Úlohy 1

1) $$\lim_{(x,y)\to(2,2)} \frac{x^3-\cancel{y}^3}{x^4-y^4} = \lim_{(x,y)\to(2,2)} \frac{(\cancel{x}-y)(x^2+xy+y^2)}{(x^2-y^2)(x^2+y)^2} =$$

$$= \lim_{(x,y)\to(2,2)} \frac{\cancel{(x-y)}(x^2+xy+y^2)}{\cancel{(x-y)}(x+y)(x^2+y^2)} = \frac{12}{32} = \underline{\underline{\frac{3}{8}}}$$

2) $$\lim_{(x,y)\to(0,0)} \frac{x+2y}{2x-3y} = \lim_{(x,y)\to(0,0)} \frac{x+2kx}{2x-3kx} = \lim_{(x,y)\to(0,0)} \frac{1+2k}{2-3k} = \frac{1+2k}{2-3k}$$

$y=kx$ limita <u>neexistuje</u> protože pro různé k je různá

3) $$f(x,y) = (x-y)\cdot\ln(2x+y)$$

$$f'_x(x,y) = \ln(2x+y) + (x-y)\frac{2}{2x+y}$$

$$f'_y(x,y) = -1\cdot\ln(2x+y) + (x-y)\frac{1}{2x+y}$$

3b) $$f(x,y) = \ln(x^2+y^2)$$

$$f'_x(x,y) = \cancel{\ln}\frac{2x}{x^2+y^2} \quad f''_{xx} = \frac{2(x^2+y^2)-2x\cdot 2x}{(x^2+y^2)^2} = \frac{2(x^2+y^2)-4x^2}{(x^2+y^2)^2} =$$

$$= \frac{2y^2-2x^2}{(x^2+y^2)^2}$$

$$f'_y(x,y) = \frac{2y}{x^2+y^2}$$

$$f'_{xy}(x,y) = \frac{-2x(2y)}{(x^2+y^2)^2} = -\frac{4xy}{(x^2+y^2)^2}$$

3c) $f''_{yy}(x,y) = \dfrac{2(x^2+y^2) - 2y\cdot 2y}{(x^2+y^2)^2} = \dfrac{2x^2-2y^2}{(x^2+y^2)^2}$ //

4) $f(x,y) = x \cdot \sin^2 y \qquad a = (1; \frac{\pi}{2})$

$f'_x = \sin^2 y$	1	$f'''_{xxx} = 0$	0
$f'_y = x \cdot 2\sin(y)\cdot\cos y = x\sin 2y$	0	$f'''_{xxy} = 0$	0
$f''_{xx} = 0$	0	$f'''_{xyy} = 2\cos 2y$	-2
$f''_{xy} = 2\sin y\cdot\cos y = \sin 2y$	0	$f'''_{yyy} = -4x\cdot\sin 2y$	0
$f''_{yy} = x \cdot 2\cos 2y$	-2		

$df(ah) = 1h_1 + 0h_2 = h_1$

$d^2f(ah) = 0h_1^2 + 2\cdot 0\, h_1h_2 + (-2)h_2^2 = -2h_2^2$

$d^3f(ah) = 0h_1^3 + 3\cdot 0\cdot h_1^2\cdot h_2 + 3\cdot(-2)\cdot h_1h_2^2 + 0h_2^3 = -6h_1h_2^2$

$f(a) = 1$

$T^3_{f}(ah) = f(a) + df(ah) + \frac{1}{2!}d^2f(ah) + \frac{1}{3!}d^3f(ah) =$

$= 1 + h_1 - h_2^2 - h_1h_2^2$