MVC

F1 def Rate of change?

14.3 # Partial Derivation:

Rate of change

1. Instantaneous R.O.C
2. Average R.O.C

$f(x) \Rightarrow f'(x)$

$f(y) \Rightarrow f'(y)$

$f(\underset{\text{vary}}{x}, \underset{\text{fixed}}{y}) \Rightarrow f_x(x, y)$

$f(\underset{\text{fixed}}{x}, \underset{\text{vary}}{y}) \Rightarrow f_y(x, y)$

E.1 Q $x^3 + x^2y^3 - 2y^2 = f(x, y)$

(i) $f_x(2, 1) = ?$

$f_x(x, y) = 3x^2 + 2xy^3 - 0$

$f_x(x, y) = 3x^2 + 2xy^3$

at $f_x(2, 1) = 3(2)^2 + 2(2)(1)^3$

$= 12 + 4$

$= 16$

(ii) $f_y(2, 1) = ?$

$\cancel{f_y(x, y) = 0^3 + 3x^2y^2 - 4y}$

$\cancel{\text{at } f_y(2, 1) = 3(4)(1) - 4(1)}$

$\cancel{= 12 - 4 + 8}$

$\cancel{= 86}$

$f_y(x, y) = 0 + 3(x)^2y^2 - 4y$

$f_y(2, 1) = 3(2)^2(1)^2 - 4(1)$

$= 3(4) - 4$

$= 12 - 4$

$= 8$

iii) $4 - x^2 - y^2$ at $(1,1)$

iv) $f(x,y) = \sin(x/1+y)$

v) $f(x,y) = y^3 - 3xy$

vi) $f(x,t) = e^{-t}\cos \pi x$

iii) $4 - x^2 - y^2$ at $(1,1)$

$f(x,y) = 4 - x^2 - y^2$

$f_x(x,y) = 0 - 2x - y^2$

$f_x(1,1) = -2 - (1)^2$

$= -2 - 1 = -3$

$f_y(x,y) = 0 - x^2 - 2y$

$f_y(1,1) = -(1)^2 - 2(1)$

$= -1 - 2 = -3$

v) $f(x,y) = y^3 - 3xy$

$f_x(x,y) = - 3y$

$f_y(x,y) = 3y^2 - 3x$

iv) $f(x,y) = \cancel{y^3 - 3}\ \sin(x/1+y)$

$f_x(x,y) = \cos\left(x\right) \cdot \left(\frac{1}{\ }\right)$

(2) MVC

Q Find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ if z is defined implicity as a function of x and y by the equation

Ex 5 $x^3+y^3+z^3+6xyz=1$

Sol

taking ∂ w.r.t x

$$3x^2+0+3z^2\frac{\partial z}{\partial x}+6y\left(x\frac{\partial z}{\partial x}+z\right)$$

$$3x^2+3z^2\frac{\partial z}{\partial x}+6xy.\frac{\partial z}{\partial x}+6yz=0$$

$$3z^2\frac{\partial z}{\partial x}+6xy\frac{\partial z}{\partial x}=-6yz-3x^2$$

$$\frac{\partial z}{\partial x}\left(3z^2+6xy\right)=-6yz-3x^2$$

$$\boxed{\frac{\partial z}{\partial x}=-6yz-3x^2}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-6xz - 3y^2}{3z^2 + 6xy}}$$

Q a) Find 2nd partial derivative of

1) $f(x,y) = x^3 + x^2y^3 - 2y^2$

2) $f(x,y,z) = x\sin(y-z)$

3) $f(x,y,z) = xy\sin^{-1}(yz)$

Sol

$$f_x(x,y) = 3x^2 + 2xy^3 - 0$$

$$\boxed{f_{xx}(x,y) = 6x + 2y^3}$$

$$f_y(x,y) = 3y^2x^2 - 4y$$

$$\boxed{f_{yy}(x,y) = 6yx^2 - 4}$$

Q b) Implicit:

1. $x^2 + 2y^2 + 3z^2 = 1$
2. $e^x = xyz$
3. $yz + x\ln y = z^2$
4. $x^2 - y^2 + z^2 - 2z = 4$

