① $f(x,y) = 4x + 2y + 1$

$g(x,y) = x^2 + x - y + \frac{1}{4} = 0$

$F(x,y) = 4x + 2y + 1 + \lambda(x^2 + x - y + \frac{1}{4})$

$F'_x = 4 + \lambda(2x+1)$

$F'_y = 2 + \lambda(-1) = 2 - \lambda$

I $\quad 4 + \lambda(2x+1) = 0 \qquad 4 + 2(2x+1) = 0 \quad 2x = -3 \quad x = \frac{-3}{2}$

II $\quad 2 - \lambda = 0 \qquad \lambda = +2$

III $\quad x^2 + x - y + \frac{1}{4} = 0 \qquad \frac{9}{4} - \frac{3}{2} - y + \frac{1}{4} = 0$

$\frac{5-3}{2} - y = 0 \quad y = 1$

$(-\frac{3}{2}, 1)$ – podezřelý bod

$\frac{9}{4} - \frac{3}{2} - 1 + \frac{1}{4} = 0 \Rightarrow$ leží na vazbě

$g'(x, h(x)) = 2x + 1 - y' = 0 \quad 2(\frac{-3}{2}) + 1 - y' = 0 \quad -2 = y'$

$g''(x, h(x)) = 2 - y'' = 0 \Rightarrow y'' = 2$

$f'(x, h(x)) = 4 + 2y' + 1$

$f''(x, h(x)) = 2y''$

$f''(\frac{-3}{2}, 1) = 2 \cdot 2 = 4 > 0$ v bode je lokalní vazný mi

maximum

② $F(x,y) = x^4 + y^4 - x^2 - y^2 = 0$

$F'(x,y) = 4x^3 + 4y^3y' - 2x - 2yy' = 0$

$$\frac{\overset{-2}{\cancel{4}}x^3 - \overset{-1}{\cancel{8}}x}{-\underset{2}{\cancel{4}}y^3 + \underset{1}{\cancel{8}}y} = y' = \frac{x - 2x^3}{2y^3 - y}$$

$y' = 0 \Rightarrow \frac{x - 2x^3}{2y^3 - y} = 0 \qquad x - 2x^3 = 0$

$x = 0 \;/\!/ \qquad 1 - 2x^2 = 0$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}} \;/\!/$

$x = 0: \quad y^4 - y^2 = 0$

$y = 0 \;/\!/ \quad y = \pm 1 \;/\!/$

$x = \pm\frac{\sqrt{2}}{2}: \quad \left(\frac{\sqrt{2}}{2}\right)^4 + y^4 - \left(\frac{\sqrt{2}}{2}\right)^2 - y^2 = 0$

$\frac{4}{16} + y^4 - \frac{1}{2} - y^2 = 0$

$-\frac{1}{4} + y^4 - y^2 = 0$

$t = y^2 \qquad t^2 - t - \frac{1}{4} = 0$

$t > 0$

$y^2 = \frac{1+\sqrt{2}}{2}$

$y = \pm\sqrt{\frac{1+\sqrt{2}}{2}} \;/\!/$

$D = 2$

$t_{1,2} = \frac{1 \pm \sqrt{2}}{2}$

$t_2 = \frac{1-\sqrt{2}}{2} < 0$

$t_1 = \frac{1+\sqrt{2}}{2}$

$(0,0), (0,1), (0,-1) \left(\frac{\sqrt{2}}{2}, \sqrt{\frac{1+\sqrt{2}}{2}}\right), \left(\frac{\sqrt{2}}{2}, -\sqrt{\frac{1+\sqrt{2}}{2}}\right), \left(-\frac{\sqrt{2}}{2}, \sqrt{\frac{1+\sqrt{2}}{2}}\right)$ (

$$F''(x,y) = 12x^2 + 4y^2 \cdot 3 \cdot y' \cdot y' + 4y^3 \cdot y'' - 2 - 2y' - 2y' \cdot y = 0$$

$y' = 0$

$$12x^2 + 4y^3 \cdot y'' - 2 - 2yy'' = 0$$

$$6x^2 + 2y^3 \cdot y'' - 1 - yy'' = 0$$

$$2y^3 \cdot y'' - yy'' = 1 - 6x^2$$

$$y'' = \frac{1 - 6x^2}{2y^3 - y} = \frac{1 - 6x^2}{y(2y^2 - 1)}$$

$y''(0,0) =$ neexistuje

$y''(0,1) = \frac{1}{1} = 1 > 0$ minimum//

$y''(0,-1) = \frac{1}{-3} = -1/3 < 0$ maximum//

$$y''\left(\pm\frac{\sqrt{2}}{2}, -\sqrt{\frac{1+\sqrt{2}}{2}}\right) = \frac{1 - \frac{6 \cdot 2}{4}}{2\left(-\sqrt{\frac{1+\sqrt{2}}{2}}\right)^3 + \sqrt{\frac{1+\sqrt{2}}{2}}} = \frac{-2}{-\sqrt{\frac{1+\sqrt{2}}{2}}\left(\frac{2 \cdot (1+\sqrt{2})}{2} - 1\right)} =$$

$$= \frac{-2}{\underbrace{-\sqrt{\frac{1+\sqrt{2}}{2}}}_{<0}\underbrace{(\sqrt{2})}_{>0}} \Rightarrow > 0 \text{ maxima//}$$

$$y''\left(\pm\frac{\sqrt{2}}{2}; \sqrt{\frac{1+\sqrt{2}}{2}}\right) = \frac{-2}{\sqrt{\frac{1+\sqrt{2}}{2}}\left(\frac{2 \cdot (1+\sqrt{2})}{2} - 1\right)} = \frac{-2}{\sqrt{\frac{1+\sqrt{2}}{2}} \cdot \sqrt{2}} < 0 \text{ maximum}$$