{bmatrix} 4-1.7216 & 2 & 3 \\ 2 & -1.7216 & 1 \\ 2 & 1 & 4-1.7216 \end{bmatrix}$$

$$\begin{bmatrix} 2.2784 & 2 & 3 \\ 2 & -1.7216 & 1 \\ 2 & 1 & 2.2784 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2.2784 & 2 & 3 \\ 0 & -3.4774 & -1.63333 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & -0.904341 \\ 0 & 1 & 0.469741 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = -0.904341x_3$$

$$x_2 = -0.469741x_3$$

$$x_3 = x_3$$

$$\Rightarrow V_2 = \begin{bmatrix} -0.904341 \\ -0.469741 \\ 1 \end{bmatrix}$$

WHEN $\lambda_3 = 7.09692$

$$\begin{bmatrix} 4 - 7.09692 & 2 & 3 \\ 2 & -7.09692 & 1 \\ 2 & 1 & 4 - 7.09692 \end{bmatrix} \rightarrow \begin{bmatrix} -3.09692 & 2 & 3 \\ 2 & -7.09692 & 1 \\ 2 & 1 & -3.09692 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -3.09692 & 2 & 3 \\ 0 & -5.90531 & 2.9374 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1.29546 \\ 0 & 1 & -0.50598 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 1.29546 x_3$$

$$x_2 = 0.50598 x_3$$

$$x_3 = x_3$$

$$V_3 = \begin{bmatrix} 1.29546 \\ 0.50598 \\ 1 \end{bmatrix}$$

$$\lambda_1 = -0.81948, \quad \lambda_2 = 1.7216, \quad \lambda_3 = 7.09692$$

$$V_1 = \begin{bmatrix} 8.10997 \\ -21.03623 \\ 1 \end{bmatrix}, \quad V_2 = \begin{bmatrix} -0.90434 \\ -0.46974 \\ 1 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 1.29546 \\ 0.50598 \\ 1 \end{bmatrix}$$

Q6) $Ax = \lambda x$

$$\hookrightarrow Av = \lambda v$$

$$\hookrightarrow Av = 2v$$

$$\rightarrow (A - A^{-1})x$$

$$= Ax - A^{-1}x$$

$$= 2x - \frac{1}{2}x$$

$$(A - A^{-1})x = \frac{3}{2}x \quad \therefore A^{-1}v = \frac{1}{\lambda}v$$

$$(A - A^{-1})^T x^T = \frac{3}{2}x^T$$

$$(A - A^{-1})x^T x = \frac{3}{2}x^T (x^{-1})^T x$$

$$(A - A^{-1})^T x = \frac{3}{2}x$$

\hookrightarrow EIGENVALUE OF

$$(A - A^{-1})^T x = \frac{3}{2}$$