Qc $u = x^3y^5 + 2x^4y$; $u_{xx} = ?$, $u_{yy} = ?$

Qd $z = \arctan\left(\frac{x+y}{1-xy}\right)$; $z_x = ?$, $z_y = ?$

Clairut's Theorem

$$f_{xy} = f_{yx}$$

* Partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is called Laplace equati

Qa2 $f(x,y,z) = x\sin(y-z)$

$f_x(x,y,z) = \sin(y-z) \cdot 1$
$f_{xx}(x,y,z) = 0$

$f_y(x,y,z) = x\cos(y-z) \cdot \frac{\partial}{\partial y}(y-z)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \Rightarrow \text{laplace equation}$$

Solution of this eqn equation is Harmonic

Q $u(x,y) = e^x \sin y$ 96.

$u_x(x,y) = \sin y \, e^x$

$u_{xx}(x,y) = \sin y \, e^x$

$u_y(x,y) = e^x \cos y$

$u_{yy}(x,y) = -e^x \sin y$

$u_{xx} + u_{yy} = 0$

$e^x \sin y - e^x \sin y = 0$

$0 = 0$

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \Rightarrow \text{wave equation}$$

Q 10 $u(x,t) = \sin(x - at)$

$u_x(x,t) = \cos(x - at) \cdot \frac{\partial}{\partial x}(x - at)$

$= \cos(x - at)(1 - 0)$

$U_x(x,t) = \cos(x - at)$

$U_{xx}(x,t) = -\sin(x-at) \frac{\partial}{\partial x}(x-at)$

$= -\sin(x-at)(1)$

$U_{xx}(x,t) = -\sin(x-at)$

$U_t(x,t) = \cos(x-at) \cdot (1-a)$

$= 1-a \cos(x-at)$

$U_{tt}(x,t) = 1 \cdot a\{-\sin(x-at)\} \cdot (0-a)$

$= -a\{a\sin(x-at)\}$

$= -a^2 \sin(x-at)$

$-a^2 \sin(x-at) = a^2 - \sin(x-at)$

$-a^2 \sin(x-at) = -a^2 \sin(x-at)$

Ex: Q1 Find directional derivative $D_u f(x,y)$ if

973

$f(x,y) = x^3 - 3xy + 4y^2$

and $\vec{u}$ is the unit vector given by angle $\theta = \pi/6$ what $D_u f(1,2) = ?$

Directional Derivative:

$f(x,y) \rightarrow$ unit vector $\hat{u} = (a,b)$

$$D_u f(x,y) = f_x(x,y)\,a + f_y(x,y)\,b$$

Case I:

$\theta = ?$

if $\theta = \pi/6$

$D_u f(1,2) = ?$

$$D_u f(x,y) = f_x(x,y)\cos\theta + f_y(x,y)\sin\theta$$

Case II: either 2D or 3D

$$D_u f(x,y,z) = f_x(x,y,z)\,a + f_y(x,y,z)\,b + f_z(x,y,z)\,c$$

gradient :-

$$\nabla \cdot f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right) \cdot f$$

$$= \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}$$

Q $f(x,y) = x^2y^2 + 2xy \; ; \; \vec{u} = (2i + 5j)$

$$\Delta f = (2xy^2 + 2y)\hat{i} + (2x^2y + 2x)\hat{j}$$

$$= \left((2xy^2 + 2y)\hat{i} + (2x^2y + 2x)\hat{j}\right) \cdot \left(\frac{2i + 5j}{\sqrt{29}}\right)$$

$$= \left(\frac{4xy^2 + 4y}{\sqrt{29}}\right)\hat{i} + \left(\frac{10x^2y + 10x}{\sqrt{29}}\right)\hat{j}$$

$$= \frac{4xy^2 + 4y + 10x^2y + 10x}{\sqrt{29}}$$

Q let $f(x,y,z) = x^2y + y^3z^2 - xyz$ in The diretion of unit vector $\vec{u} = (-1, 0, 3)$

