Solution:

Q1

First, order the points so that they are as close to and as centered about the unknown as possible

$x_0 = 7 \quad f(x_0) = 6.5$
$x_1 = 12 \quad f(x_1) = 7.0$
$x_2 = 15 \quad f(x_2) = 11.2$
$x_3 = 18 \quad f(x_3) = 10.8$
$x_4 = 19 \quad f(x_4) = 8.5$

Next, the divided differences can be computed and displayed in the recursive format,

i	x_i	$f(x_i)$	$g[x_{i+1},x_i]$	$g[x_{i+2},x_{i+1},x_i]$	$g[x_{i+3},x_{i+2},x_{i+1},x_i]$	$g[x_{i+4},x_{i+3},x_{i+2},x_{i+1},x_i]$
0	7	6.5	0.1	0.1625	-0.038	-0.000239
1	12	7.0				
2	15	11.2				
3	18	10.8				
4	19	8.5				

The first through third-order interpolations can then be implemented as

$$f_3(11) = 6.5 + 0.1*(11-7) + 0.1625*(11-7)(11-12) \\ -0.038*(11-7)*(11-12)*(11-15)$$

=5.642

The error for the third-order prediction can be computed with R_3 as

$$R_3 = 0.000239*(11-7)*(11-12)*(11-15)*(11-18)$$

=0.026768

Q2

$$a_0 = (\sum^n y_i \sum^n x_i^2 - \sum^n x_i \sum^n x_i y_i)/(n\sum^n x_i^2 - (\sum^n x_i)^2)$$

=2.01

$$a_1 = (n\sum^n x_i y_i - \sum^n y_i \sum^n x_i)/(n\sum^n x_i^2 - (\sum^n x_i)^2)$$

=0.903

Q3

$a_0 = 0.5$

$$a_k = \frac{2}{1}\left[\int_{-0.5}^{0} -2t\cos(2k\pi t)\ dt + \int_{0}^{0.5} 2t\cos(2k\pi t)\ dt\right]$$

$$= 4\left\{\left[-\frac{\cos(2k\pi t)}{(2k\pi)^2} - \frac{t\sin(2k\pi t)}{2k\pi}\right]_{-0.5}^{0} + \left[\frac{\cos(2k\pi t)}{(2k\pi)^2} + \frac{t\sin(2k\pi t)}{2k\pi}\right]_{0}^{0.5}\right\}$$

$$= \frac{8}{(2k\pi)^2}(\cos k\pi - 1) = \frac{2}{(k\pi)^2}(\cos k\pi - 1)$$

$b_k = 0$

Substituting these coefficients gives

$$f(t) = \frac{1}{2} - \frac{4}{\pi^2}\cos(2\pi t) - \frac{4}{9\pi^2}\cos(6\pi t) - \frac{4}{25\pi^2}\cos(10\pi t) + \cdots$$

Q4

For second derivative, the central finite difference with 2nd-order accuracy is as follow

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

	x	$f(x)$
x_{i-1}	-0.2	0.9607894
x_i	0.0	1
x_{i+1}	0.2	0.9607894

At x=0, h=0.2

$$f''(x) = \frac{2*0.9607894 - 2*1}{0.2^2} = -1.960528$$

The analytical result is $f''(x) = 4x^4e^{-x^2} - 2e^{-x^2} = -2$

The percentage relative error is $\frac{|2 - 1.960528|}{|-2|} \times 100\% = 1.976\%$

Q5

Analytical solution:

$$\int_0^3 x^2e^x\ dx = \left[(x^2 - 2x + 2)e^x\right]_0^3 = 98.42768$$

Trapezoidal rule (n = 4):

$$I = (3-0)\frac{0 + 2(1.190813 + 10.0838 + 48.03166) + 180.7698}{8} = 112.2684 \qquad \varepsilon_t = 14.062\%$$

Simpson's rule (n = 4):

$$I = (3-0)\frac{0 + 4(1.190813 + 48.03166) + 2(10.0838) + 180.7698}{12} = 99.45683 \qquad \varepsilon_t = 1.046\%$$