

ME46002 Assignment 1

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Q1

Question:

Find the root(s) of the equations $f(x)=0$

$$f(x) = \frac{\sin(x)}{x} + \frac{x}{1 + \tan(x)} - 1.2 = 0$$

by using the bisection method with $a = 0.1$ and $b = 1.0$

The percentage relative error of the solutions should be less than 2%.

Solution:

Iteration	a	b	c (root)	f(a)	f(c)	f(a) * f(c)	error
1	0.1	1.0	0.55	- 0.11078439 3	0.091297717 5	- 0.01011436 22	81.81818 18
2	0.1	0.55	0.325	- 0.11078439 3	0.025579596 6	- 0.00283382 009	69.23076 92
3	0.1	0.325	0.2125	- 0.11078439 3	- 0.032720909 1	0.00362496 606	52.94117 65

4	0.212 5	0.325	0.26875	- 0.03272090 91	- 0.001278280 98	4.18265157 e-5	20.93023 26
5	0.268 75	0.325	0.296875	- 0.00127828 098	0.012706406 3	- 1.62423575 e-5	9.473684 2
6	0.268 75	0.296875	0.282812 5	- 0.00127828 098	0.005854982 88	- 7.48431325 e-6	4.972375 68
7	0.268 75	0.282812 5	0.275781 25	- 0.00127828 098	0.002323844 24	- 2.97052589 e-6	2.549575 07
8	0.268 75	0.275781 25	0.272265 625	- 0.00127828 098	0.000531688 915	- 6.79647826 e-7	1.291248 2

c = 0.272265625 where the error < 2%

Q2

Question:

Determine the root of the following equation

$$f(x) = \sin^2(x) + \frac{x-1}{\cos(x)} = 0$$

using bisection method with two initial guesses of a=0 and b=1. Perform the computation until the percentage relative error is less than 2%.

Solution:

Iteration	a	b	c (root)	f(a)	f(c)	f(a) * f(c)	error
1	0.0	1.0	0.5	-1.0	-0.339898117	0.339898117	100.0
2	0.5	1.0	0.75	-0.339898117	0.122956118	-0.0417925529	33.333333
3	0.5	0.75	0.625	-0.339898117	-0.120074318	0.0408130345	20.0
4	0.625	0.75	0.6875	-0.120074318	-0.00162927498	0.000195634082	9.090909
5	0.6875	0.75	0.71875	-0.00162927498	0.059859121	-9.75269682e-5	4.34782609
6	0.6875	0.71875	0.703125	-0.00162927498	0.0289193564	-4.71175838e-5	2.222222
7	0.6875	0.703125	0.6953125	-0.00162927498	0.0135967498	-2.21528443e-5	1.12359551

c = 0.6953125 where the error < 2%

Q3

Solution:

$$f(x) = e^x + \cos(x) - 2 = 0$$

$$f'(x) = e^x - \sin(x)$$

Iteration	xi	f(x)	dF(x)	x	error
1	2	5.805202935477 79	4.6345841051 2678	1.30043918742 921	53.7941965555 289
2	1.30043918742 921	1.403832910321 24	4.6345841051 2678	0.99753543630 981	30.3652121111 539
3	0.99753543630 981	0.169216176822 879	3.5517275196 4363	0.94989209208 1687	5.01565858114 605
4	0.94989209208 1687	0.003659795771 7972	3.3987833876 652	0.94881529625 378	0.11348845577 8316
5	0.94881529625 378	1.835799425808 91e-6	3.3953739852 2268	0.94881475557 7132	5.69844266283 453e-5

x = 0.948814755577132 where the error < 0.1%

Q4i

Solution:

$$f(x) = x^3 - x - 3 = 0$$

$$f'(x) = 3x^2 - 1$$

Iteration	xi	f(x)	dF(x)	x	error
1	0.5	-3.375	506.0	-13.0	103.846153846154
2	-13.0	-2187.0	506.0	-8.67786561264822	49.8064222272831
3	-8.67786561264822	-647.811855408859	224.916054773548	-5.79762722299171	49.679606481671
4	-5.79762722299171	-192.075010071203	99.8374442503237	-3.8737497488287	49.6644749636895
5	-3.8737497488287	-57.2554953873476	44.0178113496512	-2.57301503186278	50.5529389007971
6	-2.57301503186278	-17.4613900345127	18.8612190625755	-1.64723234641804	56.2023133808588
7	-1.64723234641804	-5.82232573828777	7.14012320925765	-0.83179463369199	98.0335385301039

8	- 0.831794633699 199	- 2.743709361115 12	1.07564693795 236	1.71895809408 539	148.3894654884 98
9	1.718958094085 39	0.360248382940 782	7.86445078766 507	1.67315090517 071	2.737779884236 25
10	1.673150905170 71	0.010724544702 9332	7.39830185442 074	1.67170130972 111	0.086713783208 1673

$x = 1.67170130972111$ where the error $< 0.1\%$

Q4ii

Solution:

$$f(x) = x^3 - x - 3 = 0$$

$$f'(x) = 3x^2 - 1$$

Iteration	xi	f(x)	dF(x)	x	error
1	0.0	-3.0	26.0	-3.0	100.0
2	-3.0	-27.0	26.0	- 1.961538461538 46	52.94117647 05882
3	- 1.961538461538 46	- 8.585741920 80109	10.542899408 284	- 1.147175961403 55	70.98845578 48091

4	- 1.147175961403 55	- 3.362522157 36205	2.9480380592 6645	- 0.006579371480 71208	17335.95060 35885
5	- 0.006579371480 71208	- 2.993420913 32797	- 0.9998701356 12756	- 3.000389074071 23	99.78071605 65418
6	- 3.000389074071 23	- 27.01011728 83186	26.007003787 4181	- 1.961818175666 32	52.93920258 70166
7	- 1.961818175666 32	- 8.588691379 14838	10.546191663 1242	- 1.147430228481 6	70.97494269 97757
8	- 1.147430228481 6	- 3.363271968 90261	2.9497883877 0003	- 0.007256247552 42338	15712.99728 5331
9	- 0.007256247552 42338	- 2.992744134 51171	- 0.9998420406 14374	- 3.000473188773 22	99.75816322 64013

Cannot find solution with initial guess = 0 because the second derivative of $f(x)$ at 0 is 0, thus the point where $x = 0$ is an inflection point causing the subsequent calculations to diverge.