$D_u f(x,y,z)$

$$\nabla f = \left[(2xz + 0 - yz)\hat{i} + (0 + 3y^2z^2 - xz)\hat{j} + (x^2 + 2zy^3 - xy)\hat{z}\right] \cdot \left[\frac{-1}{\sqrt{10}}\hat{i} + \frac{0}{\sqrt{10}}\hat{j} + \frac{3}{\sqrt{10}}\right.$$

$$= -\left(\frac{2xz - yz}{\sqrt{10}}\right) + \frac{3y^2z^2 - xz}{\sqrt{10}}(0) + \frac{3(x^2 + 2zy^3}{\sqrt{10}}$$

$$D_u f(x,y,z) = \frac{-2xz + yz}{\sqrt{10}} + \cancel{3y^2z} \; \frac{3x^2 + 6zy^3 - 3xy}{\sqrt{10}}$$

How mathematics is helpful for CS student (Tools, topics)?

just write name

Chain Rule:

Case I: $z = f(x, y)$ $\quad x = g(t)$, $y = h(t)$

$$\frac{\partial z}{\partial t} = \frac{dz}{dt} = ?$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

Ex Q $z = x^2 y + 3xy^4$; $x = \sin(2t)$ $\quad$ 976

$y = \cos t$

$\frac{dz}{dt} = ?$ when $t \to 0$

Sol

$= \frac{\partial z}{\partial x}$

$$\frac{dz}{dt} = (2xy + 3y^4) \cdot (2\cos 2t) + (x^2 + 12xy^3) \cdot (-\sin t)$$

$$\frac{dz}{dt} = 2\cos(2t)(2xy + 3y^4) - \sin t (x^2 + 12xy^3)$$

$t \to 0$

$= 2\cos(0)(2xy + 3y^4) - \sin(0)(x^2 + 12xy^3)$

$= 2(2xy + 3y^4) - 0$

$= 4xy + 6y^4$

$= 0 + 6(1)$ $\quad \because x = 0; y = 1$

$= 6$

$\delta \quad \delta\, \gamma\, \gamma$

Q Implicit differentiation

E.8 $x^3 + y^3 = 6xy$ 962

$$\frac{dy}{dx} = ?$$

Sol

$$\frac{dy}{dx} = -F_x / F_y$$

$$\frac{dy}{dx} = -3x^2 - 6y / 3y^2 - 6x$$

$z = f(x, y) \qquad x = g(s, t)$

$y = h(s, t)$

Q E.3 $z = e^x \sin y \qquad x = st^2$ 980

$y = s^2 t$

$$\frac{\delta z}{\delta s} = ? \qquad \frac{\delta z}{\delta t} = ?$$

Sol

$$\frac{\delta z}{\delta s} = \frac{\delta z}{\delta x} \cdot \frac{\delta x}{\delta s} + \frac{\delta z}{\delta y} \cdot \frac{\delta y}{\delta s}$$

$$= (\sin y \cdot e^x)(t^2) + (e^x \cos y)(2st)$$

$$\frac{\delta z}{\delta s} = \sin(y) e^x \cdot t^2 + 2st e^x \cos y$$

Implicit Function

Q Find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$

if $x^3+y^3+z^3+6xyz=1$

$\frac{\partial z}{\partial x} = -F_x/F_z$; $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$

$= \partial x$

$F_x = 3x^2 + \cancel{3y} + 0 + 6yz$
$= 3x^2 + 6yz$

$F_z = 0 + 0 + 3z^2 + 6xy$
$= 3z^2 + 6xy$

$\frac{\partial z}{\partial x} = -(3x^2+6yz)/3z^2+6xy$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-x^2-2yz}{z^2+2xy}}$$

