Solution:

Q1:

Based on the bisection method, the estimation of root is given by c=(a+b)/2;
If f(a)*f(c)>0, a=c otherwise b=c in the next iteration.
Iteration
1 a=0.000000 b=2.000000 c=1.000000 Error=100.000000%
2 a=1.000000 b=2.000000 c=1.500000 Error=33.333333%
3 a=1.000000 b=1.500000 c=1.250000 Error=20.000000%
4 a=1.250000 b=1.500000 c=1.375000 Error=9.090909%
5 a=1.250000 b=1.375000 c=1.312500 Error=4.761905%
Hence the root is x=1.3125 with an error of 4.8%

Q2:

$$f(x) = \cos(x)\ln(x) + 0.2 * \exp(x)$$
$$f'(x) = -\sin(x)\ln(x) + \cos(x)/x + 0.2 * \exp(x)$$

$$Hence \qquad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Iteration=1 xi=2.000000 x=0.140058 Error=1327.9%
Iteration=2 xi=0.140058 x=0.366659 Error=61.8%
Iteration=3 xi=0.366659 x=0.569535 Error=35.6%
Iteration=4 xi=0.569535 x=0.625998 Error=9.0%
Iteration=5 xi=0.625998 x=0.628862 Error=0.46%

Hence the root is x=0.628862 with error of 0.46%

Q3:

First pivoting [A] by switching the first and second rows of $[A]$. Note that we must make the same switch for the right-hand-side vector $\{B\}$

$$[A] = A = \begin{bmatrix} 5 & 1 & 2 \\ 4 & 1 & 6 \\ 1 & 8 & 2 \end{bmatrix} \qquad \{B\} = \begin{Bmatrix} 3 \\ 6 \\ 2 \end{Bmatrix}$$

Then, forward elimination is carried out; Subsequently, pivoting should be carried out before the next elimination step by switching the second and third rows of $[A]$.

Therefore, the *LU* decomposition is A=L*U:

$$[L]=\begin{bmatrix}1 & 0 & 0\\ 0.2 & 1 & 0\\ 0.8 & 0.02564 & 1\end{bmatrix} \qquad [U]=\begin{bmatrix}5 & 1 & 2\\ 0 & 7.8 & 1.6\\ 0 & 0 & 4.35897\end{bmatrix}$$

Forward substitution to find [D] for $[L]\{D\} = \{B\}$
Back substitution to solve $[U]\{X\} = \{D\}$ for [x]

$x_1 = 0.2705$
$x_2 = 0.0117$
$x_3 = 0.8176$

Q4:

step	x_1	x_2	x_3	$e_1\%$	$e_2\%$	$e_3\%$
1	0.6000	0.1750	0.5708	100	100	100
2	0.3367	0.0652	0.7647	-78.2178	-168.3706	25.3508
3	0.2811	0.0237	0.8087	-19.7747	-175.2253	5.4380
4	0.2718	0.0139	0.8165	-3.4168	-70.9449	0.9590
5	0.2706	0.0120	0.8176	-0.4306	-15.0383	0.1320
6	0.2706	0.0118	0.8177	-0.0256	-2.2150	0.0110
7	0.2706	0.01176	0.8176	0.0060	-0.2079	-0.0008

Thus, after 7 iterations, the maximum error is 0.2% and we arrive at the result: x_1 = 0.2705782, x_2 = 0.0117624 and x_3 = 0.817654.

Q5:

The standard format of this eigenvalue problem is described by

$$AX = \lambda X, \quad or \quad (A-\lambda I)X = 0$$

$$\det(A-\lambda I)=\begin{vmatrix}4-\lambda & 2 & 3\\ 2 & -\lambda & 1\\ 2 & 1 & 4-\lambda\end{vmatrix}=0$$

$$=-\lambda^3+8\lambda^2-5\lambda-10$$

The real solutions for λ is

λ_1 =7.09692,
λ_2 =1.7215567
λ_3 =-0.81848

The corresponding eigenvectors are

$\mathbf{V}=[\mathbf{v_1}\ \mathbf{v_2}\ \mathbf{v_3}]=$

$$\begin{bmatrix} -0.756271467925860 & -0.633402021320966 & 0.359321888273646 \\ -0.295384972841325 & -0.329009081586388 & -0.932161044849301 \\ -0.583781795383168 & 0.700396247577181 & 0.044312154911722 \end{bmatrix}$$

Q6:

Because $AX=\lambda X=2X$
Therefore $(A-A^{-1})X=AX-A^{-1}X=AX-(1/2)X=2X-0.5X=1.5X$

That is
$(A-A^{-1})X=1.5X$

Therefore the eigenvalue of $(A-A^{-1})$ is 1.5, and

$(A-A^{-1})^T$ has the same eigenvalue of 1.5.