EE 380 (Control Systems) – Homework 6

Due: April 23

Problem 1. (20 points) Answer the following questions.

(a) Consider the following system $G(s)$ and sinusoidal input:

$$\begin{aligned} -3\dot{y}(t) - 2y(t) &= 7u(t) \\ u(t) &= 6\cos(t+4) \end{aligned}$$

What is the magnitude and phase of $G(1j)$? Is the steady-state output bounded? If yes, what is it?

(b) Consider the following system $G(s)$ and sinusoidal input:

$$\begin{aligned} \ddot{y}(t) + 0.1\dot{y}(t) + 4y(t) &= \dot{u}(t) + 2u(t) \\ u(t) &= -\cos(2t) \end{aligned}$$

What is the magnitude and phase of $G(2j)$? Is the steady-state output bounded? If yes, what is it?

Problem 2. (40 points) Sketch the Bode plots by hand for the following systems:

(a) A PI controller with input $e(t)$ and output $u(t)$:

$$\dot{u}(t) = K_p\dot{e}(t) + K_i e(t)$$

with $K_p = 10$ and $K_i = 1$.

(b) A "low frequency boost" controller with input $e(t)$ and output $u(t)$:

$$\dot{u}(t) + u(t) = \dot{e}(t) + 10e(t)$$

This type of controller will be encountered later in the course.

(c) A first-order system with right-half plane zero with input $u(t)$ and output $y(t)$:

$$2\dot{y}(t) + 0.6y(t) = -\dot{u}(t) + 30u(t)$$

(d) A second-order underdamped system with input $u(t)$ and output $y(t)$:

$$\ddot{y}(t) + 0.2\dot{y}(t) + 4.01y(t) = -u(t)$$

After you're done, check your results using `MATLAB`. Turn in both the hand sketches and the `MATLAB` plots.

Problem 3. (40 points) For each of the transfer functions given below, draw the Bode plots (both magnitude and phase) by hand, using the techniques discussed in Lecture 15. Explain all steps in your drawing procedures.

(a) $L(s) = \dfrac{s+8}{s(s+4)}$

(b) $L(s) = \dfrac{8s}{s^2+0.2s+4}$

(c) $L(s) = \dfrac{s^2+0.2s+1}{s(s+0.2)(s+6)}$

(d) $L(s) = \dfrac{s+10}{s(s^2+1.4s+1)}$

After you're done, check your results using `MATLAB`. (Note that the bode command in `MATLAB` plots magnitude in decibels.) Turn in both the hand sketches and the `MATLAB` plots.

Problem 4. Consider the feedback diagram in Figure 1 below. Suppose $L(s) := G(s)K(s)$. Consider two pairs of plants and controllers:

i) $K(s) = \dfrac{10(s+3)}{s}$ and $G(s) = \dfrac{-0.5(s^2 - 2500)}{(s-3)(s^2+50s+1000)}$

ii) $K(s) = \dfrac{0.4s+1}{s}$ and $G(s) = \dfrac{1}{s+1}$

iii) $K(s) = 2$ and $G(s) = \dfrac{1}{(s+1)^3}$

Perform the following calculations for each plant/controller pair:

(a) (15 points) Verify that the feedback system is stable.

(b) (15 points) Use the Bode plot of $L(s)$ to compute the gain margin(s) of the feedback system.

(c) (15 points) Use the Bode plot of $L(s)$ to figure out the phase margins of the feedback system.

You can use the `allmargin` command to check your answers.

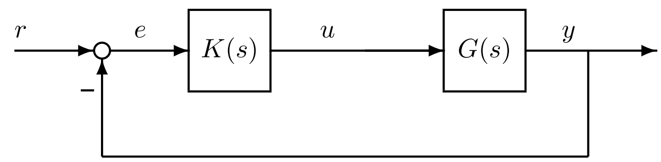

Figure 1: Feedback Loop