$F_y = 0 + 3y^2 + 0 + 6xz$
$= 3y^2 + 6xz$

$F_z = 0 + 0 + 3z^2 + 6xy$
$= 3z^2 + 6xy$

$\frac{\partial z}{...} = ... F_y$

$$= -3y^2 - 6xz \Big/ 3z^2 + 6xy$$

$$\boxed{\frac{\partial z}{\partial y} = -y^2 - 2xz \Big/ z^2 + 2xy}$$

Case I

Q1 Use chain rule to find

$\dfrac{dw}{dt} \Big/ \dfrac{dz}{dt}$, if **a)** $z = x^2 + y^2 + xy$; $x = \sin t$; $y = e^t$

b) $w = xe^{y/z}$; $x = t^2$; $y = 1 - t$; $z = 1 + 2t$

Case II

Q2 Find partial derivative by using chain rule

a) $z = x^2 y^3$; $x = s\cos t$; $y = s\sin t$

b) $z = \sin\theta \cos\phi$; $\theta = st^2$; $\phi = s^2 t$

c) $z = e^{x+2y}$; $x = s/t$; $y = t/s$

d) $z = e^r \cos\theta$; $r = st$; $\theta = \sqrt{s^2+t^2}$

Q1 $\dfrac{dz}{dt}$

a) $z = x^2 + y^2 + xy$; $x = \sin t$; $y = e^t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$= (2x + 0 + y)(\cos t) + (2y + x)(e^t)$$

$$\frac{dz}{dt} = \cos(t)\, 2x + y + e^t(2y + x)$$

Case III

General Case

$$w = f(x, y, z, t)$$

$x = x(u, v)$

$y = y(u, v)$

$z = z(u, v)$

$t = t(u, v)$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial u} + \frac{\partial w}{\partial t} \cdot \frac{\partial t}{\partial u}$$

$$\frac{\partial w}{\partial v}$$

Q

$$u = x^4 y + y^2 z^3$$

$x = r s e^t$

$y = r s^2 e^{-t}$

$z = r^2 s \sin t$

$\frac{\partial u}{\partial r} = ?$

at $r = 2, s = 1, t = 0$

Sol

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial r}$$

$$\frac{\partial u}{\partial t} = ?$$

$$\frac{\partial u}{\partial s} = ?$$

Q $f(x,y) = 4 + x^3 + y^3 - 3xy$

$f_x(x,y) \Rightarrow 3x^2 - 3y = 0$

$f_y(x,y) \Rightarrow 3y^2 - 3x = 0$

For critical point

$f_x = 0 \quad ; \quad f_y = 0$

$3x^2 - 3y = 0 \rightarrow (2) \quad ; \quad 3y^2 - 3x = 0 \rightarrow (1)$

$3x^2 = 3y \quad ; \quad 3y^2 = 3x$

$\boxed{x^2 = y} \rightarrow (*) \quad ; \quad \boxed{y^2 = x}$

Put in (1)

$3(x^2)^2 - 3x = 0$

$3x^4 - 3x = 0$

$3x(x^3 - 1) = 0$

$3x = 0 \quad , \quad (x^3 - 1) = 0$

$\boxed{x = 0} \quad \quad x^3 = 1$

$\boxed{x = 1}$

Put in (*)

$\boxed{y = 0} \quad ; \quad \boxed{y = 1}$

C.P $\rightarrow (0,0) \, , \, (1,1)$

$D = f(x,y) = f(0,0) = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= (6x)(6y) - (-3)^2$

$f(0,0) \quad = 36xy - 9$

F25

Q $f(x,y) = x^3 - 3xy + 4y^2$

(1, 2)

