Solution

Qs (1)

The iterations are as follows
Based on the bisection method, the estimation of root is given by c=(a+b)/2;
If f(a)*f(c)>0, a=c otherwise b=c in the next iteration.

Iteration=1 a=0.100000 b=1.000000 c=0.550000 Error=0.818182
Iteration=2 a=0.100000 b=0.550000 c=0.325000 Error=0.692308
Iteration=3 a=0.100000 b=0.325000 c=0.212500 Error=0.529412
Iteration=4 a=0.212500 b=0.325000 c=0.268750 Error=0.209302
Iteration=5 a=0.268750 b=0.325000 c=0.296875 Error=0.094737
Iteration=6 a=0.268750 b=0.296875 c=0.282813 Error=0.049724
Iteration=7 a=0.268750 b=0.282813 c=0.275781 Error=0.025496
Iteration=8 a=0.268750 b=0.275781 c=0.272266 Error=0.012912

Hence the root is x=0.272266 with an error of 1.29%

Qs (2)

Based on the bisection method, the estimation of root is given by c=(a+b)/2:

Iteration=1 a=0.000000 b=1.000000 c=0.500000 Error=1.000000
Iteration=2 a=0.500000 b=1.000000 c=0.750000 Error=0.333333
Iteration=3 a=0.500000 b=0.750000 c=0.625000 Error=0.200000
Iteration=4 a=0.625000 b=0.750000 c=0.687500 Error=0.090909
Iteration=5 a=0.687500 b=0.750000 c=0.718750 Error=0.043478
Iteration=6 a=0.687500 b=0.718750 c=0.703125 Error=0.022222
Iteration=7 a=0.687500 b=0.703125 c=0.695313 Error=0.011236
Hence the root is x=0. 695313 with an error of 1.1%

Qs(3):

$$f(x) = e^x - \cos(x) - 2 = 0$$

$$f'(x) = e^x + \sin(x)$$

$$Hence \qquad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Iteration=1 xi=2.000000 x=1.300439 Error=53.794197
Iteration=2 xi=1.300439 x=0.997535 Error=30.365212
Iteration=3 xi=0.997535 x=0.949892 Error=5.015659
Iteration=4 xi=0.949892 x=0.948815 Error=0.113488
Iteration=5 xi=0.948815 x=0.948815 Error=0.000057

Hence the root is x=0.948815 with error=0.0006%

Qs (4)

$$f(x) = x^3 - x - 3 = 0 \;;\; f'(x) = 3x^2 - 1$$

$$x_{i+1} = x_i - \left(\frac{x_i^3 - x_i - 3}{3x_i^2 - 1}\right)$$

(a)

F(X)=X^3-X-3

F'(x)=3X^2-1

By using Newton Raphson method,

$X_{i+1}=X_i-F(X_i)/F'(X_i)$, given that $X_1=0.5$

i	x	f(x)	f'(x)	error
1	0.5	-3.375	-0.25	
2	-13	-2187	506	1.038462
3	-8.67787	-647.812	224.9161	0.498064
4	-5.79763	-192.075	99.83744	0.496796
5	-3.87375	-57.2555	44.01781	0.496645
6	-2.57302	-17.4614	18.86122	0.505529
7	-1.64723	-5.82233	7.140123	0.562023
8	-0.83179	-2.74371	1.075647	0.980335
9	1.718958	0.360248	7.864451	1.483895
10	1.673151	0.010725	7.398302	0.027378
11	1.671701	1.05E-05	7.383756	0.000867
12	1.6717	1.02E-11	7.383741	8.54E-07
13	1.6717	0	7.383741	8.29E-13

The solution is 1.671701 with an error of 0.09%

(b) Not converged if starting from X_1=0.0

	x	f(x)	f'(x)
1	0	-3	-1
2	-3	-27	26
3	-1.96154	-8.58574	10.5429
4	-1.14718	-3.36252	2.948038
5	-0.00658	-2.99342	-0.99987
6	-3.00039	-27.0101	26.007
7	-1.96182	-8.58869	10.54619
8	-1.14743	-3.36327	2.949788
9	-0.00726	-2.99274	-0.99984
10	-3.00047	-27.0123	26.00852
11	-1.96188	-8.58933	10.5469
12	-1.14749	-3.36343	2.950167
13	-0.0074	-2.9926	-0.99984
14	-3.00049	-27.0128	26.00886
15	-1.96189	-8.58948	10.54707
16	-1.1475	-3.36347	2.950253
17	-0.00744	-2.99256	-0.99983
18	-3.0005	-27.0129	26.00895
19	-1.9619	-8.58951	10.5471
20	-1.1475	-3.36348	2.950273