THE HONG KONG POLYTECHNIC UNIVERSITY

Department of Mechanical Engineering

Programme : BEng(Hons) in Mechanical Engineering (43478)

Subject Title : Numerical Methods for Engineers **Subject Code :** ME46002

Session : Semester 2, 2017/2018

Date : **30 April 2018** **Time** : **15:15 pm – 18:15 pm**

Time Allowed : 3 hours **Subject Examiner(s)** : **Dr. G. P. Zheng**, **Dr. T. Y. Ng**

This question paper has 4 pages (attachments included).

Instructions to Candidates : This paper has 5 questions.
Answer all 5 questions.

Constants :

Others :

Available from Invigilator :

DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO

Question 1:

(a) Determine the root of the following equation

$$f(x) = \frac{1}{x} - \frac{x}{\sin(x)+2} = 0 \ ,$$

using bisection method with two initial guesses of a = 0.2 and b = 3. Perform the computation until the percentage relative error is less than 3%. x is in unit of radians.

(10 marks)

(b) Employ the Newton-Raphson method to determine the root for

$$f(x) = x^2 + \cos(2x)\cdot e^{-x} - 1 = 0 \ ,$$

using an initial guess of $x_0 = 0.5$. Perform the computation until the percentage relative error is less than 0.1%. x is in unit of radians. (10 marks)

Question 2:

(a) Solve the simultaneous linear equations $A\,\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 2 & 6 & 1 \\ 7 & 1 & 4 \\ 8 & 5 & 2 \end{bmatrix} \quad \textit{and} \quad b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \ ,$$

using the Gaussian elimination *with pivoting.*

(7 marks)

(b) Assume *any arithmetic computation* must follow the five-digit chopping arithmetic which is defined as follows: given a real number q expressed in normalized decimal form as $q = \pm(0.d_1d_2d_3d_4d_5\cdots d_kd_{k+1}\cdots)\times 10^n$ the value of this number is $q = \pm(0.d_1d_2d_3d_4d_5)\times 10^n$, where $1 \le d_1 \le 9$, and $0 \le d_k \le 9$ for $k > 1$. Based on the five-digit chopping arithmetic, solve the simultaneous equation $A\,\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = [x_1 \ \ x_2 \ \ x_3]^T$,

$$A = \begin{bmatrix} 12 & -6 & 0 \\ -3 & 1.501 & 5.626 \\ 5 & 0 & 5 \end{bmatrix} \quad \textit{and} \quad b = \begin{bmatrix} 6.9 \\ 3.9 \\ 5.375 \end{bmatrix} \ ,$$

using naïve Gaussian elimination method. (9 marks)

(c) Solve the following equations using the Gauss-Seidel method until the percent relative error falls below $\varepsilon_s = 7\%$.

$$-3x_1 + x_2 + 12x_3 = 50$$
$$6x_1 - x_2 - x_3 = 3$$
$$6x_1 + 9x_2 + x_3 = 40$$

(9 marks)

Question 3:

(a) For a set of 5 data points ($\mathbf{x}_i$, $\mathbf{y}_i$), i=1, 2, 3, 4, 5 with x_i=1.1, 2.4, 3.7, 4.6, 7.1 and y_i=0.21, 0.5, 0.69, 0.73, 0.98, linear least-squares regression is employed to find the fitting coefficients.

(i) Find the values of the coefficients a_0 and a_1 of the linear relation

$$y = (a_0 + a_1 x) .$$

(6 marks)

(ii) Find the values of the coefficients k and n of the relation, as follows

$$y = 1 - e^{-k \cdot x^n} .$$

(7 marks)

(b) For a set of three data points (x_i,y_i), i = 1, 2, 3, with x_i= -4, 0, 4 and y_i= -16, 0, 16. If the data are fitted with cubic splines $f\ (x) = a + bx + cx^2 + dx^3$ with the constrains f '(-4) = 12 and f '(4) = 12, determine the cubic functions.

(5 marks)

(c) For an nth-order Newton's divided difference interpolating polynomial $f_n(x)$, the error of interpolation can be estimated by $R_n = \left| g[x_{n+1}, x_n, x_{n-1}, \ldots, x_0] \cdot (x - x_0)(x - x_1) \cdots (x - x_n) \right|$, where ($x_0$, $f(x_0)$), (x_1, $f(x_1)$),..., (x_{n+1}, $f(x_{n+1})$) are data points; $g[x_{n+1}, x_n, x_{n-1}, \ldots, x_0]$ is the (n+1)th finite divided difference. To minimize R_n, if there are more than n+1 data points available for calculating $f_n(x)$ using the nth-order Newton's interpolating polynomial, n+1 data points (x_0, $f(x_0)$), (x_1, $f(x_1)$),..., (x_n, $f(x_n)$) should be chosen from the available data points such that $\left| (x - x_0)(x - x_1) \cdots (x - x_n) \right|$ is the smallest. Given the data

x_i	2	4	6	8	10	12
$f(x_i)$	2.2	5.1	8.1	10	7.8	3.2

Calculate $f_n(10.5)$ using Newton's divided difference interpolating polynomials with n = 3. Estimate the error R_3 of $f_3(10.5)$.

(7 marks)

Question 4:

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} .$$

(a) Determine all the eigenvalues and their corresponding eigenvectors for matrix **A** using conventional method. (4 marks)

(b) Perform 4 iterations using the Power method to estimate the dominant eigenvector and its corresponding eigenvector for the matrix with the initial vector $\mathbf{x}_0 = (1, 1, 1)^T$. Provide the details for the first two iterations. (6 marks)

Question 5:

Given the function

$$f(x) = e^{-x^3} \quad x \in R$$

(a) With $h = 0.01$, estimate df/dx at $x = 0.4$ up to 5 decimal places by using the following methods:

(i) Forward and backward difference formulas;

(ii) Richardson extrapolation;

(iii) Analytical formula.

(7 marks)

(b) Find an approximate value for the integral

$$\int_{0.1}^{0.5} f(x)dx \quad ,$$

using the following integration methods:

(i) 4-point Gaussian quadrature. (4 marks)

(ii) Trapezoidal rule and Simpson's rule. (4 marks)

(iii) Suppose that

$$\int_{0.1}^{0.5} f(x)dx = Q[f] + E[f] \quad ,$$

where the error term is given by

$$E[f] = -\frac{1}{30}h^2 f''(\mu) \quad ,$$

for some $\mu \in (0.1, 0.5)$. If $h = 2/(5n)$, determine the minimum n (integer) such that the absolute error is less than the tolerance $\varepsilon = 0.0001$.

(5 marks)

~ End ~