$D_u f(x,y) = f_x \cos\theta + f_y \sin\theta$

$\theta = \pi/6$

$f_x = 3x^2 - 3y$

$f_y = -3x + 8y$

$D_u f(x,y) = (3x^2 - 3y)\cos(\pi/6) + (3x + 8y)\sin(\pi/6)$

$= 3(1)^2 - 3(2)\ \frac{\sqrt{3}}{2} + (-3(1) + 8(2))\ \frac{1}{2}$

$= (3-6)\frac{\sqrt{3}}{2} + (-3+16)\frac{1}{2}$

$= -\frac{3\sqrt{2}}{2} + \frac{13}{2}$

$D_u f(1,2) = \frac{-3\sqrt{2} + 13}{2}$

Q $f(x,y) = x^4 + y^4 - 4xy + 1$

$f_x = 4x^3 - 4y$

$f_y = 4y^3 - 4x$

f_x

$4x^3 - 4y = 0$; $4y^3 - 4x = 0 \rightarrow (3)$

$4x^3 = 4y$; $y^3 = x \rightarrow (2)$

$x^3 = y \rightarrow (1)$

$f(1,1) = 12(1)^2 \cdot 12(1)^2 - (4)^2$
$= 144 - 16$
$= 128$ R. minime

$D = f((-1,-1)) = 12(-1)^2 12(-1)^2 - (-4)^2$
$= 144 - 16$
$= 128$

$D > 0$, $f_{xx} > 0$
R. minime

Q

$z = f(x, y)$ $x = r^2 + s^2$
$y = 2rs$

(a) $\frac{\partial^2 z}{\partial r^2}$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot 2r + \frac{\partial z}{\partial y} \cdot (2s)$$

$$\frac{\partial}{\partial r}\left(\frac{\partial z}{\partial r}\right) = \frac{\partial}{\partial r}\left(\frac{\partial z}{\partial x} \cdot 2r\right) + \frac{\partial}{\partial r}\left(\frac{\partial z}{\partial y} \cdot 2s\right)$$

$$= 2\left[\frac{\partial z}{\partial x}(1) + \frac{\partial}{\partial r}\left(\frac{\partial z}{\partial x}\right) r\right] + 2s \frac{\partial}{\partial r}\left(\frac{\partial z}{\partial y}\right)$$

$$= 2\frac{\partial z}{\partial x} + 2r \frac{\partial}{\partial r} \frac{\partial z}{\partial x} + 2s \frac{\partial}{\partial r} \frac{\partial z}{\partial y} \rightarrow (1)$$

$$\frac{\partial}{\partial r} = \frac{\partial}{\cdot} \cdot \frac{\cdot}{\partial r} \qquad z = f(x, y)$$

$$\frac{\partial}{\partial r}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{-}\left(\frac{\partial z}{\partial x}\right)\frac{-}{\partial r}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial r} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\frac{\partial y}{\partial r}$$

Q $g(s,t) = f(s^2-t^2, t^2-s^2)$

$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0$$

Sol

let $x = s^2 - t^2$

$y = t^2 - s^2$

$$\frac{\partial g}{\partial s} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$= \frac{\partial g}{\partial x}\cdot(2s) + \frac{\partial g}{\partial y}(-2s)$$

Lagrange Multiplier:

+ve = max
-ve = min

Case I → Maximum / Minimization

$\Delta f(x,y) = \lambda \Delta g(x,y)$ $f(x,y)$

$\Delta f = \lambda \Delta g$ Constraint

$\Delta f(x,y,z) = \lambda \Delta g(x,y,z)$ $g = ?$

Q) $f(x,y) = xy$ constraint $\frac{x^2}{8} + \frac{y^2}{2} = 1$

also ellipse

$f_x = \lambda g_x$

$f_y = \lambda g_y$

$f_x = \lambda g_x$	$f_y = \lambda g_y$
$y = \lambda \frac{2x}{8}$	$x = \lambda \frac{2y}{2}$
$y = \frac{x\lambda}{4} \rightarrow (1)$	$x = \lambda y \rightarrow (2)$

$x \times$ in (1) on b.s	$y \times$ in (2) on b.s
$xy = \frac{x^2\lambda}{4} \rightarrow (3)$	$xy = \lambda y^2 \rightarrow (4)$

Compare (3) and (4)

$\frac{x^2\lambda}{4} = \lambda y^2$

$\frac{x^2}{4} = y^2 \quad \div 2 \Rightarrow \frac{x^2}{8} = \frac{y^2}{2}$

Q2 Find the maxima & minima of function

$f(x,y,z) = xyz$ subject to a constraint $x^2+y^2+z^2=3 \rightarrow (1)(2)$

Sol

$f_x = \lambda g_x$	$f_y = \lambda g_y$	$f_z = \lambda g_z$
$yz = \lambda 2x$	$xz = \lambda 2y$	$xy = \lambda 2z$
x 'x' on both side	x 'y' on both side	x 'z' on B.S

$xyz = 2\lambda x^2 \qquad xyz = 2\lambda y^2 \qquad xyz = 2\lambda z^2$

$\cancel{2\lambda x^2 = 2\lambda y^2} \quad ; \quad \cancel{2\lambda y^2 = 2\lambda z^2}$

$\cancel{x^2 = y^2} \quad ; \quad \cancel{y^2 = z^2}$

~~put in (1)(2)~~

$\cancel{2}\cancel{\lambda}x^2 = \cancel{2}\cancel{\lambda}y^2 = \cancel{2}\lambda z^2$

$x^2 = y^2 = z^2$

put in (1)(2)

$x^2 + x^2 + x^2 = 3$

$3x^2 = 3$

$\boxed{x = \pm 1}$

$\boxed{y = \pm 1}$

$f(x,y,z) = xyz$

$f(1,1,1) = 1$ max

$f(-1,-1,-1) = -1$ min

$f(1,1,-1) = -1$ min

$f(1,-1,-1) = 1$ max

$f(-1,1,1) = -1$ min

$f(-1,1,-1) = 1$ max

$f(-1,-1,1) = 1$ max

$f(+1,-1,1) = -1$ min

Cas-I

Q3 $f(x,y) = x^2+y^2 \; ; \quad xy = 1$

Q4 $f(x,y) = 3x+y \; ; \quad x^2+y^2 = 10$

Q5 $f(x,y,z) = 2x+2y+z \; ; \quad g(x,y,z) = x^2+y^2+z^2 = 9$

Q6 $f(x,y,z) = x^2+y^2+z^2 \; ; \quad '' = x+y+z = 12$

Q7 $f(x,y,z) = x+y+z+t \; ; \quad g(x,y,z,t) = x^2+y^2+z^2+t^2$

Out line   Assignment #4 * 2

1) Language Multiplier
2) Double Integral
3) Triple Integral
4) triple Integral in cylinderical & spherical
5) Coordinate

Vol...

$f(x,y) = \sum f(x_i, y_i)\, \Delta x_i\, \Delta y_i$

Fubni's Theorem,

$f(x,y) \Rightarrow$ Continuous function.

$R = \{(x,y) \mid a \leq x \leq b \,;\, c \leq y \leq d\}$

$$\int_c^d \int_a^b f(x,y)\, dx\, dy \;\Bigg|\; \int_a^b \int_c^d f(x,y)\, dy\, dx$$

Eg $\int_0^2 \int_1^2 (x - 3y^2)\, dx\, dy$

$$\int_0^2 \left[\frac{x^2}{2} - 3y^2 x \right]_1^2 dy$$

$$\int_0^2 \left(\frac{4}{2} - 6y^2\right) - \left(\frac{1}{2} - 3y^2\right) dy$$

$$\int_0^2 2 - 6y^2 - \frac{1}{2} + 3y^2\, dy$$

Q1: $\int_{1}^{4}\int_{0}^{2}(6x^2y-2y)\,dy\,dx$

Q2: $\int_{0}^{2}\int_{1}^{2}(4x^3-9x^2y^2)\,dy\,dx$

Q3: $\int_{0}^{2}\int_{0}^{4} y^3 e^{2x}\,dy\,dx$

Q4: $\int_{\pi/6}^{\pi/2}\int_{-1}^{5}(\cos y)\,dx\,dy$

Q5: $\int_{-3}^{3}\int_{0}^{\pi/2}(y+y^2\cos x)\,dx\,dy$

Q6: $\int_{0}^{1}\int_{0}^{3} e^{x+3y}\,dx\,dy$

By Fubini's Theorem

Q7: $\iint_R (y+xy^{-2})\,dA$; $R\{(x,y) \mid 0\le x\le 2,\ 1\le y\le 2\}$

Q8: $\iint_R \frac{1+x^2}{1+y^2}\,dA$; $R\{(x,y) \mid 0\le x\le 1;\ 0\le y\le 1\}$

Evaluate the triple integral

Q9: $\iiint_B xyz^2\,dv$

$B=\{(x,y,z) \mid 0\le x\le 1;\ -1\le y\le 2\ ;\ 0\le z\le 3\}$